



Determination of a Desirable Inventory Policy in a three Echelon Multilayer Supply Chain with Normal Demand

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ABSTRACT

Inventory control is one of the most important issues in supply chain management. In this paper, a three-echelon production, distribution, inventory system composed of one producer, a set of wholesalers and retailers is considered. Customers' demands can be approximated by a normal distribution and the inventory policy is a kind of continuous review (R, Q). In this paper, a model based on standard cost structure of inventory systems is developed and a heuristic algorithm is designed to optimize the developed model. The application of model is examined in a series of designed experiments that are compared with simulation results. These comparisons verify the validity of the model. Regarding to real complexities in three-echelon systems analysis, the proposed method can have a wide application in practical problems with the same considerations and assumptions. In addition, this method can be used to approximate those systems that follow a Poisson demand.

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1. Introduction

Supply chain has become the major and dominant paradigm of business and competition. More than 40 years ago, Forrester in 1958 introduced the elements of a theory that today is called supply chain management (SCM) [1]. The concept of supply chain is whether that many experts believed that competition is transferred from companies to chains. The extension of SCM is

whether that in the past decade large international corporations such Cisco, Dell Computer, Gillette, Kodak, LEGO, Motorola, Sony, 3M, Xerox and Wal-Mart have implemented SCM. Moreover, international consultancy firms such as IDM business Consulting Services, A.T. Kearney, Cap Gemini, etc. have adopted SCM as an important business area.

In addition, a large number of universities and business schools have included SCM courses in their curricula [2]. Many scholars and experts gave different definitions for SCM that depend on their viewpoint and attitude. Walker [3] defined supply chain as "the global network is used to deliver goods and services from raw

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material to final consumers by a designed flow of information, physical goods and funds”.

There are many challenges that are latent in supply chain concept. The decisions made in SCM are mainly about the flows between chain stages. Therefore, many scholars express the challenges and problems that SCM have tried to answer them. The scholars point to the time dimensions and effects of these challenges over performance of supply chains. From the viewpoint of these theories, inventory control is one of the main challenges of SCM [4, 5, 6, and 7]. These theories have emphasized that the inventory control and its policies are the most important problems of supply chains. The main complexity of supply chain inventory control is to achieve coordination between chain stages.

Supply chain inventory control problem can be analyzed in the context of multi echelon inventory systems which is a generalized form of classic inventory model that its original form is proposed by Harris in 1913 [8]. According to classical inventory control, two main questions that these models try to answer are when and how much to order an item such that the warehouse costs is minimized and satisfies the demand synchronously.

The researchs in the field of multi-echelon inventory systems formally are originated in 1950s and 1960s. Clark and Scarf [9] have done one of the most prominent initial works in this field, which in many researches are known as the founders of this field. One of the other prominent works in this field is Sherbrooke's [10] METRIC method which is used greatly in further works. Continuous review models of multiechelon inventory systems in 1980s concentrated more on repairable items in a Depot-Base system than on consumable items.

Lee and Moinsade [11, 12] examined the problem of determination of optimal order quantity and inventory levels in such systems.

Axsater [13, 14, and 15] considered a two echelon system and proposed different solutions to such a system with different assumptions on its demand pattern or inventory policy. Cachon [16] examined a two-echelon system with random and discrete demand, batch ordering, periodic review and constant transferring time.

Grubbström and Wang [17] examined a multiechelon production – inventory system and considered the approximation of net present value as the objective function. Ben-Daya and Hariga [18] also considered an integrated production – inventory system with random demand and a linear relation between lead time and batch quantity. Seifbarghy and Akbari joker [19] examined a two echelon inventory system, consists of one central warehouse and N retailers, with poisson demand and lost sales. Zhao et al. [20] proposed “fixed partition and power of two” approach for a three-echelon system consists of one manufacturer, one central warehouse and several retailers.

