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Geometry Definition and Contact Analysis of Spherical Involute Straight Bevel Gears

H. Ligata* **& H.H. Zhang**

H. Ligata, *American Axle & Manufacturing, Detroit, MI 48210*
H.H. Zhang, *Purdue University, West Lafayette, IN 47907*

KEYWORDS

spherical involute surface,
forged straight bevel gears,
unloaded tooth contact analysis,
gear misalignment

ABSTRACT

A practical application of the spherical involute surface to the forged straight bevel gears is provided and demonstrated in this work. Conjugate (pure involute) theoretical surfaces are developed from the input design parameters. The surfaces are modified to suit the actual application (automotive differential). The unloaded (or low load) tooth contact analysis of modified surfaces is performed to obtain the prediction of the contact pattern. In order to verify the procedure and predictions, actual straight bevel gears are forged by using provided surfaces, and their contact pattern is compared to the predictions. Influence of the misalignments on the gear performance is investigated in order to provide more robust design.

1. Introduction

Spherical involute is a natural and probably the best suited tooth form for the straight bevel gears. These gears were traditionally manufactured by using cutting process. Such a process uses straight cutting blades and creates *octoidal* surface which is, although rather close, still different from the spherical involute. Forging process is capable of producing spherical involutes but, unlike cutting process, it requires manufacture of the forging dies with the precise representation of the tooth surfaces. The production of the dies can take a long time which usually does not leave any time for trial-and-error procedure. For that reason it is important that the forged bevel gears have robust tooth surfaces which are resistant to the common manufacturing errors, as well as assembly and application misalignments.

The spherical involute surface and its properties are known to engineers for a very long time. Grant [1] in his 1890 paper described the spherical involute and two other tooth forms (octoidal and circular) used for straight bevel gears today. In his work he stated that only octoidal tooth form could be manufactured by using relatively simple process.

In the last couple of decades, with the development of CNC machining, manufacture of the forging dies with the accurate representation of the spherical involute tooth surface became feasible and the application of such a tooth form itself became possible. Large amount of published work in the recent years [2-7] was concerned with the theoretical aspects of this tooth form, investigating its kinematic properties and working on its easier incorporation into Computer Aided Design software used for forging die manufacture.

This work gives an overview of the theory behind the spherical involute surface, and describes its application to the real-life design by describing necessary tooth surface modifications and influence of the various misalignments. The ‘classical’ approach of defining gear geometry, described by Litvin [9], was used in this work because of its clarity (and amount of available material) and relatively simple shape of the surfaces of the straight bevel gears. The design parameters were not considered here, as they were described in the authors’ previous work [10].

2. General Equation of the Spherical In Volute Surface

The general equation of a spherical involute surface can be easily obtained by using approach described by Shunmugam et al [6], and Figliolini et al [7]. The involute is crated by the point T, Figure 1, on

* Corresponding author: H. Ligata
Email: *Haris.Ligata@aam.com*
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the tangent (base) plane which rolls over the base cone without slipping (pure rolling). The trajectory (spherical involute) of the point T can be practically visualized as the trajectory of the fingers unwrapping the ice-cream cone. The coordinates of the point trajectory can be expressed as [9]:

$$\begin{aligned} x &= R\cos(\beta\sin(\alpha_b))\sin(\alpha_b)\cos(\beta) + R\sin(\beta\sin(\alpha_b))\sin(\beta) \\ y &= R\cos(\beta\sin(\alpha_b))\sin(\alpha_b)\sin(\beta) - R\sin(\beta\sin(\alpha_b))\cos(\beta) \\ z &= R\cos(\beta\sin(\alpha_b))\cos(\alpha_b) \end{aligned} \quad (1)$$

where:
R: Distance along the base cone, or radius of tangent (base) plane in Fig. 3.1,
α_b: Base cone angle,
β: Roll angle through which base cone rotates during pure rolling motion.
Complete spherical involute surface can be obtained by varying parameters *R* and β in the equation (1). *R* is varied between inner and outer cone distances (obtained in the design stage), while β is varied over the range that covers portion of the gear tooth from root (fillet) to tip. Base cone angle (α_b) is determined by using pitch angle (α_p) and pressure angle (ϕ) in the expression [7],

