

# Inventory Cost Evaluation Under VMI Program with Lot Splitting

Rasoul Haji<sup>\*</sup>, MohammadMohsen Moarefdoost & S. Babak Ebrahimi

Rasoul Haji, Department of Industrial Engineering, Sharif University of Technology, Tehran, Iran

MohammadMohsen Moarefdoost, Department of Industrial Engineering, Sharif University of Technology, Tehran, Iran

S. Babak Ebrahimi, Department of Industrial Engineering, Iran University of science & Technology, Tehran, Iran

## KEYWORDS

VMI, Supply chain management,  
Monte-Carlo simulation,  
Lot-splitting

## ABSTRACT

*This paper aims to evaluate inventory cost of a Two-echelon serial supply chain system under vendor managed inventory program with stochastic demand, and examine the effect of environmental factors on the cost of overall system. For this purpose, we consider a two-echelon serial supply chain with a manufacturer and a retailer. Under Vendor managed inventory program, the decision on inventory levels are made by manufacturer centrally. In this paper, we assume that the manufacturer monitors inventory levels at the retailer location and replenishes retailer's stock under  $(r, n, q)$  policy; moreover, the manufacturer follows make-to-order strategy in order to respond retailer's orders. In the other word, when the inventory position at the retailer reaches reorder point,  $r$ , the manufacturer initiates production of  $Q=nq$  units with finite production rate,  $p$ . The manufacturer replenishes the retailer's stock with replenishment frequency  $n$ , and the complete batch of  $q$  units to the retailer during the production time. We develop a renewal reward model for the case of Poisson demand, and drive the mathematical formula of the long run average total inventory cost of system under VMI. Then, by using Monte Carlo simulation, we examine the effect of environmental factors on the cost of overall system under VMI.*

© 2010 IUST Publication, IJIEPR, Vol. 21, No. 4, All Rights Reserved.

## 1. Introduction

In recent years, many scholars in the field of Industrial & Systems Engineering, Operations management and Business have worked on Logistics & Supply Chain Management (SCM) area. A supply chain consists of suppliers/vendors, manufacturers, distributors, and retailers interconnected by transportation, information, and financial infrastructure [1]. SCM is concerned with finding the best strategy for the whole supply chain, and is an attempt to coordinate processes involved in

producing, shipping and distributing products along the supply chain. There are two major types of flow in the supply chain: material flow including raw materials, work-in-process inventories, finished products, and returned items, and information flow [2]. To manage material flow effectively and reduce operational costs, many industries have adopted and developed new approaches and initiatives such as Continuous Replenishment, Efficient Customer Response and Vendor Managed Inventory [3, 4, 5&6]. VMI program is a coordination initiative associated with the inventory management in the supply chain. In VMI, the supplier manages inventory of its retailer by means of online messaging and Electronic Data Interchange [7&8]. VMI program can be implemented in two ways, with centralized decision making and decentralized

<sup>\*</sup> Corresponding author. Rasoul Haji

Email: [Haji@Sharif.edu](mailto:Haji@Sharif.edu) [mmohsenm@alum.sharif.edu](mailto:mmohsenm@alum.sharif.edu)  
[b\\_ebrahimi@iust.ac.ir](mailto:b_ebrahimi@iust.ac.ir)

Paper first received May. 15. 2010, and in revised form Dec. 05. 2010.

decision making utilizing coordination mechanisms such as quantity discount [1]. In this paper we consider the centralized decision making approach to present our model. The centralized decision making approach or Joint Economic Lot Sizing Problem under deterministic demand is first introduced by Goyal. He suggests a solution to the problem under the assumption of having an infinite production rate for the vendor and a lot-for-lot policy for the shipments from the vendor to the buyer [9].