Gao and Wang [21] considered a three echelon system and tried to minimize its total cost. Lejeune and Margot [22] also considered an integer linear programming model for a three-echelon inventory – production – distribution system. Hajiaghaei and Sajadifar [23] also studied a three echelon system consists of two warehouse and N retailers.

Also, Angelus [24], Niranjan and Ciarallo [25], and Hajiaghaei-Keshteli et al. [26] are instances of the latests studies in this field.

Some of authors used normal distribution as an approximation of demand in multi echelon systems. In this paper, a multiechelon–multilayer production–inventory–distribution system is considered in which customer demands in retailers can be approximated by a normal distribution. To analyse such a system, cost function of each level of the system, including manufacturer, wholesalers and retailers are estimated by using approximation of each stages demand and inventory level distribution. The rest of the paper is organized as follow.

The section 2 consists of the problem definition and introduction of modeling parameters and variables. The problem modeling and optimization are expressed in section 3. In section 4, statistical design that is used to verify the model is explained and the results of simulation studies, which are designed and done based on these experiments, are shown and compared with the proposed model's results. The paper is ended with conclusion and suggestions for future researchs in section 5.

2. Problem Definition

Figure 1 illustrates the problem under the study in this paper. As can be seen in Fig.1, the system consists of a central manufacturer that serves N wholesalers. These wholesalers themselves serve several retailers. There are no interrelationships between the retailers of one wholesaler with other wholesalers or their retailers. It is assumed that the customers demand, that retailers faced with, can be approximated based on a normal distribution.

Also, the inventory policy of retailers and wholesalers follows a continuous review (R, Q) policy. Unsatisfied demands are assumed backordered with a certain cost for each unsatisfied unit in retailers and wholesalers, and these backordered orders will be satisfied based on a first come, first serve (FCFS) approach. Also, there is no backordering cost in manufacturer.

It is assumed that the batch sizes of retailers and wholesalers are known and are determined based on a deterministic model, as many similar papers such as Axsater [13], and Seifbarghy and Akbari joker [19] done before to simplify the problem. Another assumption is that the retailers and wholesalers lead-time is constant.

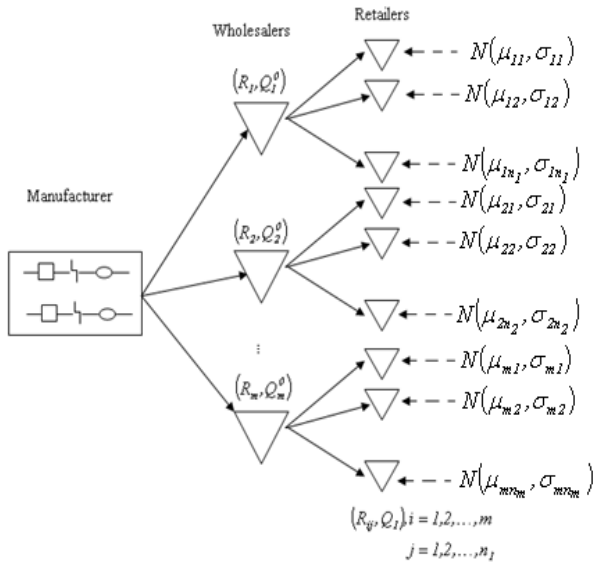


Fig. 1. Three echelon – multilayer inventory system

The following notations will be used:

- m : Number of wholesalers;
- n_i : Number of retailers that ordered to wholesaler $i = 1, 2, \dots, m$;
- $N(\mu_{ij}, \sigma_{ij})$: Demand distribution of j th retailer's of wholesaler i that follows a normal distribution with mean μ_{ij} and variance σ_{ij} ;
- R_{ij} : Reorder point of j th retailer of wholesaler i ;
- Q_i : Common batch size of retailers of wholesaler i ;
- R_i : Reorder point of wholesaler i ;
- Q_i^0 : Batch size of wholesaler i ;
- IC_{ij} : Holding cost per unit at j th retailer of wholesaler j ;
- π_{ij} : Penalty cost per backordered unit at j th retailer of wholesaler j ;
- τ_{ij} : Lead time of orders at j th retailer of wholesaler j ;
- IC_i : Holding cost per unit at wholesaler j ;
- π_i : Penalty cost per backordered unit at wholesaler j ;
- τ_i : Lead time of orders at wholesaler j ;
- IC_0 : Holding cost per unit at manufacturer;
- Q_0 : Production batch size of Manufacturer;
- P : Processing rate of manufacturer;

