

# Fractional Chance Constrained Programming: A Fuzzy Goal Programming Approach

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## ABSTRACT

*There are many cases that a nonlinear fractional programming, generated as a result of studying fractional stochastic programming, must be solved. Sometimes an approximate solution may be sufficient enough to start a new process of calculations. To this end, this author introduces a new linear approximation technique for solving a fractional chance constrained programming (CCP) problem. After introducing the problem, the equivalent deterministic form of the fractional nonlinear programming problem is developed. To solve the problem, a fuzzy goal programming model of the equivalent deterministic form of the fractional chance constrained programming is provided and then the process of defuzzification and linearization of the problem is started. A sample test problem is solved for presentation purposes.*

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## 1. Introduction

Under some circumstances the measure to be used by researcher is the division of one function of variables to another function where one or both of these functions can be linear or nonlinear. Data Envelopment Analysis (DEA) is sample situation where so far hundreds of applications of that had appeared in the literature. This is why we can say that fractional programming has attracted the attention of some researchers during past four decades. In this regard Saad (2007) indicated that: "the main reason for interest in fractional programming stems from the fact that linear fractional objective functions occur frequently as measures of performance in a variety of circumstances such as when satisfying objectives under uncertainty".

Lara and Stancu-Minasian [28] have reviewed fractional programming as a tool for studying the sustainability of agricultural systems where the

essentials of technique in both the single and the multi-objective cases are outlined as well.

Authors pointed to this reality that algorithms embedded in the programming packages for solving the models are not friendly and this shortcomings need to be overcome, however. Two procedures for avoiding this shortcoming in the multiple objective cases are discussed. Publication of five bibliographies complied by Stancu-Minasian (1999) reflect this reality that a large number of theoretical as well as algorithmic work have been done by many researchers over the years. As Lara and Stancu-Minasian [28] mentioned in their work, although output/input ratios arise naturally in many economic problems very few real applications of fractional programming have been reported, particularly in the field of agriculture. Perhaps, the lack of friendly procedures for solving the models is one of the main reasons. However, a number of fractional programming applications can be seen in the work of (Stancu-Minasian's, 1997). Lara (1993) reported an application in the field of livestock ration formulation.

There are many situations in which business deal with the linear fractional programming problem. In this regard, Steuer (1986) says that the mathematical

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optimization problems with a goal function that is a ratio with a linear numerator and a linear denominator have many applications: in finance (corporate planning, bank balance sheet management), in marine transportation, water resources, health care and so forth. Considering linear functions  $p(x)$  and  $q(x)$  and the optimization problem

$$\text{Max } f(x) = \frac{p(x)}{q(x)}$$

s.t.

$$S = \{x \mid Ax = b, x \geq 0\}$$

where  $S$  is assumed to be a nonempty bounded polyhedron. To solve this problem many authors as such as Charnes and Cooper (1962), Martos (1975), Wolf (1985) have conducted research on this problem and proposed different algorithm for different form and shape of the problem. Comparative investigations of such algorithms can be found in Arsham and Kahn (1990) and Bhatt (1989).

In their book, *Nonlinear Programming, Theory and Algorithms*, Bazaraa and Shetty (1979) have shown that the fractional type objective function shown above has several important properties - it is (simultaneously): pseudo convex, pseudo concave, quasi-convex, quasi-concave, strict quasi-convex and strict quasi-concave. This means that the point that satisfies the Kuhn-Tucker conditions for the maximization problem gives the global maximum on the feasible set. In addition, each local maximum is also a global maximum. This maximum is obtained at an extreme point of  $S$  (Metev and Gueorguieva [23]). Chadha [4] proposed a procedure for solving a fractional programming problem with absolute value functions.

$$\begin{aligned} \text{maximize } z &= \frac{\sum_1^n c_j |x_j| + \alpha}{\sum_1^n d_j |x_j| + \beta} \\ \text{subject to } Ax &= b, \end{aligned}$$

where  $S$  is defined as follows:

$$S = \left[ x : Ax = \sum_1^n A_j x_j = b \right]$$

Later Chang [3] proposed a fuzzy goal programming approach for solving fractional programming with absolute-value functions. The problem of fractional

chance constrained programming has not been studied with a structure defined by this author in this article. Metev and Gueorguieva [23] have discussed about a simple method for obtaining weakly efficient points in multi objective linear fractional programming problem. Authors show that the property of strict quasi-convexity allows to use successfully the reference point method for the analysis of MOLFP problems. Omar M. Saad [24] has proposed a solution algorithm to fuzzy multi objective fractional programming problems where fuzzy parameters are considered in the right-hand side of the constraints. Furthermore, the concept of  $\alpha$ -level set of a fuzzy number has been employed by the authors for the difuzzification purposes.

