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RESEARCH PAPER

A New Method to Optimize the Reliability of Repairable Components with A Switching Mechanism and Considering Costs and Weight Uncertainty

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ABSTRACT
The reliability of each component in a system plays a crucial role, as any malfunction can significantly reduce the system's overall lifespan. Optimizing the arrangement and sequence of heterogeneous components with varying lifespans is essential for enhancing system stability. This paper addresses the redundancy allocation problem (RAP) by determining the optimal number of components in each subsystem, considering their sequence, and optimizing multiple criteria such as reliability, cost uncertainty, and weight. A novel approach is introduced, incorporating a switching mechanism that accommodates both correct and defective switches. To assess reliability benefits, Markov chains are employed, while cost uncertainty is evaluated using the Monte-Carlo method with risk criteria such as percentile and mean-variance. The problem is solved using a modified genetic algorithm, and the proposed method is benchmarked against alternative approaches in similar scenarios. The results demonstrate a significant improvement in the Model Performance Index (MPI), with the best RAPMC solution under a mixed strategy achieving an MPI of 0.98625, indicating superior model efficiency compared to previous studies. Sensitivity analysis reveals that lower percentiles in the cost evaluations correlate with reduced objective function values and mean-variance, confirming the model's robustness in managing redundancy allocation to optimize reliability and control cost uncertainties effectively.

KEYWORDS: *Redundancy allocation problem; Repairable; Component sequencing; Markov chain; Mixed strategy; Cold standby*.

1. Introduction

Nowadays, the complexities of advanced systems have drawn significant attention to system reliability, given the potential for substantial damage resulting from component failures. Consequently, numerous methods, techniques, and models have been developed to address this issue. Approaches to enhance system reliability include: (i) improving component reliability, (ii) installing redundant components in parallel, and (iii) replacing components with substitutes. In product design and manufacturing, the evaluation of reliability can be more accurately described using stochastic models. Stochastic modeling has been implemented for different structures, redundant design types, and various distribution assumptions regarding component time-to-failure (TTF) [1].

Reliability optimization problems, which employ methods to enhance system reliability, aim to optimize objective functions related to reliability. These objectives may involve maximizing system reliability or minimizing resource requirements while adhering to specific design constraints. In heterogeneous k-out-of-n: G cold-standby system structures, the sequence in which different system elements are initiated can significantly impact both the system's mission cost and reliability. Standby techniques are commonly adopted to enhance system reliability, with three types of standby strategies: hot standby, warm standby, and cold standby [2].

In the cold standby method, the standby component is protected against the stresses caused by the system's operation, ensuring that components do not fail before use. In the warm standby strategy, components are exposed to system performance stresses to a greater extent

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than in the cold standby state. In the hot standby strategy, the occurrence of a component failure is independent of its usage or non-usage [3].

Over the years, extensive research has been conducted on redundancy allocation problems (RAPs) using meta-heuristic approaches. Coit (1996) utilized a combination of neural networks and genetic algorithms for the optimal redundancy allocation problem, while Gilani et al. (2017) employed the multi-objective Strength Pareto Evolutionary Algorithm (SPEA-II) to optimize RAPs with multiple objectives [4], [5]. Sharifi et al. (2020) proposed a series-parallel redundancy allocation problem for a system and utilized Monte Carlo simulation and genetic algorithms to solve the problem with two independent failure states [6]. Coit (2001) provided a formula for maximizing component time-to-failure reliability using the Erlang distribution and integer programming [7]. Liu and Coit (2000) developed a solution to determine optimal redundancy allocation in systems consisting of multiple k-out-of-n subsystems in series [8].

Saha and Chakrabart (2021) developed a supply chain model that scrutinizes the impact of production system reliability on product defect rates, involving a producer and retailer. The study detailed the process of reworking defective products and the implications for storage and sales, illustrating that while warehouse rental benefits the retailer, reduced production reliability negatively affects the manufacturer's profits [9]. Building on the theme of reliability, Nayeri et al. (2024) expanded the scope to evaluate raw material providers in the steel industry, incorporating resilience, digitalization, and Circular Economy (CE) factors. Their case study utilized novel stochastic methods to prioritize providers based on crucial performance indicators, including reliability, highlighting its significance alongside price and quality [10]. Further emphasizing the importance of reliability, Hermanto et al. (2024) rigorously tested the reliability of their framework through exploratory and confirmatory factor analyses. This ensured the framework's stability and consistency across various measurements, and by employing structural Eq. modeling, they confirmed the reliability of the identified capabilities. This robust validation provides a dependable basis for assessing supply chain agility in Product-Service Systems, making the framework a valuable tool for industry practitioners seeking to enhance operational resilience and agility [11].

The use of a mixed strategy, incorporating both active and standby components within subsystems, significantly increases system longevity. This paper employs a mixed redundancy strategy while also addressing cost uncertainty and using a multi-objective function for optimizing costs and weights—key aspects of effective system design. These considerations not only help achieve a specific design but also yield more realistic solutions. Furthermore, the approach integrates heterogeneous components within each subsystem, optimizes component costs and weights, manages cost uncertainty, determines the optimal sequence of components, and includes redundancy components with a switching mechanism. This comprehensive strategy provides a robust framework for system design. There are notable gaps in the literature regarding the optimization of system reliability under these constraints. This paper seeks to fill that gap by presenting an innovative approach that addresses these limitations. By employing a mixed strategy and using Markov chains, the paper facilitates the modeling and analysis of complex system states. Additionally, the model incorporates cost uncertainty, enhancing its applicability to real-world scenarios. The specific contributions of the paper can be summarized as follows:

- Development of a model for designing subsystems in complex systems with the objective of maximizing reliability while considering costs and weights.
- Introduction of a mixed strategy that combines active and standby components within subsystems to enhance system reliability.
- Utilization of Markov chains to solve the model and analyze the complex system state.
- Incorporation of cost uncertainty in the objective function to enhance the model's applicability to real-world scenarios.

The paper is structured as follows: Section 2 details the problem and the assumptions made in this study. Section 3 introduces a mixed strategy for heterogeneous components. Section 4 presents the mathematical model and proposes a genetic algorithm as the solution method. A numerical example is provided in Section 5. Sections 6 and 7 outline the research methodology and findings, respectively. Section 8 presents the model results and evaluation. Finally, Section 9 concludes the paper and offers suggestions for future research.

2. Literature Review

Some studies ([12, 13]) have explored achieving higher system reliability comparable to that attained by RAPs under the same conditions. Cha et al. (2008) modeled a general standby system and demonstrated that their model includes cold, hot, and warm standby systems with units of exponential distribution as special cases [14]. In more recent studies on redundancy allocation problems with Markov chains (RAPMC), Kim and Kim ([1, 15]) developed new stochastic models to describe the life of non-repairable systems designed with series and parallel arrangements, incorporating active or standby redundant components based on Markov theory. The application of Markov processes in switching models has been extensively researched, with the first application being in finance. Kim and Park (1994) utilized this model for a continuous-time Markov chain (CTMC) to evaluate the reliability of a phased mission system with random phase durations following general distributions [16]. Ling et al. (2019) addressed the reliability of the k-out-of-n system problem, where components are selected randomly in batches, and the reliability of each subsystem must be maximized [17]. Hawkes et al. (2011) investigated a model where switching between organizations is governed by an alternating renewal process, with underlying processes being Markovian [18]. Wang et al. (2011) extended the model proposed by Hawkes et al. to include a Markov repairable system with stochastic switching between different environments [19]. Levitin and Amari (2009) introduced optimal loading of components for a series-parallel system, considering the dependence of elements' failure rates on their load, aiming to improve system reliability as much as possible [20]. Li et al. (2020) illustrated a mixed redundancy strategy that allows for the simultaneous use of both active and cold standby components in a subsystem [18].

