**Using Imperialist competitive algorithm optimization in multi-response nonlinear programming**

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| **Key words** |  | **Abstract** |
| *Multi-response Optimization;**Imperialist competitive;**Genetic Algorithm;**Response surface methodology;* |  | *The quality of manufactured products is characterized by many controllable quality factors. These factors should be optimized to reach high quality products. In this paper we try to find the controllable factors levels with minimum deviation from the target and with a least variation. To solve the problem a simple aggregation function is used to aggregate the multiple responses functions then an imperialist competitive algorithm is used to find the best level of each controllable variable. Moreover the problem has been better analyzed by Pareto optimal solution to release the aggregation function. Then the proposed multiple response imperialist competitive algorithm (MRICA) has been compared with Multiple objective Genetic Algorithm. The experimental results show efficiency of the proposed approach in both aggregation and non aggregation methods in optimization of the nonlinear multi-response programming.* |

1. ***Introduction***

Multi Response Optimization methods can be classified into three basic groups as follow:

The first one is desirability functions proposed by Derringer and Suich [1] in which response variables are aggregated into one response and should be optimized simultaneously based on the total desirability function. In this method a score between zero and one is assigned to each response. The score value of zero indicates that the predicted response value is completely

undesirable however the value of one indicates that the corresponding response has reached its desired target value. The second group of related researches considers responses based on their importance. Actually response variables have different importance according to the decision maker opinion. So these techniques optimize the responses based on their priority factor or importance weights. The third category is called

Loss function approach. In this case all response values are aggregated and converted to a single one based on Taguchi function considering the location and dispersion effects. It is obvious that when a single objective function optimization is desired; an aggregation function should be used in most of MRO problems. Some studies use the aggregated function in Taguchi method. Some researches proposed in these three categories can be described as follow:

Recently many studies have been conducted to solve the MRO problem. Saurav et al [2] proposed a hybrid of Taguchi method and principal component analysis to solve the problem. Also they have compared the proposed method with Grey-Taguchi method. Bashiri and Hejazi [3] have converted multi response to a single response in order to analyze the robust experimental design by implementing some Multiple Attribute Decision Making (MADM) such as VIKOR, TOPSIS, PROMETHEE. Their proposed method decreased the statistical error. Chang et al [4] have generalized the Taguchi method in order to use it in different situations. The proposed model is presented using a weighted convex loss function.

Also some exact solution approaches have been proposed to solve the MRO. For example Onur koksoy[5] and Lorenz Imhof [6] have form this field of interest.

Moreover it can be understood from the literature that a few researches have been conducted in heuristic and meta-heuristic solution methods field to adjust the variables to their suitable levels. This paper tries to fill the gap.

In this paper we propose a Response Surface Methodology based on the Mean Square Error (MSE) in order to form the relations between the responses and variables. Moreover a new multi-response Imperialist Competitive Algorithm (MRICA) is proposed in order to find the controllable factors best settings. The rest of presented paper is organized as follow:

In section 2, the RSM will be discussed in details. The proposed solution method will be introduced in section 3. Experimental result comparison between proposed method and other solution method proposed in literature will be done in section 4. At last section 5 will conclude the presented paper.

**2. Problem description**

Relations between the responses and factors can be formulated as follow:

|  |
| --- |
|  (1)  |

This formula illustrates the relation between the factors () versus the process *i’th* response ( ).

The first step in RSM is finding the best approximation of the relationship between the response and a set of independent controllable variables. If there is a curvature in the total system, the **second-order** or higher polynomial models should be implemented. A second-order relation function taken from [10] can be modeled as equation (2):

|  |
| --- |
|  (2)  |

Where denotes the response variable. and denote the controllable variables and noise factors vector. In this model  is the intercept of the regression function.  is the linear effects of control variables vector. is the matrix which its main diagonals denote the regression pure factors quadratic coefficients and its off-diagonals denote the coefficient of half quadratic factors interactions. is the vector of coefficients for the linear effects in the noise variables and  is a matrix shows the interactions between the controllable variables and noise factors, followed by  which is the random error with .