A solution algorithm has been proposed by Saad and Abd-Rabo [25] for solving integer linear fractional programs where right-hand side constraints are considered to be random variables. Saad and Sharif developed a solution method for solving integer linear fractional programming problems with chance constraints assuming the independency of random parameters involved in their model building [26].

Masatoshi Sakawa and Kosuke Kato (1998) conducted a research on the interactive decision-making for multi objective linear fractional programming problems with block angular structure involving fuzzy numbers. A multi objective linear fractional programming (MOLFP) problem with the block angular structure is formulated as

$$\begin{aligned} &\text{minimize } z_1(x, c_1, d_1) \\ &\quad \vdots \\ &\text{minimize } z_k(x, c_k, d_k), \\ &\text{subject to } A_1 x_1 + \cdots + A_p x_p \leq b_0, \\ &\quad B_1 x_1 \leq b_1, \\ &\quad \quad \quad \vdots \\ &\quad \quad \quad B_p x_p \leq b_p, \\ &\quad x_j \geq 0, \quad j = 1, \dots, p, \end{aligned}$$

Where each of these objective functions is as defined below. More details on the definition of parameters used and the variables of the problem can be obtained from Masatoshi Sakawa and Kosuke Kato (1998). Through the use of the  $\alpha$ -level sets of fuzzy numbers, an extended Pareto optimality concept called the  $\alpha$ -Pareto optimality is introduced.

$$z_i(x, c_i, d_i) = \frac{p_i(x, c_i)}{q_i(x, d_i)} = \frac{c_{i1}x_1 + \cdots + c_{ip}x_p + c_{i,p+1}}{d_{i1}x_1 + \cdots + d_{ip}x_p + d_{i,p+1}}$$

Here, the basic idea is to use fuzzy goal programming (GP) as a tool for solving the fractional stochastic problem. To do that, first we define a fuzzy goal programming for the stochastic programming problem and then we apply the concept of defuzzification to convert the fuzzy model into a model that is not fuzzy. This is because there is no solution procedure available for fuzzy models. Hence, we are in need of developing an equivalent crisp model of the proposed fuzzy system. Linear goal programming was originally introduced by Abraham Charnes and William Cooper [17] in early 1961. One can solve a GP model either regularly or interactively. Goal programming and interactive goal programming [14, 15, 16] are chosen to solve multi criterion programming problems of various types for following primary reasons:

1. Computationally efficient and ease of modeling
2. Concepts can be easily communicated with the decision makers
3. It is flexible enough to address problems in a MCDM form.

The main difference between fuzzy goal programming (FGP) and goal programming (GP) is that the GP requires the DM to set definite aspiration values for each goal while in the FGP these are specified in an imprecise manner [9].

The remainder of this article is organized as follow: model development is discussed in section 2. Membership function is defined in section 3. Fuzzy goal programming modeling is discussed in section 4. Linearization technique is discussed in section 5. Compromise goal constraint is the topic of section 6. The overall linearization model is discussed in section 7. An example problem is discussed in section 8. Our conclusion is given in section 9.

## 2. Model Development

The event of a constraint violation must be regarded as a risk taking issue. The degree of constraint violation, shown by  $(1 - \alpha)$ , is called the risk level with  $\alpha$  referring to the constraint reliability. The input factors play a significant role in deteriorating systems reliability by violating one or more constraints. For instance, the required work force level for the manufacturing of a product depends upon the sufficiency of raw materials, demand fluctuations, market saturation and inflation rates. One well defined methodology for treating such problems with probabilistic constraints is known as Chance Constraint Programming (CCP). The concept was introduced into the literature of Stochastic programming mainly through the exposition of Charnes and Cooper [2] and since then developed and applied by Kataoka [13], Sengupta [20, 21], and Zare Mehrjerdi [12, 15, 16, 18, 22], to mention a few.