Furthermore, Wang et al. (2020) proposed a model for a multi-type production system, formulating it as a multi-objective optimization problem and considering the optimal redundancy strategy for both cold standby and active components. The exact reliabilities of cold standby redundant subsystems with imperfect detectors/switches were defined using an approach based on continuous-time Markov chains [22]. Li et al. (2016) analyzed non-repairable systems to model reliability problems, considering cyclic-mission switching and multi-state failure components [23]. Sadeghi et al. (2020) formulated the non-repairable redundancy allocation problem (RAP) for a series-parallel system with cold standby components and a faulty switching mechanism for heterogeneous components in each subsystem [24]. Hsieh (2020) employed a cold standby hybrid strategy to improve system reliability, optimizing the number of components used in each subsystem [25]. Gilani (2020) presented an optimization problem to investigate the impact of heterogeneous component order on overall system reliability [23]. In these papers, the consideration of reparability introduces a multi-objective aspect, where optimizing cost and weight contributes to increasing the system's lifetime.

Wu and Yang (2020) considered a repairable system in a hot standby state with a faulty switching mechanism, but their model assumes homogeneous components within each subsystem [27]. Kan and Eryilmaz (2020) examined a repairable system consisting of an active component and a standby component, with two identical components and one repairable component [28]. In these studies, the components are homogeneous. Considering components as heterogeneous will help to make the model more generalizable, so in this paper, the components are considered heterogeneous. Wu and Yang (2020) investigated a repairable system in a hot standby state with a faulty switching mechanism. However, their model assumes homogeneous components within each subsystem [29]. Kan and Zubair et al. (2023) examined a repairable system consisting of an active component and a standby component, with two identical components and one repairable component in their system [30]. Chambari et al. (2021) proposed a bi-objective simulation-based optimization model for RAP, considering heterogeneous components in each subsystem. Their objective functions aimed to minimize system costs and maximize system reliability [28]. Chowdhury et al. (2023) developed an optimal strategy for each subsystem, either active or cold standby, based on a proposed mathematical model. There are gaps in studies on optimizing the reliability of systems to reduce the constraints mentioned above. This paper tries to reduce the gap in this field by removing the above limitations. In these studies, the components were assumed to be homogeneous. Considering heterogeneous components would enhance the generalization of the model, thus in this paper, heterogeneous components are considered.

Li et al. (2023) performed a detailed analysis of a cold standby repairable system with two identical parts, considering the rest of the server during repairs [32]. Bhandari et al. (2024) investigated the steady-state availability of a series-parallel system with repairable components and redundant components. They extended the problem to include different types of components and repairers, varying failure rates of operating components, and fixed numbers of failed components and repair rates in each parallel redundant subsystem. By employing Markov model theory and matrix analysis, they obtained the steady-state probability vector of each subsystem and the steady-state availability of the entire system [33]. Shahriari et al. (2024) studied a cold standby repairable system with two dissimilar components maintained by a repairman [34]. Gholinezhad (2024) proposed a strategic model, RRAP-CM-MCCS (Markov-based cold standby combined with component mixing), to enhance system reliability [35]. Generally, there is a stark gap between previous research and this study, as shown in Table 1, which compares them.

Tab. 1. Comparison between present study and past ones

Authors	Features	Research
Kim and Park (1994)	- Maximizing reliability while considering costs and weights	Development of a model for designing subsystems
Li et al. (2019)	- Combining active and standby components to enhance reliability	Introduction of a mixed strategy within subsystems
Hawkes et al. (2011)	- Solving complex system state using Markov chains	Utilization of Markov chains to analyze system state
Wang et al. (2011)	- Enhancing model's applicability to real-world scenarios	Incorporation of cost uncertainty in objective function
Kim and Park (1994)	- Evaluation of reliability in phased mission system	Continuous-time Markov chain for evaluating system reliability
Ling et al. (2019)	- Maximizing reliability of subsystems with random component selection	Reliability analysis of k-out-of-n system with random selection
Hawkes et al. (2011)	- Model investigating switching between organizations governed by renewal process	Investigation of model with switching governed by renewal process
Wang et al. (2011)	- Extending model to include stochastic switching between environments in a repairable system	Extension of model to include stochastic switching between environments
Levitin and Amari (2009)	- Optimal loading of components considering failure rates and load dependence to improve system reliability	Optimal loading of components for series-parallel system
Li et al. (2020)	- Illustrating mixed redundancy strategy for simultaneous use of active and cold standby components in a subsystem	Mixed redundancy strategy for active and cold standby components
Wang et al. (2020)	- Formulating multi-objective optimization problem for optimal redundancy strategy in production system	Multi-objective optimization for optimal redundancy strategy
Li et al. (2016)	- Modeling reliability problems in non-repairable systems with cyclic-mission switching and multi-state failure components	Reliability modeling of non-repairable systems with cyclic-mission switching
Sadeghi et al. (2020)	- Formulating non-repairable redundancy allocation problem for a series-parallel system with cold standby components and faulty switching mechanism	Formulation of non-repairable redundancy allocation problem
Hesieh (2020)	- Employing cold standby hybrid strategy to improve system reliability, optimizing the number of components used in each subsystem	Employment of cold standby hybrid strategy
Guilani (2020)	- Investigating impact of heterogeneous component order on system reliability through optimization problem	Optimization problem to investigate impact of heterogeneous component order
Wu and Yang (2020)	- Investigating repairable system in hot standby state with faulty switching mechanism	Repairable system with hot standby state and faulty switching
Kan and Eryilmaz (2020)	- Examining repairable system consisting of active and standby components with two identical components and one repairable component	Repairable system with active and standby components
Wu and Yang (2020)	- Investigating repairable system in hot standby state with homogeneous components within each subsystem	Repairable system with homogeneous components
Li et al. (2023)	- Performing detailed analysis of a cold standby repairable	Repairable system with two identical

Authors	Features	Research
	system with two identical parts	parts
Bhandari et al. (2024)	- Investigating steady-state availability of series-parallel system with repairable and redundant components	Steady-state availability of series-parallel system
Shahriari et al. (2024)	- Studying cold standby repairable system with dissimilar components maintained by a repairman	Cold standby repairable system with dissimilar components
Gholinezhad (2024)	- Proposing strategic model (RRAP-CM-MCCS) to enhance system reliability	Strategic model for enhancing system reliability
This study	-Costs uncertainty, Markov Chain, Monte-Carlo	Optimization reliability model

2.1. Research gap

Despite significant advancements in the field of system reliability and redundancy allocation, there remain critical gaps that this study aims to address. Previous research has extensively explored various strategies for enhancing system reliability, such as improving component reliability, implementing redundant components, and using substitutes. However, several limitations persist:

1. **Limited consideration of heterogeneous components**: Many studies have assumed homogeneous components within subsystems, which does not reflect the complexity of real-world systems. For instance, Wu and Yang (2020) and Kan and Eryilmaz (2020) investigated repairable systems with homogeneous components, which limits the generalizability of their models [27, 28]. In contrast, this study incorporates heterogeneous components, providing a more comprehensive and realistic approach to system design.
2. **Inadequate integration of cost uncertainty and multi-objective optimization**: While some studies have addressed cost aspects, they often lack a robust treatment of cost uncertainty and multi-objective optimization. For example, Sadeghi et al. (2020) and Hsieh (2020) focused on optimizing redundancy allocation without fully integrating cost uncertainty [24, 25]. This study addresses this gap by employing the Monte-Carlo method to evaluate cost uncertainty and utilizing a multi-objective function to optimize both costs and weights.
3. **Insufficient application of advanced stochastic models**: Recent advancements in stochastic modeling, particularly the use of Markov chains, have shown promise in reliability analysis. However, their application in redundancy allocation problems remains underexplored. Kim and Kim (2017) and Levitin and Amari (2009) developed stochastic models, yet these have not been fully integrated with redundancy allocation strategies incorporating heterogeneous components [1, 15, 20]. This research leverages Markov chains to model the reliability benefits and system state transitions more accurately.
4. **Neglect of mixed redundancy strategies**: Although mixed redundancy strategies combining active and standby components have been suggested, they have not been thoroughly investigated in the context of heterogeneous components and cost uncertainties. Li et al. (2020) illustrated a mixed redundancy strategy but did not address these additional complexities [21]. This study fills this gap by presenting an innovative mixed redundancy strategy that considers both active and standby components within heterogeneous subsystems, while managing cost uncertainty and optimizing multiple criteria.
5. **Lack of comprehensive sensitivity analysis**: Many existing studies lack detailed sensitivity analysis to validate the robustness of their models under varying conditions. This study conducts a comprehensive sensitivity analysis to assess the impact of different percentiles in cost evaluations, ensuring the model's robustness and applicability to real-world scenarios.

