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Linear Profile Monitoring in the Presence of Non-Normality and Autocorrelation

Rassoul Noorossana*, Abbas Saghaei & Mehdi Dorri

Rassoul Noorossana, *Industrial Engineering Department Islamic Azad University, South-Tehran Branch, Tehran, Iran*
Abbas Saghaei, *Industrial Engineering Department Islamic Azad University, Science and Research Branch, Tehran, Iran*
Mehdi Dorri, *Industrial Engineering Department Islamic Azad University, South-Tehran Branch, Tehran, Iran*

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ABSTRACT

In an increasing number of practical situations, the quality of a process or product can be effectively characterized and summarized by a profile. A profile is usually a functional relationship between a response variable and one or more explanatory variables which can be modeled frequently using linear or nonlinear regression models. In this paper, we study the effect of non-normality on profile monitoring in Phase II when within or between autocorrelation is present. Different levels of autocorrelation and skewed and heavy-tailed symmetric non-normal distributions are used in our study to evaluate the performance of three existing monitoring schemes numerically. Simulation results indicate that the non-normality and autocorrelation can have a significant effect on the in-control performances of the considered schemes. Results also indicate that the out-of-control performances of the schemes are not very sensitive to low and moderate levels of autocorrelation in moderate and large shifts.

1. Introduction

In some practical situations, the quality of a process or product can be characterized by a relationship between a response variable and one or more explanatory variables. This relationship is usually known to as profile. Many authors including Mestek et al. (1994), Stover and Brill (1998), Lawless et al. (1999), Kang and Albin (2000), Mahmoud and Woodall (2004), Wang and Tsung (2005), Gupta et al. (2006), Soleimani et al. (2009), and Jensen et al. (2009) discussed real world applications in which a linear profile could be considered to represent the status of a process or product effectively. Properties of linear profile in Phase I and Phase II has been studied many researchers including Mestek et al. (1994), Stover and Brill (1998), Kang and Albin (2000), Kim et al. (2003), Mahmoud and Woodall (2004), Noorossana et al. (2004), Gupta et al. (2006), Zou et al. (2006), Mahmoud et al. (2007), Saghaei et al. (2009), and Zhu and Lin (2010). Sometimes status of a product or process can be well modeled by a nonlinear relationship. Several authors including Jin and Shi (1999), Walker and Wright (2002), Ding et al. (2006), Williams et al. (2007), Moguerza et al. (2007), and Vaghefi et al. (2009) have discussed monitoring of nonlinear profiles. Kazemzadeh et al. (2008) considered polynomial profiles and extended three phase I methods to monitoring such profiles. Another paper on profile monitoring is by Zou et al. (2007) in which they proposed a multivariate exponentially weighted moving average (MEWMA) control chart for monitoring general linear profiles in Phase II.

In all the above mentioned studies, it is implicitly assumed the error terms in the model are independent and identically distributed normal random variables with mean zero and fixed variance. However, in certain practical cases, these standard assumptions may be violated. Soleimani et al. (2009) presents an example of situation for which error terms within profiles are autocorrelated. They considered a first order

* **Corresponding author. Rassoul Noorossana**
Email: *rassoul@iust.ac.ir* *a.saghaei@srbiau.ac.ir* *me.dorri@gmail.com*
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autoregressive model to account for the autocorrelation structure between observations in each linear profile and developed four methods to monitor them in Phase II. Jensen et al. (2008) proposed a linear mixed model to account for the autocorrelation within a linear profile in Phase I. Noorossana et al. (2008a) and Kazemzadeh et al. (2009) proposed different time series based methods for monitoring linear profiles where autocorrelation exists between profiles over time. However, a few studies have been done to investigate the effect of non-normality and autocorrelation on linear or nonlinear profile monitoring. Noorossana et al. (2008b) studied the effect of non-normality of the error terms using Gamma and *t*-distributions to model the behavior of observations. Noorossana et al. (2008a) and Soleimani et al. (2009) investigated the effect of between and within linear profile autocorrelation, respectively.

In this paper, we study the effect of simultaneous violation of the normality and independency assumptions on the performances of three common methods of linear profile monitoring. To study the effect of non-normality and autocorrelation, we consider the use of both heavy tailed symmetric and skewed non-normal distributions with dependent error terms generated by a first order autocorrelation model. In addition, we consider an autoregressive model of order one to model the autocorrelation structure between error terms. Different autocorrelation values are considered in the numerical examples.

Next section discusses three methods which have been proposed by Kang and Albin (2000) and Kim et al. (2003) to monitor linear profiles in phase II. Section 3 covers some non-normality and autocorrelation issues related to linear profiles. The simultaneous effects of non-normality and autocorrelation on the performance of the three methods are evaluated in Section 4. Our concluding remarks are presented in the final section.

2. Linear Profile Monitoring Methods

As discussed in the introduction section, many researchers have contributed to the development of simple linear profile monitoring methods. One important reason is its simplicity and applicability to model real life problems. In this paper, we investigate the simultaneous effects of non-normality and autocorrelation on the performance of the control schemes developed by Kang and Albin (2000) and Kim et al. (2003). These are the common control schemes used in phase II analysis.

