RESEARCH PAPER

A Hybrid Methodology of Data Science and Decision-Making Techniques: Lessons from COVID-19 Pandemic Management

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ABSTRACT
Today, data mining and machine learning are recognized as tools for extracting knowledge from large datasets with diverse characteristics. With the increasing volume and complexity of information in various fields, decision-making has become more challenging for managers and decision-making units. Data Envelopment Analysis (DEA) is a tool that aids managers in measuring the efficiency of the units under their supervision. Another challenge for managers involves selecting and ranking options based on specific criteria. Choosing an appropriate multi-criteria decision-making (MCDM) technique is crucial in such cases. With the spread of COVID-19 and the significant financial, economic, and human losses it caused, data mining has once again played a role in improving outcomes, predicting trends, and reducing these losses by identifying patterns in the data. This paper aims to assess and predict the efficiency of countries in preventing and treating COVID-19 by combining DEA and MCDM models with machine learning models. By evaluating decision-making units and utilizing available data, decision-makers are better equipped to make effective decisions in this area. Computational results are presented in detail and discussed in depth.

KEYWORDS: *Data mining; Machine learning; Data envelopment analysis; Multi-criteria decision-making; COVID-19.*

1. Introduction

Since the end of 2019, the COVID-19 crisis (abbreviated for coronavirus disease of 2019) with the scientific name severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2) originated in China and quickly spread around the world [1]. The main symptoms of this disease are headache, dizziness, lack of appetite, lethargy, and cough [2]. Since the beginning of the disease outbreak, COVID-19 brought a high death rate and, in a period, it was considered the most important challenge for the health systems of countries. In addition, this crisis dealt a serious blow to the global economy, and all countries were looking for ways to manage this pandemic and its diverse effects on various aspects, including health, economy, politics, culture, sports, etc. [3]. In the latest published statistics on this pandemic in 2024, over 704 million confirmed cases were reported, with more than 7 million deaths recorded [4]. This catastrophic statistic justifies the use of the word crisis for COVID-19. Based on this, the first dimension of the current research problem originates and develops from the

COVID-19 crisis. Paying attention to the key measurements taken by the countries to manage the pandemic and evaluating the effectiveness of those measurements through the definition of appropriate inputs-outputs is one of the main issues of this research.

On the other hand, from a methodological point of view, one of the biggest challenges managers faces is making decisions in various situations. Researchers have developed various methods and techniques to help with decision-making and evaluating decision-making units. Today, assessing the efficiency of organizations and companies is crucial for senior managers in their strategic planning [5]. Organizations must monitor and evaluate their activities, particularly in complex and dynamic environments, to ensure effective functioning. Important methods in this area include multi-criteria decision-making (MCDM) models and data envelopment analysis (DEA) [6, 10]. Therefore, another dimension of the current research problem is from the methodological aspect, that is, how to use DEA-MCDM models and techniques to reflect the efficiency of countries as decision-making units (DMUs).

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DEA is a linear programming method that measures the relative efficiency of units within a system. MCDM is divided into two types: multi-objective decision-making (MODM) and multi-attribute decision-making (MADM). The goal is to select the best option or assign weights to decision factors. Although DEA was first introduced as an efficiency evaluation method, it is now used alongside MCDM techniques due to its wide range of applications, particularly in decision-making. Examples include evaluating the efficiency of stocks in the capital market, mutual funds, military bases, city planning, and assessing various automakers, among many others [10-13].

From another perspective, as the volume of information grows, the need for algorithms capable of extracting useful knowledge from data increases. The process of knowledge discovery involves finding valuable information and predicting future events based on that information. The term "data mining" became popular in the 1990s, and it is an interdisciplinary field that combines databases, statistics, and machine learning to extract valuable insights from large datasets. Many industries, such as retail, banking, manufacturing, telecommunications, and insurance, use data mining algorithms to uncover different patterns and relationships [14].

Machine learning, when combined with other models and algorithms, can help interpret and predict the information obtained from those models. This combination can address the limitations of DEA models and MCDM techniques. Machine learning is a subset of artificial intelligence that allows systems to learn and improve without being explicitly programmed, enabling them to interpret previously unseen data [15].

During the global COVID-19 pandemic, machine learning has been used to analyze data from various sources, helping researchers and decision-makers extract hidden knowledge. Some applications of machine learning in this area include [16]:

- **Application 1.** Analyzing radiographic images to identify COVID-19 infections.
- **Application 2.** Clustering symptoms of infected individuals.
- **Application 3.** Evaluating the effectiveness of treatments.
- **Application 4.** Predicting the spread of the virus and outcomes like recovery or mortality.
- **Application 5.** Predicting the efficiency of units in treating patients.

In summary, the way of using machine learning algorithms in this paper places its application in categories 3 and 5 mentioned above. In general, this paper pursues the development of an integrated methodology of machine learning algorithms and DEA-MCDM methods for the assessment of the management of the COVID-19 crisis as the main purpose. Now with the clarification of the research problem, we can point out the contributions and novelties of this paper, which include the following:

- Using the MADM technique and the DEA model to form a multi-objective planning model to measure the efficiency of countries and states involved in COVID-19 (Implementation through MODM technique).
- Generation of supervised learning model and regression prediction of efficiency values.
- Implementation of the methodology through the registered data of COVID-19 for 46 states of the USA and 43 European countries.

The main reason for using machine learning in this paper is to address the rapid growth of COVID-19 data. These data allow for measuring the efficiency of countries at any given point in time during the pandemic, but their rapid growth and increase make it challenging to continuously use DEA models. At this point, machine learning can be a very useful tool.

The remainder of the article is organized as follows:

Section 2 reviews the research literature in a comprehensive and combined way from the viewpoints of the case study and methodology. Section 3 describes the details of the research process, steps, data details and theories. The results and discussion are presented in depth in Section 4 and finally, Section 5 reflects the conclusions and managerial insights.

2. Literature Review

In order to highlight the gaps of articles in the literature related to the present paper, the search for studies was conducted in the period of 2020 to 2024. In this step, we tried to find articles in English form and the highest level of publications. In the following, the studies found and their details are presented.

The outbreak of the COVID-19 pandemic has

brought unprecedented challenges to various sectors worldwide. Decision makers are faced with complex decisions that require a systematic approach to effectively address the evolving situation. MADM techniques offer a structured framework to evaluate and select the best alternatives among a set of options based on multiple criteria.

Several studies have emphasized the utility of MADM techniques in healthcare systems throughout the COVID-19 pandemic. For example, Celeste et al. [17] have utilized the Analytic Hierarchy Process (AHP) for the optimal allocation and distribution of COVID-19 vaccines. This approach ensures optimal resource management and enhances decision-making efficiency in crises. The use of MADM techniques has also been explored in assessing and mitigating risks associated with COVID-19 containment measures. Decision makers can utilize techniques like Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) or ELECTRE to rank potential strategies based on criteria such as effectiveness, feasibility, and economic impact. By employing these techniques, policymakers can identify the most suitable risk mitigation strategies tailored to specific regional contexts. In this research area, Hezar et al. [18] focused on the comparative analysis of TOPSIS, VIKOR, and COPRAS methods for regional safety assessment of COVID-19. In the context of economic recovery post-COVID-19, MADM techniques play a crucial role in prioritizing policy interventions and investments. By considering various dimensions such as economic growth, employment, and social welfare, decision-makers can formulate evidence-based recovery strategies that align with long-term sustainable development goals. For example, we can refer to the research of Le et al. [19] in this field. They used the EDAS technique based on fuzzy uncertainty to determine the production strategies of Vietnam's industry in the post-COVID-19 era. The global impact of the COVID-19 pandemic has underscored the need for efficient decision-making processes to address multifaceted challenges. MODM techniques provide a structured approach to evaluating alternatives based on multiple conflicting objectives. MODM techniques have been employed in optimally allocating scarce resources within healthcare systems amidst the COVID-19 outbreak. Studies

have utilized methods such as Multi-Objective Linear Programming (MOLP) to balance conflicting objectives like minimizing mortality rates, maximizing patient throughput, and maintaining healthcare worker safety. By considering multiple objectives simultaneously, decision-makers can devise strategies that optimize resource allocation and enhance overall system efficiency. For example, Eriskin et al. [20] presented a robust multi-objective model for healthcare resource management and location planning during pandemics. The disruption caused by the pandemic has underscored the critical importance of efficient supply chain management in ensuring the timely delivery of essential goods and services. Researchers have applied Multi-Objective Optimization (MOO) techniques to optimize supply chain networks, considering objectives such as cost reduction, lead time minimization, and resilience to disruptions. By leveraging MODM approaches, organizations can build agile and robust supply chains capable of adapting to dynamic demand patterns and mitigating supply shortages. For example, Mondal and Roy [21] developed a multi-objective stable closed-loop open-loop supply chain model under mixed uncertainty during the COVID-19 pandemic situation.