By addressing these gaps, this study presents a novel approach to redundancy allocation in complex systems. It incorporates heterogeneous components, integrates cost uncertainty through advanced stochastic methods, employs a mixed redundancy strategy, and provides a thorough sensitivity analysis.

3. Problem Definition

Complex systems, due to their sensitivity, require careful design of subsystems with a specific number of components to achieve maximum reliability [32]. In the real world, these components can be utilized in multiple subsystems and can have varying costs and weights. The use of the Markov chain facilitates solving the model in complex states. This paper aims to determine the number of components used in each subsystem and their optimal sequence, ensuring that reliability is maximized while costs and weights are minimized by applying a mixed strategy in subsystems. Moreover, costs are estimated under uncertainty in the objective function, making the model more representative of real-world conditions.

According to the described methodology, a mixed strategy is obtained when components are both active and standby in the subsystem. Each subsystem generally consists of a combination of n_{Ai} active components and n_{Si} standby components. Coit (2003) revealed that the relationship between the reliabilities of active and cold-standby strategies depends significantly on the redundancy level [36]. Additionally, as Gilani et al. (2020) stated, if the maximum number of redundancy components, n_{ij}, is $n_i \leq n_{ij}$ in a subsystem, the cold-standby strategy predominates.

In general, each subsystem can employ one of four strategies:

1. **No redundancy**: If there is only one active component, the strategy does not involve redundancy.
2. **Active redundancy**: If there is more than one active component, an active redundancy strategy is used.
3. **Cold-standby redundancy**: If there is one active component and one or more cold-standby components, a cold-standby redundancy strategy is used.
4. **Mixed redundancy**: If there are multiple active components and one or more cold-standby components, a mixed redundancy strategy is applied.

Fig 1 illustrates the types of strategies (active, cold standby, and mixed) within each subsystem for a maximum of four components.

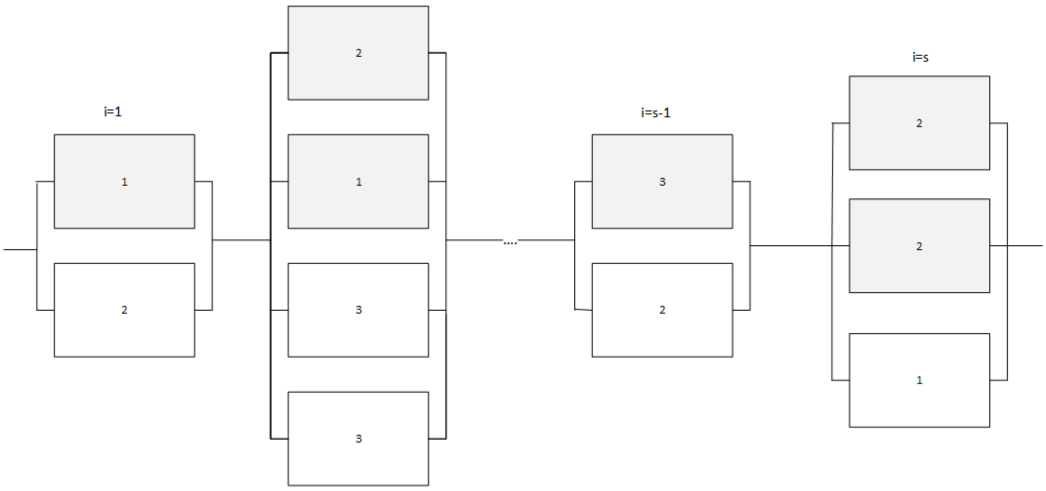

Fig. 1. A heterogeneous series-parallel system with the mixed strategy.

3.1. Model assumptions

In this study, a series-parallel system is used for modeling. The system has n components, where nA are active components, and $n_S = n - n_A$ are ready-to-use standby components. The following assumptions are considered:

1. The system includes both active components and cold standby components.
2. The components are non-repairable.
3. All subsystems are independent of each other.
4. Switching time can be ignored.
5. The components are heterogeneously located within each subsystem.
6. The lifetime of the components follows a phase-type distribution.

3.2. Definition of notations

The symbols and variables used in the study are defined as shown in Table 2.

Tab. 2. Variables and symbols of the model

Introduction	Variable	Introduction	parameter
Number of active and cold-standby components in subsystem i	n_{Ai}, n_{si}	Reliability of switching system in subsystem i	$\rho_i(t)$
Redundancy level of subsystem i	n_i	Failure rate of component *i*	λ_i
Order of redundant components in	H_{Ai}, H_{Si}	Set of state space in subsystem *i*	s_i

subsystem i for the active and cold-standby strategies, respectively			
Probability density function of the jth failure in subsystem i	$f_{ik_i}^{\max,n_{Ai}}$	Set of transition and absorbing states in subsystem i, respectively	s_{iT}, s_{iA}
Reliability of component *j* in subsystem *i* at time *t*	$r_{ij}(t)$	Initial state probability distribution vector for subsystem *i*	π_i
System reliability	$R_i(t,q)$	Infinitesimal generator matrix of the CTMC model	Q
	Symbols	Transition rate matrix among transient states	D
Subsystem	I	Failure rate column vector	d
Component	J	Number of transient states	M
Redundancy allocation problem with mixed components	RAPMC	Transition probability matrix to indicate the transitions in the switch and the operating component	p_T, p_A
Continuous-time Markov chain	CTMC	Maximum total cost	C
Transition rate matrix	TRM	total weight of the system	W
		Weight and Cost characteristics of component *j* in subsystem *i*	w_{ij}, c_{ij}

4. Mathematical Model

The mathematical model is presented for a series-parallel system. A series-parallel system is a configuration commonly used in practice (see Fig. 2). In a series configuration, all components must operate for the system to function, whereas in a parallel configuration, only one component needs to function for the system to operate [34,35].

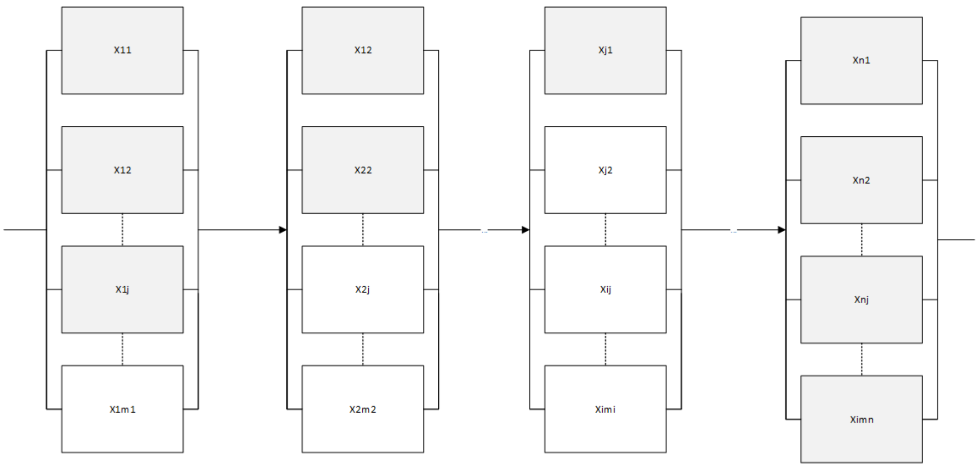

Fig. 2. Series–parallel configuration

The components in each subsystem are arranged in parallel, but each subsystem is in series with the others. These components differ in characteristics such as reliability, weight, and cost. Moreover, the model aims to achieve three objectives: minimize costs, minimize the weight of each subsystem, and maximize system reliability, all according to the optimal sequence within each subsystem.

$$Min \quad C = \sum_{i=1}^{s} c_i(q_i) \tag{1}$$

$$Min \quad W = \sum_{i=1}^{s} w_i(q_i) \tag{2}$$

$$Max \quad R_s(t,q) = \prod_{i=1}^{S} R_i(t,q) \tag{3}$$

$$F(H_{Ai}) \geq 1 \qquad i = 1,\ldots,S \tag{4}$$

$$q_i = \{H_{Ai}, H_{Si}\} \quad set \quad of \quad \text{integer} \quad number \quad i = 1,\ldots,S \tag{5}$$

Where $c_i(q_i)$, which is the total cost of the system, is determined by the optimal sequence in each subsystem (considering both active and redundant components). Additionally, the total weights of the system $(w_i(q_i))$ components are calculated according to the optimal sequence in each subsystem. R_s (t,q) indicates the reliability of the entire system (SSS being the number of subsystems), which corresponds to the reliability with the optimal sequence in each subsystem. The objective function aims to maximize the reliability of the whole system. q_i represents the selected sequence of the i-th subsystem. H(.) ensures that there is a maximum of one component in each subsystem.

