



# Supplier Selection and Evaluation by Fuzzy Multi-Criteria Decision Making Methodology

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## KEYWORDS

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## ABSTRACT

*In this study, an outsourcer evaluation and management system is developed for a manufacturing company by use of Fuzzy goal programming (FGP). A first phase of the methodology evaluation criteria for outsources and the objectives of the company are determined. Considering the fuzziness in the decision data, linguistic variables that can be expressed in generalized fuzzy number are used. The propose approach is utilized from fuzzy sets, Analytic Network Process (ANP), fuzzy TOPSIS and Preference Ranking Organization method for enrichment evaluations (PROMETHEE) approaches. Evaluation criteria for this problem are weighted by Fuzzy ANP approach then in the Fuzzy TOPSIS and Fuzzy PROMETHEE approaches. At the second phase the FGP model developed selects the most appropriate outsourcers suitable to be strategic partners with the company and simultaneously allocates the quantities to be ordered to them. At the end, gives the computational results.*

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## 1. Introduction

In today's highly competitive, companies have been forced to focus on supply chain management. In general, a supply chain consists of all links from suppliers to customers: suppliers, manufacturing plants, distribution centers and etc. supplier selection and evaluation are becoming recognized as a strategic and important component of supply chain strategy. In most of the economical, industrial, financial or political decision problems, the evaluation and selection of solution is a typical multiple criteria decision making (MCDM) problem.

In other word determine which is the best among all possible efficient alternatives, according to the decision-maker (DM) preference, taking into account several criteria. There are many techniques that have

been developed to help decision-makers rank alternatives according to many criteria. In this study, the proposed methodology is based on PROMETHEE, which is a well-known multi-criteria decision aid method.

In all previous researches for determining each weight of criteria, comments decision makers would consider. In this study, evaluation criteria is weighted by fuzzy-ANP (F-ANP) approach, then alternatives are evaluated by fuzzy-TOPSIS and Fuzzy-PROMETHEE approaches. The fuzzy goal programming (FGP) model developed selects the most appropriate outsourcers suitable to be strategic partners with the company and simultaneously allocates the quantities to be ordered to them.

The rest of the paper is organized as follow: section 2 presents the explanation about fuzzy sets, PROMETHEE method, fuzzy ANP, fuzzy TOPSIS and fuzzy PROMETHEE. In section 3 presents the explanation about fuzzy goal programming and section 4 presents the proposed integrated F-ANP and F-PROMETHEE and FGP methodology also in

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this section the F-TOPSIS is proposed. In section 5 a real application is presented and in the last section conclusion remarks are given. There are many techniques that have been developed to help decision-makers rank alternatives according to many criteria. Several authors such as Keeney [1], Vinke [2], have analyzed how to model a real-world multi-criteria situation.

Promethee (preference ranking organization method for enrichment evaluation) was first proposed by Brans et al [3], and has been applied in several areas of social sciences and management, such as project management, and military applications.

## 2. Fuzzy Set

A fuzzy set  $\tilde{A}$  in a universe of discourse  $E$  is characterized by a membership function which associates with each element  $x$  in  $E$  a real number in the interval  $[0, 1]$ .

$$\tilde{A} = \{ (x, \mu_{\tilde{A}}^{(x)}) \mid x \in X \} \quad (1)$$

The triangular fuzzy number can be denoted using  $(a^l, a^m, a^u)$ , and the membership function, can be expressed as:

$$f_{\tilde{A}}^{(x)} = \begin{cases} 0 & x < a^l \\ \frac{(x - a^l)}{(a^m - a^l)} & a^l \leq x \leq a^m \\ \frac{(a^u - x)}{(a^u - a^m)} & a^m \leq x \leq a^u \\ 0 & x > a^u \end{cases} \quad (2)$$

### 2-1. Analytical Network Process (ANP), F-ANP

ANP is a comprehensive decision-making technique that has the capability to include all the relevant criteria which have some bearing on arriving at a decision. Analytic hierarchy process (AHP) serves as the starting point of ANP, Shankar & J. Harkharia [4].

In fact, ANP uses a network without needing to specify levels as in a hierarchy.

Influence is a central concept in the ANP. In some cases, if there is vagueness for the decision problem, utilizing fuzzy sets is a useful way. For this reason, in this study, the usage of the fuzzy version of ANP is preferred ([5-6]).

