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Heuristic Method to Solve Capacitated Location-Routing Problem with Fuzzy Demands

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ABSTRACT

In this paper, the capacitated location routing problem with fuzzy demands (CLRP-FD) is considered. In CLRP-FD, facility location problem (FLP) and vehicle routing problem (VRP) are observed simultaneously. Indeed, the vehicles and the depots have a predefined capacity to serve the customers that have fuzzy demands. To model the CLRP-FD, a fuzzy chance constrained programming is designed, based on fuzzy credibility theory. To solve the CLRP-FD, a greedy clustering method (GCM) including the stochastic simulation is proposed. To get the best value of the preference index of the model and analysis its influence on the final solution of the problem, numerical experiments are carried out. Finally, to show the performance of the model, computational experiments are performed on a real case in Ardakan.

1. Introduction and Literature Review [1]

Ever increasing demand of customers for less waiting time to receive their desired products, and competitive prices between the producers, make the logistics as the main problem in supply chain management. In recent years, the efficient, reliable, and flexible decisions on location of depots and the distribution routings are of vital importance for managers (Nadizadeh, et al. [1]). Many researchers indicated that if the routes are ignored while locating the depots, the costs of distribution systems might be immoderate (Prins, et al. [2]). The location-routing problem (LRP) overcomes this disadvantage by simultaneously considering the location and routing decisions (Barreto, et al. [3]). The LRP is defined as a facility location problem (FLP) that solves the vehicle routing problem (VRP), simultaneously (Stenger, et al. [4]; Escobar, et al. [5]). More on vehicle routing problem and stochastic vehicle routing problem can be seen from the work of Zare-Mehrjerdi [6], Mehrjerdi [7] and Mehrjerdi [8].

LRP is applicable to a wide variety of fields such as food and drink distribution, newspapers delivery, waste collection, bill delivery, military applications, parcel delivery and various consumer goods distribution (Manzour-al-Ajdad, et al. [9]; Ting and Chen [10]). In capacitated LRP (CLRP), the problem is constrained with the vehicles and the depot(s) capacities to supply the customers. Furthermore, the customers have to only be supplied by a single vehicle; in the other

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words the vehicle meets every customer in a tour, once. A homogenous fleet of vehicles transports the products from the depots to the customers and return there as soon as finishing the entire tour. Moreover, the capacity of each potential depot and vehicle are predefined. The objectives in CLRP are to determine the location of depots, and a set of customers to be served by each depot as well as the distribution routes. (Bouhafs and Koukam [11] and Contardo, et al. [12]). The CLRP is an NP-hard problem, so some approximating heuristic algorithms had been developed to solve it (Marinakis and Marinaki [13], Barreto, et al. [3], Jabal-Ameli, et al. [14]). In this kind of problems, the solution times increase exponentially as with an increase in the size of the problem, while an exact algorithm is applied to solve them. For this reason, most of papers in the field of CLRP are focused on only new solution methods that are often based on heuristic or meta-heuristic approaches. Some reviews on solution methods of CLRP exist in the literature that can be found in Nagy and Salhi [15] and Prodhon and Prins [16].

Recently fuzzy logic has been used to solve many different problems. The need to use fuzzy logic in problems arises whenever there are some vague or uncertain parameters. In CLRP, some works have been done with fuzzy variables so far. Zarandi, et al. [17] presented a CLRP in which travel time between two nodes was a fuzzy variable. They used fuzzy variables and credibility theory to model the problem. A simulation-embedded simulated annealing (SA) procedure was proposed in order to solve the problem. They tested the proposed method using a standard test problem of CLRP and the results showed that the proposed method is robust and could be used in real world problems. In the second work, Zarandi, et al. [18] considered the location-routing problem with time windows under uncertainty. They assumed that demands of customers and travel times were fuzzy variables. In their work, a fuzzy chance constrained programming model was designed using credibility theory and a simulation-embedded SA algorithm was presented in order to solve the problem. To initialize solutions of SA, a heuristic method based on fuzzy c-means clustering with Mahalanobis distance and sweep method was employed. They attested the proposed solution approach with some numerical experiments.

In this paper, CLRP with fuzzy demands (CLRP-FD) is considered. In this problem, it is assumed that the demands of customers are not known. This means that the information about demand at each customer is often not precise enough. For example, based on experience, it can be concluded that demand of a customer is “around 50 units”, “between 20 and 60 units”, etc. For this reason, often there is not enough data to be used to fit a probability distribution of the demand of customers. On the other hand, based upon the expert’s judgment, one can easily estimate the demand of customers. Therefore, while using the probability theory is cumbersome and costly, fuzzy logic is worthwhile in these problems (Zarandi, et al. [17]).

This paper described a fuzzy chance constrained programming FCCP) with credibility theory to model the CLRP-FD. The method consists of four phases; in first phase, the customers are clustered using a greedy search algorithm. In second phase, with determining the gravity centers of the clusters, the most appropriate depot(s) among a set of potential depots are selected to be established. The third phase allocates the clusters to established depots. Finally, ant colony system (ACS) is applied to set up the best routs between the depot(s) and the assigned clusters. Since the actual value of demand of a customer is only known when the vehicle reaches the customer, stochastic simulation is used in fourth phase to determine the demands of customers. The verification of the FCCP model and validation of the GCM are performed on some numerical experiment and a case study in Ardakan. Both of the developed GCM to solve the CLRP-FD and the proposed real case to show the performance of the model are the main contribution of the paper.

The remainder of this paper is organized as follows: In Section 2, some basic concepts of fuzzy theory are given. Section 3 introduces the CLRP-FD and presents a FCCP model using the credibility theory. Details of the GCM to solve CLRP-FD are presented in Section 4. In Section 5, computational experiments are given to reveal the performance of the FCCP model and the proposed method. In the final Section, the conclusion remarks of the paper are presented.

2. Fuzzy Credibility Theory

The concept of the fuzzy set was initiated by Zadeh [19] via the membership function. Then it has been well developed and applied in a wide variety of real problems. In order to measure a fuzzy event, the term fuzzy variable was introduced by Kaufmann [20], and later Zadeh [21] proposed the possibility measure theory of fuzzy variable.

Although, possibility measure has been widely used, it has no self-duality property. However, a self-dual measure is absolutely necessary in both theory and practice. In order to recently a modification to possibility theory which is called credibility theory was founded by Liu [22] and studied very recently by many scholars all around the world. Since a fuzzy version of CLRP with credibility theory will be considered in this paper, a brief introduction to basic concepts and the definitions used in this paper presented as follows:

Let Θ be a nonempty set, and P the power set of Θ. Each element in P is called an event, and ϕ is an empty set. In order to present anaxiomatic definition of possibility, it is necessary to assign a number Pos $\{A\}$ to each event A, which indicates the possibility that A will occur. In order to ensure that the number Pos $\{A\}$ has certain mathematical properties, the following four axioms are approved Liu (2004):

Axiom 2.1. $\text{Pos}\{\Theta\} = 1$;

Axiom 2.2. $\text{Pos}\{\phi\} = 0$;

Axiom 2.3. For each $A_i \in p(\Theta)$, $\text{Pos}\{U_i \;\; A_i\} = \sup_i \text{Pos}\{A_i\}$;

Axiom 2.4. If Θ_i is a non-empty set, and the set function $\text{Pos}_i\{\}$; $i= 1, 2, \ldots, n$, satisfies above three axioms, and $\Theta = \Theta_1 \times \Theta_2 \times \cdots \times \Theta_n$, then for each $A \in p(\Theta)$, $\text{Pos}\{A\} = \sup_{(\theta_1 \times \theta_2 \times \ldots \times \theta n) \in A} \text{Pos}_1\{\theta_1\} \wedge \text{Pos}_2\{\theta_2\} \wedge \cdots \text{Pos}_n\{\theta_n\}$.