The lexicographic method is used to solve the problem. First, the problem is solved with the cost objective function as a single objective, and the value obtained from solving the cost model is placed as a practical value in the constraints. Then, the problem with the objective function of minimizing weights is solved. The value obtained from solving the weight model is considered a practical value and replaced as a constraint in the model. Finally, the problem is solved to maximize the reliability of the system model. The final model is shown below in Eq.s (6-11)

$$Max \quad R_s(t,q) = \prod_{i=1}^{S} R_i(t,q) \tag{6}$$

$$\sum_{i=1}^{S} \cos t(q_i) \le C \tag{7}$$

$$\sum_{i=1}^{S} Weight(q_i) \le W \tag{8}$$

$$H(H_{Ai}) \ge 1 \qquad i = 1,\ldots,S \tag{9}$$

$$q_i = \{H_{Ai}, H_{Si}\} \quad set \quad of \quad \text{int}eger \quad number \quad i = 1,\ldots,S \tag{10}$$

Where constraints 7 and 8 represent the practical values obtained from solving the cost and weight objective functions.

5. Methodology

This study is of an applied type, and the data has been collected using a well-known benchmark example from market information. In this method, first, according to the failure rate in each subsystem, instead of using the infinitesimal generator matrix, the transition rate matrix between the transient states is used. Using Markov chains, the reliability of the whole system is calculated. The main reason for using the transition rate matrix between the transient states is to simultaneously consider the two states of the switching system when it is operating properly and when it has failed, as proven in the article by [36]. To solve the model, a genetic algorithm is used. A modified genetic algorithm is also employed to optimize the problem, which maximizes the reliability of the entire system. Due to the complex form of the objective function in the developed model, which is nonlinear, non-differentiable, non-convex, and integer-based, standard optimization methods and existing techniques for convex optimization are not appropriate. The non-convexity of the cost function can cause these algorithms to find a local optimum with poor performance.

Meta-heuristic algorithms do not require the derivative and other ancillary information, allowing them to escape local optimum solutions [37]. They are therefore very suitable for solving the formulated problem. In meta-heuristic algorithms, the represented solution techniques have profoundly affected results [38]. One way to modify these algorithms is to change the searching mechanism by adding operators specific to the nature of the problem [39]. For example, a max-min mutation operator was added to the Modified-GA in [40] to improve performance. The efficiency of the min-max mutation-based GA has been proven in reliability optimization problems [41-45]. The GA algorithm is coded with MATLAB R2019b.

5.1. Research steps

The steps start with the constraints and the last objective function, which is the reliability objective function, followed by the first objective function, which is the cost objective function [50-55]. The cost objective function also has uncertainty, which is described in phase 7.

Phase 1. Calculate component reliability

Reliability for each component in each subsystem is calculated according to the Eq. (11):

$$r_{ij}(t_m) = \exp(-\lambda_j t_m) \tag{11}$$

Which λ_j is the rate of failure and t_m is work time.

Phase 2. Calculate the reliability of the subsystem

At this stage, the transfer rate matrix, which consists of D and d, shows the transition rate matrix among transient states and absorption states for each subsystem, respectively. Also, the infinitesimal generator Q for PH (π, D) is described as in Eqs. (12-13) below:

$$Q = \begin{bmatrix} D & d \\ 0 & 0 \end{bmatrix} \tag{12}$$

$$D = \begin{bmatrix} -\lambda_1 & \lambda_1 & 0 \\ 0 & -\lambda_1 & \lambda_2 \\ 0 & 0 & -\lambda_3 \end{bmatrix}, d = \begin{bmatrix} 0 \\ 0 \\ \lambda_3 \end{bmatrix} \tag{13}$$

The Markov chain is shown in Fig. 3., where the system with failure rate is λ. In the initial case, the failure rate is zero. As soon as the system encounters the first failure and the first component leaves the system, the other component is replaced, and this process continues until the system reaches the absorption state.

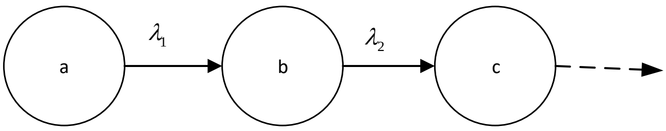

Fig. 3. A markov model

Phase 3. Calculate the transfer rate matrix for a mixed redundancy strategy

There are n_i components in each subsystem where n_A is the active component and n_s is the standby component. As soon as the last active component breaks down, the first component is on standby to go online. However, to activate the standby component, an imperfect switching mechanism is used. In case of a switch failure, it is not possible to transfer to the standby state, and the system enters the absorption state directly. So, the TTF distribution of the switching mechanism is assumed to be PH:(π_{id}, D_{id}) The infinitesimal generator of switching mechanism Q_{id} is represented as:

$$Q_{id} = \begin{bmatrix} D_{id} & d_{id} \\ 0 & 0 \end{bmatrix} \tag{14}$$

If, according to Fig.4, there are three active components and one cold-standby component. Then, according to Fig.5, all the transition states are established until the absorption state is reached. Also, it cannot continue when the system reaches the absorption state.

The probability density function, cumulative distribution, and reliability functions for a particular unit are presented by Eq. (15-17)

$$F(t) = \pi_{s_A} + 1 - \pi \exp(Dt)\vec{1}, for \quad t \geq 0 \tag{15}$$

$$f(t) = \begin{cases} \pi_{s_A} & , for \quad t = 0 \\ \pi \exp(Dt) d & , for \quad t \geq 0 \end{cases} \tag{16}$$

$$R(t) = \pi \exp(Dt)\vec{1} \tag{17}$$

Where, π_{s_A} explains the initial probability that the system starts in a failed state, which is in this paper used to be $\pi_{s_A} = 0$.

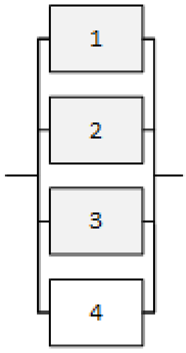

Fig. 4. A subsystem with one cold-standby components

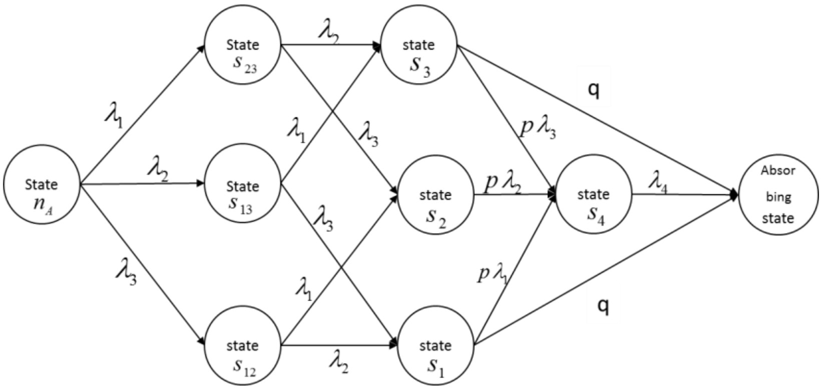

Fig. 5. Possible state transitions

- If there are four component in each subsystem, then the different strategies used for each subsystem are as follows:
- If there are one active component and three standby components, then the strategy is standby.
- If there are two active components and two standby components to work, then it applies the mixed redundancy strategy.
- If there are three active components and one standby component, then it applies the mixed redundancy strategy.
- If there are four active components and zero standby components, then the strategy is active.
- All four combinations are tested, and the best one can be selected that can maximize the reliability of the whole system with using a genetic algorithm. Also in Eq. 18 shows the operating state matrix and the P_A transition state matrix for the system

$$P_T = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \ P_A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \tag{18}$$

Also, in the paper of Pourkarim et al. (2020), the transfer rate matrix is presented as follows, which is also used to obtain the optimal amount of reliability (There is proof of this formula in their paper).