When one or more parameters are presumed to be random variable with known distribution function, then a variant of stochastic programming known as chance constrained Programming (CCP) can be used to solve the problem. On the other hand, modeling under uncertainty for dealing with uncertain parameters may be employed. The approach of CCP has shown an operational way for introducing probabilistic constraints into the collection of the LP problem constraints.

The resemblance between the GP and LP indicates that CCP can similarly apply to the MOGP model for identifying the trade-off between the risk and objective attainments. The broad application of MOGP problem and CCP demonstrates the significant and immediate role of these programming models in the analysis of real world problems. These classes of mathematical programs are accepted by the risk taking managers and also examined by a large group of researchers. Peters et al [19] employed the concept of recourse actions and chance constraints in the model concerning the water release and distribution problem of the Karun River and its tributaries in Khuzestan, Iran. This author utilized the concept of CCP to develop a MOGP model for water resources systems [18].

In the model developed by this author the following notations are used:

$X_j$  = The  $j$ th decision variable

$C_{1j}$  = A random variable with known distribution functions such that  $C_{1j} \sim N(C_j^-, \sigma_j^2)$

$C_{2j}$  = A random variable with known distribution functions such that  $C_{2j} \sim N(D_j^-, \phi_j^2)$

$F_1$  = Lower bound for probabilistic constraint 2

$F_2$  = Lower bound for probabilistic constraint 3

$a_{ij}$  = Technological coefficients

$\alpha$  = The probability that the probabilistic constraint (related to numerator) would not hold

$\beta$  = The probability that the probabilistic constraint (related to denominator) would not hold

$b_i$  = The level of the  $i^{\text{th}}$  resource

$$q_1 = N^{-1}(\alpha)$$

$$q_2 = N^{-1}(\beta)$$

$$u_{1j} = \sigma_j^2$$

$$u_{2j} = \phi_j^2$$

The remaining of the notations used in this model building is defined as the process of development progresses.

The general format of the fractional chance constrained programming problem employed in this research is as shown below:

$$\text{P1: Maximize } Z(X) = \frac{F_1 + r}{F_2 + s} \quad (1)$$

S.t.

$$P\left\{\sum_{j=1}^n C_{1j} X_j \geq F_1\right\} \geq \alpha \quad (2)$$

$$P\left\{\sum_{j=1}^n C_{2j} X_j \geq F_2\right\} \geq \beta \quad (3)$$

$$\sum_{j=1}^n a_{ij} X_j \leq b_i \quad (4)$$

$$X_j \geq 0$$

Where  $i=1, \dots, m$  and  $j=1, \dots, n$ . A major difficulty in using CCP when input-output coefficients and/or cost vectors are random variables having a known distribution functions is the need for a nonlinear computer program. When technological coefficients are independent normally distributed random variables then the EDF of problem P1 can be shown as:

P2: Max

$$\mu(X) = \begin{cases} 1 & \text{if } Z \geq u^- \\ \frac{Z - l^-}{u^- - l^-} & \text{if } l^- \leq Z \leq u^- \\ 0 & \text{if } Z \leq l^- \end{cases} \quad (9)$$

#### 4. Fuzzy Goal Programming Modeling

We can write the following goal programming model for the membership function as shown in [10]:

$$\text{Min } d^- + d^+$$

S.t.

$$\frac{Z - l^-}{u^- - l^-} + d^- - d^+ = 1 \quad (10)$$

$$L\left\{\sum_{j=1}^n C_{1j} X_j - q_1(X'VX)^{1/2} + r\right\} / \left\{\sum_{j=1}^n C_{2j} X_j - q_2(X'WX)^{1/2} + s\right\} - Ll^- + d^- - d^+ = 1 \quad (13)$$

Or

$$\frac{F_1(X) + r}{F_2(X) + s} = \frac{\sum_{j=1}^n C_{1j} X_j - q_1(X'VX)^{1/2} + r}{\sum_{j=1}^n C_{2j} X_j - q_2(X'WX)^{1/2} + s} \quad (5)$$

S.t.