In the F-ANP, to evaluate the decision-makers preferences, pair-wise comparison are structured using triangular fuzzy numbers  $(a^l, a^m, a^u)$ .

$$\tilde{A} = \begin{pmatrix} (1,1,1) & \cdots & (a_{1n}^l, a_{1n}^m, a_{1n}^u) \\ \vdots & \ddots & \vdots \\ (\frac{1}{a_{1n}^u}, \frac{1}{a_{1n}^m}, \frac{1}{a_{1n}^l}) & \cdots & (1,1,1) \end{pmatrix} \quad (3)$$

Evaluation criteria for this problem are weighted by Fuzzy ANP approach. The logarithmic least-squares method is used in this study since it is a most-used and effective method. This method for calculating triangular fuzzy weights can be given as follows (Tuzkaya & Onut [9]):

$$\begin{aligned} \tilde{w}_i &= (w_i^l, w_i^m, w_i^u) \quad i=1,2,\dots,n \\ a_{ij} &\approx \left( \frac{\tilde{w}_i}{\tilde{w}_j} \right) \\ w_i^s &= \frac{(\prod_{j=1}^n a_{ij}^s)^{1/n}}{\sum_{k=1}^n (\prod_{j=1}^n a_{kj}^s)^{1/n}} \quad s \in \{l, m, u\} \end{aligned} \quad (4)$$

The next step, the normalized weight vector can also be obtained by some other methods in the literature. One of these is Yager index, and it is simply calculated.

$$\tilde{F} = (n-a, n, n+b) = \frac{(3n-a+b)}{3} \quad (5)$$

### 2-2. Topsis, F-Topsis

TOPSIS (for the technique for order preference by similarity to ideal solution), developed by Hwang and Yoon [8], is a widely used MADM (for multiple attribute decision making) method. The basic concept of TOPSIS is that the chosen alternative should have the shortest distance from the positive-ideal solution and the farthest distance from the negative-ideal solution. There exists a large amount of literature involving TOPSIS theory and applications. For example, Lai et al. [9] applied the concept of TOPSIS on MODM problems.

Step1. The decision matrix, which consists of alternatives and criteria, is described by:

$$D = \begin{matrix} & \begin{matrix} x_1 & \cdots & x_j & \cdots & x_n \end{matrix} \\ \begin{matrix} A_1 \\ \vdots \\ A_i \\ \vdots \\ A_m \end{matrix} & \begin{bmatrix} \tilde{x}_{11} & \cdots & \tilde{x}_{1j} & \cdots & \tilde{x}_{1n} \\ \vdots & & \vdots & & \vdots \\ \tilde{x}_{i1} & \cdots & \tilde{x}_{ij} & \cdots & \tilde{x}_{in} \\ \vdots & & \vdots & & \vdots \\ \tilde{x}_{m1} & \cdots & \tilde{x}_{mj} & \cdots & \tilde{x}_{mn} \end{bmatrix} \end{matrix} \quad (6)$$

$$\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij})$$

$$w = (\tilde{w}_1, \dots, \tilde{w}_j, \dots, \tilde{w}_n)$$

$$\tilde{w}_j = (\alpha_j, \beta_j, \gamma_j)$$

Step2. Identify the positive aspect of alternative (benefits) and negative aspect alternative (costs) and do the following calculation.

$$\tilde{r}_{ij} = \begin{cases} \tilde{x}_{ij} / \tilde{x}_j^+ = (\frac{a_{ij}}{c_j^+}, \frac{b_{ij}}{b_j^+}, \frac{c_{ij}}{a_j^+}) \\ \tilde{x}_j^- / \tilde{x}_{ij} = (\frac{a_j^-}{c_{ij}}, \frac{b_j^-}{b_{ij}}, \frac{c_j^-}{a_{ij}}) \end{cases}$$

$$N = \begin{bmatrix} A_1 & \begin{bmatrix} \tilde{r}_{11} & \dots & \tilde{r}_{1j} & \dots & \tilde{r}_{1n} \end{bmatrix} \\ \vdots & \vdots \\ A_i & \begin{bmatrix} \tilde{r}_{i1} & \dots & \tilde{r}_{ij} & \dots & \tilde{r}_{in} \end{bmatrix} \\ \vdots & \vdots \\ A_m & \begin{bmatrix} \tilde{r}_{m1} & \dots & \tilde{r}_{mj} & \dots & \tilde{r}_{mn} \end{bmatrix} \end{bmatrix} \quad (7)$$

Step3. the weighted normalized decision matrix, is described by:

$$v = \begin{bmatrix} A_1 & \begin{bmatrix} \tilde{v}_{11} & \dots & \tilde{v}_{1j} & \dots & \tilde{v}_{1n} \end{bmatrix} \\ \vdots & \vdots \\ A_i & \begin{bmatrix} \tilde{v}_{i1} & \dots & \tilde{v}_{ij} & \dots & \tilde{v}_{in} \end{bmatrix} \\ \vdots & \vdots \\ A_m & \begin{bmatrix} \tilde{v}_{m1} & \dots & \tilde{v}_{mj} & \dots & \tilde{v}_{mn} \end{bmatrix} \end{bmatrix} \quad (8)$$

$$\tilde{v}_{ij} = \tilde{r}_{ij} \tilde{w}_j$$

Step4. the ranking method provided by lee & li:

$$M(v_{ij}) = \frac{-a_{ij}^2 - a_{ij} b_{ij} + b_{ij} c_{ij} + c_{ij}^2}{3(c_{ij} - a_{ij})} \quad (9)$$