On the other hand, DEA models were widely used by researchers to measure the efficiency of DMUs (including health systems or countries). For example, Ordu et al. [22] used different DEA models to evaluate and rank the performance of 16 countries in handling the COVID-19 pandemic. Since several countries were deemed efficient, they used a more advanced DEA model to rank these efficient countries. Taherinezhad and Alinezhad [23] used a two-stage output-oriented DEA model with variable returns to scale (VRS) to measure the efficiency of nations. They also used ensemble learning algorithms to predict class efficiencies. Following the previous research, Taherinezhad and Alinezhad [24] implemented the Multi-Layer Perceptron (MLP) model for accurate prediction and regression of efficiencies. The basis of the predicted efficiency values in their study was the values obtained from the DEA model with different variables from the previous research.

The COVID-19 pandemic has underscored the critical need for innovative approaches to enhance disease surveillance, diagnosis, treatment, and

public health interventions. Machine learning algorithms have emerged as powerful tools for analyzing vast amounts of data, extracting insights, and supporting decision-making processes in the context of infectious disease outbreaks. Machine learning algorithms have been instrumental in developing robust disease surveillance systems and early warning mechanisms for tracking the spread of COVID-19 and predicting potential outbreaks. Recent studies have demonstrated the effectiveness of machine learning models in analyzing epidemiological data, social media trends, and healthcare records to detect patterns, identify risk factors, and forecast disease transmission dynamics. By leveraging machine learning techniques such as deep learning [24], ensemble methods [23], and anomaly detection, researchers have been able to enhance the accuracy and timeliness of disease surveillance efforts, enabling proactive interventions to contain the spread of the virus. For example, Bathwal et al. [25] developed a hybrid model that combined machine learning and epidemiological techniques to predict daily mortality during the COVID-19 pandemic. Hashim et al. [26] generated models using neural networks and linear regression to predict COVID-19 infection rates, showing that actual infection numbers were much higher than reported in some countries. Also, Khanday et al. [27] developed a machine learning model to diagnose COVID-19 using clinical text reports. They found that logistic regression and naive Bayes algorithms provided the most accurate results. It is important to mention that the mentioned studies are only a few examples of extensive machine learning studies in COVID-19. In order to find the above studies, we tried to consider the most relevant ones with the present paper.

By reviewing the studies found [17-27], we find that MCDM, DEA, and machine learning techniques have been used separately in most of the studies. Only studies [23] and [24] have employed the combined approach of DEA and machine learning for the evaluation of the Covid-19 pandemic. The point here is that the separate use of the mentioned mathematical methods, although it is a gap in previous studies, but it is not considered their weakness. In the following study, an integrated methodology including decision-making techniques (DEA-MCDM) and data science is used to cover this gap. Therefore, this paper is one of the few papers

that simultaneously involve decision-making tools and machine learning in the computational outputs of the COVID-19 pandemic. This integration of tools offers numerous advantages in analysis and management insights related to COVID-19 and can be utilized by researchers, scientists, and policymakers.

3. Methodology

In this paper, based on architecture is given in Fig.1, we first use a group decision-making method called the group best-worst method (GBWM) [28] and the CCR DEA model [29] to create a multi-objective programming model for calculating the efficiency of each unit. The case study uses data from 46 U.S. states and 43 European countries. The dataset includes three inputs: population, total cases, and active cases, and two outputs: total recovered and total deaths. After calculating the relative efficiency of U.S. states and European countries using the CCR and GBWM-DEA models, we will predict the relative efficiency of these units using regression models. The flexibility of machine learning in combination with other models and algorithms has created this advantage to interpret and predict the information obtained from other models.

Therefore, considering the disadvantages and limitations of DEA models and MADM techniques, machine learning in combination with these two methods can solve this problem to a large extent. The above explanation is fully reflected in Fig. 1. Overall, Fig. 1 reflects the main stages of this study. The starting phase involves data extraction, while the final phase consists of evaluating the proposed model and drawing conclusions. Fig. 1 serves as a roadmap, and having it is essential for ensuring the high quality of the current research. Additionally, Fig. 1 illustrates the relationship between the DEA-MCDM models and machine learning algorithms. As shown, the output data from the DEA model (efficiencies) serves as part of the input data for the machine learning algorithms. It is worth noting that the type and number of models and algorithms are clearly specified in Fig. 1.

3.1. Selecting data and features

To achieve the best results, it's important to choose a dataset that can provide the most accurate output for data mining models.
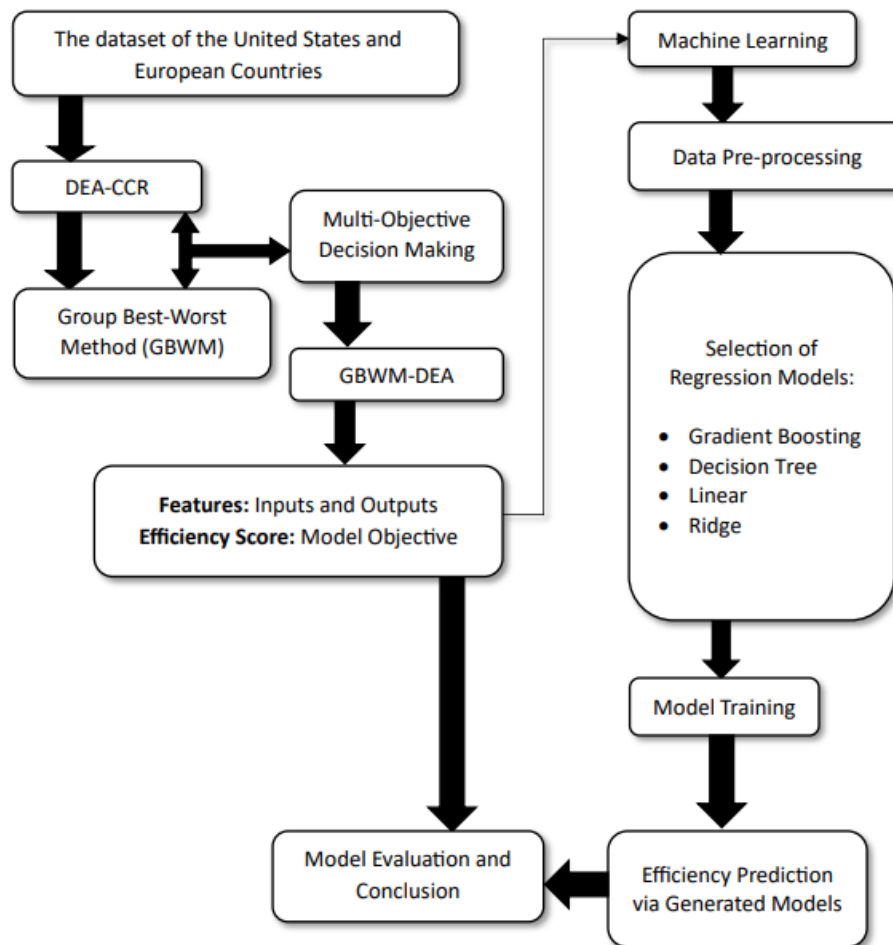
Fig.1. Architecture of the proposed methodology

Also, having features that do not have a meaningful connection or correlation with each other and the response variable can increase the error in the model. Therefore, selecting features that have a good correlation and are relevant to our goal is crucial [24]. In this study, we used information from 46 U.S. states and 43 European countries to implement DEA models and machine learning. The data, collected from the Worldometers website [4], includes five input and output variables used to measure the efficiency of units and apply regression models.