$$\sin(\alpha_b) = \sin(\alpha_p)\cos(\phi). \quad (2)$$

3. Conjugate Spherical Involute Surfaces

Spherical involute surfaces of pinion and gear must be conjugate to transfer power (motion and load) in a smooth manner. In general, for any regular surface it is possible to create a surface of the mating part (gear). If the surface Σ_1 is defined in its coordinate system S_1 by two parameters (in our case *R* and β), then the conjugate mating surface Σ_2, in its own coordinate system S_2, must satisfy [9]:

$$f(R,\beta,\varepsilon) = \left(\frac{\partial \mathbf{r}_2}{\partial R} \times \frac{\partial \mathbf{r}_2}{\partial \beta}\right) \cdot \frac{\partial \mathbf{r}_2}{\partial \varepsilon} = 0. \quad (3)$$

where
ε: Angle of rotation of surface (gear) Σ_1 in its coordinate system S_1, $\varepsilon = \varepsilon_1$,
$\left(\frac{\partial \mathbf{r}_2}{\partial R} \times \frac{\partial \mathbf{r}_2}{\partial \beta}\right) = N_2^{(1)}$: Normal to surface Σ_1 in coordinate system S_2 for a specific angle of rotation of surface Σ_1, $\varepsilon = \varepsilon_1$,
$\frac{\partial \mathbf{r}_2}{\partial \varepsilon} = \mathbf{v}_2^{(12)}$: Relative velocity of contact point on surface Σ_2 with respect to the contact point on surface Σ_1, for given angle of rotation of Σ_1, ε.
The equation (3) is known as *equation of meshing*. In the special case when gears rotate around parallel, or intersecting axes, the relative velocity of the contact points, $\mathbf{v}_2^{(12)} = \mathbf{v}_2^{(21)}$, can be represented as the velocity in rotation around instantaneous axis of rotation, and the equation can be simplified to become [9] (shown here for surface Σ_1),

$$\frac{X_1 - x_1(R,\beta,\varepsilon)}{N_{x1}(R,\beta,\varepsilon)} = \frac{Y_1 - y_1(R,\beta,\varepsilon)}{N_{y1}(R,\beta,\varepsilon)} = \frac{Z_1 - z_1(R,\beta,\varepsilon)}{N_{z1}(R,\beta,\varepsilon)} \quad (4)$$

Where x_1, y_1, z_1: points on the surfaces (z_1 is the center axis of the gear 1), from equation (1) after rotation through angle ε, and X_1, Y_1, Z_1: coordinate of a point on the instantaneous center of rotation (in fixed coordinate system.
Combining the equations (1) and (4), it is possible to obtain contact points (in terms of ε) of a surface (in this case Σ_1). In essence, equation (1) represents spherical involute surface point (for fixed parameters *R* and β) while equation (4) represents condition that the point is a contact point. Conjugate surfaces could be created in several ways.
For example, natural way would be to create one member tooth surface (equation (1)), find contact points of such surface as it goes through the rotation (equation 4)), and then express the contact points in the coordinate system of the mating member. Approach used in this work uses the concept of rack, routinely applied in generation of parallel axes (spur and helical) gear tooth forms. In bevel gear application, the 'rack' is called crown rack (or 'third member', when its infinitely thin surface is fitted between mating gear surfaces while they rotate). The contact points on the crown rack, as it rotates through the predetermined angle, are then expressed in the corresponding coordinate systems of pinion and sidegear (as they rotate together with their gears through angles determined by the gear ratio) to obtain the coordinates of their tooth surfaces. This approach will be demonstrated through several figures on the following pages. The major characteristics of the crown rack surface are that its pressure angle is equal to design pressure angle of the gears (ϕ) and pitch cone angle (α_p) is equal to 90° (its pitch surface is a flat plane tangent to the pitch cones of the mating gears). These values can be used in equation (2) to obtain base cone angle of crown rack, and then in equation (1) to find the coordinates of its surface. Such a surface is shown in Figure 2, created for the example from Table 1.