Kang and Albine (2000) proposed two control schemes for monitoring linear profiles in phase II. Their first method involves a bivariate T^2 control chart to monitor the linear regression coefficients. Their second strategy is to use a combination of an exponentially weighted moving average (EWMA) and a range (R) chart to monitor the regression residuals obtained at each sample. The EWMA control chart and R control chart are used for monitoring the average and variation of the residuals, respectively. Kim et al. (2003) proposed a combination of three EWMA control charts to monitor intercept, slope, and process standard deviation, separately.

In simple linear profiles, it is assumed that paired observations (x_i, y_{ij}) for i=1,2,…,n and j=1,2,... are collected over time and the relationship between the paired observations is best represented by a linear profile, i.e.,

$$y_{ij} = A_0 + A_1 x_i + \varepsilon_{ij}, \qquad (1)$$

where ε_{ij}'s are independent and identically distributed (i.i.d) normal random variables with mean zero and variance σ^2. It is also assumed that the X-values are fixed and values of the parameters A_0, A_1, and σ^2 are known.

The first approach we describe in this section is the bivariate T^2 control chart proposed by Kang and Albin (2000). To use this chart, it is assumed that the least squares estimators for intercept and slope follow a bivariate normal distribution. The mean vector and variance-covariance matrix for the bivariate normal distribution are defined as

$$\boldsymbol{\mu} = (A_0, A_1) \quad \text{and} \quad \boldsymbol{\Sigma} = \begin{pmatrix} \sigma_0^2 & \sigma_{01}^2 \\ \sigma_{01}^2 & \sigma_1^2 \end{pmatrix} \qquad (2)$$

For sample j, the sample statistic in the bivariate T^2 control chart is calculated using the following equation:

$$T_j^2 = (\mathbf{z}_j - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1} (\mathbf{z}_j - \boldsymbol{\mu}) \qquad (3)$$

where $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ are defined in Equation (2) and $\mathbf{z}_j$ is the vector of sample least squares estimators. It is well-known that T_j^2 follows a central chi-square distribution with 2 degrees of freedom when process is under statistical control. Therefore, upper control limit for this chart is $\mathrm{UCL} = \chi^2_{2,\alpha}$ where $\chi^2_{2,\alpha}$ is the 100(1-α) percentile of the chi-square distribution with 2 degrees of freedom. As it was stated earlier, the second approach proposed by Kang and Albin (2000) is known as EWMA/R method. In this method, the regression residuals obtained at sample j using Equation (4) are monitored by a combination of EWMA control chart and R chart, respectively.

$$e_{ij} = y_{ij} - a_{0j} - a_{1j} x_i \quad , \quad i = 1, 2, ..., n \qquad (4)$$

EWMA control chart monitors average value of these residuals and R chart is used to detect shifts in variation. For the j^{th} sample, the EWMA control chart statistics are given by Equation (5). This statistic is a weighted average of the j^{th} residual average and the previous residual averages.

$$z_j = \theta \bar{e}_j + (1-\theta) e_{j-1}, \tag{5}$$

where $\bar{e}_j$ is the average of the residuals for sample j, θ ($0 < \theta < 1$) is the smoothing constant and $z_0 = 0$. This control chart signals as soon as the value of z_j plots out of control limits. The control limits for the EWMA control chart are computed using Equation (6).

$$LCL = -L\sigma\sqrt{\frac{\theta}{(2-\theta)n}} \quad \text{and}$$
$$UCL = L\sigma\sqrt{\frac{\theta}{(2-\theta)n}} \tag{6}$$

where L ($L > 0$) is a constant selected to obtain a specified in-control ARL value. Kang and Albin (2000) proposed the range control chart in conjunction with the EWMA chart. The statistic for the range control chart is $R_j = \max(e_{ij}) - \min(e_{ij})$. The control limits for the R chart are:

$$LCL = c\,(d_2 - Ld_3) \quad \text{and} \quad UCL = c\,(d_2 + Ld_3) \tag{7}$$

respectively, where L ($L > 0$) is a constant chosen to obtain a specified in-control ARL value and the values of d_2 and d_3 are constants that depend on the sample size n.

Kim et al. (2003) propose a control scheme consisting of three independent EWMA control charts to monitor intercept and slope coefficients of the model and standard deviation of the process separately. To construct three independent EWMA control charts, they first coded the X -values. After transforming the X-values by subtracting the sample average form each observation, i.e. $X_i' = (X_i - \bar{X})$, an alternative form of the underlying linear profile is obtained as $y_{ij} = B_0 + B_1 X_i' + \varepsilon_{ij}, i = 1, 2, ..., n$, where $B_0 = A_0 + A_1\bar{X}$ and $B_1 = A_1$. The least squares estimator for the new intercept and slope coefficients are $b_{0j} = \bar{y}_j$ and $b_{1j} = a_{1j}$, respectively. For sample j, $\bar{y}_j$ is the average of the observations and a_{1j} is the least squares estimator for A_1. These estimators are independent normal random variables with means B_0 and B_1 and variances $n^{-1}\sigma^2$ and $S_{xx}^{-1}\sigma^2$, respectively. Since the covariance between b_{0j} and b_{1j} is zero now one can use two separate control charts for monitoring these coefficients. Kim et al. (2003) used three separate EWMA control charts to monitor the intercept, slope, and error variance, respectively.