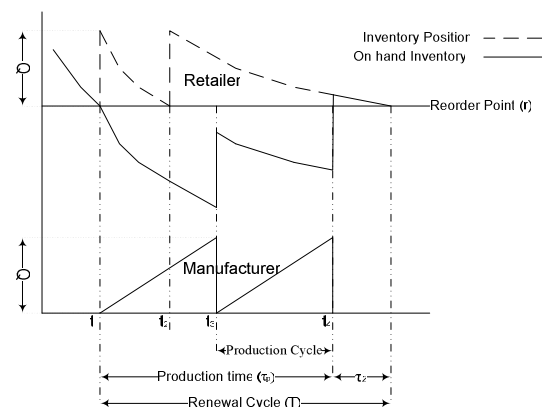
Banerjee develops Goyal's work by relaxing the infinite production rate [10]. Yulinag et al. use centralized decision making approach and present an analytical model to explore how supply chain parameters affect the cost savings realized from VMI [11]. They consider an infinite production rate for the vendor (manufacturer) and deterministic demand at the retailer. Our model differs from the one proposed by Banerjee in the sense that we assume stochastic demand at the retailer and lot-splitting is allowable. We propose an exact formula to compute the average long run cost of overall system and calculate this cost approximately by Monte-Carlo simulation.

Information sharing is an important element of VMI programs. Many of researches have studied the impact of information sharing in supply chains on the bullwhip effect. Much of this literature has shown that the bullwhip effect can be minimized through information sharing in the supply chain [12, 13 & 14]. Lee et al. (2000) [15] analyze the value of demand information sharing in a two echelon serial supply chain using analytical models. They consider a simple autocorrelated AR(1) process for the underlying demand process at the retailer and show that information sharing alone could provide significant inventory reduction and cost savings to the manufacturer, and also the underlying demand process have notable impact on the magnitudes of cost savings and inventory reductions associated with information sharing by numerical examples. Moinzadeh [16] considers a two level arborescent supply chain and analyze the value of information sharing when demand follows a Poisson process. He compares the performance of the proposed model with those that do not use information in their decision making via a numerical experiment. Based on this model, Haji and Sajadifar obtain the exact value of the expected system cost by using the idea of the one for one ordering policy [17]. In this paper we extend the recent work of Haji et al. in the manner that manufacturer can replenish his retailer's stock during the production cycle, in the other word, lot-splitting is allowable. We consider a serial two level supply chain consisting of one manufacturer and one retailer [18]. We derive the long-run average cost of overall system when the manufacturer follows a make-to-order policy under the VMI program with lot-splitting by renewal reward model for the case of Poisson demand and examine the impact of environmental factors on the cost of overall

system under VMI program via Monte-Carlo simulation.

**2. Problem Context**

In this paper we consider a serial supply chain system consist of a manufacturer and a retailer. Inventory decisions in this model are made by manufacturer centrally and manufacturer replenishes retailer's stock under  $(r, n, q)$ -policy. The manufacturer follows make-to-order strategy to respond retailer's orders. In the other word, when the inventory position at the retailer reaches reorder point,  $r$ , the manufacturer initiates production of  $Q=nq$  units with finite production rate,  $p$ . Then, the manufacturer replenishes the retailer's stock with replenishment frequency,  $n$ , and the complete batch of  $q$  units to the retailer during the production time. For simplicity, consider the case where  $Q=q$  and  $n=1$ . In this problem, when retailer's inventory position reaches reorder point,  $r$ , the manufacture would be in two possible states: Idle state or Busy state. If the manufacturer is idle, he will initiate his production immediately. If the manufacturer is in the idle state when the inventory position of the retailer triggers the reorder point (the number of his committed orders is zero), the manufacturer initiates production of  $Q$  units with finite production rate,  $p$ . On the other hand, if the manufacturer is in the busy state when the inventory position of the retailer triggers the reorder point, he initiates production of this order immediately after finishing the production of his previous committed orders. For example in Fig2, the retailer inventory position triggers reorder point,  $r$ , at time  $t_1$ . Since the manufacturer is in the idle state, he initiates production of  $Q$  units immediately, and dispatches the complete batch of  $Q$  units at time  $t_3$  to retailer. In the middle of manufacturer's production cycle, at time  $t_2$ , the retailer inventory position triggers  $r$ , since the manufacture is in the busy state, the production of this order will be postponed to the time  $t_3$  after completion of his first committed order, and will be dispatched to the retailer at time  $t_4$ . Note that the manufacturer produces until the number of his committed orders becomes greater than zero. (Fig 1)



**Fig. 1. Inventory Transaction Diagram**

In the cases where  $n$  is greater than 1, so the manufacturer's production quantity is an integer multiple of the retailer's replenishment quantity ( $Q=nq$ , where  $n$  is a positive integer) i.e. the manufacturer sends the batch size of  $q$  units to the retailer  $n$  times during the production cycle. Figure.2 illustrates the case where  $n=3$ .