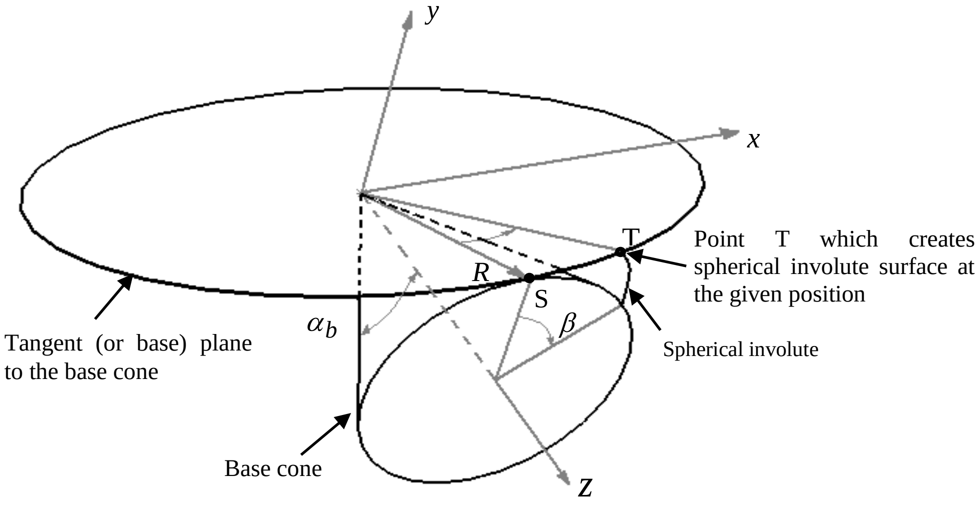


Fig. 1. Creation of spherical involute.

Tab. 1. Input parameters for an example straight bevel gear pair

	Pinion		**Gear**
Number of teeth (Ntp, Ntg)	13		17
Pressure angle (ϕ), [°]		22.5	
Outer cone distance (OCD), [mm]		60	
Pitch-line face width (FWp, FWg), [mm]	25		25
Outer normal backlash (B), [mm]		0.15	
Depth factor		2.1	
Clearance factor		0.125	