Positive ideal solution and negative ideal solution:

$$v_j^+ = (a^+, b^+, c^+), v_j^- = (a^-, b^-, c^-)$$

Step5. Calculate the distance.

$$D_0 = d(\tilde{v}_{ij}, \tilde{v}_j^+) = \left( \frac{1}{3} [(a_0 - a^+)^2 + (b_0 - b^+)^2 + (c_0 - c^+)^2] \right)^{1/2}$$

$$D_0 = d(\tilde{v}_{ij}, \tilde{v}_j^-) = \left( \frac{1}{3} [(a_0 - a^-)^2 + (b_0 - b^-)^2 + (c_0 - c^-)^2] \right)^{1/2} \quad (10)$$

Step6. Calculate the relative closeness for each alternative as given by:

$$c_i^+ = \frac{s_i^+}{s_i^+ + s_i^-} \quad (11)$$

$$s_i^+ = \sum_{j=1}^n D_{ij}^+, s_i^- = \sum_{j=1}^n D_{ij}^- \quad (12)$$

Step7. Rank the alternatives according to the relative closeness. The best alternatives are those that have higher value

### 2-3. Promethee, F-Promethee

The PROMETHEE I (partial ranking) and PROMETHEE II (complete ranking) were developed by Brans et al and presented for the first time in 1982 at a conference.

Let A be a set of alternatives and  $g_j^a$  represent the value of criterion  $g_j$  ( $j=1, 2, \dots, J$ ) of alternative  $a$ . Assuming that more is preferred to less,

$$\begin{cases} F_j(a,b)=0 & g_j(a)-g_j(b) \leq q_j \\ F_j(a,b)=1 & g_j(a)-g_j(b) \geq p_j \\ 0 < F_j(a,b) < 1 & q_j < g_j(a)-g_j(b) < p_j \end{cases} \quad (13)$$

Where  $q_i$  and  $p_i$  are indifference and preference thresholds for  $i$ th criterion, respectively. Different shapes (six types) for  $F_j$  have been suggested by brans et al [10]. If  $a$  is better than  $b$  according to  $j$ th criterion,  $F_j(a, b) > 0$ , otherwise  $F_j(a, b) = 0$ .

Using the weights  $w_j$  assigned to each criterion, one can determine the aggregated preference indicator as follows.

$$\Pi(a,b) = \sum w_j f_j(a,b) \quad (14)$$

If the number of alternatives is more than two, overall ranking is done by aggregating the measures of pair wise comparisons. For each alternative  $a$ , the following two outranking dominance flows can be obtained with respect to all the other alternatives  $x$ :

$$\phi^+(a) = \frac{1}{n-1} \sum_{x \in A} \Pi(a, x) \quad (15)$$

The leaving flow is the sum of the values of the arcs leaving node  $a$  and therefore provide a measure of the outranking character of  $a$ . the higher the leaving flow, the better the alternative  $a$ ,

$$\phi^-(a) = \frac{1}{n-1} \sum_{x \in A} \Pi(x, a) \quad (16)$$

The entering flow measures the outranked character. The smaller entering flow, the better alternative  $a$ .

According to PROMETHEE I, action  $a$  is superior to action  $b$  if the leaving flow of  $a$  is greater than the leaving flow of  $b$  and entering flow of  $a$  is smaller than the entering flow of  $b$ .

$a$  outranks  $b$  if :  $\phi^+(a) \geq \phi^+(b)$  and  $\phi^-(a) \leq \phi^-(b)$

Equality in leaving flow and entering flow indicates indifference among the two compared alternatives. In the case where the leaving flows indicate  $a$  is better than  $b$ , while the flows indicate the reverse the actions are considered incomparable.

Action  $a$  and  $b$  are incomparable if:

$$\begin{aligned} \phi^+(a) > \phi^+(b) \quad \text{and} \quad \phi^-(a) > \phi^-(b) \\ \phi^+(a) < \phi^+(b) \quad \text{and} \quad \phi^-(a) < \phi^-(b) \end{aligned}$$

The complete ranking flow given by

$$\phi(a) = \phi^+(a) - \phi^-(a) \quad (17)$$

The higher the net flow, the better the alternative. This is the PROMETHEE II version of the method. In PROMETHEE I, the partial is obtained from the leaving and entering flows. In PROMETHEE II, the consideration of net flow leads to complete ranking.

### 3. Fuzzy Goal Programming

Applying fuzzy set theory (FST) into GP has the advantage of allowing for the vague aspirations of a DM, which can then be qualified by some natural language terms. When vague information related to the objectives are present then the problem can be formulated as a fuzzy goal programming (FGP) problem. The FST in GP was first considered by Narasimhan and Tiwari et al, extended the fuzzy theory to the field of GP[11],[12].

