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RESEARCH PAPER

Control and Synchronization of The Hyperchaotic Closed-Loop Supply Chain Network by PI Sliding Mode Control

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ABSTRACT
In this paper, a closed-loop supply chain is modelled based on hyperchaotic dynamics. Then, synchronization of the two hyperchaotic closed-loop supply chains is performed with a proportional-integral (PI) sliding mode controller design method. Using Lyapunov stability theory, it has been proved that the PI sliding mode controller can converge the synchronization error to zero in a limited time. The most important issue in the design of control strategies is the behavior of the control signal. In other words, it affects the cost of design and implementation or controlling the chaotic closed-loop supply chain with minimal control strategies, discussing the cost of the hyperchaotic supply chain network control method. Numerical simulation results show that the control signal has low amplitude and fluctuations. so, the PI sliding mode control method can be implemented in the real world. Based on the numerical simulation results, the use of two controllers is proposed to reduce design costs.

KEYWORDS: *Closed loop; Supply chain; Hyperchaos; Synchronization; Sliding mode; Lyapunov.*

1. Introduction

Chaos theory is a new field in explorations of nonlinear dynamic. In the last decade, control and management of chaotic supply chain have become an important topic in industrial engineering. Chaos was first introduced by Edward Lorenz in 1963 [1]. He showed that the nonlinear model can behave chaotically by changing its parameters. The most important features of chaotic dynamics are their high sensitivity to the initial conditions (with a small change in the initial conditions of the equations, their future behavior will be different) and having at least one positive view of Lyapunov. After Lorenz, in 1976, Rossler introduced another model of chaos [2]. Then the study and analysis of chaotic models became very widespread [3,4]. There are more complex dynamic systems than the chaotic models called hyperchaotic., Hyperchaotic system, possessing more than one positive Lyapunov exponent, has more complex behavior and abundant dynamics than chaotic system. Historically, hyperchaos was introduced by Rossler in 1990 [5]. After her, the study of hyperchaotic systems received much attention. Chen et al. introduced a new 4-dimensional hyperchaotic system which had larger Lyapunov exponents [6]. After them, various models of hyperchaotic systems have been introduced [7-11]. The most important applications of hyperchaotic models in engineering are such as secure communications [12-13], image encryption [14-15], satellite [16], robotic [17] and laser [18], these hyperchaotic models are also used in economic [19-20] and finance systems [21].

Supply chain control and management are one of the most important branches in the field of industrial engineering. Given the breadth and complexity of supply chain networks, it is very difficult to decide and manage. Mathematical supply chain models help to make more successful management and decisions about how to produce, distribute and control the supply chain network. One of the major problems of supply chain research is a phenomenon known as Bullwhip Effect which indicates the situation in which demand variability is enhanced by moving upwards the supply chain. Study on the

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complexity of the bullwhip effect by LU Yingjin el at in 2004 [22].

Modelling supply chain chaos and the presence of bullwhip effect was first introduced in [23] based on the Lorenz chaotic model in 2006. Next, Bifurcation analysis and synchronization issues in a three-echelon supply chain by K.R. Anne researched [24]. The objectives of this article are three: 1- To demonstrate the potential of the concept of nonlinear dynamics and supply chain modelling. 2- To illustrate the challenge of synchronization in the supply chain. 3- provide strategic decision makers with an approach or technique for controlling and stabilizing their supply chain status in the event of uncertainty.

Alper Göksu el at [25] presents the synchronization and control of a chaotic supply chain management system based on its mathematical model. To do this, they used an active controller to control and synchronize two identical chaotic supply chain management systems with different initial conditions. They have investigated a three-tier supply chain model by disturbing the parameters. Uğur Erkin Kocamaz el at [26] investigated the issue of synchronizing and controlling a turbulent supply chain with Adaptive Neuro-Fuzzy Inference System (ANFIS) methods. Their model is also a three-level network and their goal are to synchronizaion two identical chaotic supply chains that have different initial conditions. Robust control for a chaotic supply chain network was examined in 2016 by Hamid et al. Their model is a five-tier supply chain network [27]. In this paper, a supply chain network is investigated Which has two ordering policies: 1- Smooth ordering policy and what is the new policy designed based on a proportional derivative controller. 2- are forecasting methods Used in the network: Moving Average Prediction (MA) and exponential smoothing Forecast (ES). Sayantani Mondal [28], A new supply chain model is proposed in 2019. A model assuming that the demand for a product does not increase uniformly with the increasing inventory. In this model, it is assumed that demand has a saturation level and does not increase uniformly with inventory. After presenting the model, synchronization was performed by the active control method.

A New Supply Chain System and Its Impulsive Synchronization by Yang Peng el at in 2020 investigated [29]. Their proposed model is a three-tier supply chain network. Following the Mondal idea, where demand is saturated and does not increase with uniform inventory, Yang Peng el at propose a new supply chain system that is not unique. Xiao Xu el at management and optimization of chaotic supply chain system using adaptive super-twisting (STW) sliding mode control (SMC) algorithm to manage chaotic supply chain system [30]. Their model is the same Zhang, Li, and Xu 2006. All the supply chain network models mentioned so far have had chaotic behaviors. In [31] the chaotic four-level supply chain model is introduced. subject it is computer-aided digital manufacturing process. An active control method with linear feedback has been used to control the chaotic supply chain network.

Xiao Xu el at in 2022, proposed a new dynamical behavior modelling for a four-level supply chain under FO-sliding mode control [32]. A four-echelon nonlinear supply chain system is built to describe complex dynamical behaviors. Next, the novel fractional order adaptive sliding mode control (FO-ASMC) algorithm has been implemented for ensuring efficient supply chain management. The control signal is depicted. But the use of four controllers is suggested.