The EWMA control chart for B_0 uses the statistic defined by Equation (8), where the first value of this statistic is equal to $z_{0,b_0} = B_0$.

$$z_{j,b_0} = \theta b_{0j} + (1-\theta) z_{j-1,b_0}, \; j = 1, 2, ... \tag{8}$$

The upper and lower control limits of this chart are:

$$\text{LCL} = B_0 - L_{b_0}\sigma\sqrt{\frac{\theta}{(2-\theta)n}} \quad \text{and}$$
$$\text{UCL} = B_0 + L_{b_0}\sigma\sqrt{\frac{\theta}{(2-\theta)n}} \tag{9}$$

The slope parameter B_1 can also be monitored by using EWMA statistic computed by Equation (10). It is also assumed that the value of z_{0,b_1} is equal to $z_{0,b_1} = B_1$.

$$z_{j,b_1} = \theta b_{1j} + (1-\theta) z_{j-1,b_1}, \; j = 1, 2, ... \tag{10}$$

Control limits for the EWMA control chart are given by:

$$\text{LCL} = B_1 - L_{b_1}\sigma\sqrt{\frac{\theta}{(2-\theta)S_{xx}}} \quad \text{and}$$
$$\text{UCL} = B_1 + L_{b_1}\sigma\sqrt{\frac{\theta}{(2-\theta)S_{xx}}} \tag{11}$$

where L_{b_0} and L_{b_1} are multiples chosen to obtain a specified in-control ARL. Finally to monitor the process variability, Kim et al. (2003) considered the EWMA control chart for monitoring the error variance σ^2 using the method proposed by Crowder and Hamilton (1992). The statistic for this EWMA control chart is given by Equation (12).

$$z_{j,MSE} = \max\{\theta \ln(MSE_j) + (1-\theta) z_{j-1,MSE}, \ln(\sigma_0^2)\},$$
$$j = 1, 2, ... \tag{12}$$

where $z_{0,MSE} = \ln(\sigma^2)$. The EWMA control chart signals an out-of-control condition when the value of $z_{j,MSE}$ is greater than the upper control limit calculated using:

$$\text{UCL} = L_{MSE}\sigma\sqrt{\frac{\theta}{(2-\theta)S_{xx}}\text{var}[\ln(MSE_j]} \tag{13}$$

where
$\text{var}[\ln(MSE_j] \cong 2(n-2)^{-1} + 2(n-2)^{-2} + (4/3)(n-2)^{-3} + (16/15)(n-2)^{-5}$,

θ $(0 < \theta \leq 1)$ is a smoothing parameter, and L_{MSE} is a multiple chosen to obtain a specified in-control ARL, respectively. A signal from any of these EWMA control charts leads to an out-of-control condition.

3. Non-Normality and Autocorrelation

Normality and independency of error terms are critical assumptions in linear profile monitoring. In certain situations, these assumptions, due to process nature, could be violated leading to misleading results. In this paper, we consider *t*-distribution as a heavy-tailed symmetric non-normal distribution and gamma distribution as a skewed distribution to study the effect of non-normality on the performance of linear profile monitoring methods.

Student *t*-distribution is symmetric about its mean but has more probability in its tails than the standard normal distribution. In fact, the *t*-distribution differs from normality in the fourth and higher moments that can affect the shape of a distribution while the third or the lower moments are equal to a normal distribution. The gamma distribution is far from normality. It is different from a normal distribution in the third and higher moments that can significantly affect the shape of a distribution while mean and variance are equal to a normal distribution. On the other hand, the mean value of a common gamma distribution is positive.

However, the mean of error terms in profile monitoring is often assumed equal to zero. Hence, the common gamma distribution is not suitable to use as a error term distribution. We propose the use of a generalized gamma distribution which allows us to generate both positive and negative values for error terms. The probability density function of generalized gamma distribution is given below.

$$f(x) = \frac{(x-\gamma)^{\alpha-1}}{\beta^{\alpha}\Gamma(\beta)} \exp\{-(x-\gamma)/\beta\} \quad x > \gamma \qquad (14)$$

where γ ($\gamma > 0$) is the location parameter, α ($\alpha > 0$) is the shape parameter, and β ($\beta > 0$) is the scale parameter, respectively. The generalized gamma distribution is often denoted as Gam (α, β, γ). The mean and variance of this distribution is given by $\mu = \gamma + \alpha\beta$ and $\sigma^2 = \alpha\beta^2$, respectively.

In linear profile monitoring, it is also assumed that error terms are independent. Under certain circumstances, this assumption could be also violated. Independence assumption could be failed due to between profiles autocorrelation or within profile autocorrelation.

In the between profiles autocorrelation case, it is assumed that error terms between successive linear profiles can be modeled using the following first order autoregressive model:

$$\begin{aligned} y_{ij} &= A_0 + A_1 x_i + \varepsilon_{ij} \\ \varepsilon_{ij} &= \phi\varepsilon_{i(j-1)} + a_{ij} \end{aligned} \qquad (15)$$

where ε_{ij}'s are the correlated error terms and a_{ij}'s are independent and identically distributed normal random variables with mean zero and variance σ^2.