A typical FGP problem formulation can be stated as follows:

Find  $x_i \quad i=1,2,\dots,n$   
to satisfy

$$\begin{aligned} z_m(x_i) &\lesssim \bar{z}_m & m=1,2,\dots,M \\ z_k(x_i) &\gtrsim \bar{z}_k & k=M+1,M+2,\dots,K \\ g_j(x_i) &\leq \tilde{b}_j & j=1,2,\dots,J \\ X_i &\geq 0 & i=1,2,\dots,n \end{aligned} \quad (18)$$

where  $Z_m(x_i)$  is the  $m$ th goal constraint,  $Z_k(x_i)$  the  $k$ th goal constraint,  $\bar{Z}_m(x_i)$  the target value of the  $m$ th goal,  $\bar{Z}_k(x_i)$  the target value of the  $k$ th goal.

In next phase, FGP needs max-min limits ( $u, l$ ) for each goal and after that for each goals, the fuzzy MF's can be developed as follows:

For the  $m$ th objective (approximately less than or equal to):

$$\mu_{z_m}^{(x)} = \begin{cases} 1 & z_m(x) \leq l_m \\ 1 - \frac{z_m(x) - l_m}{u_m - l_m} & l_m \leq z_m(x) \leq u_m \\ 0 & z_m(x) \geq u_m \end{cases} \quad (19)$$

For the  $k$ th objective (approximately greater than or equal to):

$$\mu_{z_k}^{(x)} = \begin{cases} 1 & z_k(x) \geq u_k \\ 1 - \frac{u_k - z_k(x)}{u_k - l_k} & l_k \leq z_k(x) \leq u_k \\ 0 & z_k(x) \leq l_k \end{cases} \quad (20)$$

Using bellman and zadeh's approach  $\mu_F(x)$  can be calculated as follow [13]:

$$\begin{aligned} \mu_F(x) &= \mu_{z_1}^{(x)} \cap \mu_{z_2}^{(x)} \dots \cap \mu_{z_k}^{(x)} \\ &= \min [\mu_{z_1}^{(x)}, \mu_{z_2}^{(x)}, \dots, \mu_{z_k}^{(x)}] \end{aligned} \quad (21)$$

Zimmermann [14] first used the maximin operator of Bellman and Zadeh. By introducing the auxiliary variable, which is the overall satisfactory level of compromise, formulation (21) can be equivalently transformed as:

$$\begin{aligned} \text{Max } Z &= \lambda \\ \lambda &\leq \mu_{z_1} \\ \lambda &\leq \mu_{z_2} \\ &\vdots \\ \lambda &\leq \mu_{z_k} \\ g_j(x_i) &\leq b_j, \quad j = 1, 2, \dots, J \\ x_i &\geq 0, \quad i = 1, 2, \dots, n \\ 0 &\leq \lambda \leq 1 \end{aligned} \quad (22)$$

Generally, the solution procedure of IFGP can be summarized in the following steps:

*Step 1:* Develop a multi objective linear programming model.

*Step 2:* Solve the first objective function as a single objective problem. Continue this process  $K$  times for the  $K$  objective functions. If all the solutions are the same, select one of them as an optimal compromise solution and go to Step 8. Otherwise, go to Step 3.

*Step 3:* Evaluate the objective function at the  $K$ th solution and determine the best lower bound ( $l_k$ ) and the worst upper bound ( $u_k$ ).

*Step 4:* Define the MF of each objective function and also the initial aspiration level.

*Step 5:* Develop problem (22) and solve it as a linear programming problem.

*Step 6:* Present the solution to the DM. If the DM accepts it, go to Step 8. Otherwise, go to step 7

*Step 7:* Evaluate each objective function of the solution. Compare the upper bound of each objective with the new value of the objective function. If the new value is lower than the upper bound, consider this as a new upper bound. Otherwise, keep the old one as is. Repeat this process  $K$  times and go to Step 4.

Step 8: Stop.

#### 4. An Integrated Methodology Proposed

In this study, F-ANP is utilized only in the decision criteria evaluation phase. In order to evaluate suppliers, the rest of the calculations are completed via F-PROMETHEE and F-TOPSIS approaches.

Based on the F-PROMETHEE method and its result, we proposed a methodology that integrates F-ANP, F-PROMETHEE and FGP approaches. Following determining the alternatives and the decision criteria phase, evaluation of criteria with F-ANP is realized. This stage begins with the DMT's linguistic preferences for the pair-wise comparisons of the criteria.

Here, the DMT is asked to compare the evaluation criteria linguistically according to their affect on the realization of the main goal (MG). Then, the linguistic preferences of the DMT are converted to triangular fuzzy numbers.

At the next step, using logarithmic least-square technique and normalization, criteria weights are calculated with the assumption of there is no interdependence between them.