Hamidzadeh el. at presents a mathematical model of a four-level supply chain under hyperchaos circumstances [33]. They proposed method for hyperchaotic supply chain control and stability is described. Also, they showed that the hyper-chaotic supply chain has a center of gravity, by focusing on this center of gravity, the entire hyper-chaotic supply chain network will be well controlled.

All models of chaotic supply chain networks introduced are traditional (or forward) network type. A closed-loop supply chain is a combination of a direct or traditional supply chain with a reverse supply chain. That is, after the production process is completed and shipped and distributed through the sales representative, the manufacturer tries to collect defective, old or second-hand goods. This is where the reverse supply chain process begins. In fact, the term closed loop refers to the fact that the chain uses these products to maintain and recover value and to help reduce environmental waste. In addition, some of the waste and obsolete products can be used as valuable sources of raw materials to build new products. In past researches, the model of closed loop supply chain with hyperchaotic behavior has not been investigated. The innovation of the current research shows that the closed loop supply chain network can be subjected to chaotic conditions. The most important issue that has not been addressed in most research is the cost of the control strategy.

In control theory, it is called the control signal. If this signal itself behaves similarly to chaotic systems, it will be very difficult and costly to implement in the real world. Table 1 shows a summary of important activities in the field of chaos control and synchronization in the supply chain. The most important issue in the control strategy designs of the chaotic and hyperchaotic supply chain network is the number of controllers and the analysis of its behavior. Also, the time to reach zero error is also important in the design costs of the control strategy.

Tab. 1. A summary of chaos and hyperchaos researches in supply chain

No. Ref	Modelling	Synchronization chaos	Elimination chaos	Strategy	Number of controllers	Control signal	Maximum stability time
[23]	Three echelon	√	None	RBF Control	Three	None	≈ 5 sec
[25]	None	√	√	Active Control	Three	None	≈ 7 sec
[26]	None	√	√	ANFIS Control	Three	None	≈ 2 sec
[28]	Three echelon	√	None	Nonlinear Control	Three	None	≈ 0.5 sec
[29]	Three echelon	√	None	Impulsive Control	Three	None	≈ 0.7 sec
[30]	Three echelon	None	√	Adaptive Sliding Mode Control	Three	Available	≈ 2.5 sec
[31]	Four echelon (Hyperchaotic)	√	√	Nonlinear Control	Four	None	≈ 6 sec
[32]	Four echelon (Hyperchaotic)	√	None	FO-ASMC	Four	Analyzed	≈ 0.3 sec
[33]	Four echelon (Hyperchaotic)	√	√	Nonlinear	Four and One	Analyzed	≈ 1 sec
This paper	Four echelon (Hyperchaotic)	√	None	PI Sliding Mode	Four and Two	Analyzed	≈ 0.5 sec

In this paper, we aim is to present a dynamic model of hyperchaotic closed-loop supply chain network. The model under study is a four-level network that can recycle some materials from the latest supply chain process and send them to the supplier stage. The illustration is behavior of the proposed control strategy (signal of control). The behavior of control strategy shows that it is low cost and can be implemented in the real world. Also, with the help of numerical simulation, one of the important methods to reduce the cost of controlling the supply chain network is illustrated.

2. Dynamic Behavior of Hyperchaotic Closed-Loop Supply Chain

Defective products and wastes have always been a major challenge for manufacturers. They have found that recycling and reusing products, wastes and crop residues not only reduce their harmful effects on the environment, but also improve their competitive position in the market. Following this strategy, supply chain networks can be significantly restructured and the economic benefits can be maximized.

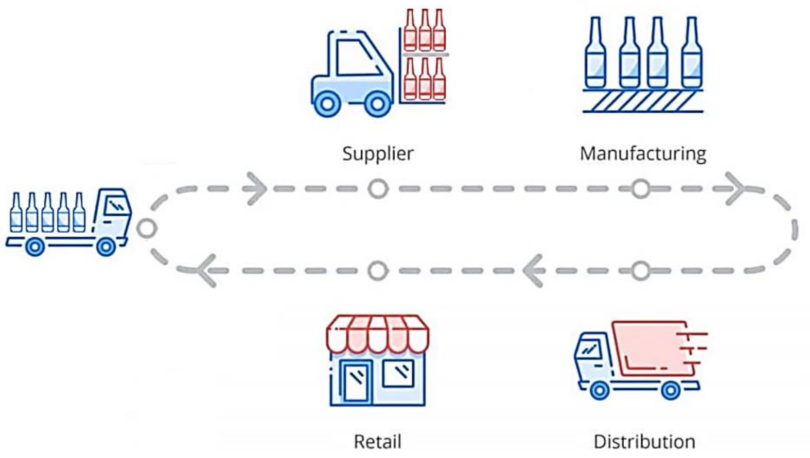

Fig. 1. Overview of a four-level closed-loop supply chain

Model parameters are:

a: Distributor delivery rate to retailer

b: Product sales rates in retail

c: Product recycling rate that is realized in the retailer

d: Inventory control rate at the distributor

e: Safety factor in the manufacturer

f: The number of recyclables that can be used in the supplier

h: Raw material supply rate in the supplier

x: The amount of the retailer request in the current period

y: The amount that the distributor can distribute in the current period

z: Number of products produced in the factory for the current period

w: The number of raw materials to produce the product at the request of the factory in the current period.

Assumption: Information is transmitted along the supply chain with a delay of the one-time unit. Thus, the behavior of the model in stage i is affected by the information in stage $i-1$.

Scenario 1: The retailer requests a rate from the distributor, and the distributor can meet the retailer's demand at a response rate. Retailers in the supply chain try to get used products from customers and send them to the supplier of raw materials for recycling. Therefor the equations:

$$x_i = a\,y_{i-1} - bx_{i-1} + cw_{i-1} \tag{1}$$

Where, a is the delivery factor of the products requested from the distributor to the retailer and b is the coefficient of sales of products in retail and c is the coefficient of receipt of products consumed to be sent to the supplier.

