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A Holding Strategy to Optimize the Bus Transit Service

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ABSTRACT

Bus systems are unstable without considering any control. Thus, we are able to consider some control strategies to alleviate this problem. A holding control strategy is one commonly used real-time control strategy that can improve service quality. This paper develops a mathematical model for a holding control strategy. The objective of this model is to minimize the total cost related to passengers at any stop. To solve the model, particle swarm optimization (PSO) is proposed. The results of the numerical examples show that the additional total cost caused by service irregularity is reduced by 25% by applying the presented holding model to the given problem.

1. Introduction

By increasing the environment pollution and traffic congestion, authorities of most cities have formed many strategies on giving some priorities to the development of an urban public transportation system [1]; therefore, they pay attention to bus transit planning. The primary concern of a bus company is how to allocate its limited public transportation resource to meet the passengers' trip demands efficiently [2]. Public transit planning covers a wide research area. The public transit planning process is usually divided in a sequence of five steps, namely (1) design of route, (2) setting of frequency, (3) timetabling, (4) vehicle scheduling and (5) crew scheduling [3].

In this paper, we focus on vehicle scheduling in the public transit system. The purpose of this scheduling is to determine the number of buses required for the given period. Chakroborty et al. [4] minimized the total waiting time spent by passengers under the constraints of fleet size, maximum headway, stopping time bounds and maximum transfer time. Laurent et al. [5] developed a solution approach for simultaneously scheduling drivers and vehicles dedicate to the transport by limousines.

In transit systems, a reliable service is the main objective of operation managers. The minimum passengers waiting time and short traveling time are some instances of reliable service. Reliability is an important measure of service quality that needs some improvements. So, it is important to use a control strategy in order to improve the reliability of services. The control strategies are divided into the planning control and real-time control strategies. Planning control strategies are useful for long-term works (e.g., restructuring of bus routes); however, real-time control strategies are useful for short-term works (e.g., adding extra buses). The real-time control strategy can help reduce the impact of accident and vehicular breakdown. This strategy is divided into three categories, namely station, inter-station, and other controls. Station control strategies include stop-skipping strategy, short-turning strategy and holding strategy. These strategies are the most popular strategies [6], [7].

Stop skipping is an operating strategy that has been frequently used in heavy demand corridors. The basic idea behind this control strategy is to allow those vehicles that are late and behind schedule to skip certain low-demand stops and increase operating speed. One disadvantage of this control strategy is that

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passengers with either their origin or destination stop being skipped have to wait for at least another vehicle to got serviced [8].

The short-turning strategy means that a vehicle turns around before it reaches the route terminals. It consists of selecting part of the fleet to serve cycles or loops on those route segments exhibiting a high demand [9]. When the headway variance or passenger waiting time in the opposite direction of the bus is high, we can use this strategy [10], [11]. However, this strategy increases the waiting time for the passengers on-board the vehicle that are requested by the driver in order to alight and transfer to the subsequent vehicle [6]. By using an aggregated demand model, Ceder [12] proposed a two-stage optimization approach. In the first stage, the fleet size minimizes for a given demand level while in the second stage, the number of trips using the short-turn is minimized in order to reduce the effect of the strategy on passenger times. Vijayaraghavan et al. [13] proposed the insertion of express services and short-turns for the service of some trips pursuing the reduction of the fleet size.