For the case of within profile autocorrelation, it is assumed that error terms within each profile can be modeled according to a first order autoregressive model. For this situation, it is assumed that paired observations (x_i, y_{ij}) are collected over time and the relationship between the paired observations can be modeled using the following relationship:

$$\begin{aligned} y_{ij} &= A_0 + A_1 x_i + \varepsilon_{ij} \\ \varepsilon_{ij} &= \rho\varepsilon_{(i-1)j} + a_{ij} \end{aligned} \qquad (16)$$

where ε_{ij}'s are the correlated error terms and a_{ij}'s are independently and identically distributed normal random variables with mean zero and variance σ^2.

In the next section, we consider the case where the assumptions of non-normality and independency of error terms no longer holds. Non-normality violation in linear profile monitoring is investigated under within and between profiles autocorrelation.

4. Performance Comparisons

Performance of a control chart is commonly evaluated by how fast the chart detects sustained shifts after they occur. Detecting these shifts quickly and with few false alarms are worthwhile features for a control chart. The ability of a control chart to detect process changes can be measured by average run length (ARL) which is defined as the number of subgroups expected to be inspected before a signal is generated by the control chart.

In this section, the in-control and out-of-control average run length values denoted by ARL(0) and ARL(1), respectively, are compared to a normal distribution, a *t*-distribution, and a Gamma distribution in the presence of between and within profile autocorrelations. If a control chart is insensitive or less sensitive to changes in these critical assumptions then the chart is said to be robust with respect to the violation of normality and independence assumptions. Performance of the control schemes described in Section 2 under simultaneous violation of normality and independence assumptions are evaluated numerically using 10,000 simulation runs to estimate the in-control and out-of-control ARL values. The underlying in-control reference model is assumed to be $y_{ij} = 3 + 2x_i + \varepsilon_{ij}$ with 2, 4, 6, and 8 as X-values. By coding the x-values, the alternative model

$y_{ij} = 13 + 2x'_i + \varepsilon_{ij}$ is obtained where X'-values are -3, -1, 1, and 3.

For normal distribution, it is assumed that error terms are i.i.d. normal random variables with mean $\mu = 0$ and variance $\sigma^2 = \frac{5}{3}$. Degrees of freedom for the *t*-distribution is chosen as $v = 5$. In this case, error terms are i.i.d. random variables with mean 0 and variance $\sigma^2 = \frac{5}{3}$. For the gamma distribution, we set $\alpha = \frac{5}{3}$, $\beta = 1$, and $\gamma = -\frac{5}{3}$ which lead to error terms having a gamma distribution with mean 0 and variance $\sigma^2 = \frac{5}{3}$. Such *t* and gamma distributions are far enough from normality to show the effect of non-normality on performance of control schemes. Of course, one could consider different values for the parameters of the distributions.

All control chart schemes were designed to have the same overall in-control ARL of 200. The smoothing constants θ in all of the EWMA control charts were set equal to 0.2. In order to investigate the effect of between or within profiles autocorrelation on the performance of the control schemes we used various values of φ and ρ.

For T^2 control chart, the upper control limit is given by $\text{UCL} = \chi^2_{2,0.005} = 10.59653$ yielding an in-control ARL of roughly 200. The EWMA/R approach uses a combination of the smoothing parameter θ and the multipliers, L_{EWMA} and L_R yielding an overall in-control ARL value approximately equal to 200. The value of multiplier L_{EWMA} is chosen as 2.8851 corresponding to smoothing parameter value of $\theta = 0.2$, yielding an in-control ARL of 400. Notice that for the R chart, a value of 3.308 is considered for the multiplier yielding an in-control ARL of approximately 400. Therefore, the combination of the two charts has an overall in-control ARL of approximately 200. Finally, for EWMA3 approach, the multipliers L_{b_0}, L_{b_1}, and L_{MSE} are designed to yield an overall in-control ARL value of roughly 200.

For monitoring the Y-intercepts, L_{b_0} and for monitoring the slopes, L_{b_1} are both set equal to 3.1144 leading to an in-control ARL of 800. The combination of these two EWMA control charts yields an overall in-control ARL of approximately 400. Besides, the multiplier L_{MSE} is chosen as 1.3016 yielding an in-control ARL of roughly 400. As a result, the overall in-control ARL for EWMA3 approach is approximately equal to 200.

4.1. In-Control Performance Comparison

In this section, we compare the effect of non-normality and autocorrelation on the in-control performance of linear profile monitoring methods. As mentioned before, we use a heavy-tailed *t*-distribution and a skewed gamma distribution for the violation of normality assumption and simultaneously induce autocorrelation in the error terms. Table 1 contains the values of ARL(0) when error terms follow normal, *t,* or gamma distribution in the presence of different levels of autocorrelation between profiles over time. It can be shown that non-normality and autocorrelation between profiles seriously affect the EWMA/R approach more than other schemes. It is also obvious that all control schemes are sensitive to normality and independence assumption especially when autocorrelation coefficients are large.