In tradition studies on RSM, The model factors optimization based on only one response have been discussed. However in practice there are more than one responses and a practitioner should optimize the model considering this situation. In this paper we assume that the responses are independent or uncorrelated.

During the quality control process, to create a robust design which will results in optimum factor level adjustment, the mean and variance of system should be considered. This procedure is called **dual-response** system (DRS). Many effective methods for DRS have been proposed recently ([7], [8], [9]).

One of the popular approaches to the DRS optimization is the so-called MSE criterion proposed by Lin and Tu[10].

They have suggested three basic approaches of MSE for DRS optimization.

The first is used when a practitioner wants to minimizes the response value. In other words the smaller value of a response is preferable.

|  |  |
| --- | --- |
|  | (3) |

The second approach implies that the response value should be maximized:

|  |  |
| --- | --- |
|  | (4) |

And the last approach is useful when the target value of the response variable is desirable:

|  |  |
| --- | --- |
|  | (5) |

In Eq(8), is the target value which the response value should be near to it.

1. ***Solution approaches***

However the main contribution of proposed paper is proposing two solution method using imperialist competitive algorithm (ICA) and Multi Response Imperialist competitive algorithm (MRICA) . These two approaches have been introduced in order to find the best levels for controllable variables in such way that target values for the corresponding responses be satisfied.

Imperialist competitive algorithm is a new meta-heuristic evolutionary algorithm that starts with a population of answers, any of them called a country [11]. These countries divided into two basic groups. The best answers or the powerful countries become the imperialists and the colonies consist of the rest of the answers. There are two competition types of inner and outer competitions.

***3-1 Inner competition:***

In each empire it is a preferable that the colonies move towards the imperialist and this creates new neighbors, so we can explore the search space by this strategy.

***3-2 Outer competition:***

The basic competition in an iteration of the algorithm is between the empires where each imperialist wants to take possession of other imperialists’ colonies. And after some iteration the weak imperialists will collapse and at the end of the algorithm only the powerful empire with the best solution will be remained and that is the optimum or at least near to optimum answer.

In each iteration, the weakest empire is selected and its weakest country is assigned to the more powerful empires. By these considerations we proposed two forms of ICA which are used in two situations described follow:

***3-3 The first approach***

The basic contribution of this research is making the optimization more flexible by using ICA.

Suppose that we have two MSE approaches should be minimized. We define the follow statement to choosing the importance coefficients:

 

Where

.

As mentioned before in this research two main solution approaches have been considered. In the first one an aggregated function is analyzed according to the two responses and in the second approach the Pareto frontier is analyzed.

The pseudo code for the proposed ICA method for optimization of the aggregated function can be written as follow:

1. *Start*
2. *Create the initial population randomly.*
3. *Select the ‘n\_imp’ best strings as the imperialists and construct the empires.*
4. *Divide the rest of the strings (countries) between the empires based on their normalized power.*
5. *If only one empire remains, go to step 8.*
6. *Start the inner competition:*
* *Apply the crossover operator to each empire.*
1. *Start the outer competition:*
* *Find the weakest empire based on the total normalized power of the empires.*
* *Find the worst country in that empire and release it from that country.*
* *Form the vector “P” consist of the possession probability of each empire*
* *Form the vector “E” with the same size of “P” vector and create the elements of “E” between zero and one randomly.*
* *Form the vector R as R=P-E;*
* *Select the empire that the index of maximum of R is relevant to it.*
* *Assign the released country to this empire.*
* *Go to step 2;*
1. *End;*

***3-4 the second approach***

For the second solution approach a Multiple response Imperialist competitive algorithm has been proposed. The main structure of the MRICA has been depicted in figure 1.