The holding control strategy is used to delay bus movement deliberately when a vehicle is ahead of the schedule. This strategy can significantly reduce the headway variance [14]. The second category of the real-time control strategies is inter-station controls using traffic signals. The signal priority mechanism reduces vehicle delays at signaled intersections. In contrast to the holding strategy, which usually delays passengers and increases running time, signal prioritization reduces running times and decrease delays for passengers on-board of buses [6]. Wang et al. [15] proposed an integrated control method with modeling of the total delay at intersections, which is based on signal priority and holding strategy. The last category of real-time control strategies includes deadheading and expressing. Deadheading consists of increasing the frequency in the most demanded direction by allowing some of the buses to skip the stops in the low demand direction. The deadheading strategy is usually considered for transit corridors that present a high demand on one direction of operation and a low demand on the other. Ceder et al. [16] considered the insertion of an empty trip between two terminals (i.e., deadheading trip) in order to reduce the fleet size subject to satisfying a given schedule of bus departure from terminals. Eberlein et al. [10] and Fu et al. [8] considered a different problem for deadheaded vehicles starting their trip empty until a station to be determined, and from that stop, these buses start their normal service until the end of their route. So, vehicles save time and reduce the headway with respect to the bus ahead. Eberlein et al. [10] considered the waiting time in the objective function. Fu et al. [8] assumed that if a vehicle is going to skip stations, necessarily the next bus, has to serve the entire route. This simplifies the problem providing a minimum level of service for passengers waiting at skipped stations. Express vehicles serve only one section of their route, and then they continue it without stopping until reaching the terminal or a pre-specified zone, in which the service is reestablished. Express services operate just stopping at a subset of the normal service stops. Leiva et al. [17] studied an expressing, in which a public transport corridor is divided into sectors where some vehicles serve selected segments or bus stops while others serve the entire corridor.

Buses follow scheduled service with schedules that are advertised to the public. To minimize the likelihood of leaving a passenger behind at a stop, buses are not permitted to depart before their scheduled departure time [18]; therefore, holding strategies can be used by public transit operators.

Amiri-Aref et al. [19] proposed a model that minimized the maximum expected barrier distance from one facility to all demand points by considering the rectilinear distance metric. Ameli et al. [20] proposed a model in transmission network expansion planning (TNEP) to deliver safe and reliable electric power to load centers during the planning horizon.

There is an extensive literature on holding control strategies. Abkowitz et al. [21] used a holding strategy to minimize the passengers waiting time estimated by Monte Carlo simulation. Dessouky et al. [22], Dessouky et al. [18] developed a control strategy to decide when to dispatch buses and when to hold them at a terminal. Zolfaghari et al. [6] presented a holding strategy minimizing the passengers waiting time at stops. Daganzo [23] proposed a headway-based dynamic holding strategy to reduce the amount of slack time in the schedule, subject to a headway variability constraint. Daganzo et al. [24] proposed a cooperative control method, in which bus speed was regulated based on the expected demand and the spacing between the current bus and the preceding and following buses. Xuan et al. [25] proposed a dynamic holding strategy that use bus arrival deviations from a virtual schedule at control points. Lo et al. [7] designed a bus holding strategy for the mass rapid transit (MRT) system with real-time information in order to reduce the bus waiting time, passengers waiting time and passengers travelling time. In this paper, we develop a mathematical programming model that minimizes the passengers waiting time and passengers travelling time at any stop by using the holding strategy. In this model, the given problem is solved simultaneously for all buses on the route, in which the holding time is considered as a decision variable.

This paper is organized as follows. Section 2 describes the problem. Sections 3 and 4 present the mathematical model and the solution methods, respectively. Section 5 discusses a numerical example, and finally Section 6 is the conclusion.

2. Problem Description

In this study, we focus on the holding strategy that can be applied on the route that consists of a number of stops and terminals. In the route, a sequence of geographical points is connected. We assume that the route is a one-way loop. The main difference between stops and terminals is that in the terminals, all passengers on-board should alight and then new passengers may board to start a new trip. However, in the stops, passengers may board, alight or continue their trip. The information, which the controllers need to know, is as follows: 1) loads on buses departing their last visited stop, 2) passenger arrival rate, 3) fraction of passengers on-board of buses alight at stops, and 4) running time of buses between stops plus acceleration and deceleration times. We assume that these parameters can be determined from the available data.

2-1. Assumption and Limitation

Following are the main assumptions of the model.

- Buses run at constant speed (i.e., the running time between stops is certain).
- During controlling the vehicle, we consider a limited number of downstream stops for each bus.
- The dwelling time of vehicles at all stops is equal.