Scenario 2: The distributor must always seek to control they inventory level in order to respond to the retailer's requests. Given that the distributor must be able to adjust they inventory according to receiving products from the manufacturer and sending them to the retailer, they are always faced with uncertainty about the retailer and the manufacturer, Therefore:

$$y_i = d\,y_{i-1} - x_{i-1}z_{i-1} \tag{2}$$

Where d is the distributor inventory control coefficient and $x_{i-1}\ z_{i-1}$ is the uncertainty in the distributor.

Scenario 3: The manufacturer produces its products with a safety stock. There is also uncertainty in the manufacturer. This uncertainty is caused by the distributor and the retailer.

$$z_i = ez_{i-1} + x_{i-1}y_{i-1} \tag{3}$$

Where, e is the safety stock in the manufacturer and $x_{i-1}\ y_{i-1}$ is uncertainty.

Scenario 4: In this model, the manufacturer requests the raw materials from the supplier with a factor. On the other hand, part of the recycled products that reach the supplier from the retailers can be returned to the production cycle. There is uncertainty in the supplier from the retailer and manufacturer. This uncertainty is indicated by the expression $x_{i-1}\ z_{i-1}$.

$$w_i = fw_{i-1} + hw_{i-1} + x_{i-1}z_{i-1} \tag{4}$$

Where, f is the recycling rate of the retailer, h is the raw material demand rate received from the manufacturer and uncertainty is indicated by the expression $x_{i-1}\ z_{i-1}$.

If equations (1) to (4) are placed next to each other, then:

$$\begin{aligned} x_i &= a\,y_{i-1} - bx_{i-1} + cw_{i-1} \\ y_i &= d\,y_{i-1} - x_{i-1}z_{i-1} \\ z_i &= ez_{i-1} + x_{i-1}y_{i-1} \\ w_i &= fw_{i-1} + hw_{i-1} + x_{i-1}z_{i-1} \end{aligned} \quad (5)$$

Discrete differential equations (5) are a model of a four-level closed-loop supply chain. If the duration of requests along the supply chain network is very short, then the discrete differential equations (5) will be as follows:

$$\begin{aligned} \dot{x} &= a\,y - bx + cw \\ \dot{y} &= d\,y - xz \\ \dot{z} &= ez + xy \\ \dot{w} &= (f + h)w + xz \end{aligned} \quad (6)$$

Where *a, b, c, d, e, f and h* are the parameters of the closed-loop supply chain and *x, y, z and w* are closed-loop supply chain variables. Equation (6) is continuous time differential equation. This network model is a four-level nonlinear supply chain. These four levels include, retailer, distributor, manufacturer and supplier of raw materials. Fig 1 shows an overview of this closed-loop supply chain.

Closed loop supply chain parameters can take on any value. But if in this nonlinear model these parameters reach the values of a = 36, b = 36, c = 1, d = 20, e = -3, f = 1, h = 0.3, the behavior of the supply chain model will be very complicated. This hyperchaotic dynamic model is known as the Lu system [34].

For example, if the initial conditions $\begin{bmatrix} x_0 & y_0 & w_0 & z_0 \end{bmatrix}^T = \begin{bmatrix} 1 & 5 & 3 & 2 \end{bmatrix}^T$ are selected, the behavior of the hyperchaotic closed-loop supply chain network can be seen in Fig 2.

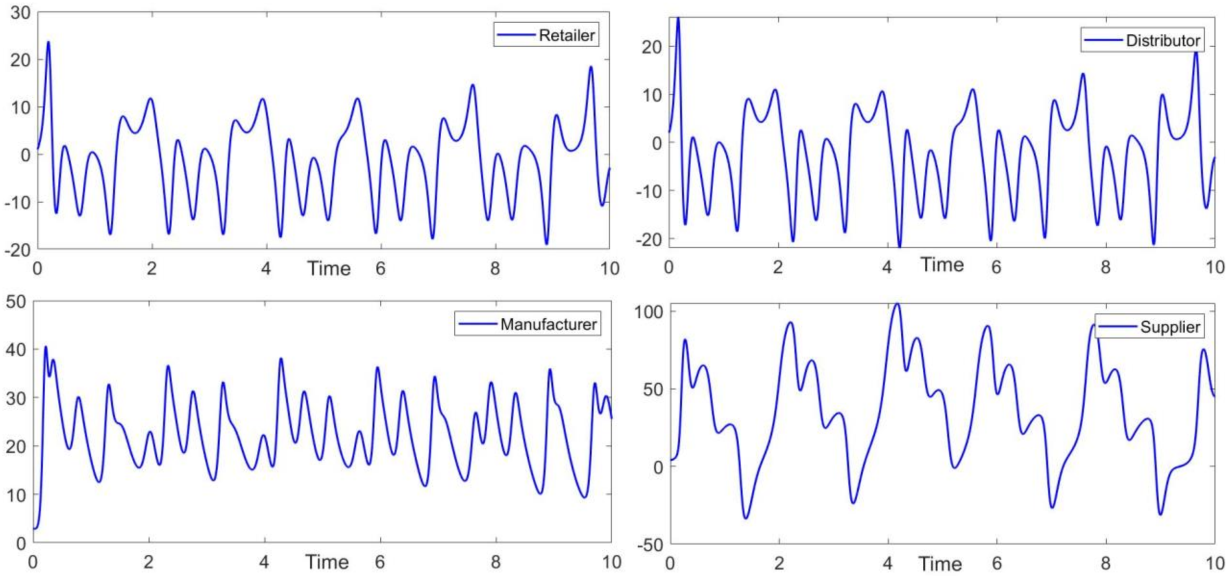

Fig. 2. Behavior of closed-loop hyperchaotic supply chain network variables

3. Control and Synchronization of Two Closed-Loop Hyperchaotic Supply Chain Networks

The problem of chaos synchronization is used in two chaotic or hyperchaotic systems. In synchronization, the first system is considered the master system, and the second system as the slave system. Fig 3 shows the concept of synchronization as a block diagram.