The problem here is to determine retailers and wholesalers reorder points, while their batch size is constant, such that their holding and backordering costs are to be minimized and to determine production batch size of manufacturer such that its costs are to be minimized.

3. Modeling Process

In this section, the modeling process that is used to model the above problem is explained.

3-1. Retailers Model

Taking into account one of the retailers in fig.1, like j th retailer of wholesaler i . the method used here is taken from axsater [27]. In this system, each retailer can be analysed as an independent depot that faces with customers demand. The following notations are defined:

$$(x)^+ = \max(x, 0) \tag{1}$$

$$(x)^- = \max(-x, 0) \tag{2}$$

By attention to inventory level (IL), holding costs will be $IC_{ij}(IL)^+$ and backordering cost will be $\pi_{ij}(IL)^-$.

Since $x^+ - x^- = x$, the sum of costs can be expressed as follows:

$$IC_{ij}(IL)^+ + \pi_{ij}(IL)^- = -\pi_{ij}(IL) + (IC_{ij} + \pi_{ij})(IL)^+ = IC_{ij}(IL) + (IC_{ij} + \pi_{ij})(IL)^- \tag{3}$$

Here, $E(IL)^-$ is the expected value of backorders. If μ shows the mean demand in per time unit, then the waiting time for each order is equal to $E(IL)^- / \mu$. This formula shows an application of little formula in queuing theory. Based on third relation in Eq. (3), the expected costs of this retailer are:

$$C(r_{ij}) = IC_{ij}E(IL) + (IC_{ij} + \pi_{ij})E(IL)^- = IC_{ij}(R_{ij} + Q_i/2 - \mu'_{ij}) + (IC_{ij} + \pi_{ij}) \int_{-\infty}^0 F_{ij}(x) dx \tag{4}$$

Where, μ'_{ij} is the average demand in lead-time of retailer ($\mu_{ij}\tau_{ij}$), and $F_{ij}(x)$ is distribution function according to Eq. (5).

$$F(x) = P(IL \leq x) = \frac{1}{Q} \int_R^{R+Q} \left[1 - \phi\left(\frac{u-x-\mu'}{\sigma'}\right) \right] du = \frac{1}{Q} \int_R^{R+Q} \left[-G'\left(\frac{u-x-\mu'}{\sigma'}\right) \right] du \tag{5}$$

In which, $G(x)$ is called normal distribution's loss function as follows and its values are given in Axsater [27]:

$$G(x) = \int_x^\infty (v-x)\phi(v)dv = \phi(x) - x(1-\phi(x)) \tag{6}$$

By inserting related relations and definitions, the following relation is achieved:

$$C(r_{ij}) = IC_{ij}(R_{ij} + Q_i/2 - \mu'_{ij}) + (IC_{ij} + \pi_{ij}) \frac{\sigma'_{ij}}{Q_i} \int_{R_{ij}}^{R_{ij}+Q_i} G\left(\frac{u-\mu'_{ij}}{\sigma'_{ij}}\right) du \tag{7}$$

That σ'_{ij} is lead-time demand's variance. Now, the $H(x)$ function is defined as follows that its values are given in Axsater [27]:

$$H(x) = \int_x^\infty G(v)dv = \frac{1}{2} [(x^2 + 1)(1-\phi(x)) - x\phi(x)] \tag{8}$$