$$\sum_{j=1}^n a_{ij} X_j \leq b_i \quad (6)$$

$$X_j \geq 0 \quad (7)$$

Without loss of generality we can assume that the denominator of problem P2 is greater than zero. Let us assume that the decision maker is able to guess an upper and lower value for the value of  $Z$ , and, on the basis of that bound he expects a reasonable solution to be determined. The upper and lower bounds of  $Z$  can be defined as below:

$$l^- \leq Z \leq u^- \quad (8)$$

#### 3. Membership Function

We can identify the membership function  $\mu$  as shown below:

$$d^- \geq 0$$

$$d^+ \geq 0$$

Let us assume that

$$L = \frac{1}{u^- - l^-} \quad (11)$$

We can write goal constraint (10) as follows:

$$LZ - Ll^- + d^- - d^+ = 1 \quad (12)$$

Now, substitute (5) into (12)

$$L\left\{\sum_{j=1}^n C_{1j} X_j - q_1(X'VX)^{1/2} + r\right\} - L\left\{\sum_{j=1}^n C_{2j} X_j - q_2(X'WX)^{1/2} + s\right\} + d^- \left\{\sum_{j=1}^n C_{2j} X_j - q_2(X'WX)^{1/2} + s\right\} - d^+ \left\{\sum_{j=1}^n C_{2j} X_j - q_2(X'WX)^{1/2} + s\right\} = \left\{\sum_{j=1}^n C_{2j} X_j - q_2(X'WX)^{1/2} + s\right\} \quad (14)$$

$$L\left\{\sum_{j=1}^n C_{1j} X_j - q_1(X'VX)^{1/2} + r\right\} + d^- \left\{\sum_{j=1}^n C_{2j} X_j - q_2(X'WX)^{1/2} + s\right\} - d^+ \left\{\sum_{j=1}^n C_{2j} X_j - q_2(X'WX)^{1/2} + s\right\} = L^0 \left\{\sum_{j=1}^n C_{2j} X_j - q_2(X'WX)^{1/2} + s\right\} \quad (15)$$

Where

$$L^0 = 1 + Ld^- \quad (16) \quad D^+ = d^+ \left\{\sum_{j=1}^n C_{2j} X_j - q_2(X'WX)^{1/2} + s\right\} \quad (18)$$

Now, let

$$D^- = d^- \left\{\sum_{j=1}^n C_{2j} X_j - q_2(X'WX)^{1/2} + s\right\} \quad (17) \quad \text{Therefore, we can write formula (15) as follows:}$$

$$L\left\{\sum_{j=1}^n C_{1j} X_j - q_1(X'VX)^{1/2} + r\right\} + D^- - D^+ = L^0 \left\{\sum_{j=1}^n C_{2j} X_j - q_2(X'WX)^{1/2} + s\right\} \quad (19)$$

Where  $D^-, D^+ \geq 0$  and  $D^-.D^+ = 0$  since  $d^-$  and  $d^+ \geq 0$  and

$$\sum_{j=1}^n C_{2j} X_j - q_2(X'WX)^{1/2} + s > 0 \quad (20)$$

When a membership goal is fully achieved,  $d^- = 0$  (i.e.,  $\mu = 1$ ) and when it is zero achieved, we have  $d^- = 1$  (i.e.,  $\mu = 0$ ). This leads to the following constraints to the model of the problem.

$$\frac{D^-}{\sum_{j=1}^n C_{2j} X_j - q_2(X'WX)^{1/2} + s} \leq 1 \quad (21)$$

Now, (21) can be written as the one shown below:

$$\sum_{j=1}^n C_{2j} X_j - q_2(X'WX)^{1/2} + s \geq D^- \quad (22)$$

## 5. Linearization Technique

Let us assume that  $C_{1j} \sim N(C_{1j}^-, \sigma_j^2)$ ,  $C_{2j} \sim N(C_{2j}^-, \varphi_j^2)$  and the variance-covariance matrix of V and W, when coefficients are independent normally distributed random variables, are as defined below:

$$V = \begin{pmatrix} \sigma_1^2 & & & \\ & \ddots & & \\ & & \ddots & \\ & & & \sigma_n^2 \end{pmatrix} \quad W = \begin{pmatrix} \varphi_1^2 & & & \\ & \ddots & & \\ & & \ddots & \\ & & & \varphi_n^2 \end{pmatrix}$$