The above four axioms form the basis of credibility measure theory, all concepts of credibility theory can be obtained from them (Liu, 2004).

Definition 2.5 Let $(\Theta, P(\Theta), \text{Pos})$ be a possibility space, and A be a set in $p(\Theta)$, then the necessity measure of A is defined by $\text{Nec}\{A\} = 1 - \text{Pos}\{A^c\}$.

Definition 2.6 Let $(\Theta, P(\Theta), \text{Pos})$ be a possibility space, and A be a set in $p(\Theta)$, then the credibility measure of A is defined by $Cr\{A\} = \frac{1}{2}$ (Pos$\{A\}$ + Nes$\{A\}$).

Considering definition 2.6, the credibility of a fuzzy event is defined as the average of its possibility and necessity. The credibility measure is self-dual. A fuzzy event may fail even though its possibility achieves 1, and hold even though its necessity is 0. However, the fuzzy event must hold if its credibility is 1, and fail if its credibility is 0. In the theory of fuzzy subsets, the law of credibility plays a role similar to that played by the law of probability in measurement theory for ordinary sets (Erbao and Mingyong [23]).

Now let consider a triangular fuzzy variable $\tilde{d} = (d_1, d_2, d_3)$, $\tilde{d}$ is denoted by its left boundary d_1, and its right boundary d_3. Thus, the dispatcher or analyst studying the problem can subjectively estimate, based on his experience and intuition and/or available data, the demand of the customer will not be less than d_1 or greater than d_3. The value of d_2 corresponding to a grade of membership of 1 can also be determined by a subjective estimate. From the definitions of possibility, necessity and credibility, it is easy to obtain (Erbao and Mingyong, 2009):

$$\text{Pos}\,\{\tilde{d} \ge r\} = \begin{cases} 1, & \text{if } r \le d_2 \\ \dfrac{d_3 - r}{d_3 - d_2}, & \text{if } d_2 \le r \le d_3 \\ 0, & \text{if } r \ge d_3 \end{cases} \quad (1)$$

$$\text{Nec}\,\{\tilde{d} \ge r\} = \begin{cases} 1, & \text{if } r \le d_1 \\ \dfrac{d_2 - r}{d_2 - d_1}, & \text{if } d_1 \le r \le d_2 \\ 0, & \text{if } r \ge d_2 \end{cases} \quad (2)$$

$$\mathrm{Cr}\,\{\tilde{d} \ge r\} = \begin{cases} 1, & \text{if } r \le d_1 \\ \dfrac{2d_2 - d_1 - r}{2(d_2 - d_1)}, & \text{if } d_1 \le r \le d_2 \\ \dfrac{d_3 - r}{2(d_3 - d_2)}, & \text{if } d_2 \le r \le d_3 \\ 0, & \text{if } r \ge d_3 \end{cases} \quad (3)$$

3. The Fuzzy Chance Constrained Program Model for the CLRP-FD

In the CLRP, demand of each customer should be supplied by a single vehicle, while total load of each route must not exceed the capacity of the vehicle. The routs starts and ends to the same depot, and total load of allocated customers must be less than or equal to the capacity of the depot. The objective is to minimize the total cost of the system including costs of depot and routing costs.

In CLRP-FD, in addition to the above assumptions, the demand of each customer is a triangular fuzzy number such as $\tilde{d} = (d_1, d_2, d_3)$. To model the problem with credibility theory, the fuzzy number representing demand at the j^{th} customer is denoted by $\tilde{d}_j = (d_{1j}, d_{2j}, d_{3j})$. Let the vehicles have equal capacity that is denoted by Q. After serving the first k customers, the available capacity of a vehicle will equal $Q_k = Q - \sum_{j=1}^{k} \tilde{d}_j$, Q_k is also a triangular fuzzy number by using the rules of fuzzy arithmetic, and

$$Q_k = \left(Q - \sum_{j=1}^{k} d_{3j},\ Q - \sum_{j=1}^{k} d_{2j},\ Q - \sum_{j=1}^{k} d_{1j} \right) = (q_{1,k},\ q_{2,k},\ q_{3,k}).$$

The credibility that the next customer demand does not exceed the remaining capacity of the vehicle can be obtained as follows:

$$\mathrm{Cr} = Cr\left\{ \tilde{d}_{k+1} \le Q_k \right\} = Cr\left\{ \left(d_{1,k+1} - q_{3,k},\ d_{2,k+1} - q_{2,k},\ d_{3,k+1} - q_{1,k} \right) \le 0 \right\} \quad (4)$$

$$\mathrm{Cr}\left\{ \tilde{d}_{k+1} \le Q_k \right\} = \begin{cases} 0, & \text{if } d_{1,k+1} \ge q_{3,k} \\ \dfrac{q_{3,k} - d_{1,k+1}}{2*(q_{3,k} - d_{1,k+1} + d_{2,k+1} - q_{2,k})}, & \text{if } d_{1,k+1} \le q_{3,k},\ d_{2,k+1} \ge q_{2,k} \\ \dfrac{d_{3,k+1} - q_{1,k} - 2*(d_{2,k+1} - q_{2,k})}{2*(q_{2,k} - d_{2,k+1} + d_{3,k+1} - q_{1,k})}, & \text{if } d_{2,k+1} \le q_{2,k},\ d_{3,k+1} \ge q_{1,k} \\ 1, & \text{if } d_{3,k+1} \le q_{1,k} \end{cases} \quad (5)$$

Similarly let the capacity of candidate depots are equal and are denoted by P. In CLRP-FD, the proper depot(s) should be opened within some candidate depots. After allocating the customers to a depot for receiving the service, the available capacity of the depot will equal $P_k = P - \sum_{j=1}^{k} \tilde{d}_j$, P_k is also a triangular fuzzy number by using the rules of fuzzy arithmetic, and

$$P_k = \left(P - \sum_{j=1}^{k} d_{3j},\ P - \sum_{j=1}^{k} d_{2j},\ P - \sum_{j=1}^{k} d_{1j} \right) = (p_{1,k},\ p_{2,k},\ p_{3,k})$$