Mathematically, synchronization means adding a term of the same variable to a chaotic or hyperchaotic slave system to follow the behavior of the variables of the master system. Also, from a management point of view, it means making decisions to force the slave system to follow the master system. Master and slave systems are referred to as the master and slave hyperchaotic closed loop supply chains respectively.

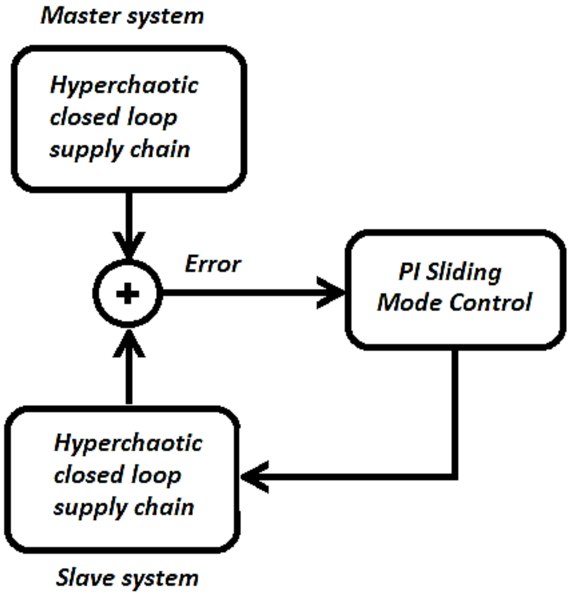

Fig. 3. block diagram of the synchronization of master and slave systems

Consider two identical hyperchaotic closed-loop supply chains as given below:

$$Master\begin{cases}\dot{x}_1 = a\, y_1 - bx_1 + cw_1\\ \dot{y}_1 = d\, y_1 - x_1 z_1\\ \dot{z}_1 = ez_1 + x_1 y_1\\ \dot{w}_1 = (f+h)w_1 + x_1 z_1\end{cases} \tag{7}$$

$$Slave\begin{cases}\dot{x}_2 = a\, y_2 - bx_2 + cw_2 + u_1\\ \dot{y}_2 = d\, y_2 - x_2 z_2 + u_2\\ \dot{z}_2 = ez_2 + x_2 y_2 + u_3\\ \dot{w}_2 = (f+h)w_2 + x_2 z_2 + u_4\end{cases} \tag{8}$$

Where x_1, y_1, z_1, w_1 are states of the master system and x_2, y_2, z_2, w_2 are states of the slave system and a, b, c, d, e, f, h are parameters of the hyperchaotic systems. In the slave system u_1, u_2, u_3, u_4 are the controller to be designed. In other words, management decisions force the slave system output to the master system outputs. Fig 4. shows the behaviors of two closed-loop hyperchaotic supply chain networks. As mentioned earlier, if the initial conditions of super-chaotic (and chaotic) dynamic systems change, their future behavior will be different. So here, the initial conditions of Equations (7) and (8) will be different.

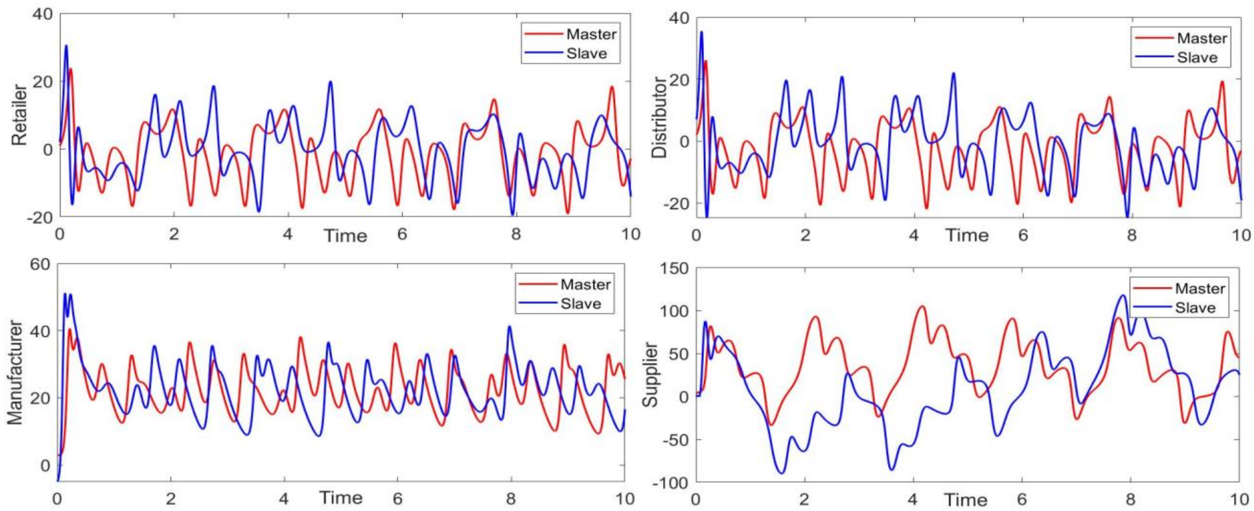

Fig 4. Behavior of closed-loop hyperchaotic closed-loop supply chains with initial conditions $[x_{10}, y_{10}, z_{10}, w_{10}]^T = [1,2,3,4]^T$ **and** $[x_{20}, y_{20}, z_{20}, w_{20}]^T = [2,7,-5,1]^T$

For Equations (7) and (8) the synchronization error is defined as follows:

$$\begin{aligned} e_1(t) &= x_2(t) - x_1(t) \\ e_2(t) &= y_2(t) - y_1(t) \\ e_3(t) &= z_2(t) - z_1(t) \\ e_4(t) &= w_2(t) - w_1(t) \end{aligned} \tag{9}$$

That is sense $\lim_{t\to\infty} |e(t)| = 0$ or synchronization of complete.