Following the F-ANP calculations, using the criteria weights, F-PROMETHEE calculations are realized. As a first step of F-PROMETHEE, the DMT is asked to determine the generalized criterion type of each criterion as level criterion and the  $q$  and  $p$  values.

At the next step, the differences between each alternative pair for each criterion are calculated. Using these differences, criteria weights and the previous Equations, alternatives' leaving, entering and net flows are calculated.

At the last step of calculations, the complete ranking of alternatives Based on the PROMETHEE II calculations is determined.

The final step of the proposed methodology is to select the outsourcers and to allocate the ordered quantities to them using FGP approach. PROMETHEE II net flows that represent overall scores of suppliers are used as coefficients of an objective function in FGP model. In addition, other objectives which are determined at the beginning of the methodology (e.g. total cost) are included into the model.

By including all objective functions and constraints, the fuzzy model can allocate order quantities among the favorable suppliers.

#### 5. An Application for a Manufacturing Company

This application is realized in a manufacturing company which is located in Tehran, Iran. The capacity of this company becomes insufficient to satisfy orders most of the time. Therefore, the company works with outsourcing firms for some of its products. There are 8

different outsourcing suppliers in the data taken from the system.

There are four different evaluation criteria to evaluate these outsourcers. In the evaluate phase, the decision makers is asked to evaluate alternative considering each criterion. These include qualitative measures that performance are rated by company managers with a five-point scale; {very bad, bad, medium, good, very good}. the definitions are given in table 1.

Tab. 1. Linguistic Scale

Linguistic scale for evaluation	Triangular fuzzy scale
(Very Bad) VB	(0,0,0.15)
(Bad) B	(0.15,0.3,0.5)
(Medium) M	(0.3,0.5,0.65)
(Good) G	(0.5,0.65,0.8)
(Very Good) VG	(0.8,1,1)

The company managers have agreed to evaluate their outsourcers under four main categories. The first evaluation criteria ( $c_1$ ) are the economical considerations.

Purchasing cost, establishment cost, etc. are some of the indicators of this criterion. Second one ( $c_2$ ) is the operational considerations. Reliability, operational feasibility, etc. are some of the indicators of this criterion. Third one ( $c_3$ ) is the environment consideration and the last one ( $c_4$ ) is the strategically considerations. This criterion is related with the strategically issues appears with the selection of a specific alternative.

Following determining the alternatives and the decision criteria phase, evaluation of criteria with F-ANP is realized. This stage begins with the DMT's linguistic preferences for the pair-wise comparisons of the criteria.

Here, the DMT is asked to compare the evaluation criteria linguistically according to their affect on the realization of the main goal (MG). Here, utilizing from table 2, the DMT is compared the criteria which are shown in table 3.

Tab. 2. Linguistic scale for the pair-wise comparisons of the criteria

linguistic scale for importance	Triangular fuzzy scale
just equal(E)	(1,1,1)
equally important( EI)	(0.5,1,1.5)
weakly more important(VMI)	(1,1,5,2)
strongly more important(SMI)	(1.5,2,2.5)
very strongly more important(VSMI)	(2,2.5,3)
absolutely more important(AMI)	(2.5,3,3.5)

**Tab. 3. Pair-wise comparisons of evaluation criteria**

	c <sub>1</sub>	c <sub>2</sub>	c <sub>3</sub>	c <sub>4</sub>
c <sub>1</sub>	E	WMI	SMI	WMI
c <sub>2</sub>	WLI	E	SMI	SMI
c <sub>3</sub>	SLI	SLI	E	SLI
c <sub>4</sub>	WLI	SLI	SMI	E

Then, the linguistic preferences of the DMT are converted to triangular fuzzy numbers using from table 2.

**Tab. 4. Triangular fuzzy values of DMT linguistic comparisons.**

	c <sub>1</sub>	c <sub>2</sub>	c <sub>3</sub>	c <sub>4</sub>
c <sub>1</sub>	(1,1,1)	(1,1.5,2)	(1.5,2,2.5)	(1,1.5,2)
c <sub>2</sub>	(0.5,0.67,1)	(1,1,1)	(1.5,2,2.5)	(1.5,2,2.5)
c <sub>3</sub>	(0.4,0.5,0.67)	(0.4,0.5,0.67)	(1,1,1)	(0.4,0.5,0.67)
c <sub>4</sub>	(0.5,0.67,1)	(0.4,0.5,0.67)	(1.5,2,2.5)	(1,1,1)

Using the logarithmic least-squares method, for calculating triangular fuzzy weights can be given as follows:

**Tab. 5. Quantity that calculated by using of Eq. 10.**

b	c <sub>1</sub>	c <sub>2</sub>	c <sub>3</sub>
c <sub>1</sub>	1.106	1.45	1.77
c <sub>2</sub>	1.03	1.28	1.5
c <sub>3</sub>	0.503	0.6	1.19
c <sub>4</sub>	0.74	0.9	1.13

At the next step, using logarithmic least-square technique (Eq.4) and normalization, criteria weights are calculated with the assumption of there is no interdependence between them (table 6).