Tab. 1. In-control ARL values for different distributions and different levels of autocorrelation between profiles

φ	T^2			EWMA/R			EWMA3		
	N	*t*	*gam*	*N*	*t*	*gam*	*N*	*t*	*gam*
0.0	200.0	59.3	49.2	200.4	55.9	57.9	199.7	119.2	111.4
0.1	189.2	58.8	48.4	141.9	49.8	54.2	126.7	89.9	89.0
0.3	125.8	52.8	45.0	58.2	36.0	36.8	41.7	40.0	40.0
0.5	60.0	39.6	37.5	24.9	20.9	20.5	15.8	17.2	16.6
0.7	23.8	23.0	24.9	11.5	11.4	11.2	7.9	8.6	8.2
0.9	10.3	11.0	10.9	6.3	6.5	6.4	5.1	5.4	5.2

Table 2 also shows the effect of non-normality and autocorrelation on the performance of T^2, EWMA/R, and EWMA3 control schemes while the process is in control. In this comparison, we consider different levels of within profile autocorrelation when non-normality is present. It can be shown that the performance of T^2 control chart is seriously affected by non-normality and autocorrelation. The results of this study show that simultaneous violation of normality and independence assumptions affects the performance of all three control schemes. Small values of ARL (0) lead to large number of false alarms while process is in control.

Tab. 2. In-control ARL values for different distributions and different levels of autocorrelation between profiles

ρ	T^2			EWMA/R			EWMA3		
	N	*t*	*gam*	*N*	*t*	*gam*	*N*	*t*	*gam*
0.0	200.0	59.3	49.2	200.4	55.9	57.9	199.7	119.2	111.4
0.1	107.8	44.1	38.1	140.1	53.9	58.2	186.9	116.3	117.3
0.3	33.6	23.6	23.3	56.5	38.8	41.0	75.8	66.2	70.6
0.5	13.7	13.0	14.8	26.7	23.9	24.2	33.3	32.9	34.8
0.7	7.0	7.5	8.3	14.7	14.2	14.1	17.5	18.1	18.4
0.9	4.3	4.8	5.0	8.9	9.1	8.8	10.2	11.1	10.8

Figure 1 shows the effect of autocorrelation values (either autocorrelation between profiles (φ) or autocorrelation within profile (ρ)) on the in control ARL performance of the control methods under different error distribution. It can be shown that EWMA3 method by Kim et al. (2003) is less sensitive to the departure from normality and independence assumptions especially for small values of autocorrelation coefficient.

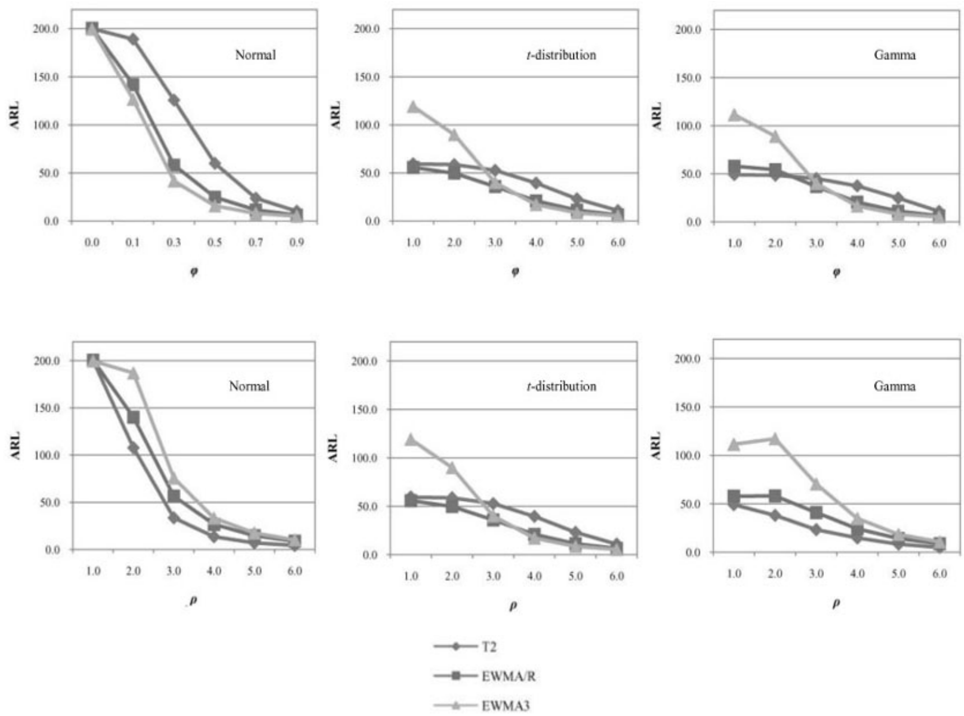

Fig. 1. ARL(0) values for various autocorrelation coefficients and distributions

4.2. Out-of-Control Performance Comparison

In this section, we investigate the simultaneous effects of non-normality and autocorrelation on the out-of-control performance of the schemes. Out-of-control ARL performance evaluation of the schemes is conducted using 10,000 simulation runs and different values of autocorrelation coefficient allowing shifts in the intercept, slop, and error standard deviation. First, we consider the case when error terms between successive profiles are not independent.

The results are summarized in Tables 3, 4, and 5 using autocorrelation coefficient values of ρ = 0.1, 0.3, and 0.9. In these tables, λ, β, and γ define shifts in the intercept, slope, and standard deviation, respectively. The results show that the non-normality and autocorrelation affects the out-of-control ARL performance significantly and as the value of autocorrelation coefficient gets larger out-of-control performance deteriorates. This impact is small in large shifts.