The integrated total sub-TRM D_i^T in subsystem *i* may be calculated using Eq. (19). As well as, table 3. shows the reliability values of each component.

$$D_i^T = Q_{id} \oplus D_1 + p_A \otimes (D_2 - D_1) \tag{19}$$

Tab. 3. Reliability values of each component

Component number	$r_{1,j}(100)$
1	0.6
2	0.9
3	0.8
4	0.7

To use the formula D_i^T the D_2, D_1 matrix must first be formed.

1- Matrix formation D_1

The matrix shows the working condition in which the standby components are passed through the switch, and the switch always works well. And faulty components cannot be repaired. For example, the matrix for a subsystem as shown in (20) is as follows:

$$D_1 = \begin{bmatrix} -\lambda_1-\lambda_2-\lambda_3 & \lambda_1 & \lambda_2 & \lambda_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\lambda_2-\lambda_3 & 0 & 0 & \lambda_2 & \lambda_3 & 0 & 0 & 0 \\ 0 & -\lambda_1-\lambda_3 & 0 & 0 & \lambda_1 & 0 & \lambda_3 & 0 & 0 \\ 0 & -\lambda_1-\lambda_2 & 0 & 0 & 0 & \lambda_1 & \lambda_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\lambda_3 & 0 & 0 & \lambda_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\lambda_2 & 0 & \lambda_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\lambda_1 & \lambda_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\lambda_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{20}$$

2- Matrix formation D_2

During the switching system has failed, the system has failed quickly after the last active or standby component. Besides, the switch is not capable of replacing the failed component with a cold standby system. Also, system failure transpires in two ways: First, when the last active component reaches the absorption states. Second, when the cold standby component is not activated, so is as Eq.(21):

$$D_2 = \begin{bmatrix} -\lambda_1-\lambda_2-\lambda_3 & \lambda_1 & \lambda_2 & \lambda_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\lambda_2-\lambda_3 & 0 & 0 & \lambda_2 & \lambda_3 & 0 & 0 & 0 \\ 0 & -\lambda_1-\lambda_3 & 0 & 0 & \lambda_1 & 0 & \lambda_3 & 0 & 0 \\ 0 & -\lambda_1-\lambda_2 & 0 & 0 & 0 & \lambda_1 & \lambda_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\lambda_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\lambda_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\lambda_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\lambda_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{21}$$

Phase 4. Monte carlo simulation for cost uncertainty

In this phase, the uncertainty in costs is calculated for the cost objective function, which Monte Carlo simulation is used to calculate.

In this method, the following steps are performed:

1. First, the desired number of iterations to obtain the value of the objective function in different iterations is determined.
2. For uncertain cost parameters, random values of probability distributions are considered. (e.g.

uniform distribution over a specified period)
3. For a specific answer, the objective function is calculated, and its value is recorded for each iteration in the production of random parameters.
4. The average (or other torque of) the values of the objective function is calculated and introduced as the optimization function considering the uncertainty in the parameters.
The Monte Carlo simulation [46,47] objective function used in this paper is mathematically given below in Eqs (22, 23):

$$f_j = c_i(q_i) \quad , \forall \mathrm{i} = 1,2,3,..,s, \forall j = 1,...,u \tag{22}$$

$$F = \left(\frac{\sum_{j=1}^{u} \mathrm{f}_j}{u}\right) \tag{23}$$

In this relation, *i* is the number of subsystems and *j* is the number of iterations, the objective function in each iteration, and F is the average cost of the subsystems per *u* number of iterations of the objective function. Moreover, The percentile and mean-variance methods (measuring risk criteria) are also used to compare the results. In percentile method, like Monte Carlo method, random numbers are generated, and the objective function is calculated during different iterations. Finally, the optimal objective function is obtained from the following formula:

$$o = \frac{h * u}{100} \tag{24}$$

$$F = \mathrm{f}_o \tag{25}$$

According to the above formula, o represents the hth percentile in the total number of the generated objective function (*u*), and f_o is the value of the objective function of the percentile [48]. Also, In the mean-variance method [46], the objective function is optimized, and the deviation of the calculated criteria is obtained as follows:

$$\bar{x} = \left(\frac{\sum_{j=1}^{N} \mathrm{f}_j}{u}\right) \tag{26}$$

$$F = \bar{x} + 1.96\left(\sqrt{\frac{\sum_{j=1}^{N} (\mathrm{f}_j - \bar{x})^2}{u-1}}\right) \tag{27}$$

In this regard, j is the number of iterations and F is the optimal objective function, obtained from the sum of the average iterations f_j and the level of risk f_j. The value of z =1.96 is considered here, which is equivalent to the upper limit of 95% two-way confidence interval.

Phase 5. Calculate system costs, weights, and reliability

First, the model is solved with the cost objective function, and the value obtained is replaced in the constraints. Then the model is solved with the weight objective function and finally by adding two cost and weight constraints, the model is solved to maximize reliability (as before it was explained). Moreover, the reliability of the $R_i(t)$ subsystem for a mixed redundancy strategy with heterogeneous components is represented by the following Eq. (28).

$$R_i(t) = \pi_i^T \exp(D_i^T, t)\vec{1} \quad for \quad t \geq 0 \tag{28}$$

Reliability is obtained by multiplying D_i^T each subsystem by time *t* and the initial probability vector π_i^T for each subsystem.

Phase 6. Investigate the effect of component order on reliability

Using heterogeneous components in a parallel system requires predicting the optimal sequence of components. Therefore, different sequences for the subsystem should be considered to examine the effect of component sequence on system reliability.

Phase 7. Solve the model with a genetic algorithm

The modified Genetic Algorithm (GA), derived from Holland's initial concept, borrows from biological genetics to represent solutions as finite-length strings. Unlike classical exact methods, GA generates a population of solutions and selects the most promising ones. GA has shown efficacy in optimizing large-scale combinatorial problems, and recent studies demonstrate its performance in reliability optimization. The algorithm's manipulation for the proposed model involves synchronizing its features with those of the problem.

The algorithm procedure entails several steps:

1. Chromosome Design: Each chromosome is encoded as a $Max(n_i) \times S$ matrix, defining the redundancy level for subsystems and the sequence order of selected components.
2. Initial Population Generation: Nstart chromosomes are randomly generated as the initial population.
3. Fitness Evaluation: The fitness function evaluates each chromosome's fitness, considering the objective function and constraint violations.
4. Selection: The tournament procedure selects candidate chromosomes based on fitness function values.
5. Crossover: Crossover operators combine selected chromosomes to produce offspring, inheriting properties from parents.
6. Mutation: Mutation operators introduce slight changes to chromosome structures, ensuring solution diversity and avoiding local optima.
7. Fitness Evaluation: The fitness values of newly generated chromosomes are evaluated, and a rank-based selection procedure selects premium chromosomes for the next generation.
8. Iteration: Steps 4 to 7 are repeated for a defined number of genetic cycles (Maxiter).
9. Termination: The algorithm terminates after completing the specified number of genetic cycles.

The modified GA compactly represents chromosome size to reduce CPU time, utilizes crossover and mutation operations for solution diversity, and employs a rank-based selection procedure for premium chromosome selection. These adaptations enhance its effectiveness in solving reliability optimization problems. The optimal parameters for the Modified-GA are determined using the Taguchi method.