It is worth mentioning that the collected data pertains to the COVID-19 statistics recorded up until the end of June 2022. The efficiency score obtained from the GBWM-DEA-CCR method will be added as a new feature to the dataset. Descriptive statistics of the dataset of the United States and European countries are shown separately in Table 1 and Table 2. These datasets are described through statistical indicators including count, mean, standard deviation (std), minimum (min), quartiles and maximum (max).

Tab.1. Description of the U.S. dataset

	Population	Total Cases	Active Cases	Total Recovered	Total Deaths
count	46	46	46	46	46
mean	6687223	2001851	33607.91	1946611	11274516
std	7658678	2309824	49690.09	2249802	23962.92
min	578759	147657	1207	145285	11198053
25%	1827712	562412	4886.25	554605.3	11271500
50%	4342705	1270805	20424	1239839	11281816
75%	8305363	2168815	35374	2112277	11290956
max	39512223	11646262	252020	11296147	11295367

Tab.2. Description of the European countries' dataset

	Population	**Total Cases**	**Active Cases**	**Total Recovered**	**Total Deaths**
count	43	43	43	43	43
mean	17358612	5605234	93767.19	5465598	37571932
std	29009575	9446110	236197.3	9213247	76361.97
min	33704	15866	81	15720	37224694
25%	2079669	498283	1982	441372	37577747
50%	5834950	1684717	9809	1670101	37601582
75%	11202531	5425559	39509.5	5282908	37612449
max	1.46E+08	38899905	1121488	37617800	37617738

3.2. Basic DEA model

The CCR model is one of the most widely recognized DEA models. It was first introduced by Charnes, Cooper, and Rhodes in 1978 [29]. In this model, each DMU is assigned a virtual input and output using weights v_i and u_r. These weights are then calculated using linear programming to maximize the ratio of virtual output to virtual input.

The optimal weights typically vary from one DMU to another. Therefore, in DEA, the weights are derived from the data and are not fixed in advance. The best set of weights is calculated for each DMU, which may differ from the weights used for other DMUs. The input-oriented fractional CCR linear programming model is written as Eq. (1) [29]:

$$\max \ z_p = u_1 y_{1p} + u_2 y_{2p} + \cdots + u_r y_{rp}$$

$$s.t.:$$

$$v_1 x_{1o} + v_2 x_{2p} + \cdots + v_m x_{mp} = 1$$

$$(u_1 y_{1j} + \cdots + u_r y_{rj}) - (v_1 x_{1j} + \cdots + v_i x_{ij}) \leq 0$$

$$v_1, v_2, \dots, v_i \geq 0, u_1, u_2, \dots, u_r \geq 0$$

$$j = 1, 2, \dots, p, \dots, n \tag{1}$$

Generally, DMUs aim to increase efficiency by reducing inputs and increasing outputs. However, in practice, reducing inputs and increasing outputs does not always improve performance because some inputs and outputs may be undesirable. If y_{rp}^g and y_{rp}^b represent desirable and undesirable outputs respectively, the first step is to transform the variables based on Eq. (2) [23]:

$$-y_{rp}^b + k_r = y_{rp}^{g'}, (r \in b) \tag{2}$$

Then, we have Eq. (3) as follows:

$$k_r = \max_{1 \leq j \leq n}\{y_{rj}\} + 1, r = 1, 2, 3, \dots, s \tag{3}$$

In this study, the Total Deaths parameter is treated as an undesirable output. Therefore, before applying the CCR model, a variable transformation will be performed according to Eq. (2) and other variables remain unchanged.

3.3. Best-worst method (BWM)

This method was presented by Rezaei [30] in 2015. The purpose of this method is to assign weights to various criteria based on the decision-maker's preferences. The decision-maker or expert identifies the best and worst criteria, and then a pairwise comparison is made between these two criteria (best and worst) and the other criteria. After this, using the min-max method, a problem is set up to determine the weights of the criteria. The steps of BWM are as follows [30]:

- **Step 1:** Identify the set of criteria for decision-making. Consider the set of criteria $\{c1, c2, c3, \dots, cn\}$ needed for decision-making.
- **Step 2:** Determine the best (most desirable, most important) and worst (least desirable, least important) criteria. In this step, the decision-maker only identifies the best and worst criteria without making any comparisons.
- **Step 3:** Rate the preference of the best criterion over the other criteria using numbers between 1 and 9 in a set called A_B, as written Eq. (4):

$$A_B = \{a_{B1}, a_{B2}, \dots, a_{Bn}\} \tag{4}$$

a_{Bj} represents the preference level of the best criterion over the j-th criterion. It is clear that the value of a_{BB} is equal to one.

- **Step 4:** The preference level of the other criteria relative to the worst criterion is represented using numbers between 1 and 9 in a set called A_w, written as Eq. (5):

$$A_w = \{a_{1w}, a_{2w}, \dots, a_{nw}\} \tag{5}$$

a_{JW} represents the preference level of the j-th criterion relative to the worst criterion. It is clear that the value of a_{WW} is equal to one.

- **Step 5:** find the optimal weight values $A_w = (w_1^*, w_2^*, \dots, w_n^*)$ To determine the optimal weight for each criterion, pairs $W_B / W_j = a_{Bj}$ and $W_j / W_w = a_{jw}$ are formed,

and to satisfy these conditions for all j, a solution must be found that minimizes the maximum absolute difference between the expressions $\left|\frac{W_B}{W_j} - a_{Bj}\right|$ and $\left|\frac{W_j}{W_w} - a_{jw}\right|$. Considering the non-negativity of the sum of the weights, the problem will take the following form.

- **Step 6:** Finally we have Eq. (6), which written as follows:

$$\min \max_j \left\{ \left|\frac{W_B}{W_j} - a_{Bj}\right| - \left|\frac{W_j}{W_w} - a_{jw}\right| \right\}$$

s. t.:
$$\sum_j w_j = 1$$
$$w_j \geq 0, \forall j \tag{6}$$

The above problem can also be written as Eq. (7):

$$\min \xi$$

s. t.:
$$\left|\frac{W_B}{W_j} - a_{Bj}\right| \leq \xi$$
$$\left|\frac{W_j}{W_w} - a_{jw}\right| \leq \xi$$
$$\sum_j w_j = 1$$
$$w_j \geq 0, \forall j \tag{7}$$

If the number of decision-makers or experts (k) is more than one, then the BWM will be of the group type. The linear form of the Group BWM also be written as Eq. (8):

$$\min \xi_1 + \xi_2 + \cdots + \xi_k$$

s. t.:
$$\left\{|W_B - a_{Bj}W_j|\right\}_k \leq \xi_k$$
$$\left\{|W_j - a_{jw}W_w|\right\}_k \leq \xi_k$$
$$\sum_j w_j = 1, w_j \geq 0, \forall j \tag{8}$$

3.4. GBWM-DEA model

In the DEA-CCR model, the constraint $\sum_{i=m+1}^{m+s} w_i y_{ij} = 1$ is different from the constraint $\sum w_i = 1$ in the BWM model. We consider the DEA model (Eq. (9)) as follows:

$$\max \sum_{i=m+1}^{m+s} w_i y_{ip} + \theta_p \sum_{i=1}^{m} w_i x_{ip}$$

s. t.:
$$\sum_{i=1}^{m+s} w_i = 1$$
$$\sum_{i=m+1}^{m+s} w_i y_{ij} - \theta_j \sum_{i=1}^{m} w_i x_{ij} \leq 0$$
$$j = 1, \dots, n, w_i \geq 0 \tag{9}$$