2.2. Notations

In this study, the variables defined as follows:

i	Index of vehicles, $i = 1, \dots, I$
k	Index of stops, $k = 1, \dots, N$
$L_{i,k}$	Load in bus i departing stop k
$d_{i,k}$	Departure time of bus i from stop k
$a_{i,k}$	Arrival time of bus i at stop k
$D_{i,k}$	Demand for bus i at stop k
$b_{i,k}$	Passengers left behind by bus i at stop k
$dt_{i,k}$	Dwell time of bus i at stop k
r_k	Passenger arrival rate at stop k
$x_{i,k}$	1, if bus i is loaded to capacity when it departs stop k; 0, otherwise
F_k	Passenger alighting fraction at stop k
$R_{i,k}$	Running time of bus i from stop $k-1$ to stop k
2δ	Acceleration and deceleration time between two consecutive stops
L^{max}	Passenger capacity of bus
M	Sufficiently large number
K	Set of stops on the route
I	Set of buses operating on the route
N^i	Total number of downstream stops of bus i
n^i	Index of the last visited stop by bus i
c_1	Value of passenger waiting cost
c_2	Value of passenger travelling cost
$A_{i,k}$	Number of passengers in bus i alighting at stop k
$B_{i,k}$	The number of passengers willing to board the bus i at stop k
$H_{i,k}$	Holding time of bus i at stop k

3. Mathematical Formulation

The model presented in this paper can be viewed as an extension to that of [6]. This requires extending the objective function and adding or altering some of constraints and equations. In this model, we consider the holding strategy to the waiting and travelling times of passengers in order to minimize the total cost. This paper attempts to answer the question which bus has to be held at any stop in order to minimize the total passengers cost. The mathematical programming model is presented below.

Minimize z = $f_1 + f_2$

$$f_1 = c_1 \sum_{i=1}^{I} \sum_{k=n^i+1}^{n^i+N^i} [\, {}^{1}\!/_{2} r_k \, (d_{i,k} - d_{i-1,k})^2 + b_{i,k}(d_{i+1,k} - d_{i,k})] \tag{1}$$

$$f_2 = c_2 (\sum_{i=1}^{I} \sum_{k=n^i+1}^{n^i+N^i} [\, L_{i,k}(R_{i,k} + 2\delta) + L_{i,k-1}(1 - F_k) \times dt_{i,k}] + \sum_{i=1}^{I} [L_{i,k} \times H_{i,k}]) \tag{2}$$

s.t.

$$a_{i,k} = d_{i,k-1} + R_{i,k} + 2\delta \quad \forall i,k \in (I,K) \tag{3}$$

$$B_{i,k} = r_k(d_{i,k} - d_{i-1,k}) + b_{i-1,k} \quad \forall i,k \in (I,K) \tag{4}$$

$$A_{i,k} = F_k . L_{i,k-1} \quad \forall i,k \in (I,K) \tag{5}$$

$$L_{i,k} \le D_{i,k} \quad \forall i,k \in (I,K) \tag{6}$$

$$L_{i,k} \ge D_{i,k} - M . x_{i,k} \quad \forall i,k \in (I,K) \tag{7}$$

$$L_{i,k} \le L^{max} \quad \forall i,k \in (I,K) \tag{8}$$

$$L_{i,k} \ge L^{max} . x_{i,k} \quad \forall i,k \in (I,K) \tag{9}$$

$$D_{i,k} = (1 - F_k) . L_{i,k-1} + B_{i,k} \quad \forall i,k \in (I,K) \tag{10}$$

$$d_{i,k} = a_{i,k} + dt_{i,k} + H_{i,k} \quad \forall i,k \in (I,K) \qquad (11)$$

$$d_{i,k} < d_{i+1,k-1} + R_{i+1,k} + 2\delta \quad \forall i,k \in (I,K) \qquad (12)$$

$$b_{i,k} = \max\{0, D_{i,k} - L^{max}\} \quad \forall i,k \in (I,K) \qquad (13)$$

$$0 \leq F_k \leq 1 \qquad \forall k \in K \qquad (14)$$

$$x_{i,k} \in \{0, 1\} \qquad \forall i,k \in (I,K) \qquad (15)$$

$$d_{i,k}, L_{i,k}, b_{i,k}, B_{i,k}, A_{i,k}, H_{i,k} \geq 0 \ \Lambda \ int \quad \forall i,k \in (I,K) \qquad (16)$$

The objective function consists of the passengers waiting cost and the passengers travelling cost. The passengers waiting cost sums the passengers waiting times at all stops along a route multiple the value of passengers waiting cost. If bus i is loaded to capacity when departing stop k, then there is an additional waiting time for passengers who can not board that bus. These passengers are assumed to wait for the next bus.