The last three tables show the in-control and out-of-control ARL values. Here, it is assumed that the error terms which follow non-normal distributions are correlated in each profile. The simulation results in

Tables 6, 7, and 8 show the significant effect of non-normality and autocorrelation on the out-of-control performance of the schemes. When shift size becomes larger, the effect of non-normality for smaller values of autocorrelation coefficient is minimal.

Tab. 3. Comparison of ARL (1) with normal, *t*, and gamma distributions for different values of between profiles autocorrelation under *Y*-intercept shift from A_0 to $A_0 + \lambda\sigma$

		λ														
	φ	0.2			0.6			1.0			1.4			1.8		
		T2	EWMAR	EWMA3	T2	EWMAR	EWMA3	T2	EWMAR	EWMA3	T2	EWMAR	EWMA3	T2	EWMAR	EWMA3
Normal	0.0	139.2	51.5	66.9	27.9	7.3	8.4	6.9	3.6	4.0	2.6	2.5	2.7	1.5	2.0	2.1
	0.1	131.8	43.6	52.6	28.2	7.4	8.4	7.0	3.7	4.0	2.6	2.5	2.7	1.4	2.0	2.1
	0.5	50.1	19.4	14.0	19.1	7.8	7.5	7.2	4.0	4.3	3.1	2.7	2.9	1.6	2.1	2.2
	0.9	10.2	6.2	5.1	8.9	5.3	4.6	6.6	4.2	3.9	4.1	3.1	3.1	2.2	2.3	2.4
t	0.0	53.2	33.7	58.2	23.4	7.0	8.4	7.6	3.5	4.0	2.7	2.4	2.7	1.4	2.0	2.1
	0.1	51.3	30.2	46.9	23.4	7.0	8.4	7.8	3.6	4.0	2.8	2.5	2.7	1.5	2.0	2.1
	0.5	35.6	17.0	15.0	18.6	7.3	7.6	7.8	3.9	4.3	3.4	2.6	2.8	1.7	2.0	2.2
	0.9	10.9	6.4	5.3	9.4	5.6	4.8	7.1	4.2	3.9	4.3	3.0	3.0	2.1	2.3	2.3
Gamma	0.0	33.8	33.7	52.5	14.6	7.4	8.8	6.3	3.6	4.0	2.9	2.5	2.7	1.6	2.0	2.1
	0.1	34.1	30.7	44.8	15.1	7.7	8.9	6.6	3.7	4.1	3.0	2.5	2.7	1.6	2.0	2.1
	0.5	28.1	19.2	16.6	14.4	8.3	8.8	7.1	4.2	4.6	3.6	2.7	2.9	1.9	2.1	2.2
	0.9	10.9	6.6	5.4	9.8	6.2	5.2	7.7	4.9	4.4	5.0	3.4	3.4	2.7	2.3	2.4

Tab. 4. Comparison of ARL (1) with normal, *t*, and gamma distributions for different values of between profiles autocorrelation under slope shift from A_1 to $A_1 + \beta\sigma$

		β														
	φ	0.025			0.075			0.125			0.175			0.225		
		T2	EWMAR	EWMA3	T2	EWMAR	EWMA3	T2	EWMAR	EWMA3	T2	EWMAR	EWMA3	T2	EWMAR	EWMA3
Normal	0.0	168.0	97.4	111.9	60.7	16.2	18.8	19.9	6.8	7.6	7.8	4.2	4.6	3.7	3.1	3.4
	0.1	159.2	75.9	79.6	60.4	15.8	17.4	19.9	6.9	7.5	7.9	4.3	4.7	3.8	3.2	3.4
	0.5	54.6	22.3	15.2	31.4	12.8	10.4	15.6	7.3	6.7	8.0	4.7	4.7	4.5	3.4	3.6
	0.9	10.6	6.2	5.1	9.7	5.7	4.9	8.6	5.2	4.5	6.8	4.4	4.0	5.3	3.7	3.4
t	0.0	55.8	45.6	84.6	36.8	14.4	18.7	19.3	6.5	7.6	8.9	4.1	4.6	4.2	3.0	3.4
	0.1	55.6	40.2	65.3	36.7	14.1	17.6	19.1	6.6	7.5	8.9	4.2	4.6	4.2	3.1	3.4
	0.5	38.1	18.9	16.0	27.3	11.6	10.8	15.7	6.8	6.9	8.5	4.6	4.7	4.7	3.3	3.5
	0.9	11.0	6.5	5.4	10.3	6.1	5.1	9.1	5.4	4.6	7.4	4.6	4.1	5.4	3.7	3.4
Gamma	0.0	38.2	43.9	73.4	22.5	15.3	19.2	12.7	7.0	8.1	7.1	4.3	4.8	4.1	3.1	3.5
	0.1	37.9	40.7	60.4	23.0	15.4	18.2	12.9	7.1	8.0	7.3	4.4	4.8	4.2	3.2	3.5
	0.5	30.8	20.8	16.8	20.1	13.5	12.1	12.8	7.8	7.7	7.8	5.0	5.1	4.9	3.5	3.7
	0.9	11.0	6.5	5.3	10.5	6.4	5.3	9.5	6.0	5.0	7.9	5.2	4.5	6.0	4.3	3.9