Here, θ_j represents the efficiency score obtained from Eq. (1). In the model, the constraint $\sum_{i=1}^{m} w_i x_{ip}$ has been replaced with the constraint $\sum_{i=1}^{m+s} w_i = 1$. The constraint $\sum_{i=1}^{m} w_i = 1$ is identical to the normalized weights constraint in GBWM. In 2019, Alinezhad and khalili [31]

demonstrated that the optimal solutions of Eq. (1) and Eq. (9) are the same. In other words, the weights generated by Eq. (1) and Eq. (9) are identical. Considering the identical constraint $\sum_{i=1}^{m+s} w_i = 1$ in both DEA and GBWM models, Eq. (9) is considered for integration with GBWM. The multi-objective decision-making model GBWM-DEA under consideration is written as Eq. (10):

$$\max f_1 = \sum_{i=m+1}^{m+s} w_i y_{ip} + \theta_p \sum_{i=1}^{m} w_i x_{ip}$$
$$\max f_2 = -\xi_1 - \xi_2 - \cdots - \xi_k$$

s. t:
$$\sum_{i=1}^{m+s} w_i y_{ij} - \theta_j \sum_{i=1}^{m} w_i x_{ij} \leq 0, \quad j = 1, \dots, n$$
$$\sum_{i=1}^{m+s} w_i = 1$$
$$\left\{|W_B - a_{Bi}W_i|\right\}_k \leq \xi_k, i = 1, \dots, m+s, k = 1, \dots, k$$
$$\left\{|W_i - a_{iw}W_w|\right\}_k \leq \xi_k, i = 1, \dots, m+s, k = 1, \dots, k$$
$$w_i \geq 0, i = 1, \dots, m+s \tag{10}$$

The constraint $\sum_{i=1}^{m+s} w_i = 1$ is shared between the two objective functions. The first constraint belongs to the first objective function, while the third and fourth constraints belong to the second objective function. Considering Eq. (10), the multi-objective decision-making model GBWM-DEA is written as Eq. (11):

$$\min \alpha$$

s. t:
$$f_1^* - \left(\sum_{i=m+1}^{m+s} w_i y_{ip} + \theta_p \sum_{i=1}^{m} w_i x_{ip}\right) \leq \alpha$$
$$f_2^* - (-\xi_1 - \xi_2 - \cdots - \xi_k) \leq \alpha$$
$$\sum_{i=1}^{m+s} w_i y_{ij} - \theta_j \sum_{i=1}^{m} w_i x_{ij} \leq 0, \quad j = 1, \dots, n$$
$$\sum_{i=1}^{m+s} w_i = 1$$
$$\left\{|W_B - a_{Bi}W_i|\right\}_k \leq \xi_k, i = 1, \dots, m+s, k = 1, \dots, k$$
$$\left\{|W_i - a_{iw}W_w|\right\}_k \leq \xi_k, i = 1, \dots, m+s, k = 1, \dots, k$$
$$w_i \geq 0, \quad i = 1, \dots, m+s, \alpha \text{ free} \tag{11}$$

Here, f_1^* and f_2^* are the optimal values of the first and second objective functions, respectively, which have been calculated separately. By solving the Eq. (11), the optimal values w_i^* are obtained, and by substituting them into the following Eq. (12), the efficiency scores of the units are determined. Here, the inputs and outputs are represented by index i, where the number of inputs is i= 1, 2, …, m and the number of outputs is i= m+1, …, m+s [32].

$$\theta_j^{GBWM-DEA}{}_{CCR} = \frac{\sum_{i=m+1}^{m+s} w_i^* y_{ip}}{\sum_{i=1}^{m} w_i^* x_{ij}} \tag{12}$$

In summary, it can be stated that Eq. (9) represents a basic DEA model, namely CCR, which is transformed into the GBWM-CCR model through the relationships defined in the GBWM method, as shown in Eq. (10) and (11). Eq. (12) also measures the efficiency of DMUs based on the

developed GBWM-CCR model.

3.5. Machine learning

The traditional method of turning data into knowledge relies on manual analysis and interpretation. These manual analyses of datasets are slow, costly, and often subjective. As the volume of data grows rapidly, this type of analysis has become impractical in many fields. When the amount of data and the reasoning needed go beyond human capacity, we turn to computer technology. Extracting knowledge from large databases involves many steps, including data handling, retrieval, preprocessing, and mathematical and statistical analysis.

Machine learning aims to help systems learn and improve automatically. Generally, machine learning methods are divided into two types: supervised and unsupervised learning. Supervised learning uses labeled data for training, while unsupervised learning works with unlabeled data. Regression models are a type of supervised learning method [33].

3.5.1. Data integration

The use of various sources for gathering information has led analysts to use multiple databases in their studies, as not all the necessary features are available in a single database. Additionally, the raw data obtained may need some calculations and processing before it can be used in machine learning algorithms, and the results of these calculations can be added as features to the database. In this research, after calculating the relative efficiency of two datasets separately, the efficiency scores were added as features to both datasets. These two datasets were then merged for implementing machine learning models [33].

3.5.2. Data normalization

In many cases, numerical features in a dataset can have different scales. When this happens, numbers with larger scales can have a greater influence on the results of a machine learning model. To address this issue, data normalization is applied to scale the values of each feature within a fixed range between 0 and 1. Since the relative efficiency score already falls within the range of 0 to 1, there is no need to normalize the efficiency- score feature. The min-max scaling normalization equation is defined as Eq. (13) [23-24]:

$$\frac{(x - min_{data})}{(max_{data} - min_{data})} \tag{13}$$

3.5.3. Selection of algorithms

The aim of this paper in using machine learning models is to predict the efficiency obtained from the GBWM-DEA model. To do this, regression models, which are a type of supervised learning method, have been used. The four models used in this paper are described below. Among these, the first two are linear regression models, while decision tree regression and gradient boosting regression are non-linear [34].

- *Regression:*

Predicting the value of a continuous variable based on the values of other variables, using a model with either a linear or non-linear relationship, is known as regression. Regression is widely studied in statistics and neural networks. Essentially, a vector xx is given as input, which is mapped to an output variable y. The goal is to calculate y or F(x) based on an estimated function. The objective is to determine the exact value of y for a given vector xx. This task, like classification, is a type of supervised prediction. Linear regression assumes that the relationship between variables is linear, while non-linear regression allows for non-linear relationships. Additionally, linear regression is less complex than non-linear regression. In linear regression, it is also assumed that errors are normally distributed, while non-linear regression does not require this assumption. Moreover, in linear regression, the effect of each variable on the dependent variable is considered in conjunction with other independent variables, whereas in non-linear regression, the effect of each independent variable is assessed independently.

- *Linear regression:*

In linear regression, it is assumed that the relationship between input and output is linear. This assumption limits the modeling method, but it is fast and efficient. The multiple linear regression model is written as Eq. (14) [34]:

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_n x_n \tag{14}$$

- *Ridge regression:*

Ridge regression applies a penalty to the regression coefficients, making them more constrained [34]. The ridge regression problem is formulated as follows:

$$\hat{\beta} = \underset{\beta}{\text{argmin}} \ \sum_{i=1}^{N} (y_i - \beta_0 - \sum_{j=1}^{p} x_{ij} \beta_j)^2$$

$$\text{s.t: } \sum_{j=1}^{p} \beta_j^2 \leq c^2 \tag{15}$$

When a linear regression model includes many correlated variables, their coefficients may be estimated with high variance and may continue to

grow without bound. A large positive coefficient on one variable might be offset by a large negative coefficient on a correlated variable. This issue can be addressed by applying a size constraint (c) on the coefficients [35].