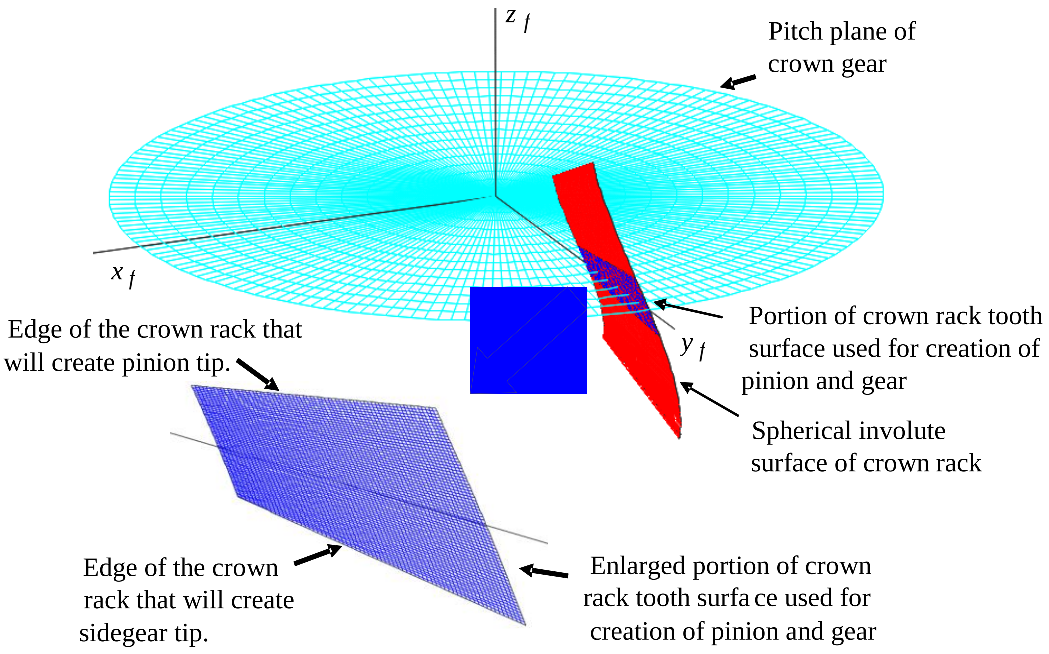


Fig. 2. Spherical involute surface of crown gear for creation of straight bevel gears.

Figure 2 shows that the crown rack (used for straight bevel gear generation) deviates from the plane as it gets further from the pitch plane. An approximation of the spherical involute form (octoidal tooth form) which uses plane as a crown rack surface is practically used in cutting of straight bevel gears.

The approximation is usually suitable, because a rather small portion of the spherical involute surface is used in generation of the gears (Figure 2). The necessary size of the crown rack (in profile direction) to create complete active surfaces (for pinion and sidegear) stretches from the side which creates gear tip, to the side that creates pinion tip (Figure 2). A kinematic relationship between a position on the face cone (tip) of a gear and parameters R, and β, can be easily obtained.

It can be then used to determine the necessary rotation of the crown rack for the creation of the tip. The size of the crown rack (i.e. its rotation angle) can be increased by a small amount to avoid non-conjugate action in the fillet region of the gears when they are pushed closer into the contact.

Figure 3 shows the contact points (green color) obtained by simultaneously using equations (1) and (4) as the crown rack rotates around axis z_f. Instantaneous center of rotation (pitch line) in this work is always positioned along the axis y_f while the local coordinate systems rotate together with the gears. Surface coordinates of pinion and sidegear would be obtained by expressing the coordinates of contact points in their local coordinate systems. It must be noted that any surface created directly by using equation (1) is not properly oriented with respect to its center axis (z axis).

A small rotation is necessary to bring the intersection of the surface with yz plane to coincide with pitch line of the fixed coordinate system (y_f axis). The pitch cones and orientation of the local coordinate systems of the pinion and sidegear are shown in Figure 4 for the starting position when they coincide with the fixed coordinate system.

As an example of the motion of the gears (and their coordinate systems), if crown rack rotates through angle ε in counterclockwise (CCW) direction around axis x_f then pinion rotates around its center axis z_p through angle $\varepsilon_{pin} = \varepsilon / \sin(\gamma)$ in clockwise (CW) direction, and sidegear rotates around z_g in CCW direction through angle $\varepsilon_g = \varepsilon / \sin(\Gamma)$. Tooth surfaces (several teeth) created in such a way are shown in Figure 5a and 5b, and the tooth surfaces ('point clouds') of the complete gears in mesh are shown in Figure 5c.

The root portion of the gear teeth in this work is not created by using equation (1). Instead, a smooth transition surface is created that connects the fillet edge of the active surface (created by using above outlined procedure) to the root cone of the design (blank) geometry.