The derivative of the equation in Equation (9) yields:

$$\begin{aligned} \dot{e}_1 &= a\,y_2 - bx_2 + cw_2 + u_1 - a\,y_1 + bx_1 - cw_1 \\ \dot{e}_2 &= d\,y_2 - x_2 z_2 + u_2 - d\,y_1 + x_1 z_1 \\ \dot{e}_3 &= ez_2 + x_2 y_2 + u_3 - ez_1 - x_1 y_1 \\ \dot{e}_4 &= (f+h)w_2 + x_2 z_2 + u_4 - (f+h)w_1 - x_1 z_1 \end{aligned} \tag{10}$$

For sliding mode design control two principles must be considered. The first is the establishment of a sliding surface for optimal performance and the second is the design of a controller to ensure sliding mode is achieved.

First, selected the sliding surface of the integral sliding mode controller is defined as:

$$s_i = \sum_{i=1}^{4} \left(e_i + \int_0^T \lambda_i e_i(\tau)\, d\tau\right) \tag{11}$$

Equation (11) establishes the control law which guarantees the existence of sliding mode s = 0. which $\lambda_i, i = 1,2,3,4$ are positive constants specified by the designer to operate the system in the sliding mode.

The derivative of the equation in Equation (11) results:

$$\dot{s}_i = \sum_{i=1}^{4} (\dot{e}_i + \lambda_i e_i) \tag{12}$$

Now, considering $\dot{s}_i = 0, i = 1,2,3,4$, the following equations are obtained:

$$\begin{aligned} \dot{s}_1 &= \dot{e}_1 + \lambda_1 e_1 = 0 \\ \dot{s}_2 &= \dot{e}_2 + \lambda_2 e_2 = 0 \\ \dot{s}_3 &= \dot{e}_3 + \lambda_3 e_3 = 0 \\ \dot{s}_4 &= \dot{e}_4 + \lambda_4 e_4 = 0 \end{aligned} \tag{13}$$

Equation (13) by considering the exponential reaching law presented by:

$$\dot{s}_i = \sum_{i=1}^{4} (\varepsilon_i sign(s_i) + \eta_i s_i) \tag{14}$$

Where sign function is:

$$sign(s) = \begin{cases} 1 & for \quad s > 0 \\ 0 & for \quad s = 0 \\ -1 & for \quad s < 0 \end{cases}$$

Therefore:

$$\begin{aligned} \dot{e}_1 + \lambda_1 e_1 &= \varepsilon_1 sign(s_1) + \eta_1 s_1 \\ \dot{e}_2 + \lambda_2 e_2 &= \varepsilon_2 sign(s_2) + \eta_2 s_2 \\ \dot{e}_3 + \lambda_3 e_3 &= \varepsilon_3 sign(s_3) + \eta_3 s_3 \\ \dot{e}_4 + \lambda_4 e_4 &= \varepsilon_4 sign(s_4) + \eta_4 s_4 \end{aligned} \tag{15}$$

Then:

$$\begin{aligned} & a\,y_2 - bx_2 + cw_2 + u_1 - a\,y_1 + bx_1 - cw_1 + \lambda_1 e_1 = \varepsilon_1 sign(s_1) + \eta_1 s_1 \\ & d\,y_2 - x_2 z_2 + u_2 - d\,y_1 + x_1 z_1 + \lambda_2 e_2 = \varepsilon_2 sign(s_2) + \eta_2 s_2 \\ & ez_2 + x_2 y_2 + u_3 - ez_1 - x_1 y_1 + \lambda_3 e_3 = \varepsilon_3 sign(s_3) + \eta_3 s_3 \\ & (f+h)w_2 + x_2 z_2 + u_4 - (f+h)w_1 - x_1 z_1 + \lambda_4 e_4 = \varepsilon_4 sign(s_4) + \eta_4 s_4 \end{aligned} \tag{16}$$

Final, the results of the following sliding mode control laws are:

$$\begin{aligned} u_1 &= -a\,y_2 + bx_2 - cw_2 + a\,y_1 - bx_1 + cw_1 - \lambda_1 e_1 + \varepsilon_1 sign(s_1) + \eta_1 s_1 \\ u_2 &= -d\,y_2 + x_2 z_2 + d\,y_1 - x_1 z_1 - \lambda_2 e_2 + \varepsilon_2 sign(s_2) + \eta_2 s_2 \\ u_3 &= -ez_2 - x_2 y_2 + ez_1 + x_1 y_1 - \lambda_3 e_3 + \varepsilon_3 sign(s_3) + \eta_3 s_3 \\ u_4 &= -(f+h)w_2 - x_2 z_2 + (f+h)w_1 + x_1 z_1 - \lambda_4 e_4 + \varepsilon_4 sign(s_4) + \eta_4 s_4 \end{aligned} \tag{17}$$

Theorem1: the response of the slave hyperchaotic closed-loop supply chain in equation (10) asymptotically converge to master hyperchaotic closed-loop supply chain with an arbitrary initial condition $[x_{10}, y_{10}, z_{10}, w_{10}]^T, [x_{10}, y_{10}, z_{10}, w_{10}]^T \in \Re^4$, by using the law sliding mode control in equation (17) if all parameters sliding mode control are $\lambda_i, \varepsilon_i, \eta_i < 0$.