The passengers travelling cost sums the passengers travelling times along a route, multiple the value of passengers travelling cost. The passengers travelling time includes the passengers' on-board time between the stops, passengers dwelling time at stops and on-boarded passengers time when the bus is hold.

Constraint (3) shows that the arrival time of vehicle *i* at stop *k* is equivalent to the departure time of vehicle *i* at stop *k*-1 plus the running time of the vehicle between stop *k*-1 and *k* plus the acceleration and deceleration time. Constraint (4) shows the number of passengers willing to board the vehicle *i* at stop *k* equal to the passenger arrival rate multiplied by the headway between two consecutive vehicles plus the number of passengers that can not arrive vehicle *i*-1 at stop *k*. Constraint (5) shows the number of passengers alight vehicle *i* at stop *k* equal to passenger alighting fraction at stop *k* multiple the load in vehicle *i* departing stop *k*-1.

Constraints (6) to (9) restrict the load in bus *i* departing stop *k*. When bus *i* at stop *k* is at the full capacity (i.e., $x_{i,k} = 1$), Constraints (8) and (9) are the binding ($L_{i,k} = L^{max}$). When bus *i* at stop *k* is not at the full capacity (i.e., $x_{i,k} = 0$), Constraints (6) and (7) are the binding $(L_{i,k} = D_{i,k})$. Constraints (6) represents that the load in bus *i* departing stop *k* is equal to or less than the demand for bus *i* at stop *k*. Constraint (8) represents that the load in bus *i* departing stop *k* is equals to or less than the passenger capacity of the bus.

Constraint (10) shows that the demand for bus *i* at stop *k* is equal to the load on the bus *i* departing stop *k*-1 less the number of passengers alighting at stop *k* added to the number of passengers willing to board the bus. Constraint (11) restricts the departure time of bus *i* at stop *k*. To reduce headway irregularities in the service, the holding strategy assumed herein allows for each bus to be held at any stop for a finite period. Constraint (12) shows that the departure time of vehicle *i* at stop *k* is less than the departure time of vehicle *i*+1 at stop *k*-1 plus the running time between stop *k* and *k*-1 and acceleration and deceleration time. Constraint (13) shows that the number of passengers left behind at stop *k* is equal to the demand at that stop for vehicle *i* less the capacity of bus *i*. Otherwise, when the load in bus *i* departing stop *k* is less than the capacity of vehicle *i*, then the number of passengers left behind at stop *k* is equal to zero.

4. Particle Swarm Optimization

Particle swarm optimization (PSO) is a swarm-based intelligence algorithm influenced by the social behavior of animals [26]. It simulates the behavior of bird flocking. Suppose that when a group of birds are randomly searching for food in an area, there is only one piece of food in an area being searched and all of birds do not know where the food is; however they know how far the food is. So the best strategy that helps them to find the food is that follow the bird that is nearest to the food. A particle in PSO is analogous to a bird flying through a search space [27]. PSO starts with a population of random solutions (i.e., particles) in a *D*-dimension space. The position of particle *i* is represented by $X_i = (x_{i1}, x_{i2}, \dots, x_{iD})$.

Each particle knows its best position (i.e., *pbest*) and the best position so far among the entire group of particles (i.e., *gbest*). The *pbest* of a particle is the best result (i.e., fitness value) reached so far by the particle, and stored as $P_i = (p_{i1}, p_{i2}, \dots, p_{iD})$, whereas the *gbest* is the best particle in terms of fitness in an entire population, and its location, obtained by any particle in the population. At each step, PSO consists of changing the velocity of each particle toward its *pbest* and *gbest* according to Eq. (17). The velocity of particle *i* is represented as $V_i = (v_{i1}, v_{i2}, \dots, v_{iD})$. Acceleration is weighted by a random term with separate random numbers being generated for acceleration toward *pbest* and *gbest*. The position of the *i*th particle is the updated according to Eq. (18) [28].