The credibility that the next allocated customer demand does not exceed the remaining capacity of the depot can be shown as follows:

$$\mathrm{Cr} = Cr\left\{ \tilde{d}_{k+1} \le P_k \right\} = Cr\left\{ \left(d_{1,k+1} - p_{3,k},\ d_{2,k+1} - p_{2,k},\ d_{3,k+1} - p_{1,k} \right) \le 0 \right\} \quad (6)$$

$$\mathrm{Cr}\left\{\tilde{d}_{k+1} \le P_k\right\} = \begin{cases} 0, & \text{if } d_{1,k+1} \ge p_{3,k} \\ \dfrac{p_{3,k} - d_{1,k+1}}{2*(p_{3,k} - d_{1,k+1} + d_{2,k+1} - p_{2,k})}, & \text{if } d_{1,k+1} \le p_{3,k},\ d_{2,k+1} \ge p_{2,k} \\ \dfrac{d_{3,k+1} - p_{1,k} - 2*(d_{2,k+1} - p_{2,k})}{2*(p_{2,k} - d_{2,k+1} + d_{3,k+1} - p_{1,k})}, & \text{if } d_{2,k+1} \le p_{2,k},\ d_{3,k+1} \ge p_{1,k} \\ 1, & \text{if } d_{3,k+1} \le p_{1,k} \end{cases} \quad (7)$$

In according to formulation (5), if the vehicle's remaining capacity is high and the demand at the next customer is low, then the vehicle's chance of being able to finish the next customer's service become greater. That is to say, the greater the difference between available capacity of the vehicle and demand at the next customer, the greater our preference to send the vehicle to serve the next customer. The preference index is described by *Cr*, which denotes the magnitude of the preference to send the vehicle to the next customer after it served current customer in according to formulation (5). Obviously, $Cr \in [0,1]$. When *Cr*= 0, the vehicle is completely sure that should return to the depot. When *Cr*=1, the vehicle is absolutely certain that is able to serve the next customer. Let the dispatcher preference index equal $Cr^*, Cr^* \in [0,1]$. So, according to the dispatcher preference index value and the credibility that the next customer demand does not exceed the remaining capacity of the vehicle, a decision must be made as to whether to send it to the next customer or return it to depot. Thus, if the relation $Cr \ge Cr^*$ is fulfilled, then the vehicle should be sent to the next customer; otherwise, the vehicle should be returned to the depot, and send it again to the next customer after loading. The process does not terminate until all of the customers' demands are fulfilled.

Similarly, in formulation (7) if the depot's remaining capacity is greater and the demand at the next customer is less, then the depot's chance of being able to allocate the next customer become greater. The assignment preference index is described by *Cr* that it's value is $Cr \in [0,1]$. When *Cr*= 0, the depot is completely sure that should not accept the next customer to give service it. When *Cr*=1, the depot is absolutely certain that is able to serve the next customer. The assignment preference index for allocating of the customers to a depot is considered Cr^*, $Cr^* \in [0,1]$. Thus, if the relation $Cr \ge Cr^*$ is fulfilled, then the depot should serve the next customer; otherwise, the customer should receive service from another opened depot. This procedure does not end until all of the customers are allocated.

Moreover, the vehicle routes (or planned routes) are designed in advance by applying the proposed method. But the actual value of demand of each customer is only known when the vehicle reaches the customer. Due to the uncertainty of demand at the customers, a vehicle might not be able to service a customer once it arrives there due to insufficient capacity when the vehicle implements the planned route. It is assumed in such situations the vehicle returns to the depot to load itself and then returns to the customer where it had a "failure" and continues service along the rest of the planned route. This arises additional distance due to route failure. So, the additional distance should be considered that the vehicle makes due to "failure" arising at some customers along the route when evaluating the planned route.

Parameter Cr^* and Cr^{**} which are subjectively determined have an extremely great impact on both the total length of the planned routes and on the additional distance. For example, lower values of parameter Cr^* express the dispatcher's desire to use vehicle capacity the best he can. These values result in shorter planned routes. But lower values of parameter Cr^* increase the number of situations in which vehicles arrive at a customer and are unable to service them, thereby increasing the total distance they cover due to the "failure" (Erbao and Mingyong [23]).In this work, stochastic simulation is used to evaluate the additional distance due to route failure.

The following notations are used to represent the mathematical programming formulation for the CLRP-FD.

Sets and parameters:

J: Set of customers indexed by j

I: Set of candidate depot sites indexed by i

K: Set of vehicles indexed by k

V: Set of all points; $V = J \cup I$

E: Set of arcs (i,j) connecting every pair of nodes $i, j \in V$

c_{ij}: Cost of traveling associated with arc $(i, j) \in E$

d_j: Demand of customer j

O_i: Fixed cost of opening a depot at candidate site i

F_i: Fixed cost of employing a vehicle at candidate site i

P: Capacity of depots; here it is assumed that all depots have equal capacity.

Q: Capacity of vehicles; here it is assumed that all vehicles are homogeneous.

Decision variables:

$$Z_i = \begin{cases} 1 & \text{if a depot at candidat site } i \text{ is opend} \\ 0 & \text{other wise} \end{cases}$$

$$Y_{ij} = \begin{cases} 1 & \text{if demand at customer } j \\ & \text{is served by the depot at candidate site } i \\ 0 & \text{other wise} \end{cases}$$

$$X_{ijk} = \begin{cases} 1 & \text{if vehicle } k \\ & \text{goes directly from} \\ & \text{customer } i \text{ to customer } j \\ 0 & \text{other wise} \end{cases}$$

$$W_{jk} = \begin{cases} 1 & \text{if customer } j \\ & \text{is served by vehicle } k \\ 0 & \text{other wise} \end{cases}$$

The corresponding fuzzy chance constrained programming (FCCP) mathematical formulation of CLRP-FD based on credibility theory is given by:

$$Minimize \quad \sum_{i \in I} O_i Z_i + \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} F_i X_{ijk} + \sum_{i \in V} \sum_{j \in V} \sum_{k \in K} c_{ij} X_{ijk} \tag{8}$$

$$Minimize \quad f \tag{9}$$

Subject to

$$\mathrm{Cr}\left(\sum_{j \in J} \tilde{d}_j W_{jk} \le Q \right) \ge Cr^{*} \qquad \forall \ k \in K \tag{10}$$

$$\mathrm{Cr}\left(\sum_{j \in J} \tilde{d}_j Y_{ij} \le PZ_i \right) \ge Cr^{**} \qquad \forall \ i \in I \tag{11}$$

$$\sum_{i \in V} \sum_{k \in K} X_{ijk} = 1 \qquad \forall \ j \in J \tag{12}$$

$$\sum_{i\in S}\sum_{j\in S} X_{ijk} \le |S|-1 \qquad \forall\, S \subseteq J;\ \forall\, k \in K \tag{13}$$

$$\sum_{i\in V} X_{ijk} - \sum_{i\in V} X_{jik} = 0 \qquad \forall\, j \in V;\ \forall\, k \in K \tag{14}$$

$$\sum_{i\in I}\sum_{j\in J} X_{ijk} \le 1 \qquad \forall\, k \in K \tag{15}$$

$$\sum_{m\in V} X_{imk} + \sum_{h\in V} X_{jhk} \le 1 + Y_{ij} \qquad \forall\, i \in I;\ \forall\, j \in J;\ \forall\, k \in K \tag{16}$$