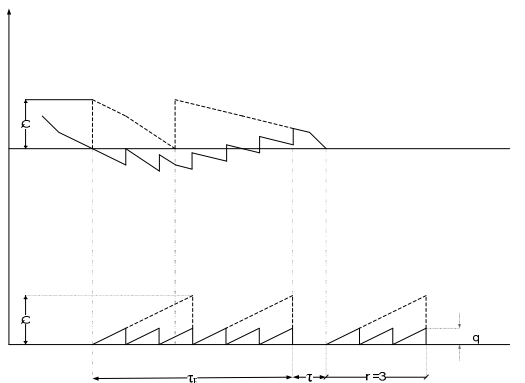


Fig. 2. Detailed Inventory Transaction Diagram

We assume that the reorder point,  $r$ , is a multiple of batch size,  $Q$ , (i.e.  $r = mQ$ ) and determined in a way that the probability of having backorder is approximately zero, in the other word, shortage is not allowed at the retailer.

### 3. The Model

Before describing the mathematical formulation, we explain some parameters and assumptions in the following sentences.

#### 3.1. Parameters

- $X_j$  = The retailer's demand during the production cycle  $j$  ( $j = 1, 2, 3, \dots$ ) in a renewal period.
- $Q$  = The manufacturer production batch size.
- $q$  = Retailer's order quantity.
- $n$  = Replenishment frequency in production cycle.
- $Y$  = The number of production cycle per a renewal cycle.
- $\bar{I}_r$  = Long-run average on-hand inventory at the retailer party.
- $\bar{I}_p$  = Long-run average on-hand inventory at the manufacturer party.
- $A_s$  = The Manufacturer's setup cost
- $r$  = Reorder point
- $T_j$  = Renewal cycle time  $j$  ( $j=1, 2, 3, \dots$ )
- $T_p$  = The manufacturer's production time in a renewal cycle. (Period of time in a renewal cycle in which the manufacturer is producing)

$T_z$  = The manufacturer's idle time in a renewal cycle. (Period of time in a renewal cycle in which the manufacturer is idle)

$\tau_p$  = Production cycle time

$C(Q, q)$  = Long-run average cost of overall system.

$H_p$  = Long-run average holding cost of manufacturer

$h_p$  = Unit holding cost/time at the manufacturer

$h_r$  = Unit holding cost/time at the retailer

$H_r$  = Long-run average holding cost of retailer.

$F$  = Long-run average setup cost of manufacturer.

#### 3.2. Assumptions

- Inter arrival time between successive demands at the retailer is exponential random variable with rate  $\lambda$ , thus the number of demands at the retailer during production cycle is Poisson random variable with mean  $\lambda \tau_p$ .
- The transportation time from manufacturer to retailer is negligible.
- Lot-splitting at the manufacturer is allowed.
- Shortage is not allowed.

Note that other parameters and assumptions will be discussed at appropriate points.

#### 3.3. Modeling Framework

In this section, we intend to drive the long-run average inventory cost of overall supply chain. This is the sum of the long-run average holding and setup cost at the manufacturer and long-run average holding cost at the retailer.

$$C(Q, q) = F + H_p + H_r \tag{1}$$

We apply renewal reward theorem to drive these costs. Then the long-run average setup and holding cost for the manufacturer, respectively, are:

$$F = \frac{\text{Expected setup Cost in a renewal cycle}}{\text{Expected duration of a renewal cycle}} \tag{2}$$

$$H_p = \frac{\text{Expected holding Cost in a renewal Cycle}}{\text{Expected duration of a renewal Cycle}} \tag{3}$$

In this study, the process is renewed when the inventory level of retailer and the inventory level of manufacturer are  $Q$  and  $0$ , respectively. Because of stochastic demand, the number of production cycle in each renewal cycle is a random

variable, (Fig1) then the renewal cycle time is stochastic. Now, we compute the expected cycle time and the expected manufacturer and retailer's holding and manufacturer's setup costs in a renewal cycle.