- *Gradient boosting regression:*

This method, introduced by Jerome Friedman in 2001 [36], is a type of non-linear regression that aims to predict the values of a dependent variable by sequentially building decision trees. Gradient Boosting Regression assigns penalties to incorrect predictions through a cost function. Additionally, it uses a learning rate (a numerical value between zero and one) to balance the contribution of decision trees in predicting the values of the dependent variable. The steps of the Gradient Boosting Regression algorithm are as follows:

input: Data $\{(x_i, y_i)\}_{i=1}^n$, and a differentiable **loss Function** $L(y_i, F(x))$

Step 1: initialize model with a constant value:
$F_0(x) = \underset{\gamma}{\text{argmin}} \sum_{i=1}^n L(y_i, \gamma)$

Step 2: For m= 1 to M:

(**A**) Compute $r_{im} = -\left[\frac{\partial L(y_i, F(x_i))}{\partial\, F(x_i)}\right]_{,F(x)=F_{m-1}(x)}$ for $i = 1, \dots, n$

(**B**) Fit a regression tree to the r_{im} values and create terminal regions, R_{jm}, for $j = 1, \dots, J_m$

(**C**) for $j = 1 \dots, j_m$ compute $\gamma_{jm} = \underset{\gamma}{\text{argmin}} \sum_{x_i \in R_{jm}} L(y_i, F_{m-1}(x_i) + \gamma)$

(**D**) update $F_m(x) = F_{m-1}(x) + v \sum_{j=1}^{J_m} \gamma_{jm} I(x \in R_{jm})$

end For
end Algorithm

- *Decision tree regression:*

Decision Tree Regression is a type of decision tree where the leaves represent numerical values, meaning each leaf provides a prediction for the dependent variable. A decision tree is made up of a root, branches, nodes, and leaves. In regression trees, the goal is to find the best threshold that minimizes the sum of squared residuals and use it as the root of the tree [34].

3.5.4. Model accuracy and error evaluation

To evaluate how well the regression models predict, two metrics are used include the R² score and the Mean Squared Error (MSE) [34]:

$$MSE(y, \hat{y}) = \frac{1}{n_{samples}} \sum_{i=1}^N (y_i - \hat{y})^2 \qquad (16)$$

In Eq. (16):

$\hat{y}$: is the estimated target value.

y: is the observed target value.

$$R^2(y - \hat{y}) = \frac{\sum_{i=1}^N (y_i - \hat{y}_l)}{\sum_{i=1}^N (y_i - y')} \qquad (17)$$

In Eq. (17):

y_i: is the observed target value for the i-th sample.

$\hat{y}_l$: is the estimated target value for the i-th sample.

y': is the mean of the estimated target values.

4. Results and Discussion

In 2020, with the increasing number of people infected with the COVID-19 disease, the death toll of infected people increased dramatically in all countries. In particular, developed countries suffered more from the effects of this disease. It is clear that one of the goals of government decision-makers is to control the disease process and manage resources to fight the COVID-19 disease. Therefore, it is very important to know the performance of the units under the order. According to this point, by using decision-making methods in combination with machine learning, it is possible to provide decision-makers with the efficiency status of units in the shortest possible time. As mentioned earlier, this study uses two datasets: one from the United States and one from European countries. General information about these datasets is provided in Tables 1 and 2. In the following, there is a step-by-step description of the implementation of the research stages and their calculation results.

4.1. Traditional DEA

In this subsection, the analysis of the information obtained from the DEA-CCR model coded in the GAMS program [37] is discussed. At first, based on the Eq. (1), we write the mathematical Eq. (18) to measure the efficiency of one of the 46 states studied in U.S. It should be mentioned that the undesirable variable (total deaths) is controlled by the Eq. (2) and (3). In this case we have:

$\max zp = \sum_{r=1}^2 u_r\, y_{rWyoming}$

$s.t:$

$\sum_{r=1}^2 u_r\, y_{rWyoming} = 1$

$\sum_{r=1}^2 u_r\, y_{rWyoming} - \sum_{i=1}^3 v_i x_{iWyoming} \le 0, \quad j = 1$

$\sum_{r=1}^2 u_r\, y_{rColorado} - \sum_{i=1}^3 v_i x_{iColorado} \le 0, \quad j = 44$

$\sum_{r=1}^2 u_r\, y_{rCalifornia} - \sum_{i=1}^3 v_i x_{iCalifornia} \le 0, j = 45$

$\sum_{r=1}^2 u_r\, y_{rArkansas} - \sum_{i=1}^3 v_i x_{iArkansas} \le 0, \quad j = 46$

$u_1, u_2, v_1, v_2, v_3 \ge 0 \qquad (18)$

Note that the Eq. (18) must be written for all the states and solved in the software. Therefore, 46 mathematical programming models determine the efficiency values. The efficiency values obtained for the states in the traditional way by solving 46 mathematical models in GAMS software are compiled in Table 5.

Based on the trend mentioned for the United States data, Eq. (19) shows one of the 43 DEA models corresponding to the data of European countries:

$$\max \quad zp = \sum_{r=1}^{2} u_r y_{rAlbania}$$
$$s.t:$$
$$\sum_{r=1}^{2} u_r y_{rAlbania} = 1$$
$$\sum_{r=1}^{2} u_r y_{rAlbania} - \sum_{i=1}^{3} v_i x_{iAlbania} \le 0, j = 1$$
$$\sum_{r=1}^{2} u_r y_{rSwitzerland} - \sum_{i=1}^{3} v_i x_{iSwitzerland} \le 0, j = 41$$
$$\sum_{r=1}^{2} u_r y_{rUK} - \sum_{i=1}^{3} v_i x_{iUK} \le 0, \qquad j = 42$$
$$\sum_{r=1}^{2} u_r y_{rUkraine} - \sum_{i=1}^{3} v_i x_{iUkraine} \le 0, j = 43$$
$$u_1, u_2, v_1, v_2, v_3 \ge 0 \tag{19}$$

The efficiency values measured for 43 European countries by GAMS software are also collected in Table 5.

Fig. 2 shows the scatter plot of the measured efficiency values for the United States (a) and European countries (b). These results are based on the traditional CCR method. The results show that out of 46 American states, only one state (Indiana) is far from other states and have more inefficiency. 12 efficient states whose efficiency value is equal to one are: 1) Wyoming; 2) West Virginia; 3) Washington; 4) Vermont; 5) Utah; 6) Rhode Island; 7) North Dakota; 8) North Carolina; 9) New York; 10) Minnesota; 11) Kentucky; 12) Florida. These efficient states can be used as a reference set for other inefficient DMUs. By examining the patterns in the inputs and outputs of the reference set, health policymakers can make decisions to influence the inputs and outputs of other inefficient units with the aim of increasing efficiency. Out of

43 European countries, 3 countries, Estonia, Moldova and Poland, are far from other units and have more inefficiency. 13 efficient states whose efficiency value is equal to one are: 1) Andorra; 2) Belarus; 3) Denmark; 4) Gibraltar; 5) Liechtenstein; 6) Monaco; 7) Montenegro; 8) Netherlands; 9) North Macedonia; 10) Norway; 11) Portugal; 12) San Marino; 13) Slovakia. As mentioned earlier, these efficient countries can be used as a reference set for other inefficient DMUs.