$$\begin{aligned} v_i^{k+1} = wv_i^k &+ c_1 rand_1 (P_{id}^k - x_{id}^k) \\ &+ c_2 rand_2 (P_{gd}^k \\ &- x_{gd}^k) \end{aligned} \qquad (17)$$

$$x_{id}^{k+1} = x_{id}^k + cv_{id}^{k+1} \qquad (18)$$

where P_{id} and P_{gd} are *pbest* and *gbest*, respectively. $rand_1$ and $rand_2$ are random real numbers drawn from *U*(0,1). Thus, the particle flies through potential solutions toward *pbest* and *gbest* in a navigated way while still exploring new areas by the stochastic mechanism to escape from local optima. c_1 express how much the particle trusts its own past experience, it is called the cognitive parameter and c_2 express how much it trusts the swarm, it is called the social parameter. The inertia weight w is responsible for dynamically adjusting the velocity of the particle, so it is responsible for trading-off between local and global searches and hence requiring a less iteration for the algorithm to converge.

5. Numerical Example

Consider a typical example where there are 15 buses operating on a route consisting of 18 intermediate stops and 2 terminals as depicted in Fig. 1. There are $N^i = 15$ stops for all buses and c_1= 0.7 and c_2= 0.3. In this paper, two cases may arise, namely either all buses are running on a schedule or there is one or more headway irregularity in the system. These cases are discussed below.

Case 1 (Regular service): Assume that the position of buses is described in Fig. 1.

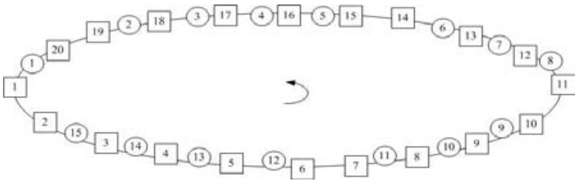

Fig. 1. Regular case with vehicles ○ and stops ☐

Table 1 summarizes information including (1) the passenger arrival rate, (2) the passenger alighting fraction, and (3) the running times of buses between two consecutive stops plus acceleration and deceleration time. We assume that this information can be determined from the available data.

Tab. 1. Information required for a controller

Stops	Passenger alighting fraction	Passenger arrival rate	Running time between stops plus acceleration and deceleration
1	1	4	3
2	0.1	1.5	3
3	0.1	1.5	4
4	0.2	1.5	3
5	0.2	2	3
6	0.2	1.5	3
7	0.3	2	2
8	0.4	1.5	3
9	0.2	2.5	3
10	0.1	2	2
11	1	4	3
12	0.1	2	3
13	0.2	1.5	2
14	0.15	1.5	3
15	0.3	2.5	3
16	0.1	1.5	3
17	0.1	2	3
18	0.15	2	3
19	0.2	2	2
20	0.1	2	3

Tab. 2. Optimum departure time of vehicles from stops and the load on those when departing the last visited stop.

Buses	Stops	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Bus 1	Departure time	5	9	13	18	22	26	30	33	37	41	44	48	52	55	59	-	-	-	-	1
	Load on the bus																				21
Bus 2	Departure time	16	20	24	29	33	37	41	44	48	52	55	59	63	-	-	-	-	5	9	12
	Load on the bus																		20		
Bus 3	Departure time	20	24	28	33	37	41	45	48	52	56	59	63	-	-	-	-	5	9	13	16
	Load on the bus																	19			
Bus 4	Departure time	20	24	28	33	37	41	45	48	52	56	59	-	-	-	-	1	5	9	13	16
	Load on the bus																18				
Bus 5	Departure time	24	28	32	37	41	45	49	52	56	60	-	-	-	-	1	5	9	13	17	20
	Load on the bus															17					
Bus 6	Departure time	31	35	39	44	48	52	56	59	-	-	-	-	1	4	8	12	16	20	24	27
	Load on the bus													19							
Bus 7	Departure time	39	43	47	52	56	60	64	-	-	-	-	5	9	12	16	20	24	28	32	35
	Load on the bus												20								
Bus 8	Departure time	39	43	47	52	56	60	-	-	-	-	1	5	9	12	16	20	24	28	32	35
	Load on the bus											21									

Tab. 2. Optimum departure time of vehicles from stops and the load on those when departing the last visited stop.