Proof1: Let us consider the following Lyapunov function:

$$\begin{aligned} V(s) &= \frac{1}{2}\sum_{i=1}^{4} s_i^2 \\ &\Rightarrow = \frac{1}{2}(s_2^2 + s_2^2 + s_3^2 + s_4^2) \end{aligned} \tag{18}$$

The derivative of Equation (18) gives:

$$\dot{V}(s) = \sum_{i=1}^{4} s_i \dot{s}_i \qquad (19)$$
$$\Rightarrow = s_1\dot{s}_1 + s_2\dot{s}_2 + s_3\dot{s}_3 + s_4\dot{s}_4$$

By substituting Equation (15) into Equation (19)

$$\begin{aligned}\dot{V} &= s_1\dot{s}_1 + s_2\dot{s}_2 + s_3\dot{s}_3 + s_4\dot{s}_4 \\ &\Rightarrow s_1(a\,y_2 - bx_2 + cw_2 + u_1 - a\,y_1 + bx_1 - cw_1 + \lambda_1 e_1) \\ &\quad + s_2(d\,y_2 - x_2 z_2 + u_2 - d\,y_1 + x_1 z_1 + \lambda_2 e_2) \\ &\quad + s_3(ez_2 + x_2 y_2 + u_3 - ez_1 - x_1 y_1 + \lambda_3 e_3) \\ &\quad + s_4((f+h)w_2 + x_2 z_2 + u_4 - (f+h)w_1 - x_1 z_1 + \lambda_4 e_4)\end{aligned} \qquad (20)$$

Now, by substituting Equation (17) into Equation (20)

$$\begin{aligned}\dot{V} &= s_1(\varepsilon_1 sign(s_1) + \eta_1 s_1) \\ &\quad + s_2(\varepsilon_2 sign(s_2) + \eta_2 s_2) \\ &\quad + s_3(\varepsilon_3 sign(s_3) + \eta_3 s_3) \\ &\quad + s_4(\varepsilon_4 sign(s_4) + \eta_4 s_4) \\ &\Rightarrow \eta_1 s_1^2 + \varepsilon_1 |s_1| + \eta_2 s_2^2 + \varepsilon_2 |s_2| + \eta_3 s_3^2 + \varepsilon_3 |s_3| + \eta_4 s_4^2 + \varepsilon_4 |s_4|\end{aligned} \qquad (21)$$

If all parameters sliding mode control are $\lambda_i, \varepsilon_i, \eta_i < 0$ then $\dot{V} < 0$

Consequently, according to the Lyapunov law, a slave hyperchaotic closed-loop supply chain, is globally asymptotically converge to master hyperchaotic closed-loop supply chain, and as a results proof is completed.

4. Numerical Simulation

MATLAB software is used for numerical simulation. With the help of this software, chaotic differential equations with initial conditions are solved under the proposed controller. Solution method in the software by using forth-order Runga-Kutta for solving the dynamic system (7), and (8) and controller (17) and simulating the result for arbitrary time is performed. The initial conditions for master and slave hyperchaotic closed-loop supply chain are $[x_{10}, y_{10}, z_{10}, w_{10}]^T = [1,2,3,4]^T$ and $[x_{20}, y_{20}, z_{20}, w_{20}]^T = [2,7,-5,1]^T$. The parameters sliding mode control are $\lambda_i = -5, \varepsilon_i = -0.001, \eta_i = -5$ for $i = 1,2,3,4$. Fig 4 illustrates the response of the hyperchaotic closed-loop supply chain system for master and slave systems. Fig 5 illustrates the complete synchronization with sliding mode control. As shown in Fig 5, the sliding mode control has been applied to the slave hyperchaotic closed-loop supply chain since time T = 2. Fig 6 shows the synchronization error. One of the most important factors in the design of controllers is how long it takes for the error to reach absolute zero. As shown in Fig 6, this time is acceptable. Finally, the cost of the sliding mode design method should be analyzed. This means determining the cost of implementing the method. In other words, it is the cost of decision-making. Fig 7 shows the sliding mode controller behavior. In some dynamic system control design methods, the controller behavior does not go to zero after sufficient time. This can increase the cost of controller design. It is also important how long it takes for the controller behavior to converge to zero. Large amplitude in controller behavior can also increase design costs. In Fig 7, the control signal u_4 has a large amplitude. For other control signals, they have large amplitudes, respectively u_2, u_3 and u_1.

To reduce the cost of designing and implementing a control (or decision-making) method, reduce the number of controllers. According to the simulation results, if the first controller is removed, synchronization is performed without any delay. According to the simulation results, if the first and third controllers are removed, the synchronization error moves asymptotically to zero. Equation (8) is now modified as follows:

$$Slave\begin{cases}\dot{x}_2 = a\,y_2 - bx_2 + cw_2 \\ \dot{y}_2 = d\,y_2 - x_2 z_2 + u_2 \\ \dot{z}_2 = ez_2 + x_2 y_2 \\ \dot{w}_2 = (f+h)w_2 + x_2 z_2 + u_4\end{cases} \qquad (22)$$

As can be seen, the controller for the slave system applies only to the distributor and supplier. Fig 8 shows the numerical simulation results of the synchronization error for the two hyperchaotic closed-loop supply chains. Fig 9 illustrates the controller behavior. The first (u_1) and third (u_3) controllers are zero at all times steps. To simulate this section, the controller parameters have not been changed.