$$\sum_{i\in I} X_{ijk} = W_{jk} \qquad \forall\, j \in J;\ \forall\, k \in K \tag{17}$$

$$\sum_{j\in J} X_{ijk} = W_{ik} \qquad \forall\, i \in I;\ \forall\, k \in K \tag{18}$$

$$Z_i \in \{0,1\} \qquad \forall\, i \in I \tag{19}$$

$$Y_{ij} \in \{0,1\} \qquad \forall\, i \in I;\ \forall\, j \in J \tag{20}$$

$$X_{ijk} \in \{0,1\} \qquad \forall\, i \in V;\ \forall\, j \in V;\ \forall\, k \in K \tag{21}$$

$$W_{jk} \in \{0,1\} \qquad \forall\, j \in J;\ \forall\, k \in K \tag{22}$$

The objective function (8) represents the sum of the fixed depot location cost and routing costs including the fixed costs of employing vehicles and the travel costs, respectively. The objective function (9) seeks to minimize total additional travel distance due to routes failure. The value of f can be obtained by stochastic simulation algorithm in Section 4.4.Chance constraints (10) and (11) assure that all customers are visited within vehicle capacity and are allocated within depot capacity with a confidence level, respectively. Each customer should be served within one route only and the customers should have only one predecessor, which is stated by constraint (12). Constraints (13) are the standard sub-tour elimination constraints which indicate that for any subset S of the set of customers J and for any route k, the number of arcs belonging to route k that connect the members of S, must not exceed the cardinality of S minus one. The continuity of the routes and return to the original depot are guaranteed through constraints (14) and (15). Constraints (16) ensure that a customer must be assigned to a depot if there is a route connecting them. Constraints (17) and (18) express the relation between two decision variables. Finally, constraints (19), (20), (21), and (22) specify the binary variables used in the formulation.

4. Proposed Heuristic Method for The CLRP-FD

A greedy clustering method, named GCM, is presented in this Section to solve the CLRP-FD. In general, GCM consists of four phases, which is illustrated in Fig. 1. In first phase, the customers are clustered using a greedy search algorithm (Fig. 1(a)). The nearest customer to last added customer to the cluster is selected to be included in the cluster. This is the same as to form a tour in traveling salesman problem (TSP), in which the nearest city to the current city (in a “greedy” search algorithm) is selected as next destination.

Each cluster includes as much as customers until the next customer demand does not exceed the remaining capacity of the vehicle, according to the dispatcher preference index value and the credibility of the next customer. In second phase, the gravity center of each cluster is calculated which is used to select depot(s) to be established (Fig. 1(b)). Due to the near distance between the opened depot(s) and the gravity center of clusters, the clusters are allocated to the opened depot(s) in third phase. On the other hand, each depot serves as much as clusters until the next cluster demand does not exceed the remaining capacity of the depot, according to the assignment preference index value and the credibility measure (Fig. 1(c)). Finally, in fourth phase, ACS forms an admissible tour between each cluster and depot (Fig. 1(d)). In this phase, the stochastic simulation is used to determine the demands of customers. This helps that the planned routs is evaluated and additional distance is obtained. The problem is initialized by defining a plane comprising the set of customers, depots, and their coordinate points, namely *CUST* and *DEP*, respectively. The heuristic method is repeated for a predefined number of iterations. When the algorithm obtained a better solution, it is replaced to the last best known solution. Details of heuristic method are described in following Sections.

4-1. Clustering The Customers

The first phase of the GCM for CLRP-FD is the clustering of the customers. The customers are grouped considering their intra distance, their fuzzy demands and the capacity of the vehicles. A greedy search algorithm is used to select a set of customers. In first step, to form a cluster, a customer is selected randomly from the set of non-clustered customers belongs to *CUST*. The algorithm searches for the nearest customer to the last selected customer of the current cluster. The nearest customer is not included in the cluster if the next customer demand exceeds the remaining capacity of the vehicle, considering the dispatcher preference index value and the credibility that the next customer demand does not exceed the remaining capacity of the vehicle. Once a new customer is selected to be included in a cluster, total fuzzy demand of current members of the cluster is calculated and it is compared with the capacity of the vehicle. If the relation $Cr \geq Cr^*$ is fulfilled -according to the formulation (7) and (10)-, then the new customer will allow to include in current cluster. Otherwise, last selected customer is withdrawn from the cluster. This customer is removed from the current search space of the algorithm. The greedy search algorithm searches for a new customer close to the last added member of the cluster among the ungrouped customers. This is to use the maximum capacity of a vehicle. The algorithm forms a new cluster if there are no more customers to be included in current cluster considering the capacity of vehicle and fuzzy demand of customers. When there are no more customers without a cluster, this phase stops. Fig. 2 illustrates the greedy search algorithm.

4-2. Choosing The Depots

The second phase of the GCM searches in potential sites to establish depot(s). In first step of the phase, the gravity center of the clusters is calculated according to equation (23), in which $(X_{(I)}, Y_{(I)})$ is the coordinates of gravity center of I^{th} cluster, (x_j, y_j) is the coordinates of j^{th} customer of I^{th} cluster, and n_I is the number of customers in I^{th} cluster. The gravity center of the cluster is used as a representative to allocate it to the proper depot.

$$\left(X_{(I)}, Y_{(I)}\right) = \left(\frac{\sum_{j \in I} x_j}{n_I}, \frac{\sum_{j \in I} y_j}{n_I}\right) \tag{23}$$

Choosing the potential site(s) for depot(s) is the same as single facility location problem (SFLP). In second step of this phase, the sum of distances between the gravity center of the clusters and each potential site is calculated. The potential sites are sorted in ascending according to their Euclidean distance with gravity center of clusters, which is calculated by equation (24). In this equation, (x^*, y^*) is the coordinates of desired potential site among all the candidates. Moreover, w_i is the total Euclidean distance between i^{th} potential site and the gravity center of clusters, (x_i, y_i) is the coordinates of i^{th} site, (a_I, b_I) is the coordinates of gravity center of I^{th} cluster, m is the number of clusters, and n is the number of potential sites.

$$\left(x^*,y^*\right):\quad Min \quad w_i=\sum_{I=1}^{m}\left[\left(x_i-a_I\right)^2+\left(y_i-b_I\right)^2\right]^{1/2} \qquad \forall i=1,\dots,n \tag{24}$$

In sorted list of potential sites, the first site is selected to be established to serve clusters. As will be mentioned in next phase, if the capacity of the current opened depot will not be able to serve all clusters considering to the credibility of each cluster, depends on the total demands of each cluster and the assignment preference index value, the next potential sites of the sorted list is selected to serve the remaining clusters. This procedure (establishing the depot(s)) is repeated until all clusters are covered.