**Tab. 6. Criteria weights neglecting the interdependences between them.**

	c <sub>1</sub>	c <sub>2</sub>	c <sub>3</sub>
c <sub>1</sub>	0.26	0.34	0.42
c <sub>2</sub>	0.24	0.3	0.37
c <sub>3</sub>	0.12	0.14	0.28
c <sub>4</sub>	0.17	0.21	0.27

At the last step of F-ANP, using least-square technique and Yager Index, the weights of the criteria is calculated. According to the results, the weights of the C1, C2, C3, and C4 are 0.34, 0.3, 0.18 and 0.22, respectively. Following the F-ANP calculations, using

the criteria weights, F-PROMETHEE and F-TOPSIS calculations are realized. First, for evaluating suppliers, Fuzzy TOPSIS is used. And then in order to use integrated approach, F-PROMETHEE is presented. As a first step, the DMT is asked to evaluate the alternatives linguistically (table 7) and then the linguistic evaluations are converted to triangular fuzzy numbers (table 8).

**Tab. 7. Alternatives linguistic evaluations by DMT.**

	c <sub>1</sub>	c <sub>2</sub>	c <sub>3</sub>	c <sub>4</sub>
s <sub>1</sub>	VB	M	G	M
s <sub>2</sub>	G	G	G	G
s <sub>3</sub>	VG	VG	G	VG
s <sub>4</sub>	M	VG	VG	G
s <sub>5</sub>	VB	M	M	M
s <sub>6</sub>	VG	M	M	VG
s <sub>7</sub>	M	M	M	M
s <sub>8</sub>	B	VB	M	M

**Tab. 8. Triangular fuzzy values of alternatives' linguistic evaluations**

	c <sub>1</sub>	c <sub>2</sub>	c <sub>3</sub>	c <sub>4</sub>
s <sub>1</sub>	(0,0,0.15)			(0.3,0.5,0.65)
s <sub>2</sub>	(0.5,0.65,0.8)	---		(0.5,0.65,0.8)
s <sub>3</sub>				
s <sub>4</sub>				
s <sub>5</sub>				
s <sub>6</sub>				
s <sub>7</sub>				
s <sub>8</sub>	(0.15,0.3,0.5)	...		(0.3,0.5,0.65)

According to the formulas and equations presented in section 2.2. A FUZZY TOPSIS method used to evaluate suppliers. Calculation and evaluation of suppliers are shown below.

**Tab. 9. The weighted normalized decision matrix**

	c <sub>1</sub>	c <sub>2</sub>	c <sub>3</sub>	c <sub>4</sub>
s <sub>1</sub>	(0,0,0)			(0.051,0.1,0.21)
s <sub>2</sub>	(0,0,0.126)	---		(0.08,0.13,0.27)
s <sub>3</sub>				
s <sub>4</sub>				
s <sub>5</sub>				
s <sub>6</sub>				
s <sub>7</sub>				
s <sub>8</sub>	(0,0,0.42)	---		(0.051,0.1,0.21)

Lee & Li ranking method used and the results are shown in table 10.



Tab. 10. Lee &amp; li ranking method

	$c_1$	$c_2$	$c_3$	$c_4$
$s_1$	0			0.12
$s_2$	0.042	...		0.16
$s_3$				
$s_4$	...		...	...
$s_5$				
$s_6$				
$s_7$				
$s_8$	0.14	...		0.12

Finally, in order to determine the best supplier will calculate the relative closeness for each alternative. The result show that the value  $c_3^+ = 0.89$ , so we can conclude that the third supplier is the best provider.

Because the proposed integrated approach based on F-ANP, F-PROMETHEE and FGP approaches, calculation and tables related to an F-PROMETHEE is described.

In order to find the overall performance of outsourcers, the performances belonging to each criterion should be integrated.

For integration, company managers (DMS) determined the weights, preference functions, and indifference and preference thresholds for each criterion which are listed in table 11. At the next step, the differences between each alternative pair for each criterion are calculated. Using these differences, criteria weights and leaving, entering and net flows are calculated (table 12).