4.2. BWM implementation

In this stage, the worst and best indicators are identified based on inputs and outputs, according to the opinions of two experts. The total number of infected individuals was selected as the best indicator, and the total number of deaths was chosen as the worst indicator by the experts. The experts' assessments of the priority of the best indicator relative to other indicators, and the priority of other indicators relative to the worst indicator, are shown in Tables 3 and 4. The GBWM model is formed based on the opinion of the first and second expert in the form of Eq. (20):

$$\min \xi_1 + \xi_2$$
$$|\{ w_4 - (w_1 * 4)\}|_1 \le \xi_1$$
$$|\{ w_4 - (w_2 * 3)\}|_1 \le \xi_1$$
$$|\{ w_4 - (w_3 * 5)\}|_1 \le \xi_1$$
$$|\{ w_4 - (w_5 * 8)\}|_1 \le \xi_1$$
$$|\{ w_1 - (w_5 * 5)\}|_1 \le \xi_1$$
$$|\{ w_2 - (w_5 * 2)\}|_1 \le \xi_1$$
$$|\{ w_3 - (w_5 * 3)\}|_1 \le \xi_1$$
$$|\{ w_4 - (w_1 * 5)\}|_2 \le \xi_2$$
$$|\{ w_4 - (w_2 * 4)\}|_2 \le \xi_2$$
$$|\{ w_4 - (w_3 * 6)\}|_2 \le \xi_2$$
$$|\{ w_4 - (w_5 * 9)\}|_2 \le \xi_2$$
$$|\{ w_1 - (w_5 * 6)\}|_2 \le \xi_2$$
$$|\{ w_2 - (w_5 * 3)\}|_2 \le \xi_2$$
$$|\{ w_3 - (w_5 * 4)\}|_2 \le \xi_2$$
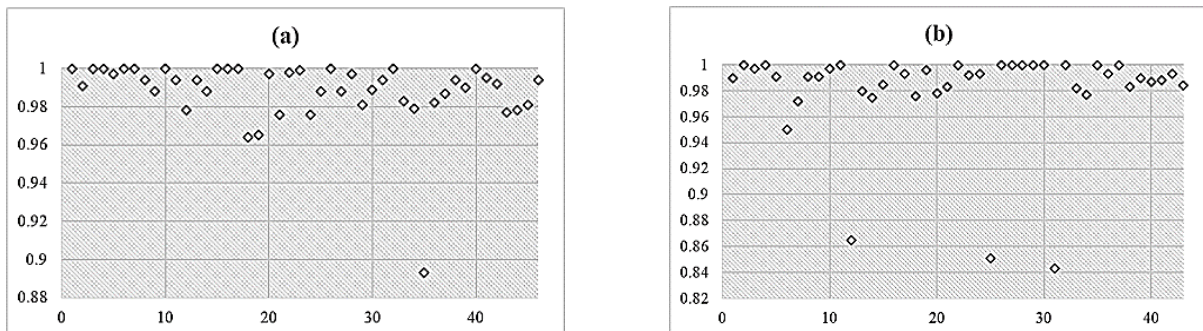$$\sum_{i=1}^{5} w_i = 1, w_i \ge 0 \tag{20}$$

Fig.2. Scatter plot of the computational results of the CCR model for two data sets of American states (a) and European countries (B) (X-axis indicates DMUs and Y-axis indicates efficiency values)

Tab.3. A_B and A_W sets based on the opinion of the first expert

criteria	Population (w_1)	Total Case (w_2)	Active Cases (w_3)	Total Recovered (w_4)	Total Death (w_5)
Best (4)	4	3	5	1	8
Worst (5)	5	2	3	8	1

Tab.4. A_B and A_W sets based on the opinion of the second expert

criteria	Population (w_1)	Total Case (w_2)	Active Cases (w_3)	Total Recovered (w_4)	Total Death (w_5)
Best (4)	5	4	6	1	9
Worst (5)	6	3	4	9	1

4.3. Multi-objective optimization via GBWM-DEA

In the next stage, we form the GBW-MDEA multi-objective model (based on Eq. (18) and Eq. (20)). Therefore, considering the first data set, we have the Eq. (21) for a DMU:

$$\max f_1 = \sum_{i=4}^{5} w_i y_{iWyoming} + \theta_{Wyoming} \sum_{i=1}^{3} w_i x_{iWyoming}$$

$$\max f_2 = -\xi_1 - \xi_2$$

s. t:

$$\sum_{i=4}^{5} w_i y_{iWyoming} - \theta_{Wyoming} \sum_{i=1}^{3} w_i x_{iWyoming} \leq 0 , \quad j = 1$$

$$\sum_{i=4}^{5} w_i y_{iArkansas} - \theta_{Arkansas} \sum_{i=1}^{3} w_i x_{iArkansas} \leq 0, \quad j = 46$$

$$|\{ w_4 - (w_1 * 4)\}|_1 \leq \xi_1$$
$$|\{ w_4 - (w_2 * 3)\}|_1 \leq \xi_1$$
$$|\{ w_4 - (w_3 * 5)\}|_1 \leq \xi_1$$
$$|\{ w_4 - (w_5 * 8)\}|_1 \leq \xi_1$$
$$|\{ w_1 - (w_5 * 5)\}|_1 \leq \xi_1$$
$$|\{ w_2 - (w_5 * 2)\}|_1 \leq \xi_1$$
$$|\{ w_3 - (w_5 * 3)\}|_1 \leq \xi_1$$
$$|\{ w_4 - (w_1 * 5)\}|_2 \leq \xi_2$$
$$|\{ w_4 - (w_2 * 4)\}|_2 \leq \xi_2$$
$$|\{ w_4 - (w_3 * 6)\}|_2 \leq \xi_2$$
$$|\{ w_4 - (w_5 * 9)\}|_2 \leq \xi_2$$
$$|\{ w_1 - (w_5 * 6)\}|_2 \leq \xi_2$$
$$|\{ w_2 - (w_5 * 3)\}|_2 \leq \xi_2$$
$$|\{ w_3 - (w_5 * 4)\}|_2 \leq \xi_2$$

$$\sum_{i=1}^{5} w_i = 1, \quad w_i \geq 0 \qquad (21)$$

Now, after calculating the optimal values of f_1 and f_2, according to the min-max method [6], we form the Eq. (22) for the first unit:

$$\min \alpha$$

s. t:

$$f_1^* - \left(\sum_{i=4}^{5} w_i y_{iWyoming} + \theta_{Wyoming} \sum_{i=1}^{3} w_i x_{iWyoming} \right) \leq \alpha$$

$$f_2^* - (-\xi_1 - \xi_2) \leq \alpha$$

$$\sum_{i=4}^{5} w_i y_{iWyoming} - \theta_{Wyoming} \sum_{i=1}^{3} w_i x_{iWyoming} \leq 0, \quad j = 1$$

$$\sum_{i=4}^{5} w_i y_{iArkansas} - \theta_{Arkansas} \sum_{i=1}^{3} w_i x_{iArkansas} \leq 0, \quad j = 46$$

$$\sum_{i=1}^{m+s} w_i = 1$$

$$|\{ w_4 - (w_1 * 4)\}|_1 \leq \xi_1$$
$$|\{ w_4 - (w_2 * 3)\}|_1 \leq \xi_1$$
$$|\{ w_4 - (w_3 * 5)\}|_1 \leq \xi_1$$
$$|\{ w_4 - (w_5 * 8)\}|_1 \leq \xi_1$$
$$|\{ w_1 - (w_5 * 5)\}|_1 \leq \xi_1$$
$$|\{ w_2 - (w_5 * 2)\}|_1 \leq \xi_1$$
$$|\{ w_3 - (w_5 * 3)\}|_1 \leq \xi_1$$
$$|\{ w_4 - (w_1 * 5)\}|_1 \leq \xi_2$$
$$|\{ w_4 - (w_2 * 4)\}|_1 \leq \xi_2$$
$$|\{ w_4 - (w_3 * 6)\}|_1 \leq \xi_2$$
$$|\{ w_4 - (w_5 * 9)\}|_1 \leq \xi_2$$
$$|\{ w_1 - (w_5 * 6)\}|_1 \leq \xi_2$$
$$|\{ w_2 - (w_5 * 3)\}|_1 \leq \xi_2$$
$$|\{ w_3 - (w_5 * 4)\}|_1 \leq \xi_2$$

$$\sum_{i=1}^{5} w_i = 1, w_i \geq 0, \ i = 1, \dots, m + s, \alpha \text{ free} \qquad (22)$$

Eq. (22) was implemented in GAMS software for both data sets of this paper. As mentioned earlier, this model was formed by the number of DMUs and the efficiency results were extracted (Table 5). Table 5 have collected the results of traditional DEA and GBWM-DEA for the data sets of American states and European countries, respectively. It should be mentioned that all the analyzes in subsection 4.1 regarding the reference set are also true for the GBWM-DEA results.