Tab. 5. Comparison of ARL (1) with normal, *t*, and gamma distributions for different values of between profiles autocorrelation under slope shift from σ to $\gamma\sigma$

		γ														
	φ	1.2			1.6			2.0			2.4			2.8		
		T2	EWMAR	EWMA3	T2	EWMAR	EWMA3	T2	EWMAR	EWMA3	T2	EWMAR	EWMA3	T2	EWMAR	EWMA3
Normal	0.0	39.2	38.0	31.6	7.9	6.7	7.0	3.8	3.0	3.9	2.5	2.0	2.8	2.0	1.6	2.2
	0.1	38.0	32.5	26.1	7.9	6.4	6.6	3.7	3.0	3.7	2.5	2.0	2.7	1.9	1.6	2.2
	0.5	19.2	11.3	8.5	6.0	4.3	4.4	3.4	2.5	3.1	2.4	1.9	2.4	1.9	1.5	2.1
	0.9	6.8	4.6	4.2	4.0	2.9	3.1	2.8	2.1	2.6	2.2	1.7	2.2	1.8	1.5	1.9
t	0.0	24.6	21.9	37.9	13.1	12.0	18.0	10.6	9.4	14.0	9.6	8.6	12.2	9.1	7.9	11.6
	0.1	24.2	20.6	33.0	13.2	11.6	16.4	10.7	9.0	12.8	9.6	8.3	11.3	9.1	7.8	10.7
	0.5	18.9	12.0	11.0	11.1	8.1	7.9	9.2	6.8	7.0	8.3	6.4	6.5	8.0	6.1	6.2
	0.9	8.4	5.4	4.8	6.5	4.5	4.2	5.8	4.2	4.0	5.5	4.0	3.8	5.3	3.9	3.8
Gamma	0.0	25.4	24.3	32.5	8.8	6.7	7.6	4.1	3.2	4.0	2.6	2.0	2.8	2.0	1.6	2.2
	0.1	25.8	21.9	27.2	8.8	6.5	7.1	4.0	3.1	3.9	2.6	2.0	2.7	2.0	1.6	2.2
	0.5	19.2	10.7	9.0	6.4	4.4	4.4	3.6	2.6	3.1	2.4	1.9	2.5	2.0	1.5	2.1
	0.9	7.2	4.7	4.2	4.2	3.0	3.2	2.9	2.2	2.5	2.2	1.8	2.2	1.9	1.5	2.0

Tab. 6. Comparison of ARL (1) with normal, *t*, and gamma distributions for different values of within profiles autocorrelation under *Y*-intercept shift from A_0 to $A_0 + \lambda\sigma$

		λ														
	φ	0.2			0.6			1.0			1.4			1.8		
		T2	EWMAR	EWMA3	T2	EWMAR	EWMA3	T2	EWMAR	EWMA3	T2	EWMAR	EWMA3	T2	EWMAR	EWMA3
Normal	0.0	139.2	51.5	66.9	27.9	7.3	8.4	6.9	3.6	4.0	2.6	2.5	2.7	1.5	2.0	2.1
	0.1	77.5	42.7	55.2	19.8	7.2	8.3	6.0	3.6	4.0	2.5	2.5	2.7	1.5	2.0	2.1
	0.5	12.3	18.5	22.2	7.1	6.7	7.5	3.8	3.7	4.0	2.3	2.6	2.8	1.6	2.0	2.2
	0.9	4.2	8.2	9.5	3.6	5.4	6.1	2.7	3.6	4.0	2.1	2.7	2.9	1.7	2.1	2.2
t	0.0	53.2	33.7	58.2	23.4	7.0	8.4	7.6	3.5	4.0	2.7	2.4	2.7	1.4	2.0	2.1
	0.1	39.9	30.2	50.3	18.2	6.9	8.2	6.4	3.6	4.0	2.6	2.5	2.7	1.5	2.0	2.1
	0.5	12.0	17.2	22.5	7.5	6.5	7.5	4.0	3.6	4.0	2.4	2.6	2.8	1.6	2.0	2.1
	0.9	4.7	8.3	10.0	3.8	5.5	6.2	2.9	3.6	4.0	2.2	2.6	2.9	1.7	2.1	2.2
Gamma	0.0	33.8	33.7	52.5	14.6	7.4	8.8	6.3	3.6	4.0	2.9	2.5	2.7	1.6	2.0	2.1
	0.1	27.0	30.6	45.2	12.4	7.5	8.5	5.8	3.7	4.1	2.9	2.5	2.7	1.6	2.0	2.1
	0.5	11.6	18.7	23.3	6.9	7.1	7.9	4.1	3.8	4.2	2.6	2.6	2.8	1.8	2.1	2.2
	0.9	5.1	9.1	10.9	4.5	6.3	7.1	3.4	4.0	4.4	2.5	2.8	3.0	1.9	2.2	2.3

Tab. 7. Comparison of ARL (1) with normal, *t*, and gamma distributions for different values of within profiles autocorrelation under slope shift from A_1 to $A_1 + \beta\sigma$