Since, $H'(x) = -G(x)$, $H(x)$ is decreasing and convex. By inserting Eq. (8), the cost function (7) can be rewritten as follow:

$$C(r_{ij}) = IC_{ij} \left(R_{ij} + \frac{Q_i}{r} - \mu'_{ij} \right) + (IC_{ij} + \pi_{ij}) \frac{\sigma'_{ij}}{Q_i} \left[H\left(\frac{R_{ij} - \mu'_{ij}}{\sigma'_{ij}}\right) - H\left(\frac{R_{ij} + Q_i - \mu'_{ij}}{\sigma'_{ij}}\right) \right] \tag{9}$$

Since $H(x)$ is a convex function, the above function is convex related to R_{ij} . Then:

$$\frac{dC}{dR_{ij}} = IC_{ij} + (IC_{ij} + \pi_{ij}) \frac{\sigma'_{ij}}{Q_i} \left[G\left(\frac{R_{ij} + Q_i - \mu'_{ij}}{\sigma'_{ij}}\right) - G\left(\frac{R_{ij} - \mu'_{ij}}{\sigma'_{ij}}\right) \right] \tag{10}$$

Because the dC/dR_{ij} is increasing related to R, C is a convex function of R that its optimal value can be derived by letting $dC/dR_{ij} = 0$ that can be achieved based on G function's value and by a simple search algorithm. For each retailer, the fill rate can be measured as follows:

$$FR = 1 - F(0) = 1 - \frac{\sigma'_{ij}}{Q_i} \left[G\left(\frac{R_{ij} - \mu'_{ij}}{\sigma'_{ij}}\right) - G\left(\frac{R_{ij} + Q_i - \mu'_{ij}}{\sigma'_{ij}}\right) \right] \tag{11}$$

Based on Eq. (10) and (11), the fill rate in optimal solution is:

$$FR = \pi_{ij} / (IC_{ij} + \pi_{ij}) \tag{12}$$

3-2. Wholesalers Model

The second stage under consideration is related to wholesalers. Suppose the wholesaler i which serves n_i retailers. The main point here is to determine the wholesaler's demand distribution. If $IL(t+L)$ is the wholesaler's inventory level at time $t+L$, $IP(t)$ will be its inventory position at time t , and $D(t,t+L)$ is random demand at time interval $(t, t+L)$, then the following relation is hold:

$$IL(t+L) = IP(t) - D(t, t+L) \tag{13}$$

In fact, by letting an arbitrary time t , inventory level at time $t+L$ will be inventory position at time t minus demand in $(t, t+L)$ interval. Also, according to Axsater [13, 15, 27], inventory position of wholesaler at time t , $IP(t)$, uniformly is distributed on $[R_0 + 1, R_0 + 2, \dots, R_0 + Q_0]$.

Now, let the j th retailer that orders to wholesaler i . this retailer's demand follows a normal distribution $N(\mu_{ij}, \sigma_{ij})$ and its inventory policy is (R_{ij}, Q_j) . Now, the probability that a certain retailer fill an order in a period of τ_i time unit is equal to $F(R_{ij})$ that can be calculated from Eq. (5) by letting $\mu' = \tau_i \mu_{ij}$ and $\sigma' = \tau_i \sigma_{ij}$. Now, the random variable O_j is defined as follows:

$$O_j = \begin{cases} Q_i & \text{if retailer } j \text{ fill an order in a} \\ & \text{time interval with length } \tau_i \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

The probability that a certain retailer fill an order in this time interval is equal to $F(R_{ij})$. In fact:

$$P_{ij} = P(O_j = Q_i) = 1 - F(R_{ij}) \tag{15}$$

Now, the random variable X_j is defined as follow:

$$X_j = \begin{cases} 1 & O_j = Q_i \\ 0 & O_j = 0 \end{cases} \tag{16}$$

This random variable has a bernolli probability function with success probability P_{ij} . So, the number of wholesaler i 's order is equal to $D_i = \sum_{j=1}^{n_i} X_j$, which is the sum of n_i bernolli random variables with different success probabilities. Probability distribution of D_i is in fact the n-fold convolution of a set of bernollian variables. The probability that n retailers ordered is:

$$\prod_{j=1}^{n_i} p_j^{x_{ij}} \cdot (1 - p_j)^{1-x_{ij}}, \sum_{j=1}^{n_i} x_{ij} = n \tag{17}$$