4.3-Allocating Clusters to Depot(s)

The vehicles start their journey from a depot, move to all the customers of a cluster, and return to the depot once finishing the service to the customers. Each cluster is supplied from exactly one depot. In third phase of the GCM, the clusters are respectively allocated to the sorted depots in second phase. According to the assignment preference index value and the credibility that the next cluster demand does not exceed the remaining capacity of the depot, each depot is able to serve some clusters. To allocate the clusters, the Euclidian distance of gravity center of each cluster to the first depot in sorted list is calculated. Based on the close distance, the clusters are sorted in ascending. The first cluster in sorted list is allocated to the first depot, if the relation $Cr \geq Cr^{**}$ is fulfilled. If yet there is remaining capacity for the first depot, the next cluster in sorted list is allocated to the depot considering the mentioned relation. The allocation process to the first depot will be finished once there is not enough depot capacity to allocate any cluster. In this situation, the allocating procedure is repeated for the remainder of the depots and unallocated clusters until all clusters are allocated.

4-4. Routing

In fourth and last phase of the GCM, the routing problem is solved for each cluster. The routing problem of CLRP-FD is the same as TSP, which is solved by using ACS. ACS is referred to ants' treatment to find food. The ants spread a material called pheromone and put it on their way so that other ants can pass the same route. The pheromone of shorter route increases and therefore, more ants move from that way. Artificial ants construct a solution by selecting a customer to visit sequentially, until all the customers in a route have been visited. Ants select the next city to visit using a combination of heuristic and pheromone information. A local updating rule is applied to modify the pheromone on the selected arc, during the construction of a route. Once all ants have constructed their tours, the amount of pheromone of the best selected route and the global best solution, are updated according to the global updating rule. More details on ACS can be found in Drigo, et al. [24] and Bouhafs, et al. [25].

As pointed before, because the demand of each customer is a triangular fuzzy number, it cannot be directly considered as a deterministic number by applying other algorithms that solve the deterministic CLRP. Since the "actual" value of each customer demand is identified as the vehicle reaches the customer, the simulation experiment is used to determine the deterministic value of the demands. For each feasible planned route that the solution of the GCM stands for, additional distances due to route failures (f) are obtained by a stochastic simulation algorithm. The steps of the stochastic simulation are summarized as follows:

Step 1: For each customer, estimate the additional distances by simulating "actual" demands. The "actual" demands were generated by following processes: (1) randomly generate a real number D in the interval between the left and right boundaries of the triangular fuzzy number representing demand at the customer, and compute its membership m; (2) generate a random number $r; r \in [0,1]$; (3) compare r with m, if $r \leq m$, then "actual" demand at the customer is adopted as being equal to D; in the opposite case, if $r > m$, it is not accepted that demand at the customer equals *D*. In this case, random numbers *D* and *r* are generated again and again until random number *D* and *r* are found that satisfy relation $r \le m$; (4) check and repeat (1) till (3), and terminate the process when all customers have a simulation “actual” demand quantity.

Step 2: Move along the route designed by ACS and calculate the additional distance due to route failures in terms of the “actual” demand.

Step 3: Repeat Step 1 and Step 2 *M* times.

Step 4: Compute the average value of additional distance by *M* times simulation, and it is regarded as the additional distances (*f*).

Note that, the routing cost of CLRP-FD consists of two amounts: additional distances and planned routs distances. In CLRP-FD, each planned routs distances between the depots and allocated clusters are obtained by ACS without considering the demands of customers.

5. Computational Results

In this Section, some examples and a case study are given to show the verification of the described FCCP model, efficiency of the GCM and how they work in real world. At first two types of test problems with different conditions are created based on the size of problem. It is assumed that there are 30 customers and 5 candidate depots for a small size problem, and 100 customers and 7 candidate depots for a large size problem. In each experiment, the coordinates of all customers and depots are generated randomly in $[100 \times 100]$. Moreover, the fuzzy demands of customers, that are triangular fuzzy numbers, are generated within [10,110] randomly. The relative values for two test problems are listed in Table 1.

The GCM was encoded in MATLAB 7.10.0. The value of dispatcher preference index Cr^* varied with the interval of 0.1 to 1 with a step of 0.1. In this work, the assignment preference index Cr^* is considered 1 due to convenience and reducing the number of different investigative statues. The average computational results of 10 times are given in Tables 2 and 3 for the small size and the large size problems, respectively. The columns of the tables respectively labeled: the dispatcher preference index (Cr^*), the planned routes, the additional distances, the routing costs that includes the planned routes and additional distances, the depot costs, the vehicle costs, the total costs that consist of routing costs as well as depots and vehicles costs.

For convenience, the results of Tables 2 and 3 are depicted by Figs. 3 and 4, respectively. As is shown in Tables 2 and 3 and also in Figs. 3 and 4, when the value of dispatcher preference index equals 0.6, the total cost has a minimum value. According to Figs. 3 and 4, lower values of parameter Cr^* denote a tendency to use total vehicle capacity. These values are associated with the routes with the shorter planned distances. On the other hand, lower values of parameter Cr^* increase the number of cases in which vehicles visit customers but are unable to serve them, thereby increase the total additional distance due to the “failure”. Higher values of parameter Cr^* are characterized by less utilization of vehicle capacity along the planned routes and less additional distance to cover due to failures. Therefore, the proper Cr^* is approximately around 0.6, considering the total cost.

In the second computational experiment, a real case is performed to reveal the verification of the FCCP model and the efficiency of the GCM. A distribution center (DC) of grocery in Ardakan city is selected. This DC serves 20 retailers that the demands of them are not precise and they can be assumed as triangular fuzzy numbers. Table 4 indicates the data related to the retailers that includes the coordination of retailer’s location and fuzzy demand of them. Moreover, data from the candidate DCs is shown in Table 5. In addition to the current DC location, there are 4 other candidate DC locations that their coordination, capacity and fixed cost of them are shown in Table 5. Finally, data about the vehicles available are indicated in Table 6. As shown in Table 6, there are two vehicles that are different in terms of the capacity and fixed cost.

Table 7 gives the summary of results on the real case. Table 7 includes two rows: first row shows the solution of current situation without using

FCCP model and second row indicates solution obtained from GCM and FCCP model. It is important to note that the transportation system of both the current status of the case study and the obtained solution of the GCM are depicted by Figs. 5 and 6, respectively. As is shown in final column of Table 7, using FCCP model and proposed GCM has achieved better solution. This result indicates the verification of the FCCP model and the efficiency of GCM in real world.

6. Conclusion and Future Research

This paper contributed to the capacitated location routing problem with fuzzy demands in the following directions: (a) a fuzzy chance constrained programming model of CLRP-FD was described based on credibility theory; (b) greedy clustering method and stochastic simulation were integrated to solve this problem, focusing on minimizing the total costs that consists of routing cost and fixed cost of depots and vehicles; (c) the dispatcher and assignment preference index greatly influenced on the planned routes, additional distances and fixed costs of depots and vehicles, and the “best” value of parameter Cr^* was obtained by the proposed method; (d) the verification of the model and effectiveness of the greedy clustering method was shown by both numerical examples and a case study. In this work, because of the large number of investigative statues, the effect of assignment preference index (Cr^{**}) did not checked on total costs and it would be tested in future researches.