Buses	Stops	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Bus 9	Departure time	50	54	58	63	-	-	-	-	**5**	9	12	16	20	23	27	31	35	39	43	46
	Load on the bus									23											
Bus 10	Departure time	54	58	62	-	-	-	-	**5**	9	13	16	20	24	27	31	35	39	43	47	50
	Load on the bus								20												
Bus 11	Departure time	57	61	-	-	-	-	**5**	8	12	16	19	23	27	30	34	38	42	46	50	53
	Load on the bus							19													
Bus 12	Departure time	-	-	-	-	**1**	5	9	12	16	20	23	27	31	34	38	42	46	50	54	57
	Load on the bus					19															
Bus 13	Departure time	-	-	-	**3**	7	11	15	18	22	26	29	33	37	40	44	48	52	56	60	-
	Load on the bus				18																
Bus 14	Departure time	-	-	**5**	10	14	18	22	25	29	33	36	40	44	47	51	55	59	63	-	-
	Load on the bus			17																	
Bus 15	Departure time	-	**5**	9	14	18	22	26	29	33	37	40	44	48	51	55	59	63	-	-	-
	Load on the bus		18																		

Tab. 3. Computational results of Case 1.

Waiting cost	28795
Travelling cost	6823
Total cost	22203

Tab. 4. Departure time of Bus 12 from stops in Case 2.

Stops	*Departure time*
1	61
2	66
3	-
4	-
5	-
6	-
7	**4**
8	12
9	16
10	20
11	23
12	27
13	31
14	34
15	38
16	42
17	46
18	50
19	54
20	57

Case 2 (Irregular service): Assume that Bus 12 has been running ahead of the schedule. The position of the buses on the route is depicted in Fig. 2. The results show that Bus 12 should hold four minutes in Stop 8 to enhance service irregularity. After that, the bus proceeds as regular one. Under such conditions, the new transmitted information is summarized in Table 4. Furthermore, we solve the presented model by our proposed PSO with the same data as used for Case 1 and the new results for Bus 12. Then, the related results with the minimum objective function values are illustrated in Table 5.

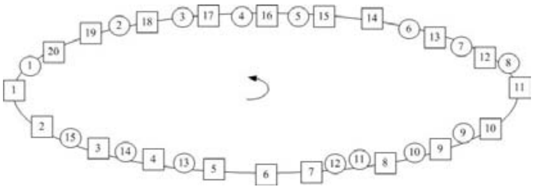

Fig. 2. Irregular case with vehicles ○ and stops

Tab. 5. Computational results for Case 2.

	Irregular service	**Holding strategy**
Waiting cost	28950	28912
Travelling cost	6816	6845
Total cost	22310	22283

If the controller does nothing, the total cost will be increased by 107=22310-22203. Applying the proposed holding model to the given problem, the best solution found that Bus 12 holds at Stop 8 for four minutes. By doing so, the additional total cost incurred by service irregularity is reduced by 25%.

6. Conclusion

In this paper, we have explained the real-time control strategies (e.g., holding, deadheading and short-turning). Then, we have presented a mathematical model for holding strategy applicable to transit the system. The cost function of the model has measured the passengers waiting cost and the passengers travelling cost.

The first function is the sum of the waiting time for the passenger who waits for the bus to come and the passengers are left behind by a full-load bus. The second one includes passengers on-board time between the stops, passengers dwelling time at stops and on-boarded passengers time when the bus is hold. We have proposed a met-heuristic algorithm, namely particle swarm optimization (PSO), for solving the model. In the first case, we have attempted to minimize the objective function without using a holding strategy. In the second case, we have assumed two decisions, first there is no holding strategy and the buses run along a route with this way. With the second decision, we correct the irregularity of the service by using holding strategy. The results indicate that using a holding strategy has improved the cost function.

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