Tab. 11. Parameters for PROMETHEE analysis

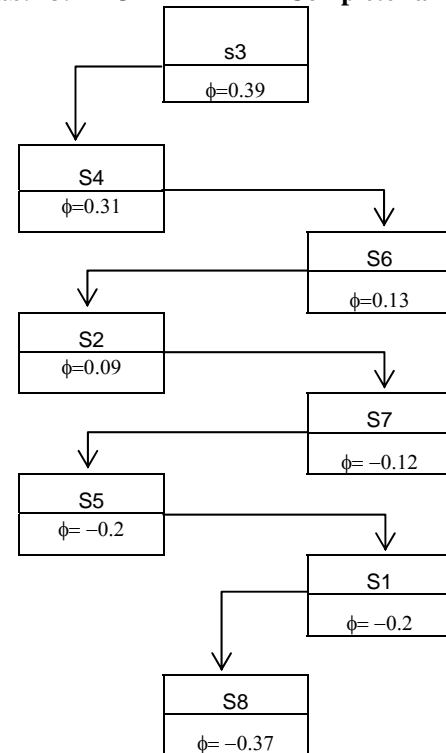
	$C_1$	$C_2$	$C_3$	$C_4$
Weights	0.34	0.3	0.18	0.22
Preference function	Level	linear	v-shape	v-shape
Indifference threshold	0.5	0.03	...	...
Preference threshold	2	0.5	0.05	0.2

Tab. 12. Alternatives' leaving, entering and net flows.

	ROWS					
	S <sub>1</sub>	S <sub>2</sub>		S <sub>7</sub>	S <sub>8</sub>	$\phi^+$
S <sub>1</sub>	0	0	...	0	0.27	0.04
S <sub>2</sub>	0.17	0		0	0.3	0.09
S <sub>3</sub>	0.675	0		0.505	0.654	0.39
S <sub>4</sub>	0.285	0		0.465	0.48	0.31
S <sub>5</sub>	0	0	...	0	0.28	0.04
S <sub>6</sub>	0.39	0		0.22	0.67	0.23
S <sub>7</sub>	0	0		0	0.28	0.04
S <sub>8</sub>	0.17	0		0.17	0	0.048
$\phi^-$	0.24	0	...	0.16	0.42	

At the last step of calculations, the complete ranking of alternatives is determined (table13). Based on the PROMETHEE II calculations, S3 is the best, S8 is the worst alternative.

Tab. 13. PROMETHEE II-Complete ranking



The overall scores achieved by PROMETHEE II are set as the weights of outsourcers and integrated in an additive fashion. The objective function developed is the objective of FGP model in final selection phase.

### 5-1. Modeling and Final Selection

In the modeling phase, four objectives are developed by the company managers. The first objective function is simply the weighted sum of quantities ordered from each outsourcer.

In other words, it is a measure of working with good suppliers which are candidate strategic partners. Hence this objective is named as total value of strategic partnership (TVSP).

The weight set is the set of net flows calculated by PROMETHEE. The goal is to maximize this summation. In other words to set the ordered quantities to the highest performing suppliers as much as possible.

PROMETHEE II net flow of the outsourcer  $j$ ,  $W_j$ :

$W_j[-0.2, 0.09, 0.39, 0.31, -0.2, 0.13, -0.12, -0.37]$

$Y_{ijk}$ : units of items  $i$  ordered from outsourcer  $j$ , month  $k$

$$\max z_1 = \sum_i \sum_j \sum_k w_j y_{ijk}$$

$i: 1, 2, \dots, 14$ : Number of items

$j:1,2,\dots,8$ : Number of outsourcing suppliers

The second objective function gives the number of units accepted in the incoming quality control. All received lots go through inspection in the incoming quality control. Some lots are rejected here. The objective is to maximize the number of accepted units as much as possible. This objective function is calculated through the ratio of accepted units in the incoming quality control ( $K$ ):

$$K_{ij} = \frac{\text{Number of accepted units of item } i \text{ (delivered by outsourcer } j)}{\text{Total units (of item } i) \text{ delivered by outsourcer } j}$$

$$\max z_2 = \sum_i \sum_j \sum_k k_{ij} y_{ijk}$$

The third objective is the measure of units arriving on-time.

$$l_{ij} = \frac{\text{Number of units of item } i \text{ on-time (delivered by outsourcer } j)}{\text{Total units (of item } i) \text{ delivered by outsourcer } j}$$

$$\max z_3 = \sum_i \sum_j \sum_k l_{ij} y_{ijk}$$

The fourth and the last objective is to minimize the total purchasing cost of all orders.

Cost<sub>ij</sub> : purchasing cost of item  $i$  from outsourcer  $j$ .

$$\min z_4 = \sum_i \sum_j \sum_k \text{cost}_{ij} y_{ijk}$$

Constraints:

This constraint assures that demands are satisfied. The sum of ordered quantities to the suppliers should exactly be equal to the quantity demanded for all materials.

$$\sum_i y_{ijk} = QD_{ik}$$

This Constraint is the set of capacity constraints. The quantity ordered to a supplier in a month should not be greater than its monthly capacity.

$$\sum_i y_{ijk} \leq MR_j$$

This constraint ensures that two outsourcers should be selected for every item in each month. If a supplier is not selected, quantity ordered to that supplier should be zero.

$$\sum_j x_{ijk} = 2$$

These constraints in 31 ensure this property ( $M$  is a very large number.). Also, if a supplier is selected.

$$x_{ijk} \times M \geq y_{ijk}$$

Once the integer programming model is developed, it is solved with each of the objective functions by themselves. In other words first  $Z_1$  is set as the objective and the model is solved. Then,  $Z_2$ ,  $Z_3$  and  $Z_4$  are all set as objective one by one and solved. For each solution the value of the objective and the other  $Z$  function values are recorded. By this way, the payoff table is constructed which is given in table 13.