Tab. 4. Specify values at different levels of GA parameters

Level	Npop	Iteration	Mutation	Crossover	Selection
1	20	50	0.2	0.6	100% the best
2	50	50	0.3	0.7	10% random
3	100	50	0.4	0.8	20% random
4	20	100	-	-	-
5	50	100	-	-	-
6	100	100	-	-	-

In this way, Mini-tab software has been used to analyze the answers. According to the number of levels, 18 suitable ones have been identified, and the optimal level of reliability has been calculated for all 18 levels. After analyzing the software, Since in this paper more reliability is of utmost desire, the value of the ratio is calculated from Eq. (29) where y_i denotes the response in i^{th} the experiment, and n denotes the number of orthogonal arrays, on which the performance of the experiments is based. The highest *S/N* ratio determines the optimal level for each factor. Its output is shown in Table 5 and Fig. 6 and 7.

$$S / N = -10 \log\left(\frac{1}{n}\sum_{i=1}^{n}\frac{1}{y_i^2}\right) \tag{29}$$

Tab. 5. Calculate reliability according to the levels of GA parameters

npop& iteration	mutation	Crossover	Selection	R
1	1	1	1	0.9
1	2	2	2	0.921
1	3	3	3	0.962
2	1	1	2	0.9632
2	2	2	3	0.9801
2	3	3	1	0.9716
3	1	2	1	0.9012
3	2	3	2	0.9842
3	3	1	3	0.95147
4	1	3	3	0.9812
4	2	1	1	0.9791
4	3	2	2	0.9341
5	1	2	3	0.98341
5	2	3	1	0.9751
5	3	1	2	0.96143
6	1	3	2	0.919
6	2	1	3	0.98625
6	3	2	1	0.959

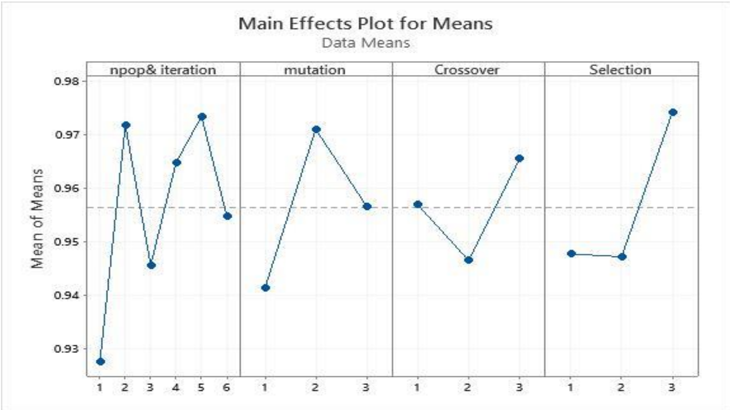

Fig. 6. Effective means for different levels of GA parameters

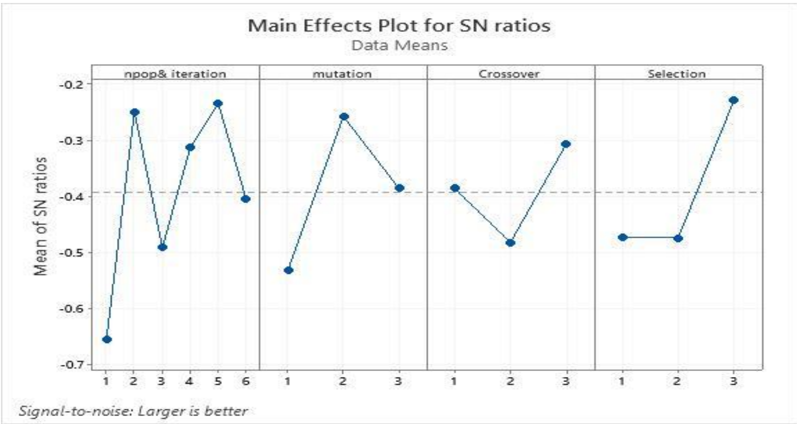

Fig. 7. S / N ratio for different levels of GA parameters

According to Fig. 6 and 7, the higher the mean and *S/N* value, the more acceptable the level. In Table 6, the levels with the highest values are selected, and the results are reported.

Tab. 6. Optimal values for GA parameters

	npop	iteration	mutation	Crossover	Selection
Level	5	5	2	3	3
Value	50	100	0.3	0.8	20% random

6. Research Findings

6.1. Numerical experiment

In this section, the proposed model of RAPMC is examined. In addition, numerical experiments are conducted on a well-known benchmark problem to realize the ability of the introduced CTMC method to model a mixed strategy with heterogeneous components and improve system reliability. The proposed formula is based on heterogeneous systems 1-out-n:G with a mixed redundancy strategy. The purpose of this model is to achieve improved system reliability by sequencing the optimal component, and the most suitable strategy for each subsystem. As well as a reliability optimization benchmark is used to examine the advantages of the proposed method. The problem consists of 14 subsystems that each subsystem can have a maximum of 4 components. The cost and weight of all components in each subsystem are limited. A mixed redundancy strategy has been used to maximize system reliability. It is also assumed that the switch is imperfect, with a reliability of 0.99 at 100 h for each subsystem. There are also heterogeneous components in each subsystem. Ultimately, the TTF of the components and the switch is supposed to support an exponential distribution [50,51]. In general, the problem is modeled and solved by considering: 1- multi-objective function 2- Heterogeneity of components 3-Optimal order of components in the subsystem 4- mixed redundancy components, and 5- cost uncertainty.

6.2. Model solution results

As a result, the algorithm was solved for 50 generations (npop), and 100 iterations and the optimal reliability value for the whole system was 0.98625, Shown in Fig.10. The number of optimal components in subsystems is obtained according to the following table 7. In addition, After solving the model with the cost objective function, using the modified genetic algorithm, its elimination function is optimized, as shown in Fig.8. It should be noted that the costs are considered as uncertainties, and the Monte Carlo simulation is used.

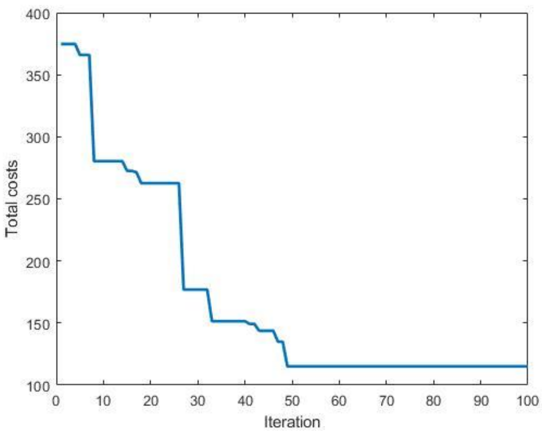

Fig. 8. The optimal amount of total subsystem costs

As determined from Fig.8, the optimal cost of the whole system is 115. The optimal objective function of the total weight of the system, as shown in Fig. 9, is obtained after calculating the cost objective function and placing it as a constraint in the model.

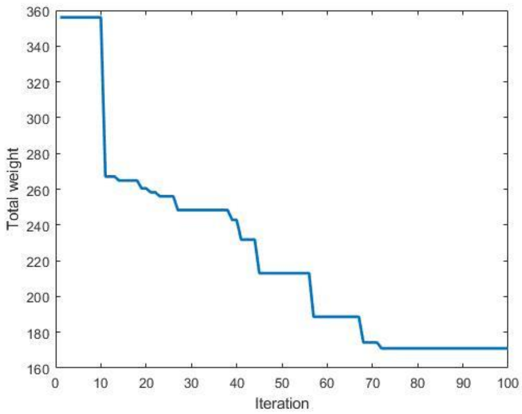

Fig. 9. The optimal amount of total subsystem weights

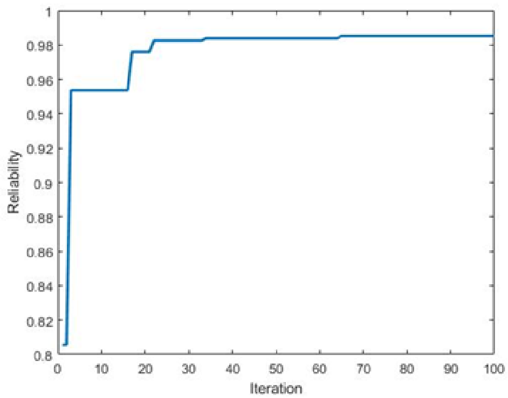

Fig. 10. The optimal value of total system reliability in a series-parallel system