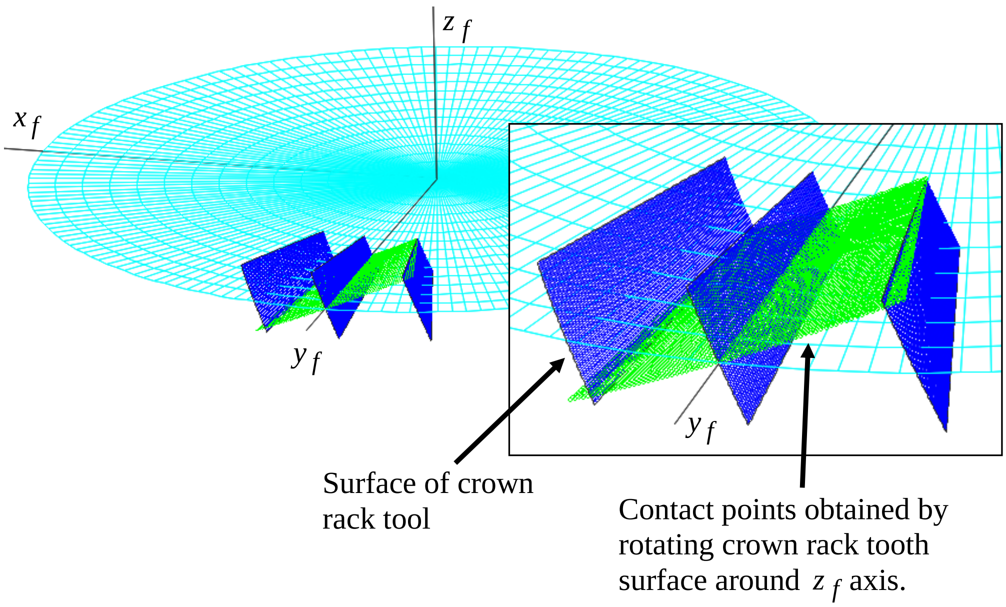


Fig. 3. Contact solution points for crown rack as it rotates to create pinion and gear surfaces

4. Modification of Involute Surfaces

The contact area of loaded gears spreads towards edges of the teeth, more heavily towards their back (heel). It is also sensitive to the improper position of the mating gears, caused either by initial (assembly) misalignment or by deformations.

Excessive (hard) contact under loaded and unloaded conditions should never reach any tooth edge (front, back, tip or root). Edge contact, caused by misalignment or deformation, results in very high contact stresses (surface failures) and uneven power transfer (vibrations and possible premature failures).
Due to the rather slow speed of the straight bevel gears, the noise is usually not an issue in the applications which use these gears.
Excessive contact at the edges could reduces contact ratio which is already rather low for these gears (ranges from 1.1~1.5) and cause rough operation. Surface modification is used to prevent excessive edge contacts by removing the material from the theoretical involute surface close to the edges of the teeth. Two main surface modifications are lead crowning and profile crowning. Lead crowning is used to obtain a localized contact in lengthwise direction.
Figure 6b shows unloaded contact after applying lead crown. Lead crown modification usually has a certain shape of the second or third order curve. Profile crowning is used to prevent excessive (`hard`) contact at the tip and root of the gear teeth. Radius of the surface curvature decreases at the roots of the mating gears (particularly pinions), causing high contact stresses in these regions, even when the gears are not misaligned. Such a modification reduces these surface stresses, especially at the high loads. Figure 6c shows the impact of profile crowning on contact pattern. The material is most often removed from the tooth form in a parabolic manner.
In this work only the portion of the tooth above the pitch cone (or a different position between root and tip of a tooth) was modified, according to the method proposed by Litvin [9]. The lead and profile modifications can be applied through the coordinate transformations which, in a form applicable to gears in this work, were proposed by Litvin [9].
If the theoretical (pure involute) coordinates of a gear *i* (represented in general form by equation (1)) are designated by $\boldsymbol{r}_{i,th}$, then the final (modified) coordinates $\boldsymbol{r}_i$ (in its local coordinate system) can be obtained as:

$$\boldsymbol{r}_i = \boldsymbol{M}_{i,prof} \cdot \boldsymbol{M}_{i,lead} \cdot \boldsymbol{r}_{i,th} \cdot \tag{5}$$

where $M_{i,prof}$ and $M_{i,lead}$ are coordinate modification matrices.