**3.3.1. Expected Renewal Cycle Time**

The renewal cycle time is the elapsed time between two successive renewal points. As it is shown in Fig.1, the cycle time includes the manufacturer production and idle time during the renewal cycle, then the renewal cycle time  $T_j$ , is:

$$T_j = T_z + T_p \tag{4}$$

Where,  $T_p$  and  $T_z$  are the manufacturer's production time and the manufacturer's idle time during a renewal cycle, respectively. If the number of production cycles during renewal cycle is  $Y$ , then the manufacturer's production time during a renewal cycle is:

$$T_p = Y \tau_p = \frac{Q}{p} Y \tag{5}$$

Where,  $p$  is the production rate. As it is shown in Fig.1, at the beginning of idle time, the retailer's inventory position is equal to its on hand inventory and its value is  $r+z$ , thus the manufacturer's idle time is the time that  $z$  units are demanded. If the number of production cycle during the renewal cycle is  $Y$ , the value of  $z$  is:

$$z = YQ - \sum_{i=1}^Y x_i \tag{6}$$

It can be shown simply that the expected idle time is:

$$E(T_z) = E\left(\frac{YQ - \sum_{i=1}^Y x_i}{\lambda}\right) \tag{7}$$

Since  $Y$  is the stopping time for  $x_i$ 's sequence ( $i=1, 2, \dots, Y$ ) and  $x_i$  ( $i=1, 2, \dots, Y$ ) are poisson random variables with mean  $\frac{\lambda Q}{p}$ , then

$$E(T_z) = \frac{Q}{\lambda} E(Y) \left(1 - \frac{\lambda}{p}\right) \tag{8}$$

With regard to equations (4 to 8), the expected value of renewal cycle time is [19]:

$$E(T) = \frac{Q}{\lambda} E(Y) \tag{9}$$

We need the expected value of the number of production cycles,  $Y$ , to compute the expected renewal cycle time. While the number of production cycles during the renewal cycle is a stochastic variable, we are required to compute its probability distribution.

If the production cycle during a renewal cycle time is  $Y$ , then its probability distribution is computed as the following:

In this case, the probability that the system having precisely  $Y=1$  production cycle is equal to the probability that less than  $Q$  units is demanded during the first production cycle time, i.e.

$$\Pr\{Y = 1\} = \Pr\{X_1 \leq Q - 1\} \tag{10}$$

We have at least another additional production cycle; if demand during the first production cycle is greater than  $Q$ . The number of production cycles is 2, if the aggregated demand at the end of the second production cycle is less than  $2Q$  units, i.e.

$$\Pr\{Y = 2\} = \Pr\{X_1 \geq Q, X_1 + X_2 \leq 2Q - 1\} \tag{11}$$

With this logic, the probability that the system having precisely  $Y=y$  production cycles in a renewal cycle is:

$$\Pr\left\{X_1 \geq Q, \sum_{i=1}^2 X_i \geq 2Q, \sum_{i=1}^3 X_i \geq 3Q, \dots, \sum_{i=1}^y X_i \leq yQ - 1\right\} \tag{12}$$

Since demand is a poisson variable then the probability that having,  $x_j$  units demand during the production cycle is:

$$\Pr(X_j = x_j) = \frac{e^{-\lambda Q/p} (\lambda Q/p)^{x_j}}{x_j!} \tag{13}$$

Where  $X_j$  is demand during the production cycle  $j$ . finally the average production cycle during a renewal cycle is:

$$E(Y) = \sum_{y=1}^{\infty} y \Pr(Y = y) \tag{14}$$

And then the expected renewal cycle is calculated.

**3.3.2. Long-Run Average Setup Cost**

When the number of production cycles during the renewal cycle and setup cost are  $Y$  and  $A_s$ , respectively, the long run average setup cost is:

$$F = \frac{E(A_s \times Y)}{E(T)} = \frac{\lambda A_s}{Q} \tag{15}$$

**3.3.2. Long-Run Average Manufacturer's Inventory Holding Cost**

As it is shown in Fig.3, the on-hand inventory level of manufacturer during the production time and idle time is,  $q/2$  and zero respectively, thus the manufacturer inventory level ( $S_p$ ) is:

$$S_p = T_p \times \frac{q}{2} + T_z \times 0 = \frac{Yq^2}{2p} \tag{16}$$

When the unit holding cost/time at the manufacturer is  $h_p$ , with regard to the equation (9), the long-run average manufacturer's inventory holding cost is:

$$H_p = h_p \frac{E(S_p)}{E(T)} = h_p \frac{\lambda q}{2p} \tag{17}$$

**3.3.3. Long-Run Average Retailer's Inventory Holding Cost**

When the unit holding cost/time at the retailer is  $h_r$ , then the average retailer's inventory cost is:

$$H_r = h_r \times \frac{E(T_p \times \bar{I}_{rp} + T_z \times \bar{I}_{rz})}{E(T)} \tag{18}$$