In this case the main purpose is finding the best solutions for more than one response in a situation that the aggregated function cannot be obtained. Actually the algorithm’s goal is to form the non-dominated solution frontier. However the inner and outer competition is in such a way described in algorithm pseudo code

In the next section the proposed approaches will be discussed in details using a numerical example taken from the literature.

|  |
| --- |
| Create the initial population and create the empiresApplying the inner and outer competitionSave the best countries in the Pareto solutions poolIs remain only one empire?NoYesForm the best frontier based on the constructed pool  |

Fig 9. The flow chart of proposed MRICA

1. ***Numerical example:***

In this paper we use the example reported by Onur koksoy[5]. In mentioned industrial case study there are three controllable variables named  describe the element configuration and also two noise factors named . Table 1 illustrates the coding of various factors. Two basic responses have been introduced by . In this example it is assumed that there is no correlation between responses.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Table 1. Levels of factors for the example

|  |  |  |  |
| --- | --- | --- | --- |
| LevelsFactors | **-1** | **0** | **1** |
|  | 15 | 30 | 45 |
|  | 8 | 11 | 14 |
|  | 7 | 9 | 11 |
|  | -1.5 | 0 | 1.5 |
|  | -.25 | 0 | .25 |

 |

In this example the first response should be minimized while the second should achieve to target value of one. Table 2 illustrates the Central Composite Design (CCD) design and experimental data for the example.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Table 2 the experimental results of Central Composite Design for the example

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| x1 | x2 | x3 | z1 | z2 | y1 | y2 |
| -1 | -1 | -1 | -1 | 1 | 1.810 | 1.10 |
| -1 | -1 | -1 | 1 | -1 | 1.690 | 1.11 |
| -1 | -1 | 1 | -1 | -1 | 1.900 | 1.07 |
| -1 | -1 | 1 | 1 | 1 | 1.780 | 1.07 |
| -1 | 1 | -1 | -1 | -1 | 1.800 | 1.47 |
| -1 | 1 | -1 | 1 | 1 | 1.630 | 1.18 |
| -1 | 1 | 1 | -1 | 1 | 1.920 | 1.41 |
| -1 | 1 | 1 | 1 | -1 | 1.780 | 1.58 |
| 1 | -1 | -1 | -1 | -1 | 1.360 | 1.57 |
| 1 | -1 | -1 | 1 | 1 | 1.220 | 2.03 |
| 1 | -1 | 1 | -1 | 1 | 1.480 | 1.38 |
| 1 | -1 | 1 | 1 | -1 | 1.440 | 1.68 |
| 1 | 1 | -1 | -1 | 1 | 0.693 | 3.37 |
| 1 | 1 | -1 | 1 | -1 | 0.616 | 3.75 |
| 1 | 1 | 1 | -1 | -1 | 0.950 | 2.81 |
| 1 | 1 | 1 | 1 | 1 | 0.817 | 2.83 |
| -1 | 0 | 0 | 0 | 0 | 1.790 | 1.24 |
| 1 | 0 | 0 | 0 | 0 | 1.030 | 2.46 |
| 0 | -1 | 0 | 0 | 0 | 1.530 | 1.23 |
| 0 | 1 | 0 | 0 | 0 | 1.220 | 1.73 |
| 0 | 0 | -1 | 0 | 0 | 1.300 | 1.63 |
| 0 | 0 | 1 | 0 | 0 | 1.440 | 1.67 |
| 0 | 0 | 0 | 0 | 0 | 1.380 | 1.73 |
| 0 | 0 | 0 | 0 | 0 | 1.390 | 1.74 |
| 0 | 0 | 0 | 0 | 0 | 1.400 | 1.74 |

 |

The estimated regression models using the Minitab 16 for the two responses are:

|  |  |
| --- | --- |
|  | (6) |

|  |  |
| --- | --- |
|  | (7) |

The mean and variance of responses can be obtained:

|  |  |
| --- | --- |
|  |  (8) |

|  |  |
| --- | --- |
|  | (9) |

|  |  |
| --- | --- |
|  | (10) |
|  | (11) |

As mentioned before the first response minimization and the second response achieving to its target value of 1 will be preferable. So the Eq(3) and Eq(5) are implemented to this example. In this stage the first aggregated solution approach is used for the example.