Since we know that

$$(a + b)^{1/2} \leq a^{1/2} + b^{1/2} \quad (23)$$

Therefore,

$$(X'VX)^{1/2} < \sum_{j=1}^n u_{1j}^{1/2} X_j \quad (24)$$

$$(X'WX)^{1/2} < \sum_{j=1}^n u_{2j}^{1/2} X_j \quad (25)$$

Where  $u_{1j} = \sigma_j^2$  and  $u_{2j} = \varphi_j^2$  for all  $j=1,2,\dots, n$ . Therefore, we have

$$\sum_{j=1}^n C_{1j}^- X_j - q_1 \sum_{j=1}^n u_{1j}^{1/2} X_j < F_1(X) < \sum_{j=1}^n C_{1j}^- X_j + q_1 \sum_{j=1}^n u_{1j}^{1/2} X_j \quad (26)$$

$$\sum_{j=1}^n C_{2j}^- X_j - q_2 \sum_{j=1}^n u_{2j}^{1/2} X_j < F_2(X) < \sum_{j=1}^n C_{2j}^- X_j + q_2 \sum_{j=1}^n u_{2j}^{1/2} X_j \quad (27)$$

Since

$$F_1(X) < \sum_{j=1}^n C_{1j}^- X_j \quad (28)$$

$$F_2(X) < \sum_{j=1}^n C_{2j}^- X_j \quad (29)$$

$$f_1(X) = \sum_{j=1}^n C_{1j}^- X_j \quad (30)$$

$$f_2(X) = \sum_{j=1}^n C_{1j}^- X_j - q_1 \sum_{j=1}^n u_{1j}^{1/2} X_j \quad (31)$$

$$f_3(X) = \sum_{j=1}^n C_{2j}^- X_j \quad (32)$$

$$f_4(X) = \sum_{j=1}^n C_{2j}^- X_j - q_2 \sum_{j=1}^n u_{2j}^{1/2} X_j \quad (33)$$

We will use the following new definitions:

Therefore we can write (19) as shown below:

$$L\left\{\sum_{j=1}^n C_{1j}^- X_j - q_1 \sum_{j=1}^n u_{1j}^{1/2} X_j + r\right\} + D^- - D^+ < L\left\{\sum_{j=1}^n C_{1j}^- X_j - q_1 (X'VX)^{1/2} + r\right\} + D^- - D^+ = L^0\left\{\sum_{j=1}^n C_{2j}^- X_j - q_2 (X'WX)^{1/2} + s\right\} < L^0\left\{\sum_{j=1}^n C_{2j}^- X_j + s\right\} \quad (34)$$

Therefore,

$$L\left\{\sum_{j=1}^n (C_{1j}^- - q_1 u_{1j}^{1/2}) X_j + r\right\} + D^- - D^+ < L^0\left\{\sum_{j=1}^n C_{2j}^- X_j + s\right\} \quad (35)$$

or

$$\sum_{j=1}^n [L(C_{1j}^- - q_1 u_{1j}^{1/2}) - L^0 C_{2j}^-] X_j + D^- - D^+ < L^0 s - Lr \quad (36)$$

Now, we can linearize (22) in a similar fashion.

Since

$$\sum_{j=1}^n C_{2j}^- X_j - q_2 (X'WX)^{1/2} \leq \sum_{j=1}^n C_{2j}^- X_j \quad (37)$$

Therefore, we have

$$D^- < \sum_{j=1}^n C_{2j}^- X_j - q_2 (X'WX)^{1/2} + s \leq \sum_{j=1}^n C_{2j}^- X_j + s \quad (38)$$

Or

$$-\sum_{j=1}^n C_{2j}^- X_j + D^- \leq s \quad (39)$$

## 6. Compromise Goal Constraints

A goal constraint incorporating the optimum value of the upper and lower bound functions of the numerator and denominator of  $F_1(X)$  and  $F_2(X)$  respectively are also of tremendous value for problem solving.