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Figures:

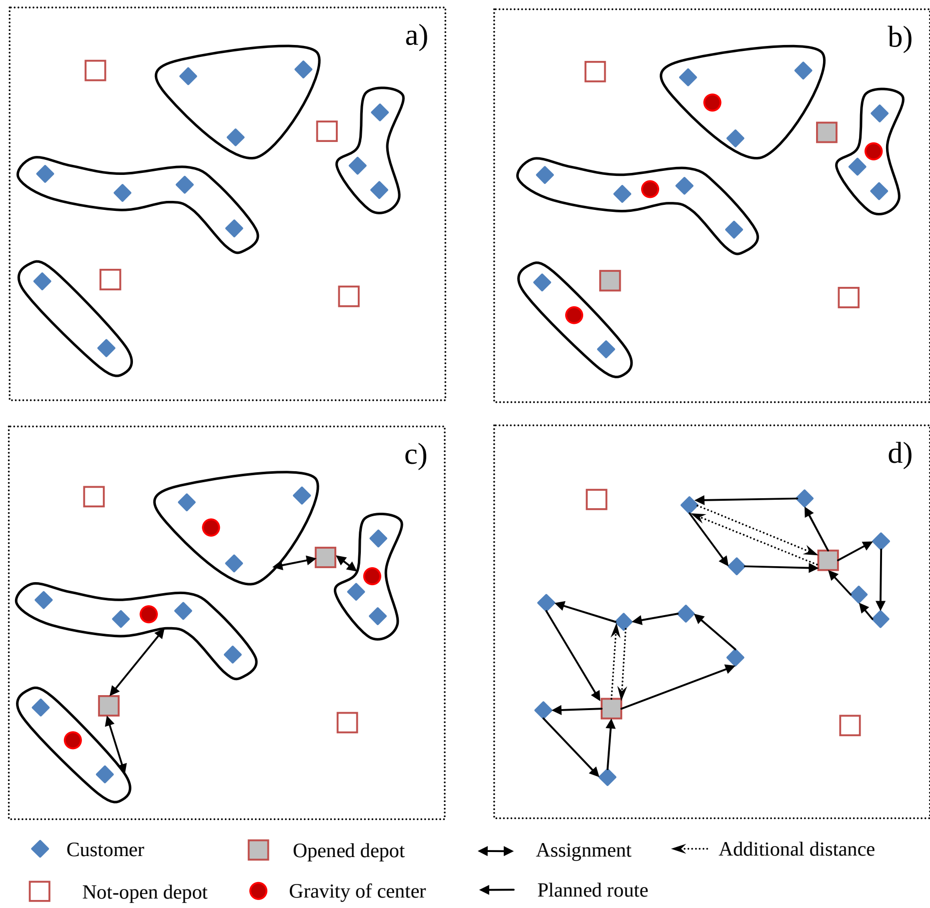

Fig. 1. Illustrative example for the proposed GCM.

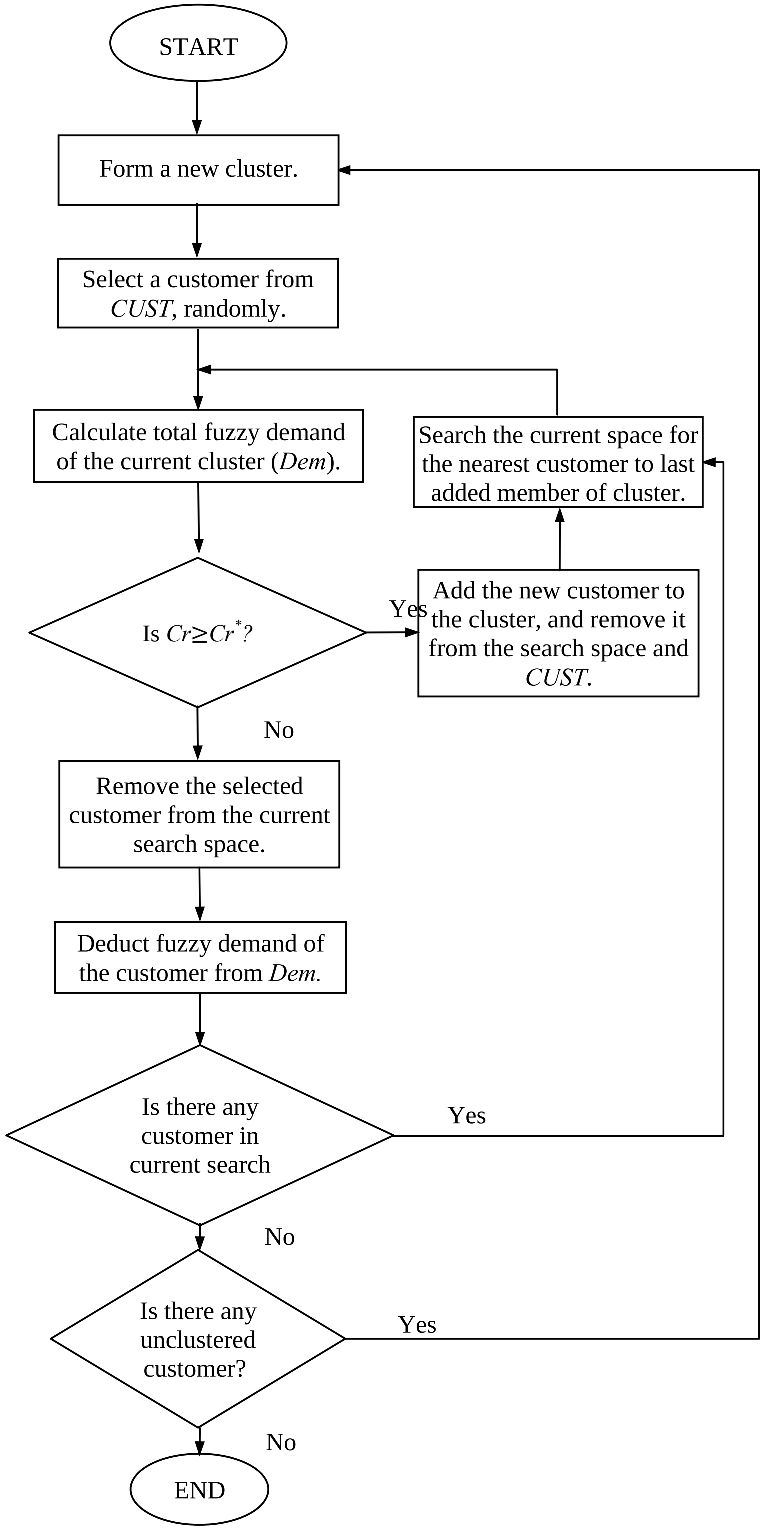

Fig. 2. The proposed greedy search algorithm.