Where  $\bar{I}_{rp}$  and  $\bar{I}_{rz}$  are average retailer's inventory level during production time and average retailer's inventory level during the idle time in a renewal cycle, respectively. As it is illustrated in Fig.3,  $\bar{I}_{rp}$  and  $\bar{I}_{rz}$  are (Hadley 1963):

$$\bar{I}_{rp} = r + \frac{q}{2} ; \bar{I}_{rz} = r + \frac{z}{2} \tag{19}$$

So, regarding equations (5, 6, 8 & 9) the long run average retailer's inventory holding cost would be:

$$H_r = h_r \left( \frac{q\lambda}{2p} + r + \frac{\lambda E(zT_z)}{2QE(Y)} \right) \tag{20}$$

It can be shown simply that

$$E(z \times T_z) = \frac{E(z^2)}{\lambda} \tag{21}$$

Where  $z$  is equal to:  $YQ - \sum_{i=1}^Y X_i$

Finally, by replacing equations (15), (17) and (20) in the equality (1) and with regard to equation (21), the long-run average cost of overall supply chain formula is derived.

$$C(n, q, Q) = \frac{A_s \lambda}{Q} + \frac{h_p \lambda q}{2p} + h_r \left( \frac{q\lambda}{2p} + r + \frac{E(z^2)}{2QE(Y)} \right) \tag{22}$$

Regarding the model assumptions  $r = mQ$ ,  $Q = nq$ , equation (22) will be modified as follow:

$$C(n, Q) = \frac{\lambda A_s}{Q} + \frac{Q h_p \lambda}{2np} + h_r \left( \frac{Q\lambda}{2np} + mQ + \frac{E(z^2)}{2QE(Y)} \right) \tag{23}$$

**4. Monte-Carlo Simulation**

Since the terms  $E(Y)$  and  $E(z^2)$  are complex and they could not be simplified as the function of the system's parameters ( $\lambda, p, Q, A_s, h_p, h_r, n$ ), we apply Monte-Carlo simulation to estimate these terms in order for examining the effect of environmental factors on the system cost. To do so, we considered 1755 problems made up of all the combinations of the following parameters:

$\alpha = A_s / h_r$	$\frac{1}{3}, \frac{1}{2}, 1, 2, 3$
$\alpha = h_p / h_r$	$\frac{1}{3}, \frac{1}{2}, 1, 2, 3$
$\rho = \lambda / p$	$\frac{1}{3}, \frac{1}{2}, 1, 2, 3$
$Q$	1, 2, 3, 4, 5
$n$	1, 2, 3, ..., Q

For each problem, we approximately calculate the long-run average cost of overall supply chain by Monte-Carlo simulation and analyze the results.

**4.1. Data Analysis**

By running the simulation study, we obtain overall system cost for different levels of parameters. We summarize these data as the following graph:

**4.1.1. The Effect of Changes in the Value of  $\rho$  on the Overall Supply Chain Cost**

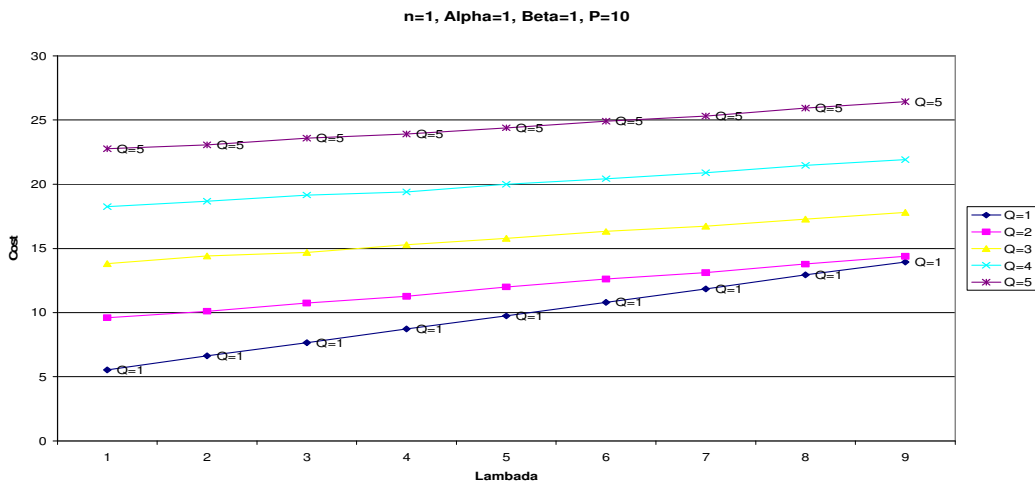


Fig. 3. Cost and Demand rate Interaction (n=1, Alpha=1, Beta=1, P=10)