$$H_1(X) = \frac{w_1}{\sqrt{\sum_{j=1}^n C_{2j}^2}} \left( \sum_{j=1}^n C_{2j} X_j - f_2^* \right) - \frac{w_2}{\sqrt{\sum_{j=1}^n C_{1j}^2}} \left[ \sum_{j=1}^n C_{1j} X_j - f_1^* \right] + d_1^- - d_1^+ = 0 \quad (40)$$

and

$$H_2(X) = \frac{v_1}{\sqrt{\sum_{j=1}^n C_{4j}^2}} \left( \sum_{j=1}^n C_{4j} X_j - f_4^* \right) - \frac{v_2}{\sqrt{\sum_{j=1}^n C_{3j}^2}} \left[ \sum_{j=1}^n C_{3j} X_j - f_3^* \right] + d_2^- - d_2^+ = 0 \quad (41)$$

where following inequalities would hold:

$$f_4^* \leq F_2^* \leq f_3^* \quad (45)$$

$$f_2(X) \leq F_1(X) \leq f_1(X) \quad (42)$$

$$f_4(X) \leq F_1(X) \leq f_3(X) \quad (43)$$

$$f_2^* \leq F_1^* \leq f_1^* \quad (44)$$

where  $f_1^*$  and  $f_2^*$  represent the optimum values of the objective functions of the LP problems of  $P_4$  and  $P_5$ , respectively. It should be noted that  $F_1^*$  is the optimal value of  $F_1(X)$  over the defined feasible region of  $S$ . However,  $P_4$  and  $P_5$  are defined below:

$$P_4: f_1^* = \{ \text{Maximize } f_1(X) = \sum_{j=1}^n \bar{C}_{1j} X_j \mid \sum_{j=1}^n a_{ij} X_j \leq b_i, X_j \geq 0 \} \quad (46)$$

$$P_5: f_2^* = \{ \text{Maximize } f_2(X) = \sum_{j=1}^n \bar{C}_{1j} X_j - q_1 \sum_{j=1}^n u_{1j}^{1/2} X_j \mid \sum_{j=1}^n a_{ij} X_j \leq b_i, X_j \geq 0 \} \quad (47)$$

In a similar fashion, we can introduce problems  $P_6$  and  $P_7$  are defined as below:

$$P_6: f_3^* = \{ \text{Maximize } f_3(X) = -\{ \sum_{j=1}^n C_{2j}^- X_j \} \mid \sum_{j=1}^n a_{ij} X_j \leq b_i, X_j \geq 0 \} \quad (48)$$

$$P_7: f_4^* = \{ \text{Maximize } f_4(X) = -\{ \sum_{j=1}^n C_{2j}^- X_j - q_2 \sum_{j=1}^n u_{2j}^{1/2} X_j \} \mid \sum_{j=1}^n a_{ij} X_j \leq b_i, X_j \geq 0 \} \quad (49)$$

## 7. The Overall Linearization Model

The overall linearization model of the problem is as shown below:

$$\sum_{j=1}^n [L(C_{1j}^- - q_1 u_{1j}^{1/2}) - L^0 C_{2j}^-] X_j + D^- - D^+ < L^0 s - Lr \quad (51)$$

**Minimize:**

$$D^- + D^+ + d_1^- + d_1^+ + d_2^- + d_2^+ \quad (50)$$

$$-\sum_{j=1}^n C_{2j}^- X_j + D^- \leq s \quad (52)$$

Subject to:

$$H_1(X) = \frac{w_1}{\sqrt{\sum_{j=1}^n C_{2j}^2}} (\sum_{j=1}^n C_{2j}^- X_j - f_2^*) - \frac{w_2}{\sqrt{\sum_{j=1}^n C_{1j}^2}} (\sum_{j=1}^n C_{1j} X_j - f_1^*) + d_1^- - d_1^+ = 0 \quad (53)$$

$$H_2(X) = \frac{v_1}{\sqrt{\sum_{j=1}^n C_{4j}^2}} (\sum_{j=1}^n C_{4j} X_j - f_4^*) - \frac{v_2}{\sqrt{\sum_{j=1}^n C_{3j}^2}} (\sum_{j=1}^n C_{3j} X_j - f_3^*) + d_2^- - d_2^+ = 0 \quad (54)$$

$$\sum_{j=1}^n a_{ij} X_j \leq b_i \quad (55)$$

$$X_j \geq 0 \quad (56)$$

S.t

$$2X_1 + X_2 \leq 18$$

$$X_1 + 2X_2 \leq 16$$

$$X_1, X_2 \geq 0$$

## 8. Example

Max

$$Z(X) = \frac{F_1(X) + 5}{F_2(X) + 10} = \frac{8X_1 + 7X_2 - 2.33(9X_1^2 + 4X_2^2)^{1/2} + 5}{20X_1 + 12X_2 - 2.33(3X_1^2 + 2X_1X_2 + 4X_2^2)^{1/2} + 10}$$