Fig. 3 shows a comparison diagram of the calculation results obtained from CCR-DEA and GBWM-DEA models. With deep precision, we find that the level of GBWM-DEA results is lower than that of CCR. This means that the model proposed in this paper is a more rigorous approach than the traditional DEA. For a catastrophic crisis like COVID-19, stricter approaches should definitely be used, because the economic and human losses of COVID-19 have been extensive. Therefore, the GBWM-DEA model can be a suitable model to reflect the performance of DMUs.

Tab.5. Measured efficiency values from CCR-DEA and GBWM-DEA models

| | U.S. dataset | | | | European countries dataset | | |
| | | Efficiency score | | | | | Efficiency score | |
Row	DMU	CCR-DEA	GBWM-DEA	Row	DMU	CCR-DEA	GBWM-DEA
1	Wyoming	1	0.985	1	Albania	0.99	0.979
2	Wisconsin	0.991	0.989	2	Andorra	1	0.988
3	West Virginia	1	1	3	Austria	0.997	0.979
4	Washington	1	0.990	4	Belarus	1	1
5	Virginia	0.997	0.980	5	Belgium	0.991	0.977
6	Vermont	1	0.999	6	Bosnia and Herzegovina	0.95	0.923
7	Utah	1	0.998	7	Bulgaria	0.972	0.967
8	Texas	0.994	0.987	8	Channel Islands	0.991	0.965
9	South Carolina	0.988	0.985	9	Croatia	0.991	0.988
10	Rhode Island	1	1	10	Czechia	0.997	0.995
11	Pennsylvania	0.994	0.986	11	Denmark	1	0.992
12	Oregon	0.978	0.978	12	Estonia	0.865	0.672
13	Oklahoma	0.994	0.992	13	Finland	0.98	0.942
14	Ohio	0.988	0.982	14	France	0.975	0.920
15	North Dakota	1	1	15	Germany	0.985	0.957
16	North Carolina	1	0.996	16	Gibraltar	1	0.847
17	New York	1	0.993	17	Greece	0.993	0.983
18	New Mexico	0.964	0.962	18	Hungary	0.976	0.965
19	New Jersey	0.965	0.962	19	Ireland	0.996	0.965
20	New Hampshire	0.997	0.998	20	Italy	0.978	0.940
21	Nevada	0.976	0.975	21	Latvia	0.983	0.955
22	Nebraska	0.998	0.996	22	Liechtenstein	1	0.992
23	Montana	0.999	0.976	23	Lithuania	0.992	0.982
24	Missouri	0.976	0.989	24	Malta	0.993	0.983
25	Mississippi	0.988	1	25	Moldova	0.851	0.670
26	Minnesota	1	0.982	26	Monaco	1	0.986
27	Michigan	0.988	0.995	27	Montenegro	1	0.999
28	Massachusetts	0.997	0.981	28	Netherlands	1	0.993
29	Maryland	0.981	0.990	29	North Macedonia	1	1
30	Maine	0.989	0.992	30	Norway	1	0.994
31	Louisiana	0.994	0.983	31	Poland	0.843	0.655
32	Kentucky	1	0.980	32	Portugal	1	0.996
33	Kansas	0.983	0.980	33	Romania	0.982	0.976
34	Iowa	0.979	0.891	34	Russia	.977	0.957
35	Indiana	0.893	0.977	35	San Marino	1	0.987
36	Illinois	0.982	0.989	36	Serbia	0.993	0.983
37	Idaho	0.987	0.994	37	Slovakia	1	0.997
38	Hawaii	0.994	0.985	38	Slovenia	0.983	0.943
39	Georgia	0.99	0.993	39	Spain	0.99	0.976
40	Florida	1	0.997	40	Sweden	0.987	0.966
41	District of Columbia	0.995	1	41	Switzerland	0.988	0.962
42	Delaware	0.992	0.977	42	UK	0.993	0.984
43	Connecticut	0.977	0.976	43	Ukraine	0.984	0.985
44	Colorado	0.978	0.977				
45	California	0.981	0.979				
46	Arkansas	0.994	0.993				

The computational basis of machine learning in the future stages is the results of GBWM-DEA. The GBWM-DEA multi-objective model integrates the strengths of both GBWM and DEA methodologies. This model is particularly effective for evaluating the efficiency of countries during the COVID-19 pandemic, as it enables the consideration of multiple objectives simultaneously.

By utilizing this multi-objective approach, the model allows for a more comprehensive assessment of efficiency, as it accommodates the complex, multifaceted nature of the pandemic's impact. The inclusion of multiple objectives ensures that countries' performances are not evaluated based on a single criterion, but rather through a balanced consideration of various relevant factors. This method is particularly valuable in dynamic, real-world situations like the COVID-19 crisis, where decisions need to account for both short-term outcomes and long-term sustainability.
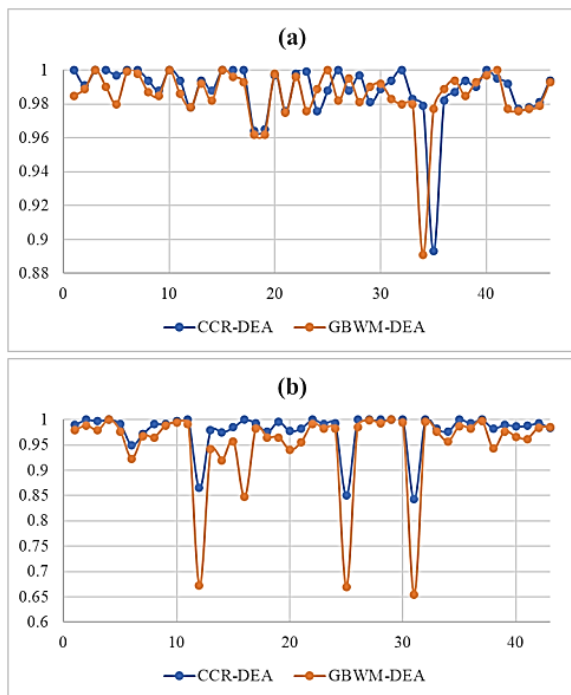

Fig.3. Comparative scatter plot of CCR-DEA and GBWM-DEA results for two data sets of American states (a) and European countries (B) (X-axis indicates DMUs and Y-axis indicates efficiency values)

4.4. Evaluation of machine learning models

In this stage, we first combine the two data sets of the paper and consider them as one integrated data set. Since supervised learning models have both a training and a test set, data aggregation can help increase the number of training samples. The more training samples, the less the challenge for more accurate prediction. After aggregating the data, the efficiency value measured by the GBWM-DEA model is considered as a target for machine learning models. On the other hand, the inputs and outputs of DEA can be used as features of machine learning models so that they can accurately predict efficiency values.

Fig. 4 shows pair diagrams for each feature and target. In Fig. 4, the lines of the regression equations are drawn for each pair of features and targets, which clearly shows the linear relationship between them. In order to implement machine learning models, it is necessary to normalize the aggregated data set values. This makes all the features have the same scale and a more accurate prediction model is produced. It should be noted that data normalization does not affect the decision tree model, because the decision tree sets each node on a feature separately. Since the measured efficiency values are numbers between zero and one, the target will not need to normalize the data. Normalization was done according to the Eq. (13) mentioned earlier. Finally, in this stage, the four models (linear regression, ridge regression, decision tree regression, and gradient boosting) are implemented in sequence. The efficiency-score feature is considered as the response (dependent) variable, and the other features are considered as exploratory (independent) variables. The models are trained on 70% of the data, and then the trained models are tested on the remaining 30% of the data. The R^2 score and MSE indicators will be used to evaluate the performance of the regression models. The details of evaluation criteria for all 4 implemented models are given in Table 6. Based on Table 6, Gradient Boosting Regression has the highest value in terms of the R^2 criterion and the lowest value in terms of the MSE criterion compared to other methods. All the calculation steps in the machine learning s tage have been done in the Python programming language [38]. For more details, Table 7 lists the predicted efficiency values by the 4 models implemented on the test data. In fact, Table 7 presents the difference between the actual efficiency values of countries and the values predicted by each of the machine learning algorithms. These data pertain to the countries included in the test dataset.