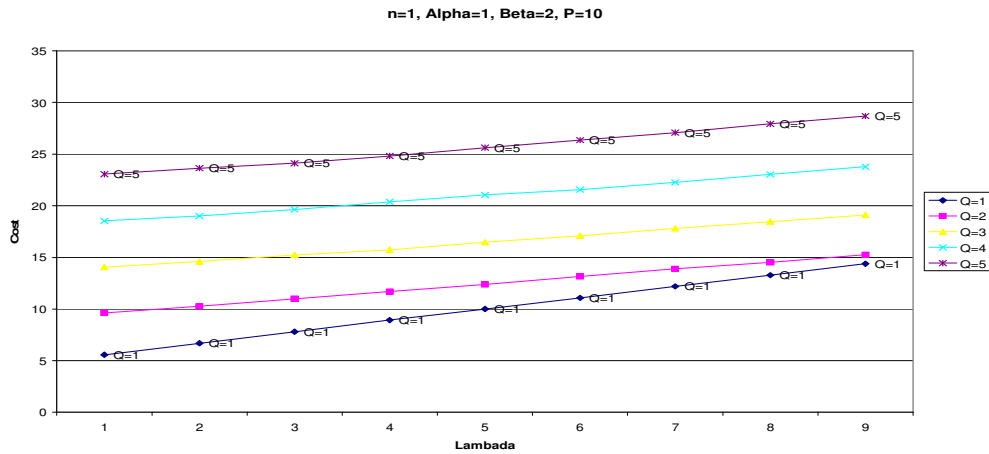


Fig. 4. Cost and Demand rate Interaction ( $n=1, \text{Alpha}=1, \text{Beta}=2, P=10$ )

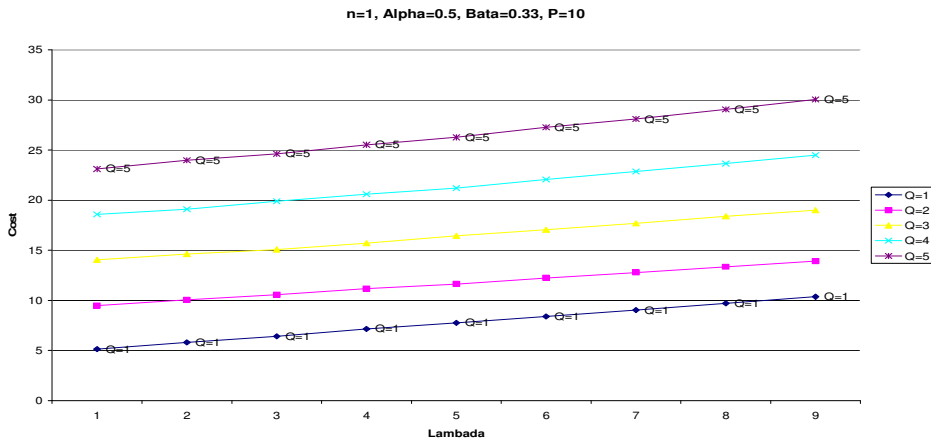


Fig. 5. Cost and Lot size Interaction ( $n=1, \text{Alpha}=0.5, \text{Beta}=0.33, P=10$ )

As it is shown in figures 4 through 6, when the value of  $\rho$  increases, the overall cost of system increases. So, we can infer that when the demand rate increases or the production rate decreases, the cost of system will increase, too.

**4.1.2. The Effect of Changes in the Value of  $n$  on the overall Supply Chain Cost**

It is worth mentioned that  $n$  is an integer and varies between 1 and  $Q$ . The following figures show the effect of  $n$  on the cost of system.