Let us assume that the value of  $Z$  is requested to be as follows:

$$1.5 \leq Z \leq 2.5$$

Considering the above information we can set up following information:

$$l^- = 1.5$$

$$u^- = 2.5$$

$$L = \frac{1}{u^- - l^-} = \frac{1}{2.5 - 1.5} = 1$$

$$L^0 = 1 + Ll^- = 1 + 1 * 1.5 = 2.5$$

r=5, and s=10

$$L^0 s - Lr = (2.5)(10) - (1)(5) = 20$$

To generate goal constraint (51) we need to calculate factors  $\{L(C_{1j}^- - q_1 u_{1j}^{1/2}) - L^0 C_{2j}^-\}$  for

j=1 and 2 in this case of dealing with two decision variables X1 and X2. The value of these factors for j=1 and j=2 are 51.01 and 32.34, respectively. Having such

information at hand we can set up goal constraint (51) and (52) as shown below:

$$51.01 X_1 + 32.34 X_2 + D^- - D^+ < 20$$

$$-20X_1 - 12X_2 + D^- \leq 10$$

The upper and lower bound functions of  $f_1(X)$  and  $f_2(X)$  for the numerator are as given below:

$$f_1(X) = 8X_1 + 7X_2$$

$$f_2(x) = 1.01X_1 + 2.34X_2$$

For the second function, the denominator, the upper and lower bound functions are as shown below:

$$f_3(X) = 20X_1 + 12X_2$$

$$f_4(X) = 15.96432X_1 + 7.34X_2$$

The following table gives optimum solution points and optimum values for upper and lower bound functions shown above

**Tab. 1. The optimum values of upper and lower bound functions**

	Upper function $f_1(X)$ (6.6667, 4.6667)	Lower function $f_2(X)$ (0, 8)	Upper function $f_3(X)$ (6.6667, 4.6667)	Lower function $f_4(X)$ (9, 0)
Optimum Solution Point				
Optimum Value	86	18.72	189	143.6789

To generate the compromise goal constraints  $H_1(X)$  and  $H_2(X)$  let assume that  $w_1=w_2=0.5$  and  $v_1=v_2=0.5$ . With the information available to us we can set up compromise goal constraints as shown below:

$$H_1(X) = 0.17815X_1 - 0.129811X_2 = 0.372591$$

and

$$H_2(X) = 0.025538X_1 - 0.04838X_2 = 0.0369$$

Taking all these information into consideration we can set up the following linearization goal programming model:

**Minimize:**

$$D^- + D^+ + d_1^- + d_1^+ + d_2^- + d_2^+$$

Subject to:

$$51.01 X_1 + 32.34 X_2 - D^- - D^+ < 20$$

$$-20X_1 - 12X_2 + D^- \leq 10$$

$$H_1(X) = 0.17815X_1 - 0.129811X_2 + d_1^- - d_1^+ = 0.372591$$

$$H_2(X) = 0.025538X_1 - 0.04838X_2 + d_2^- - d_2^+ = 0.0369$$

$$2X_1 + X_2 \leq 18$$

$$X_1 + 2X_2 \leq 16$$

$$X_1, X_2 \geq 0$$

By solving the above problem by LINDO we obtain  $X_1=0.392080$  and  $X_2=0$ . Then, after substituting the optimum solution point of the above model into the fractional function we have

$Z = \frac{(0.396001+5)/(6.259291+10)}{5.396001/16.25929} = 0.31872$ . We expect to see the value of Z in the range of (1.5, 2.5). But, it is not. The reasons why it is not happening are listed below:

- We are using a linear approximation all over the model
- The final model is a linear parametric goal programming in terms of parameters  $w_1$  and  $v_1$
- We added compromise constraints  $H_1(X)$  and  $H_2(X)$

All these together causing that the final solution to be deteriorated and stay away from the expected solution and the range that we expect that to belong to.



## 9. Conclusion

In this paper, we have introduced a new linearization technique for solving fractional chance constrained programming by employing the theory of fuzzy set and applying the fuzzy goal programming concept. The paper contributes to both fields of fractional programming and chance constrained programming giving an unlimited power to these programming tools and providing a new approximation methodology for finding a near approximate solution for a complex nonlinear type problem. The linearization goal programming model can be solved easily by many commercialized optimization computer packages.

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