Now, the probability $P(D_i = n)$ is the summation of $\binom{n_i}{n}$ different combinations that in which n retailers ordered and each combination's probability is calculated based on Eq. (17). Note that variable D_i can take its values between $\{1, 2, \dots, n_i\}$ and it is notable that D_i can take these values in 2^{n_i} different configurations. So, the wholesaler i 's demand, Wd_i , take its values from $\{Q_i, 2Q_i, \dots, n_i Q_i\}$ and its expected value is:

$$E(Wd_i) = Q_i \sum_{j=1}^{n_i} p_j \tag{18}$$

Then, the distribution of inventory level will be obtained. First, it is necessary to note that inventory position takes its values from the set of $J = \{R_i + 1, R_i + 2, \dots, R_i + Q_i^0\}$. Based on Eq. (13), the probability function $P(IL = j)$ will be obtained as:

$$P(IL = j) = P(IP = k) \sum_{k=\max(R_i+1, j)}^{R_i+Q_i^0} P(D_i(L) = k - j) = \frac{1}{Q_i^0} \sum_{k=\max(R_i+1, j)}^{R_i+Q_i^0} P(Wd_i = k - j) \tag{19}$$

by attending to first relation in EQ. (3), wholesaler's average cost is equal to:

$$C(w_i, R_i) = IC_i \cdot E(IL)^+ + \pi_i \cdot E(IL)^- = IC_i \sum_{j \in J^+} jP(IL = j) - \pi_i \sum_{j \in J^-} jP(IL = j) \tag{20}$$

Note that in Eq. (20), $E(IL)$ is the difference between mean inventory position and average demand in lead-time. Also, $J^+ \subseteq J$ is the set of positive values and $J^- \subseteq J$ is the set of negative values of inventory

position. In this case, the lower bound of wholesaler's reorder point is $-n_i Q_i$ and the optimal reorder point can be found by a simple search procedure and letting different values of $R_i \geq -n_i Q_i$ in Eq. (20) that in each step, reorder point is increased by Q_i^0 unit and finally the reorder point with lowest cost is chosen. For per wholesaler, the fill rate can be calculated as follows:

$$FR = P(IL > 0) = \sum_{j \in J^+} jP(IL = j) \tag{21}$$

It is noted that distribution function (17) can be approximated by a binomial distribution, when all retailers have a same demand pattern. In addition, this distribution can be approximated based on a normal distribution with mean μ_i (Eq. (22)) and variance σ_i^2 (Eq. (23)), when the number of retailers is increased.

$$\mu_i = Q_i \sum_{j=1}^{n_i} p_j \tag{22}$$

$$\sigma_i^2 = \sum_{j=1}^{n_i} p_j (1 - p_j) \tag{23}$$

3-3. Manufacturer Model

The last stage in model development is manufacturer model. According to classical economic production quantity (EPQ), the production costs of manufacturer will be [27]:

$$C_P = \frac{Q_0(1 - \tilde{d}/p)}{2} IC_0 + \frac{\tilde{d}}{p} A_0 \tag{24}$$

Where, \tilde{d} is the expected value of manufacturers demand and, according to Eq. (22), obtained

as $\tilde{d} = \sum_{i=1}^m \mu_i$. By replacing this value and taking

derivative of Eq. (24) and let it equal zero, the optimal production batch quantity will be obtained:

$$Q_0^* = \sqrt{2A_0 \cdot E(\tilde{d}) / IC_0 (1 - E(\tilde{d})/p)} \tag{25}$$

4. Model Verification

In order to determine the power of approximations that are used in this paper, a set of problems are designed and solved with proposed approach. Since there are not any previous numerical problems as a reference to compare presented approach with, a problem set is developed to verify the presented approach. These experiments are designed based on

two factorial designs in the context of design of experiments (DOE). Thirteen factors are considered as affective on the problem. Among these factors, the holding cost of wholesalers and retailers is considered equal to one and the lead-time of them is considered as one unit of time. Table1 illustrates the high and low levels of remaining 9 factors.