Table 3 illustrates a solutions obtained by ICA. This solution are compared with GRG algorithm proposed by Onur koksoy[5].

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Table 3 the solution comparison between ICA and GRG method

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Responsecoefficient | X1 | X2 | X3 | MSE1(ICA) | MSE2(ICA) |  | MSE1(GRG) | MSE2(GRG) | Efficiency of MSE1 | Efficiency of MSE2 |
| =.91 | 0.0768 | 0.3470 | 0.995 | 0.11693 | 2.6809 | 0.34769 | 0.12 | 2.44 | %100 | 90% |

 |

In this stage a multi-objective concept is applied during the optimization stage, so the proposed MRICA approach is applied for the example. Then the proposed MRICA is compared to the multi-objective Genetic Algorithm. The priority of the MRICA approach than the aggregated function approach is the independency of the proposed approach to the response coefficients. Hence in this section the Pareto optimal solution method has been implemented to solve the problem. In this case the ability to find more and near to real non dominated solutions show the algorithms efficiency. Actually the ability to find a better frontier depends on the number of solutions found by the algorithm. Moreover the diversity of determined non dominated solutions can be as another quality measure which is called to Crowding distance. It counts the distances between two solutions in both sides of a solution. It is clear that, a high value of this factor is desired. This factor can be calculated using equations 12 and 13.

|  |  |
| --- | --- |
|  | (12) |
|  | (13) |

Where

 is the crowding distance of *i’th* solution considering the objective *m*.

 is the *m’th* objective function value of *(i+1)’th* solution.

 is total crowding distance of an algorithm.

Mentioned measures have been reported for both algorithms of MRICA and MOGA in Table 5.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Table5. The comparison between ICA and genetic algorithm.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Algorithm | Population size | Computational time(s) | Number of non dominated solutions obtained by algorithm | Divergence of algorithms  |
| MRICA | 200 | 5.02 | 132 | 1.923 |
| MOGA | 200 | 9.78 | 51 | 1.574 |

 |

According to the reported results of Table 5, it is clear that the proposed MRICA is more efficient than MOGA. It has better crowding distance and determined non dominated solutions in a predefined population and computation time. The reason can be described as follow; In imperialist competitive algorithm we have *n\_imp* imperialists in which the inner competition is running simultaneously and this process yields to obtain better solutions in lower computational time than the genetic algorithm. Also the outer competition quarantines the diversity in solution space by admitting an unrelated but new solution to an empire competition.

Figure 2 illustrates the pareto frontier obtained by each algorithm. It is obvious that the proposed algorithm has the ability to obtain good solutions as well as genetic algorithm. It is worth to mention that the comparison has been done with an equal population size.

|  |
| --- |
| untitled.pngFig 2. The pareto optimal frontier obtained by ICA and GA |

However both of the MRICA and GA approaches can reach to the solution that was the GRG and ICA methods ultimate solution and this consideration shows the algorithms ability in the optimal or close to optimal solutions finding.

***Conclusion:***

Imperialist competitive algorithm is a new meta-heuristic algorithm which tries to optimize the complicated problems. Literally, many problems especially in industrial fields have many factors which contradict each other. The optimization of these models is difficult and need to a high value of computational time. In this research a new meta-heuristic based approach implemented to optimize a nonlinear multi-response programming model. The experimental results show that the efficiency of the proposed MRICA is comparable with those which obtain to an optimum solution. The proposed approach achieve to the optimum solution or at least near to optimum solution in a reasonable computational time. Furthermore the flexibility of this approach will permit the practitioner to choose the factor which is more important to be optimized. However the study of correlated responses will be worth for the future studies.

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