Tab. 7. Solution result for each subsystem

Components	Subsystems													
X_1	3	1	4	1	2	3	4	1	1	2	4	3	4	3
X_2	3	2	2	2	2	2	1	2	1	2	1	2	1	1
X_3	3	1	4	2	1	1	0	1	0	1	1	1	0	0
X_4	0	0	0	0	0	0	0	0	0	0	0	1	0	0

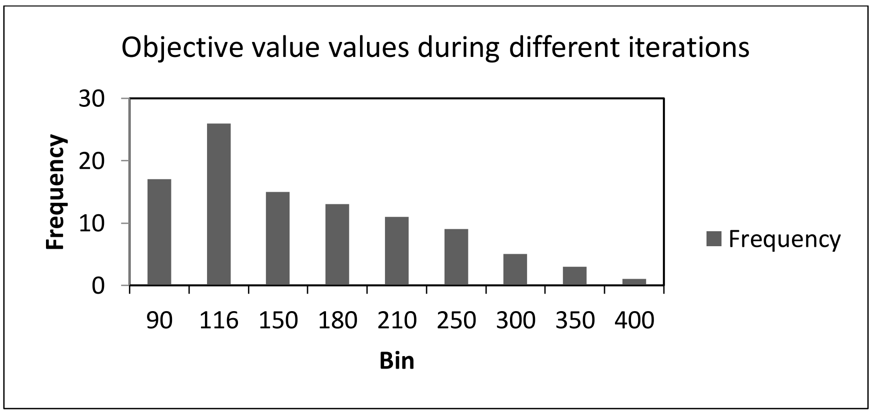

Fig. 11. Histogram of objective function values during different iterations using Monte Carlo simulations

As can be seen, the table above shows the results solved by the genetic algorithm for each subsystem. Table 4. is obtained by considering the optimal sequence for the system and using a mixed strategy that shows the sequence of components used in each subsystem. In one hundred times of Monte Carlo simulation iterations, by keeping the other variables constant and only considering the uncertainty state for the cost parameters, the value of the objective function is recorded during these iterations, and its histogram is shown in Fig. 11. Cost parameters have a uniform distribution. According to the central limit theorem, if a new random variable is defined from the linear combination of the initial variables, then regardless of the probability distribution of the original random variables, the probability distribution of the new variable will be somewhat normal. pursuant to this theorem, as shown in Fig. 11, it is expected that the resulting distribution will be closer to normal as the sample size increases. Fig.12 shows the result of comparing the objective function considering the uncertainty with the three methods of Monte Carlo, percentile, and mean-variance as follows.

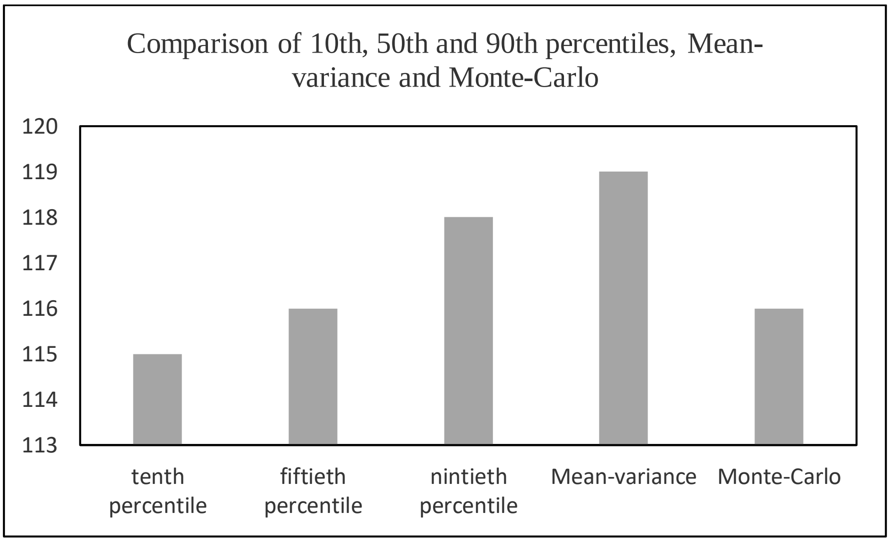

Fig. 12. Comparison of the optimal objective function in the three methods of Monte Carlo, percentile and mean-variance

According to Fig.12, the optimal objective function is compared in the three methods of Monte Carlo, 10-50-10th percentile. The objective function is repeated one hundred iterations for each percentile, which can be concluded that the smaller the selected percentile. The smaller the selected percentile, the smaller the value of the objective function and mean-variance in 100 iterations. According to Fig.12, the costs of the 10th percentile have the lowest cost, and the method of mean variance has the highest cost.

6.3. Model performance evaluation

According to the Eq. of maximum improvement index for the optimal value of reliability in this paper, considering the multi-objective function compared to Pourkarim et al. (2020), it has improved by 5.55%. The maximum possible improvement (MPI) index is used, which is a common method for evaluating the performance of the model and includes the calculation of the maximum possible improvement of the index to evaluate the results and the resulting improvement. it is expressed by the following Eq. (30):

$$MIP = (R_S(this\ \ paper) - R_S(other\ \ paper)) \times 100 / (1 - R_S(other\ \ paper)) \tag{30}$$

Thus, the reliability of the system in this article (R_s (this paper)) is measured against R_s (other studies). The MIP obtained for this paper is shown in Table.8, compared to some studies, as presented in the paper by Puorkarim et al. (2020). The result of this evaluation indicates that the MIP of this paper is a significant improvement over all the papers presented, for example, the famous benchmark. Because this paper has considered multi-objective optimization, so it has more paid attention to detail.

Tab. 8. Results from 34 sub-problems

Row	Weight	Reliability in other papers	Reliability in this paper	MPI
1	159	0.961131	0.97743	41.93316
2	160	0.976856	0.97908	9.609402
3	161	0.975577	0.97902	14.097367
4	162	0.976856	0.98051	15.788109
5	163	0.964463	0.98102	46.590877
6	164	0.978959	0.98109	10.127846
7	165	0.980243	0.98204	9.0955105
8	166	0.979356	0.98358	20.461151
9	167	0.981607	0.98404	13.227858
10	168	0.983298	0.98458	7.6757275
11	169	0.982005	0.98503	16.810225
12	170	0.983696	0.98604	14.37684
13	171	0.982498	0.98797	31.264998
14	172	0.982971	0.9886	33.055376
15	173	0.982971	0.98901	35.463034
16	174	0.98325	0.98951	37.373134
17	175	0.9839	0.98975	36.335404
18	176	0.985275	0.9901	32.767402
19	177	0.986247	0.9907	32.37839
20	178	0.986247	0.99102	34.705155
21	179	0.987218	0.99155	33.89141
22	180	0.987218	0.99238	40.384916
23	181	0.987714	0.99392	50.512779
24	182	0.987686	0.99322	44.940718
25	183	0.987686	0.99343	46.646094
26	184	0.988648	0.99303	38.601128
27	185	0.988658	0.99415	48.421795
28	186	0.988544	0.99469	53.648743
29	187	0.988585	0.99491	55.409549
30	188	0.989551	0.99469	49.18174
31	189	0.98924	0.99521	55.483271
32	190	0.989558	0.99525	54.51063
33	191	0.990452	0.99526	50.356096
34	170	0.985442	0.98625	5.5502129

As can be seen, the MPI index in this paper has improved compared to previous studies, which indicates the model's efficiency. In the present study, the best solution for RAPMC under a mixed strategy is 0.98625. The comparison between the present study and some recent RAP pioneers shows a significant effect of all the conditions used.