5. Conclusion

In the CCR method, the weights of variables are estimated by solving the model, but in real-world situations, the preferences of decision-makers in assigning weights are important. The GBWM-DEA method, which combines the Group Best-Worst Method with DEA, takes these preferences into account, making it more practical than traditional DEA methods.

This model was applied to two datasets, one from 46 U.S. states and another from 43 European countries. The results showed that the efficiency scores obtained using the GBWM-DEA method were mostly less than or equal to those obtained using the CCR method. Additionally, fewer units were considered efficient with the GBWM-DEA method. After calculating the efficiency scores, the two datasets were merged, and the efficiency score was used as the target variable for the regression models. Seventy percent of the data was used for training the models, and the remaining 30 percent was used for testing and evaluation. The results showed that the Gradient Boosting Regression model, a non-linear approach, performed the best. In DEA models, when a new decision-making unit is added, the model needs to be re-run.
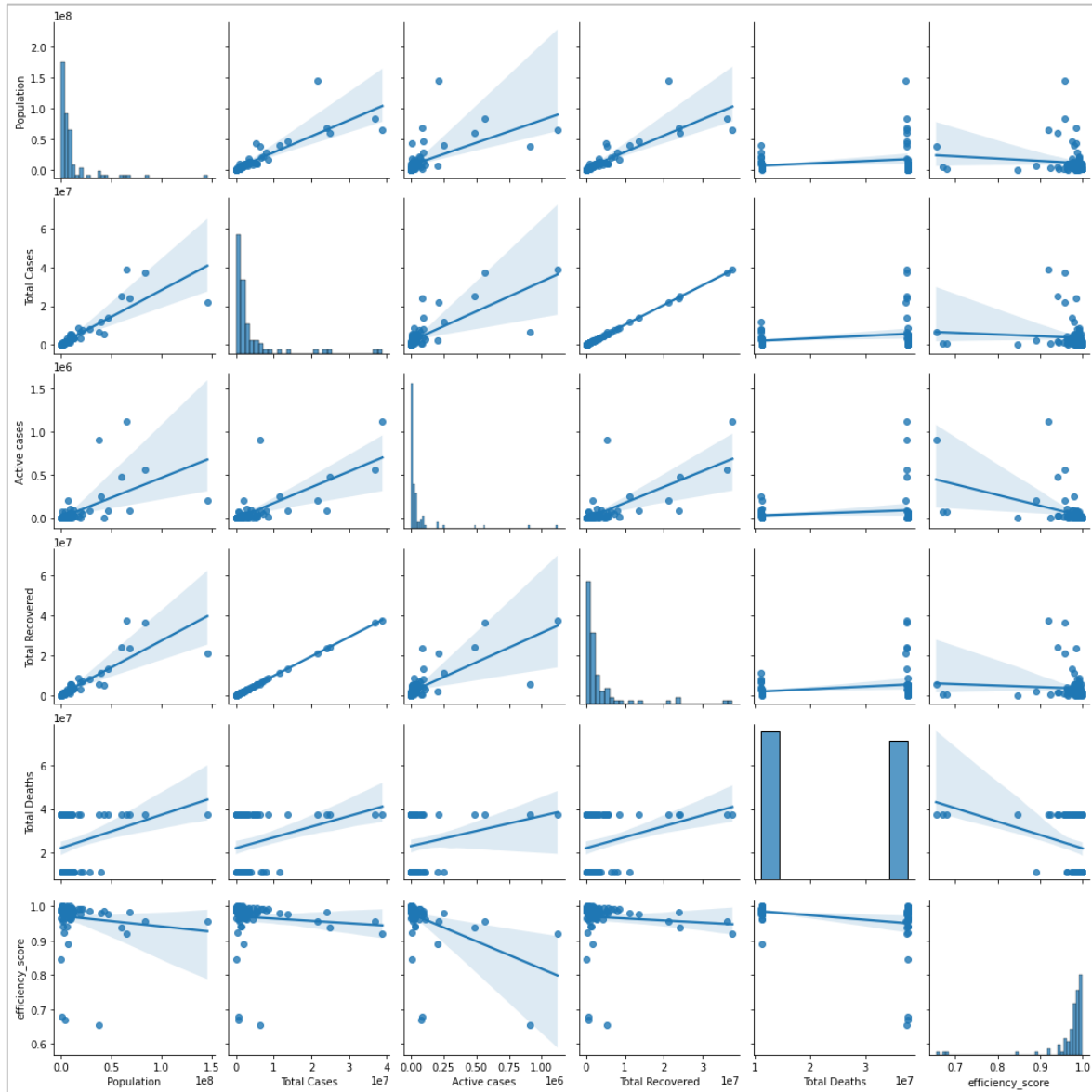

Fig.4. Paired regression plots for features and target

Tab.6. Evaluation results of regression models based on R^2 and MSE indices

Regression Model	R^2	MSE
Gradient Boosting Regression	0.736495	0.001893
Tree Regression	0.505178	0.003555
Ridge Regression	0.363511	0.004572
Linear Regression	0.363511	0.004572

Tab.7. Comparison of predicted values of test data by regression models and the GBWM-DEA model

Row	GBWM-DEA		Gradient Boosting Regression	Linear Regression	Ridge Regression	Tree Regression
1	Wisconsin	0.989	0.985	0.988	0.988	0.985
2	Pennsylvania	0.986	0.992	0.987	0.987	0.982
3	Poland	0.655	0.863	0.664	0.664	0.92
4	Finland	0.942	0.937	0.947	0.947	0.943
5	Utah	0.998	0.99	0.991	0.991	0.993
6	Albania	0.979	0.983	0.95	0.95	0.997
7	Nebraska	0.996	0.999	0.991	0.991	1
8	Ireland	0.989	0.95	0.958	0.958	0.923
9	Denmark	0.992	0.956	0.972	0.972	0.923
10	Lithuania	0.98	0.951	0.959	0.959	0.923
11	Belgium	0.977	0.986	0.968	0.968	0.983
12	Russia	0.957	0.987	0.755	0.755	0.976
13	Ukraine	0.985	0.993	0.898	0.898	0.976
14	Hawaii	0.994	0.991	0.99	0.99	0.99
15	Georgia	0.985	0.984	0.988	0.988	0.982
16	Sweden	0.966	0.968	0.949	0.949	0.979
17	Indiana	0.891	0.931	0.936	0.936	0.977
18	New Jersey	0.962	0.978	0.969	0.969	0.977
19	Connecticut	0.977	0.979	0.987	0.987	0.98
20	Serbia	0.983	0.965	0.955	0.955	0.923
21	Montenegro	0.999	0.989	0.956	0.956	0.997
22	Virginia	0.98	0.984	0.981	0.981	0.985
23	Portugal	0.996	0.987	0.986	0.986	0.965
24	Estonia	0.679	0.706	0.934	0.934	0.67
25	Montana	0.999	1.003	0.992	0.992	1
26	Macedonia	1	0.984	0.953	0.953	0.997
27	Michigan	0.982	0.983	0.988	0.988	0.982

As the number of units and variables increases, the calculations can become time-consuming and difficult. Predicting the efficiency of units using machine learning models helps avoid repeatedly running the model, significantly reducing the computational load and speeding up data analysis. We encourage researchers to the following ideas for future studies:

- Combining other machine learning models such as classification models in combination with DEA models to predict effective and ineffective units.
- Using data sets related to other decision-making units such as hospitals in combination with the model presented in this research.
- Combining other DEA models with BWM technique and using other MODM solving methods.
- Using other regression models and comparing their results with the models used in this research.
- Using the methodology proposed in this

research under conditions of uncertainty such as fuzzy, etc.
- The use of other forecasting methods, including online learning and time series methods.

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