When the quantity of factors is high, the numbers of experiments that must be designed are increased dramatically.

In this experiment, by 9 factors, the numbers of designed experiments based on 2 factorial designs are 512 experiments. Therefore, a fractional factorial design of 2^{9-4}_{IV} is used. Such a design is constituted by development of a 2^5 full factorial design by A, B, C, D and E factors as the basic design and then the four columns of F=BCDE, G= ACDE, H=ABDE, and J=ABCE are attached to design. The generators of this design are I=BCDEF=ACDEG=ABDEH=ABCEJ [28]. In the next step, 32 problems are designed and solved

with proposed approach. Every problem's solution includes the values of reorder points for each retailers and suppliers with the production batch size of manufacturer (because of the limitation in space, these results are not presented here). On the other hand, problems with above configurations are simulated using ARENA and the resulting costs of simulated systems are compared with costs of proposed method. Table2 illustrates the costs of proposed method and resulted costs of problems simulation.

According to table2, all of the resulted costs based on proposed method are in the interval of "average simulation cost \pm half width" that can be interpreted as an evidence of proposed method's credit.

Also, if a 95% confidence interval of paired observation is calculated for the mean of difference between proposed method and simulation costs, this interval will be $-7.43 \leq \mu_d \leq 7.35$ that includes zero and shows that the assumption of difference between costs is nonsignificant:

Tab. 1. Positive and negative levels of experiments' factors

Code	Factor	Positive level (+)	Negative level (-)
A	Number of manufacturer's production lines	6	1
B	Manufacturer's holding cost	10	1
C	Number of work stations in each line	15	5
D	Production capacity of each station	50	10
E	Number of wholesalers	13	2
F	Wholesalers' backordering cost	100	50
G	Number of retailers for each wholesaler	$uni(6,15)^*$	$uni(2,5)$
H	Demand rate for each retailer	$uni(2,5)$	$uni(0.5,2)$
J	Retailers' backordering cost	150	75

**uni*: uniform distribution

Tab. 2. Comparison of the results of proposed method and simulation

Run	Costs			Run	Costs		
	Approximation	Simulation	Half width		Approximation	Simulation	Half width
1	8336.17	8358.02	45.39	17	640.1635	635.13	4.32
2	4530.403	4502.01	72.25	18	923.0758	919.05	9.85
3	4274.048	4290.20	45.49	19	667.9362	667.1	5.20
4	4681.601	4688.50	39.05	20	497.1857	492.18	9.34
5	3104.086	3099.20	12.30	21	396.823	398.823	8.85
6	2999.623	2980.23	68.19	22	352.0156	357.14	12.48
7	2768.365	2753.00	32.32	23	542.23	541.23	4.21
8	3899.498	3890.40	24.10	24	273.8484	271.84	3.60
9	2873.53	2905.40	64.03	25	475.6265	477.64	4.49
10	2595.284	2634.04	45.12	26	543.5973	542.12	4.76
11	2203.094	2235	37.18	27	553.5316	559.65	7.22
12	3381.947	3401.00	53.23	28	333.3769	336.00	5.23
13	2653.491	2660.61	39.16	29	216.3747	215.34	4.32
14	1803.115	1816.11	43.12	30	330.558	332.5	3.21
15	3161.663	3179.53	29.10	31	310.0177	309.2	4.56
16	2513.898	2492	35.92	32	241.0304	238.3	61

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