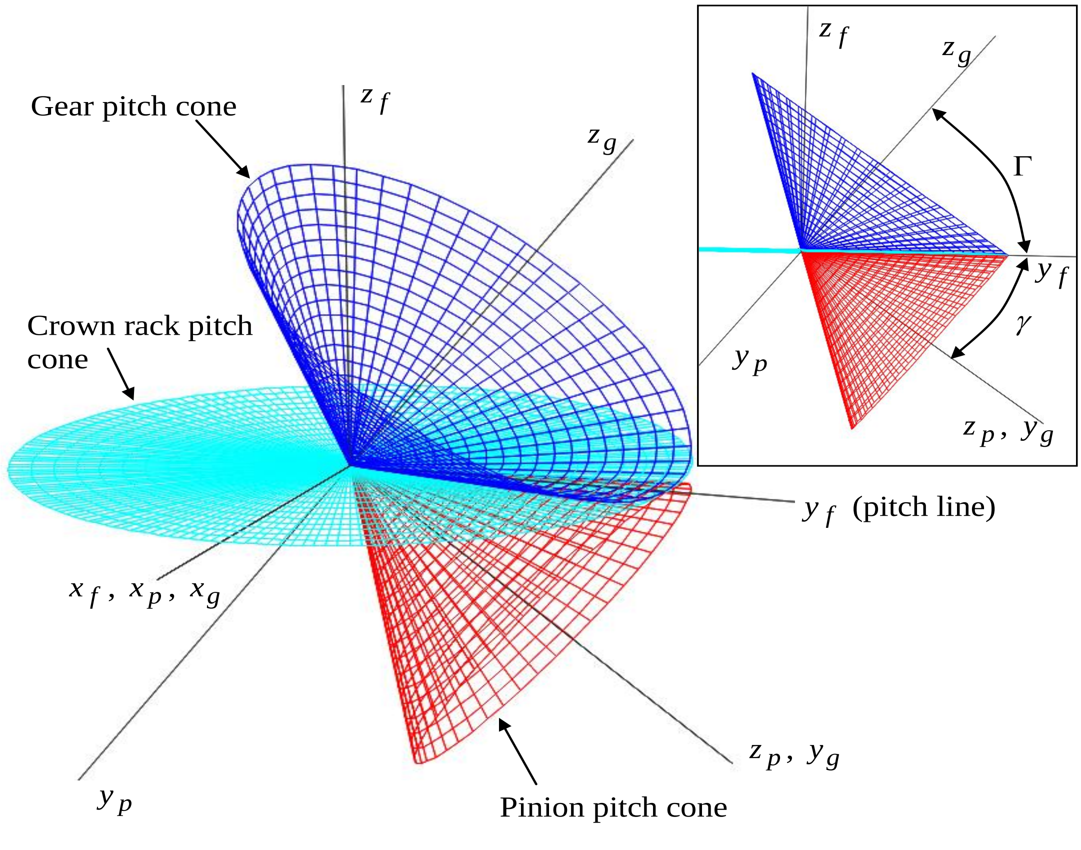


Fig. 4. Pitch cones of pinion, gear and crown rack with pitch angles of pinion (γ) and gear (sidegear, Γ) shown in the small figure (pitch angle of crown rack is equal to $\pi/2$)