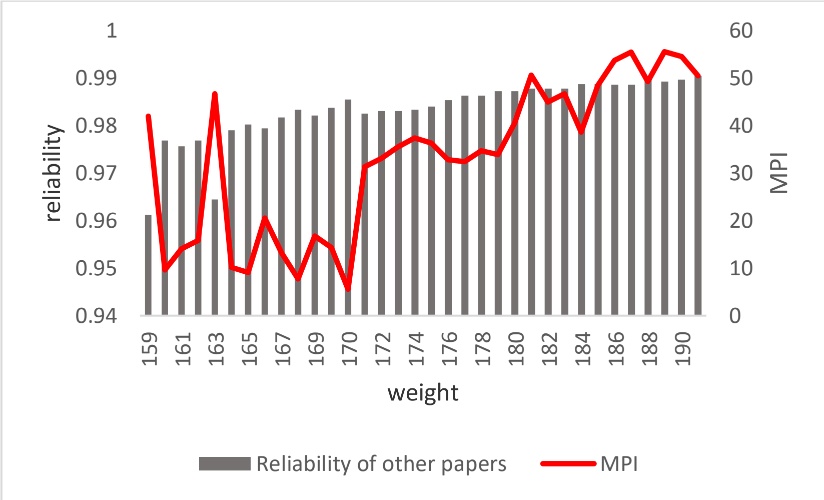

Fig. 13. Comparison of MPI index for this article and other papers

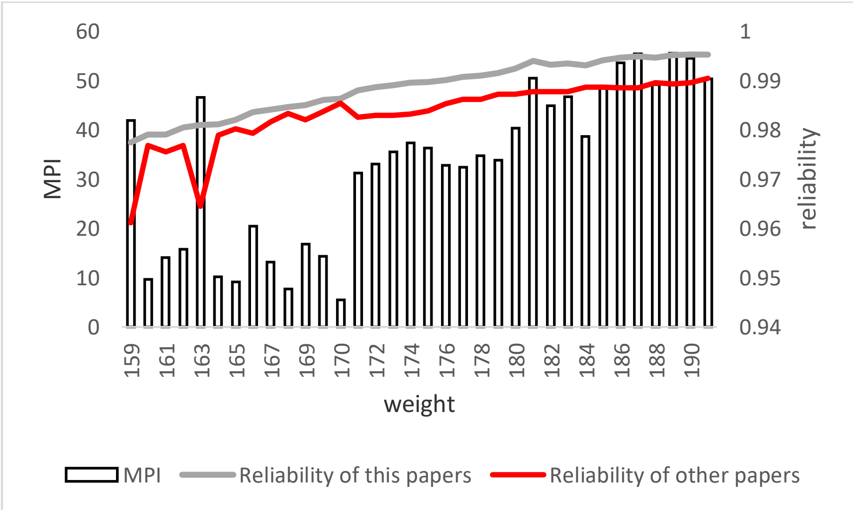

Fig. 14. Numerical results of implementing the RAPMC model with different weights

Fig. 13-14 shows a comparison of the reliability of this paper compared to other papers in different weights, and the MPI index. As it is clear, the superiority of this model over other research can be seen.

Based on the results and sensitivity analyses conducted in this study, several key managerial insights can be derived to guide decision-making in system design and redundancy allocation. These insights provide actionable recommendations for enhancing system reliability, managing cost uncertainties, and optimizing overall performance.

6.4. Managerial insights

Incorporating heterogeneous components within subsystems significantly improves system reliability compared to homogeneous components. The study's optimal reliability value of 0.98625 under a mixed strategy demonstrates the effectiveness of this approach. Managers should prioritize the use of diverse components in subsystem designs to enhance overall system resilience and longevity. By selecting components with varied lifespans and failure rates, systems can better withstand individual component failures, leading to greater overall reliability. This approach requires careful planning and integration of different component types to maximize the benefits of heterogeneity.

Utilizing a mixed redundancy strategy, which combines active and standby components, has been shown to maximize system reliability. This approach also accounts for the imperfections in switches, with a reliability of 0.99 over 100 hours. Implement mixed redundancy strategies in system designs to ensure higher reliability and efficient resource utilization. Mixed strategies allow for the seamless transition between active and standby components, reducing downtime and maintaining system performance. Managers should evaluate the specific needs of their systems and incorporate mixed redundancy where appropriate to balance reliability and cost-effectiveness.

The use of the Monte-Carlo method to handle cost uncertainties allows for more accurate financial planning and risk management. The optimal cost of the entire system was determined to be 115, with sensitivity analyses confirming the robustness of cost evaluations. Apply stochastic modeling techniques such as Monte-Carlo simulations to evaluate cost uncertainties and incorporate these insights into budgeting and financial decision-making processes. By simulating a range of cost scenarios, managers can better prepare for financial variability and make informed decisions that minimize risk and optimize resource allocation.

Balancing multiple objectives, such as reliability, cost, and weight, leads to more comprehensive and effective system designs. The study's multi-objective function optimization provided a balanced approach to achieving high reliability while managing costs and weight. Use multi-objective optimization frameworks to simultaneously address different performance criteria, ensuring that system designs meet various operational requirements. Managers should adopt tools and methodologies that facilitate the consideration of multiple factors, leading to well-rounded and efficient system designs that do not sacrifice one aspect of performance for another.

Determining the optimal sequence of components within subsystems plays a critical role in enhancing system reliability. The study's approach to sequencing components in a 1-out-n system model ensures optimal performance. Carefully sequence components within subsystems to optimize reliability and performance, using data-driven methodologies to inform these decisions. Proper sequencing can mitigate the impact of individual component failures and ensure that the most critical components are prioritized. Managers should use historical data and predictive modeling to determine the best component order. Conducting detailed sensitivity analyses ensures that system designs remain robust under varying conditions. The study's sensitivity analysis on cost parameters using Monte-Carlo simulations provided valuable insights into the system's performance under uncertainty.

Regularly perform sensitivity analyses to identify potential vulnerabilities and adapt system designs to maintain robustness and reliability. Sensitivity analysis helps managers understand the impact of different variables on system performance and make proactive adjustments to improve system resilience. The use of Markov chains for modeling system reliability and state transitions enhances the accuracy of reliability assessments. This approach provides a more detailed understanding of system behavior over time.

Integrate advanced stochastic models like Markov chains into reliability assessments to better predict and manage potential system failures. Markov models allow for a dynamic analysis of system states and transitions, offering a comprehensive view of reliability over the system's lifecycle. Managers should leverage these models to develop more accurate maintenance schedules and failure mitigation strategies. By applying these insights, managers can enhance system reliability, effectively manage cost uncertainties, and optimize overall system performance. These recommendations provide a roadmap for implementing advanced strategies and methodologies that align with the study's findings, leading to more resilient and efficient operations.

7. Conclusion

This study investigates a multi-objective system that incorporates uncertain costs and a mixed redundancy strategy with heterogeneous components in each subsystem. The primary objective is to optimize system performance and align the models with real-world conditions. To achieve this, a mixed redundancy strategy is implemented, allowing standby components to activate through a switching mechanism following the failure of active components. Additionally, component reparability is employed to enhance the system's overall lifespan and is integrated into the transfer rate matrix. The inclusion of heterogeneous components in each subsystem enhances the model's applicability and its ability to reflect real-world scenarios.

This study presents a comprehensive approach to maximizing system reliability by proposing a Markov chain-based reliability model that incorporates cost and weight constraints for each subsystem. The model effectively optimizes the sequencing of components and addresses cost uncertainties through advanced stochastic methods. By employing a modified genetic algorithm and using Monte Carlo simulation to handle cost uncertainties, the study evaluates the model's performance with a benchmark example. The use of risk criteria such as the percentile and mean-variance approaches allows for a detailed analysis, with the 10th percentile demonstrating the lowest cost objective function. The results of this study indicate a significant improvement in system reliability when compared to other methods. The proposed mixed strategy, which includes both cold standby and active components, effectively enhances system robustness and performance. The model achieves an optimal reliability value of 0.986 for the entire system, showcasing its efficiency and superiority over traditional approaches. The sensitivity analysis further underscores the model's robustness in managing cost uncertainties, providing a reliable framework for practical applications. Future research should aim to expand the model to incorporate mixed redundancy strategies, including warm and hot standby configurations. This expansion will broaden the model's scope and enhance its applicability across different types of systems and operational conditions. Further studies should also consider varying reliability parameters for switches and include a wider range of benchmark examples to validate the model's effectiveness in different contexts. Additionally, integrating real-world data and conducting field tests will provide deeper insights into the practical implementation and performance of the proposed strategies.

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