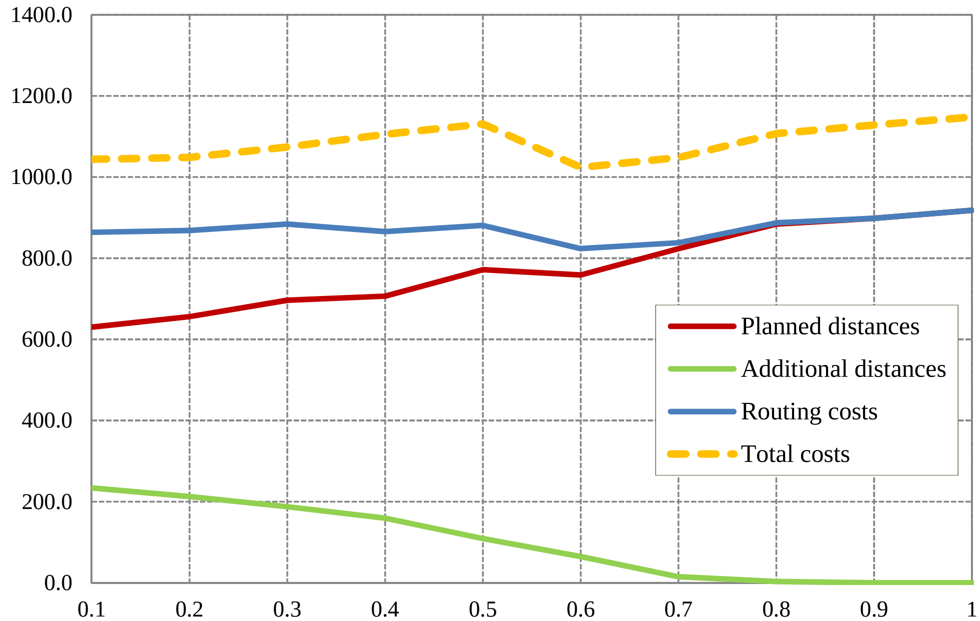

Fig. 3. The costs change tendencies with *Cr varied when number of customers are 30**.

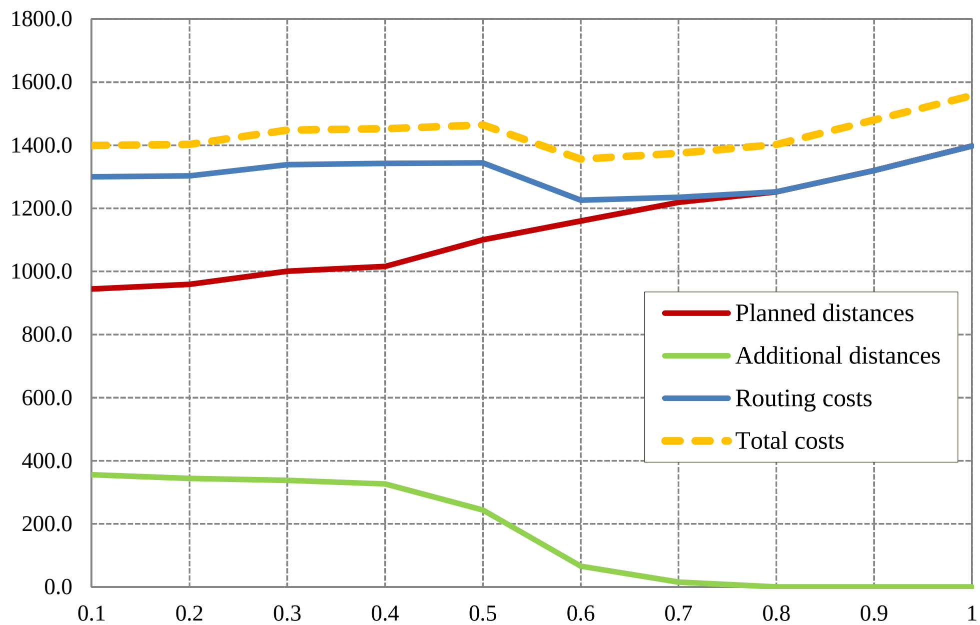

Fig. 4. The costs change tendencies with *Cr varied whennumber of customers are 100.**

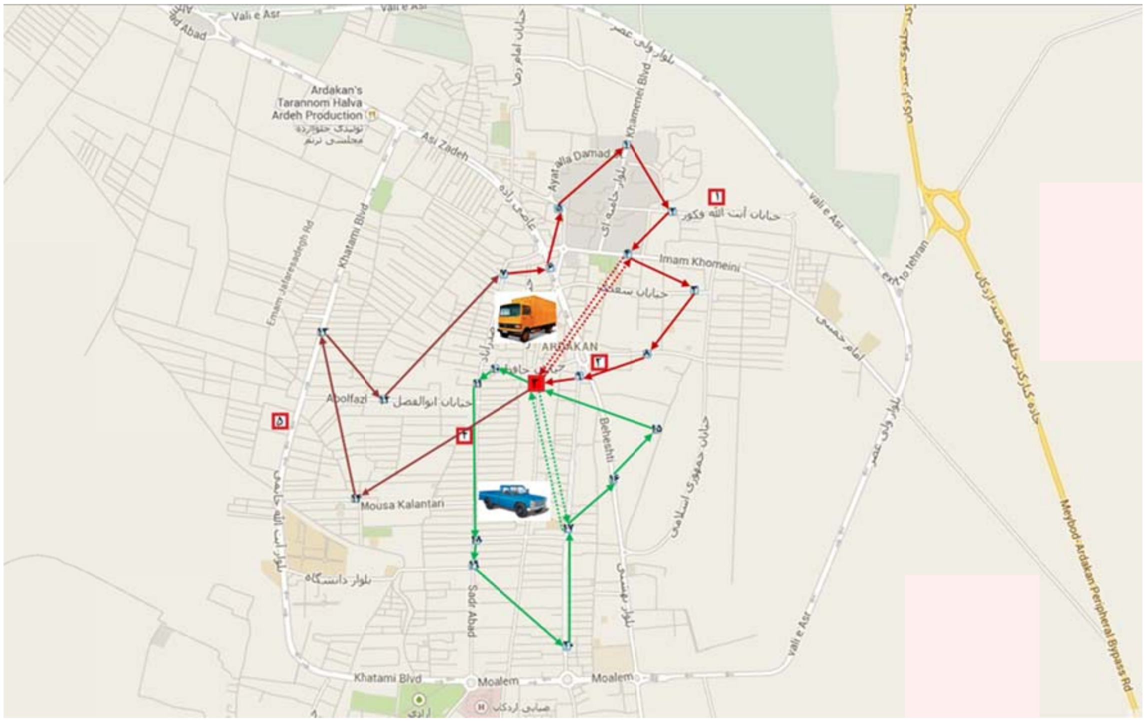

Fig. 5. A view of the current status in case of Ardakan.