		β														
	φ	0.025			0.075			0.125			0.175			0.225		
		T2	EWMAR	EWMA3	T2	EWMAR	EWMA3	T2	EWMAR	EWMA3	T2	EWMAR	EWMA3	T2	EWMAR	EWMA3
Normal	0.0	168.0	97.4	111.9	60.7	16.2	18.8	19.9	6.8	7.6	7.8	4.2	4.6	3.7	3.1	3.4
	0.1	90.6	73.5	92.7	38.5	15.2	17.5	15.0	6.8	7.4	6.6	4.3	4.6	3.5	3.1	3.4
	0.5	13.3	22.4	27.5	9.5	11.1	12.5	6.1	6.4	6.8	4.0	4.3	4.6	2.8	3.2	3.5
	0.9	4.2	8.5	10.0	4.0	6.9	8.0	3.4	5.4	5.9	2.8	4.0	4.5	2.4	3.3	3.5
t	0.0	55.8	45.6	84.6	36.8	14.4	18.7	19.3	6.5	7.6	8.9	4.1	4.6	4.2	3.0	3.4
	0.1	41.7	42.6	75.6	28.6	13.7	17.5	14.9	6.5	7.5	7.3	4.1	4.7	3.7	3.1	3.4
	0.5	12.4	20.7	27.5	9.6	10.8	12.6	6.5	6.2	6.9	4.2	4.2	4.7	2.9	3.2	3.5
	0.9	4.8	8.9	10.4	4.3	7.1	8.3	3.7	5.3	6.0	3.0	4.1	4.4	2.4	3.3	3.5
Gamma	0.0	38.2	43.9	73.4	22.5	15.3	19.2	12.7	7.0	8.1	7.1	4.3	4.8	4.1	3.1	3.5
	0.1	30.7	42.0	66.7	18.3	14.7	17.9	10.8	7.0	7.7	6.3	4.4	4.8	3.8	3.2	3.5
	0.5	12.5	21.9	28.7	8.7	11.6	13.4	6.3	6.6	7.3	4.4	4.5	4.9	3.2	3.3	3.6
	0.9	5.1	9.1	11.1	4.9	8.1	9.3	4.2	6.1	6.6	3.5	4.5	4.9	2.9	3.5	3.8

Tab. 8. Comparison of ARL (1) with normal, *t*, and gamma distributions for different values of within profiles autocorrelation under slope shift from σ to $\gamma\sigma$

		γ														
	φ	1.2			1.6			2.0			2.4			2.8		
		T2	EWMAR	EWMA3	T2	EWMAR	EWMA3	T2	EWMAR	EWMA3	T2	EWMAR	EWMA3	T2	EWMAR	EWMA3
Normal	0.0	39.2	38.0	31.6	7.9	6.7	7.0	3.8	3.0	3.9	2.5	2.0	2.8	2.0	1.6	2.2
	0.1	26.7	35.7	36.2	6.4	6.9	7.6	3.3	3.1	4.0	2.3	2.0	2.8	1.8	1.6	2.3
	0.5	6.4	13.8	16.4	3.0	5.3	6.9	2.0	2.9	3.9	1.6	2.0	2.7	1.5	1.6	2.1
	0.9	3.0	5.8	6.8	2.0	3.3	3.9	1.6	2.2	2.8	1.4	1.7	2.1	1.3	1.5	1.8
t	0.0	24.6	21.9	37.9	13.1	12.0	18.0	10.6	9.4	14.0	9.6	8.6	12.2	9.1	7.9	11.6
	0.1	19.7	22.0	39.8	11.5	11.9	19.2	9.0	9.8	14.6	8.4	8.7	12.9	7.9	8.3	11.9
	0.5	8.1	13.9	19.2	5.7	9.2	12.3	5.0	7.6	10.3	4.7	7.0	9.3	4.6	6.7	8.9
	0.9	3.8	6.9	8.3	3.2	5.2	6.4	3.0	4.8	5.7	2.9	4.5	5.5	2.8	4.4	5.2
Gamma	0.0	25.4	24.3	32.5	8.8	6.7	7.6	4.1	3.2	4.0	2.6	2.0	2.8	2.0	1.6	2.2
	0.1	20.7	24.3	36.2	7.1	7.0	8.2	3.4	3.3	4.2	2.3	2.1	2.9	1.9	1.6	2.3
	0.5	7.6	12.5	17.1	3.2	5.4	6.9	2.1	3.0	3.9	1.7	2.0	2.7	1.4	1.6	2.1
	0.9	3.2	5.9	7.1	2.0	3.3	4.0	1.6	2.3	2.8	1.4	1.8	2.2	1.3	1.5	1.8

5. Conclusions

In this paper, simultaneous effect of non-normality and autocorrelation on the performance of three common methods proposed in the literature for linear profile monitoring was investigated. Student *t* and gamma distributions under the assumption of within and between profiles autocorrelation were considered to perform the analyses. In-control and out-of-control

ARL results show that violation of these common assumptions affects the performance of T^2, EWMA/R, and EWMA3 approaches and could lead to serious misjudgment of process status by operators or process engineer. Among the methods considered in our study, EWMA3 is relatively less sensitive to the violation of the assumptions than the other schemes.

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