**Tab. 13. Pay-off table**

	$Z_1$	$Z_2$	$Z_3$	$Z_4$
Max $z_1$	5123 <sup>a</sup>	2242	4220	1959 <sup>b</sup>
Max $z_2$	20235	21243 <sup>a</sup>	18820	16950 <sup>b</sup>
Max $z_3$	9152	5244 <sup>b</sup>	16437 <sup>a</sup>	6145
Min $z_4$	16323 <sup>b</sup>	17150	18672	19542 <sup>a</sup>

Looking at the figures in table 12, the best lower bound ( $lk$ ) and the worst upper bound ( $uk$ ) are determined. Then the membership functions of each objective can be defined as follows:

$$\mu_{z_1}^{(x)} = \begin{cases} 1 & z_1(x) \geq 5123 \\ 1 - \frac{5123 - z_1(x)}{5123 - 1959} & 1959 < z_1(x) < 5123 \\ 0 & z_1(x) \leq 1959 \end{cases}$$

$$\mu_{z_2}^{(x)} = \begin{cases} 1 & z_2(x) \geq 21243 \\ 1 - \frac{21243 - z_2(x)}{21243 - 16950} & 16950 < z_2(x) < 21243 \\ 0 & z_2(x) \leq 16950 \end{cases}$$

$$\mu_{z_3}^{(x)} = \begin{cases} 1 & z_3(x) \geq 16437 \\ 1 - \frac{16437 - z_3(x)}{16437 - 5244} & 5244 < z_3(x) < 16437 \\ 0 & z_3(x) \leq 5244 \end{cases}$$

$$\mu_{z_4}^{(x)} = \begin{cases} 1 & z_4(x) \leq 16323 \\ 1 - \frac{z_4(x) - 16323}{19542 - 16323} & 16323 < z_4(x) < 19542 \\ 0 & z_4(x) \geq 19542 \end{cases}$$

Then the FGP model is developed.

$$\text{Max } Z = \lambda$$



s.t

$$\lambda \leq \mu_{z_1}$$

$$\lambda \leq \mu_{z_2}$$

$$\lambda \leq \mu_{z_3}$$

$$\lambda \leq \mu_{z_4}$$

## 6. Computational Results

At first iteration of the solution approach the results achieved are given in table 14.

**Tab. 14. Final Results**

Objective function	Value
$Z_1$	4932
$Z_2$	20481
$Z_3$	15344
$Z_4$	18940

After this iteration, if the DM is not satisfied with the TVSP objective. At this step, the lower bound is revised with the value achieved for TVSP. The model is resolved with the new parameters. The procedure is followed until the DM is satisfied.

## 7. Discussion

Validation of model using real data, it took many attempts but unfortunately the necessary conditions for implementing the method was not provided.

Explain the superiority of the method presented in the following article was added to the existing, the issue of choice of suppliers in various business areas. The Organization for the achievement of quality objectives in the decision-making and the use of fuzzy methods in terms of quality, using these methods can help in choosing a supplier of quality goals and objectives so as to be effective simply choose the supplier.

As mentioned, various measures in question are the supplier of choice. Among the various approaches presented in recent papers using fuzzy network analysis can be seen. The method presented in the article is trying to use the first phase of network analysis and evaluation of fuzzy criteria weights given to them and then PROMETHEE fuzzy methods used in industry and in various articles.

The proposed method uses fewer calculations and yet Comprehensive deals to rank suppliers.

## 8. Conclusion

In today's highly competitive and global operating environment, due to the high variety of customer demands companies have been forced to focus on supply chain management. Supplier selection and evaluation are becoming recognized as a strategic and important component of supply chain strategy.

In this study, an outsourcer evaluation and management system is developed for a manufacturing company by use of Fuzzy goal programming (FGP). A first phase of the methodology evaluation criteria for outsources and the objectives of the company are determined. Considering the fuzziness in the decision data, linguistic variables that can be expressed in generalized fuzzy number are used. The propose approach is utilized from fuzzy sets, Analytic Network Process (ANP) and Fuzzy TOPSIS and Preference Ranking Organization method for enrichment evaluations (PROMETHEE) approaches.

At the last step for this phase, base on the PROMETHEE II, the complete ranking of alternatives is determined. At the second phase the FGP model developed selects the most appropriate outsourcers suitable to be strategic partners with the company and simultaneously allocates the quantities to be ordered to them. A comparison study may be realized with the other proposed approaches such as ELECTRE, fuzzy ELECTRE, fuzzy AHP, etc. can be appropriate in future research.

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