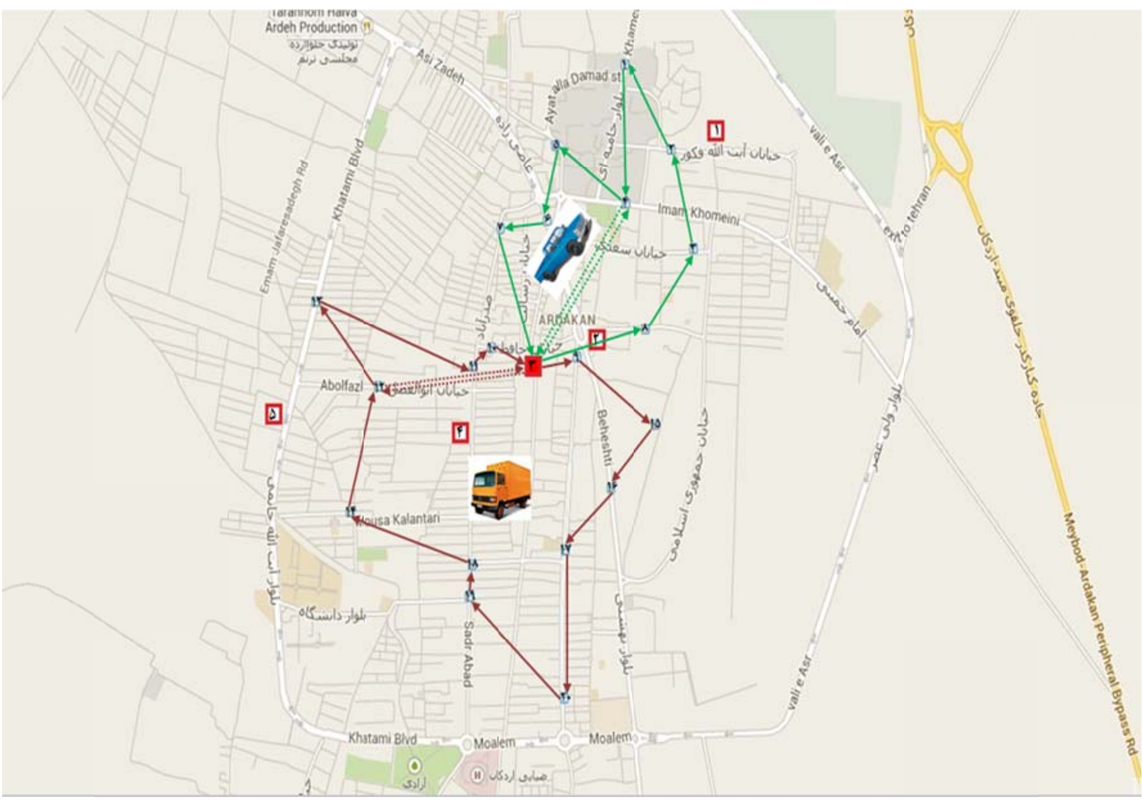

Fig. 6. The obtained solution of the GCM in case of Ardakan.

Tables:

Tab. 1. The relative values of the test problems.

No. of customers	No. of candidate depots	Vehicle capacity	Depot capacity	Fixed cost of depots	Fixed cost of vehicles
30	5	300	900	50	10
100	7	800	10000	50	10

Tab. 2. The average of results with different Cr^* when number of customers are 30.

Cr^*	Planned routes	Additional distances	Routing costs	Depot costs	Vehicle costs	Total costs
0.1	630.1	234.0	864.1	150	30	1044.1
0.2	656.0	212.6	868.6	150	30	1048.6
0.3	696.5	187.7	884.2	150	40	1074.2
0.4	706.5	159.2	865.7	200	40	1105.7
0.5	771.9	109.1	881	200	50	1131
0.6	759.0	64.9	823.9	150	50	1023.9
0.7	823.4	14.9	838.3	150	60	1048.3
0.8	884.3	3.0	887.3	150	70	1107.3
0.9	898.4	0.0	898.4	150	80	1128.4
1	918.3	0.0	918.3	150	80	1148.3

Tab. 3. The average of results with different Cr^* when number of customers are 100.

Cr^*	Planned routes	Additional distances	Routing costs	Depot costs	Vehicle costs	Total costs
0.1	944.1	355.4	1299.5	50	50	1399.5
0.2	958.9	344	1302.9	50	50	1402.9
0.3	1000.5	337.8	1338.3	50	60	1448.3
0.4	1015.9	326.6	1342.5	50	60	1452.5
0.5	1100.2	244	1344.2	50	70	1464.2
0.6	1160	65.9	1225.9	50	80	1355.9
0.7	1219.3	15.5	1234.8	50	90	1374.8
0.8	1252.1	0.4	1252.4	50	100	1402.4
0.9	1319.9	0	1319.9	50	110	1479.9
1	1397.5	0	1397.5	50	110	1557.5

Tab. 4. Data related to the retailers in case of Ardakan.

No. of retailer	Coordination of retailer's location as (*x*,*y*)	Fuzzy demand of retailer
1	(54.022217814941435, 32.323722372291314)	(25,30,40)
2	(54.02590853454593, 32.319261642228646)	(15,20,30)
3	(54.02805430175784, 32.3136763533639)	(15,30,35)
4	(54.02153116943362, 32.31639650459991)	(40,45,50)
5	(54.01655298950198, 32.319588044561684)	(30,40,50)
6	(54.0155659365845, 32.31563487048783)	(20,55,80)
7	(54.01213270904544, 32.31483696121567)	(20,25,40)
8	(54.02329069854739, 32.309505296152466)	(5,5,10)
9	(54.01831251861575, 32.307945634128224)	(25,35,40)
10	(54.01127440216067, 32.308453434013955)	(30,35,50)
11	(54.01002985717776, 32.307292744378806)	(10,25,30)
12	(54.00183302642825, 32.306313400933725)	(30,60,90)
13	(53.997005050201444, 32.31086544460456)	(5,15,20)
14	(54.00188667060855, 32.2991130880544)	(40,50,55)
15	(54.023494546432524, 32.300119715310295)	(35,55,70)
16	(54.021230762023954, 32.299394219313974)	(5,15,20)
17	(54.01733619453433, 32.29729931703078)	(35,45,60)
18	(54.00964361907962, 32.29645590114226)	(30,50,70)
19	(54.00955778839114, 32.2946420769475)	(25,45,60)
20	(54.017196719665556, 32.28930922361734)	(30,40,45)

Tab. 5. Candidate DC's status in case of Ardakan.

No. of DC	Coordination of DC's location as (*x*,*y*)	Capacity of DC	Fixed cost of DC
1	(54.029191558380155, 32.31954271097332)	4000	70000
2	(54.01927811386111, 32.30906097656545)	3500	96000
3*	(54.01458961250308, 32.308362755670466)	4000	84000
4	(54.009493415374784, 32.303366238749966)	4500	88000
5	(53.994515960235624, 32.30449070578434)	5000	72000

*Data related to the current DC

Tab. 6. Conditions of vehicles in case of Ardakan.

No. of vehicle	Type of vehicle	Capacity of vehicle	Fixed cost
1	NISSAN	280	2500
2	BENZ 808	500	4000

Tab. 7. Summary of computational results using FCCP model and GCM in case of Ardakan.

Situation	Cr^*	Cr^{**}	Planned routes	Additional distances	Routing costs	Fixed cost of DC	Fixed cost of vehicle	Total cost
Current status	-	-	16926.1	4403.3	21329.4	84000	2500	107829.4
Using FCCP and GCM	0.6	1	13429.1	2860.9	16290	84000	2500	102790