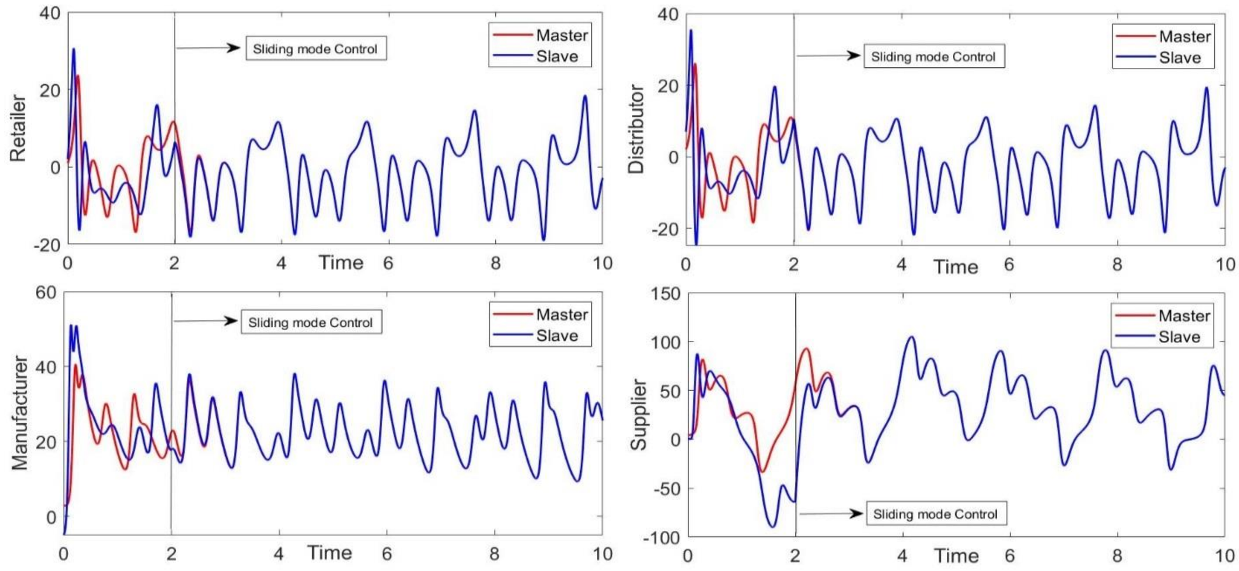

Fig. 5. Complete synchronization of master and slave hyperchaotic closed-loop supply chain using deferent initial conditions by PI sliding mode control

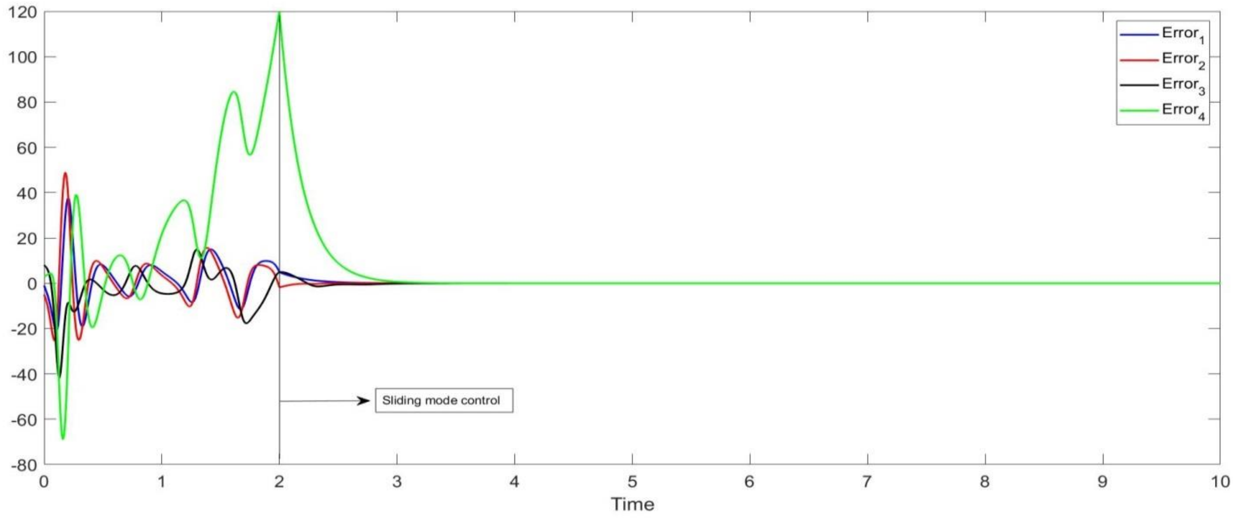

Fig. 6. Synchronization error for master and slave hyperchaotic closed-loop supply chain