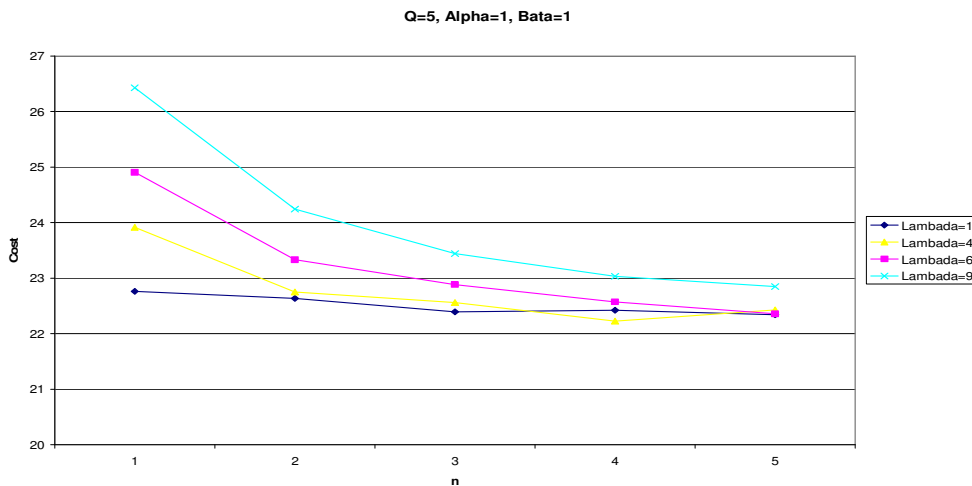


Fig. 6. Cost and Lot Split size Interaction ( $Q=5, \text{Alpha}=1, \text{Beta}=1$ )

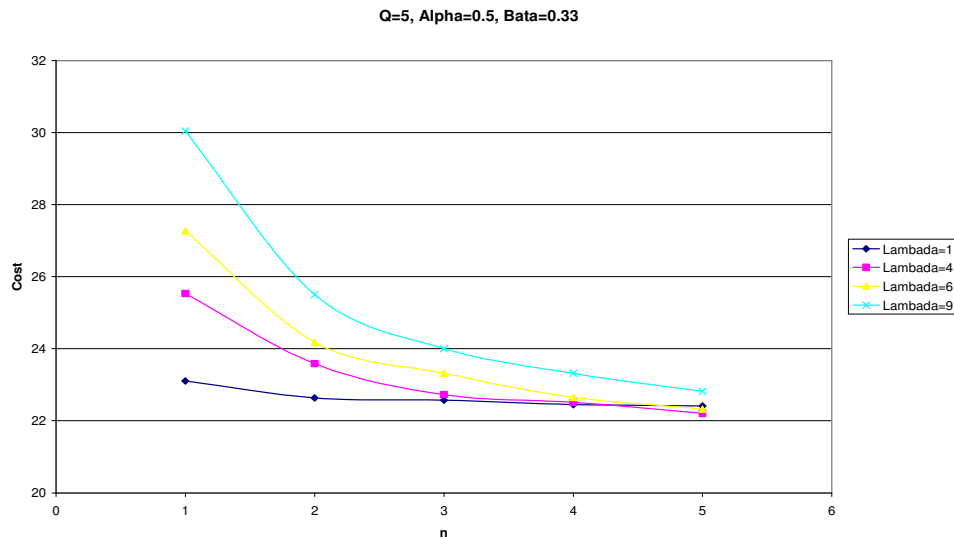


Fig. 7. Cost and Lot Split size Interaction(Q=5, Alpha=0.5, Beta=0.33)

Regarding figures 7&8, we can infer when the value of  $n$  (number of replenishment) increases the overall cost of system decreases. It is sensible, because we assumed that the transportation cost is negligible.

### 5. Conclusion

In this paper, we studied the inventory cost of a Two-echelon serial supply chain system under vendor managed inventory program, and examined the effect of environmental factors on the cost of overall system. We considered a two-echelon serial supply chain with a manufacturer and a retailer. Under Vendor managed inventory program, the decision on inventory levels were made by manufacturer centrally. In this paper, we assumed that the manufacturer monitors inventory levels at the retailer location and replenishes retailer's stock under  $(r, n, q)$  policy; moreover, the manufacturer followed make-to-order strategy in order to respond retailer's orders. We developed a renewal reward model for the case of Poisson demand, and derived the mathematical formula of the long run average total inventory cost of system under VMI. Then, by using Monte Carlo simulation, we examined the effect of environmental factors on the cost of overall system under VMI. Regarding the estimated cost of system, we found that the cost of system is sensitive to the demand rate, production rate. Therefore, we can invest on increasing the production rate in order to decrease the cost of system. Moreover, we found that by increasing the replenishment frequency, the cost will decrease. Since the transportation cost is negligible, it is a plausible conclusion.