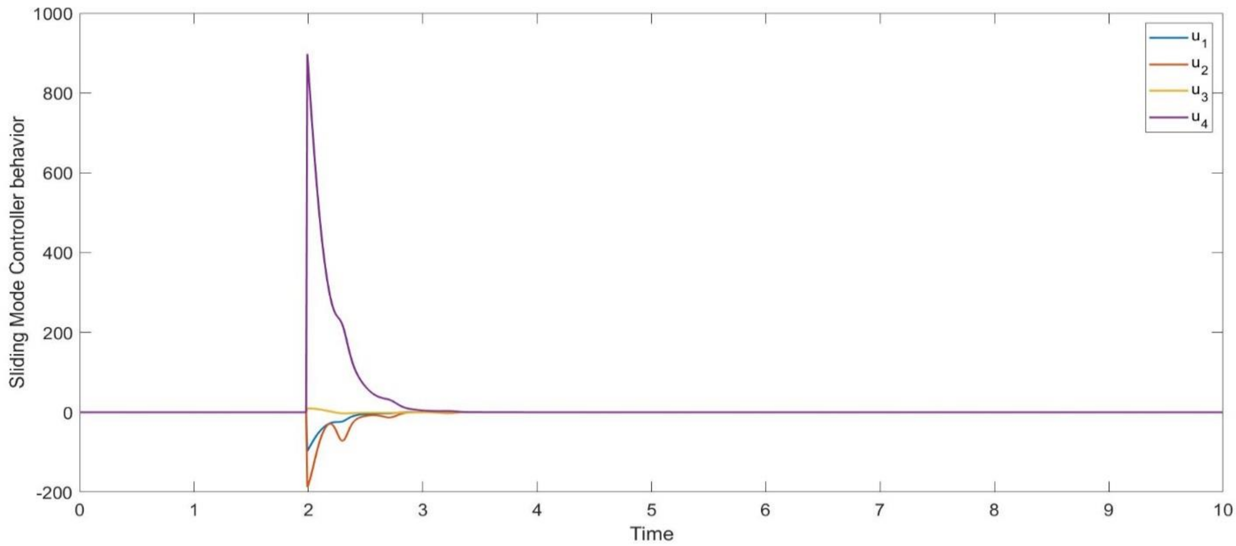

Fig. 7. PI sliding mode controller behavior

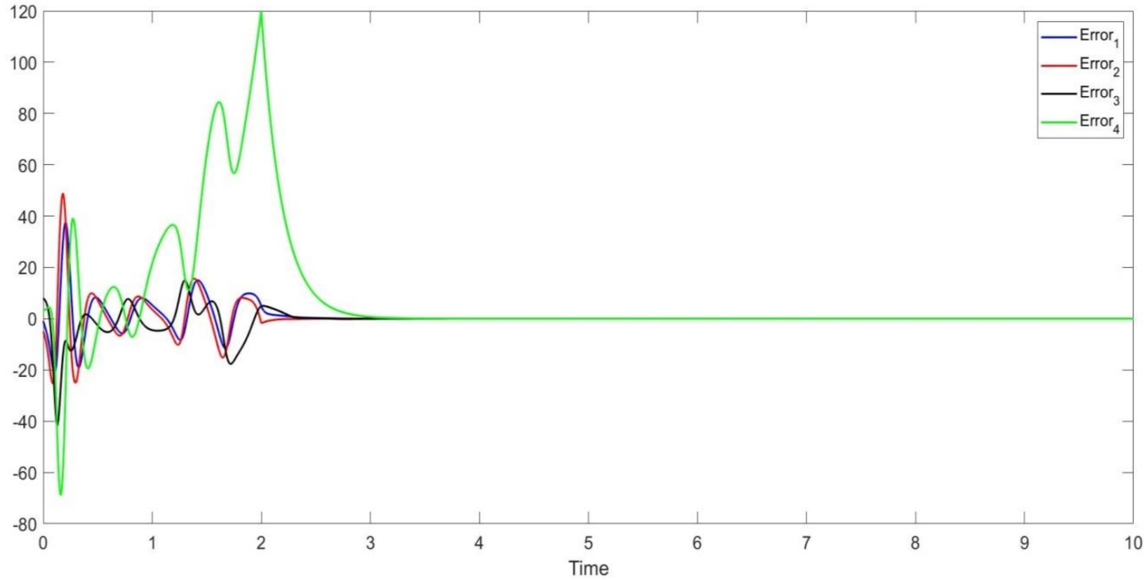

Fig. 8. Synchronization error for master and slave hyperchaotic closed-loop supply chain with two sliding mode control strategies (u_1 and u_2)

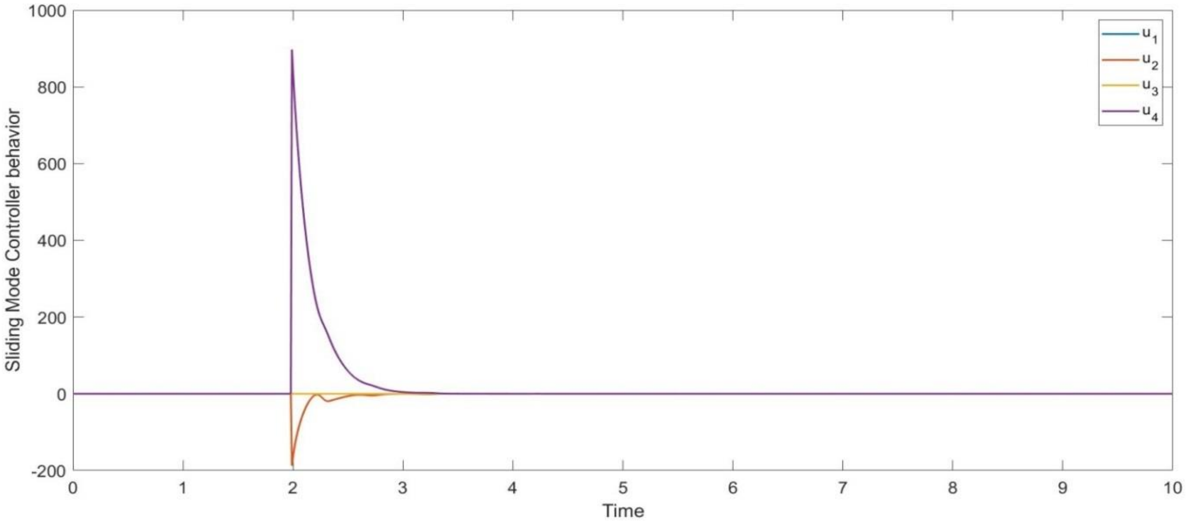

Fig 9. Sliding mode controller behavior that is u1=0 and u3=0

5. Conclusion

In this paper, a hyperchaotic closed-loop supply chain is modelled. This model includes four levels of a retailer, distributor, manufacturer and supplier of raw materials. In the retailer element, wastes of the used products can be recycled collected. They can be then sent to the supplier of raw materials. A sliding mode control based on PI method was proposed to synchronize the two hyperchaotic closed-loop supply chain networks. Numerical simulations based on Lyapunov stability theory, results have been conducted to validate and illustrate the effectiveness of the PI sliding controller design for the global synchronization of hyperchaotic closed-loop supply chain systems. The most important issue in controller design is to achieve zero error in a short time. Besides, the lower amplitude and fluctuations are, the better control signal are observed. meaning the capability to be implemented in the real-world. In most of the complex supply chain systems, it is possible to direct the entire supply chain network to the desired value with minimal controller design. If this issue is not considered, the design costs can be too high or impossible to implement. Finally, the cost of controller design was evaluated a proposed method for reducing controller design costs was demonstrated. Employing two controllers for synchronization can considerably reduce implementation costs. This approach was used in the design. Our proposal for future research is to use intelligent methods such as fuzzy control, fuzzy neural and meta-heuristic methods to optimize the control and synchronization the studied closed-loop supply chain.

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