For the further study, we leave finding the value of  $E(Y)$  and considering the transportation costs in the model as the open questions.

### Reference

- [1] Sahin, F., Robinson, E.P., *Flow Coordination and Information Sharing in Supply Chains: Review, implications, and directions for future research*. Decision Sciences 33 (4), 2002, pp. 505–536
- [2] Simchi-Levi, D., Kaminsky, P., Simchi-Levi, E., *Designing and Managing the Supply Chain*, second ed. McGraw-Hill, IL, 2003.
- [3] Stalk, G., P. Evans, L.E. Shulman. *Competing on capabilities: The new rules of corporate strategy*. Harvard Bus. Rev. 70(2), 1992, pp. 57-69
- [4] Kurt Salmon Associates, Inc. *Efficient Consumer Response: Enhancing Consumer Value in the Grocery Industry*. Food Marketing Institute, Washington, DC, 1993.
- [5] Clark, T.H., McKenney, J.L., *Procter & Gamble: Improving Consumer Value Through Process Design*. HBS Case 9-1995, 195-126.
- [6] Hall, R.W., *Consolidation Strategy: Inventory, vehicles and terminals*. J. Bus. Logist. 8(2), 1987, 57-73.
- [7] Cetinkaya, S., Lee, C.Y., *Stock Replenishment and Shipment Scheduling for Vendor-Managed Inventory Systems*, Management Science 46 (2), 2000, pp. 217–232.
- [8] Achabal, S., McIntyre, S., Smith, K., Kalyanam, A *Decision Support System for Vendor Managed Inventory*, Journal of Retailing 76 (4), 2000, pp. 430–454.
- [9] Goyal, S.K., *An Integrated Inventory Model for a Single Supplier–Single Customer Problem*. International Journal of Production Research 15, 1977, pp. 107–111.
- [10] Banerjee, A., *A Joint Economic-Lot-Size Model for Purchaser and Vendor*. Decision Sciences 17, 1986, pp. 292–311.

- [11] Yao, Y., Evers, P.T., Dresner, M.E., *Supply Chain Integration Under Vendor Managed Inventory*, Decision Support Systems, 2007, pp. 663– 674.
- [12] Cachon, G., Fisher, M., *Supply Chain Inventory Management and the Value of Shared Information*, Management Science 46 (8), 2000, pp. 1032– 1048.
- [13] Chen, F., Drezner, Z., Ryan, J.K., Simchi-Levi, D., *Quantifying the Bullwhip Effect in a Simple Supply Chain: the Impact of Forecasting, lead times, and information*, Management Science 46 (3), pp. 436– 443.
- [14] Lee, H.L., So, K.C., Tang, C., *The Value of Information Sharing in a Two-Level Supply Chain*. Management Science 46 (5), 2000, pp. 626–643.
- [15] Lee, H.L., Padmanabhan, V., Whang, S., Information distortion in a supply chain: the bullwhip effect, Management Science 43 (4), 1997, pp. 546–559.
- [16] Moinzadeh, K., *A Multi-Echelon Inventory System with Information Exchange*. Management Science, 48(3), pp. 2002, 414 – 426
- [17] Haji, R., Sajadifar, S.M., *Deriving the Exact Cost Function for Two-Level Inventory System with Information Sharing*, Journal of Industrial and Systems Engineering, Vol.2, No.1, 2008, pp 41-50
- [18] Haji, R., Moarefdoost, M., Ebrahimi, S.B, "*Finding the Cost of Inventory in a Make to Order Supply Chain Under Vendor Managed Inventory Program*", Industrial Management Journal, Vol. 1, No. 3, 2010, pp.21 –36.
- [19] Ross S. M., "*Introduction to Probability Models*", 9th edition, Elsevier Inc, 2007.
- [20] Hadley, G., T., Whitin. X., "*Analysis of Inventory Systems*". Prentice- Hall, Englewood Cliffs, NJ, 1977.