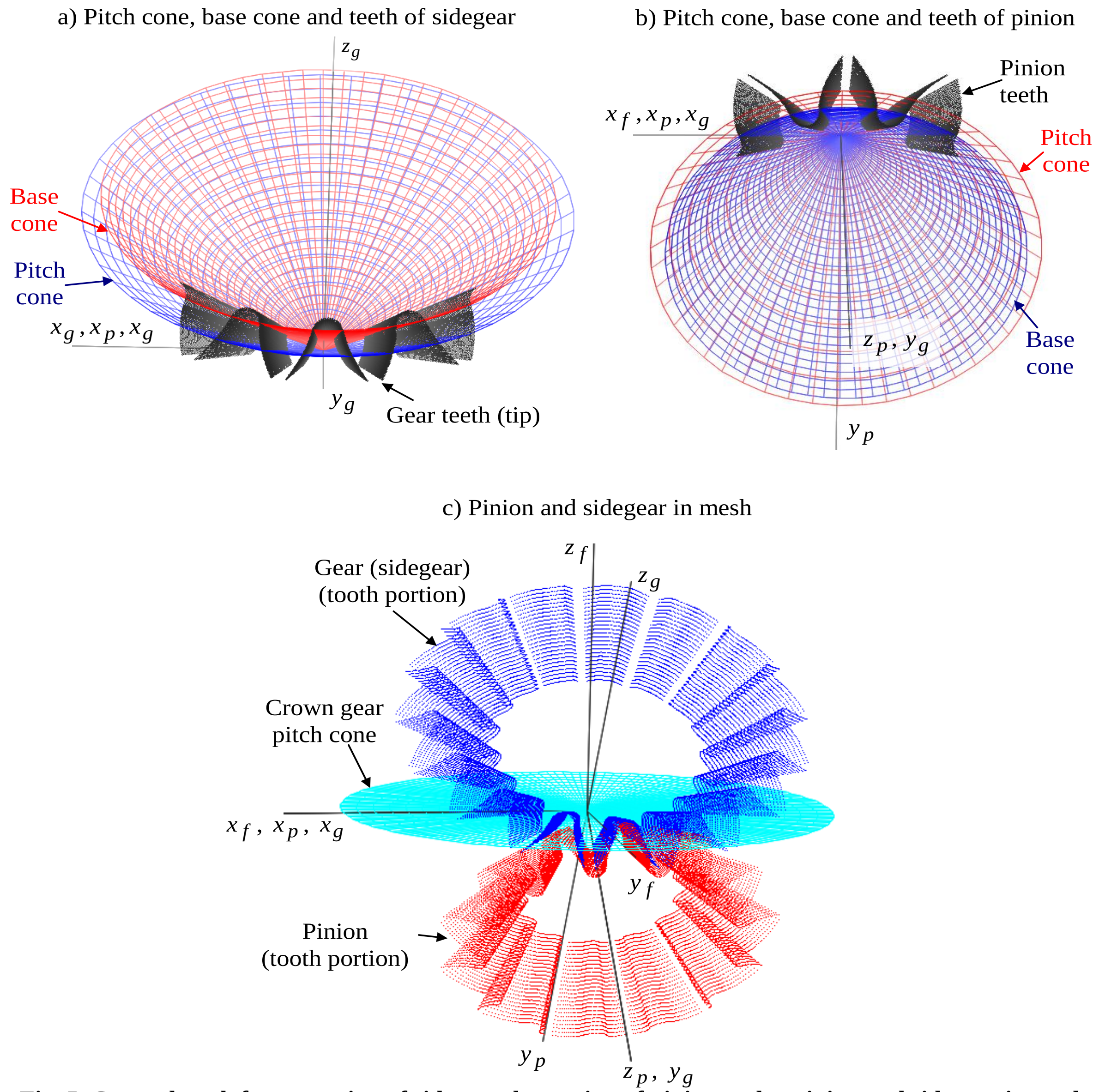


Fig. 5. Created teeth for a. portion of sidegear, b. portion of pinion and c. pinion and sidegear in mesh

4. Unloaded Tooth Contact Analysis

Tooth contact analysis of the modified surfaces is used to find the size and position of the contact area (patch) of the mating gears. It is used to verify that the position, shape and extent of the modification are suitable. The analysis would ideally be performed with loaded gears and it would include the deflection of the teeth and portion of the gears under the teeth, and show the influence of webbing at the front (toe) and back (heel) of the gears. It would also show the influence of variation in tooth thickness (much thicker at the heel then at the toe) and contact ratio (higher at the heel than at the toe). At the end, it should also include the influence of tooth truncation (reduction of the outside diameter) on the tooth deflection. Such a program would require quite involved general Finite Element Analysis (or a similar analytical tool) module and it was considered too complicated and time consuming to include in this work.

Instead, *unloaded tooth contact analysis* module for straight bevel gears was developed in this work. It considers the influence of geometry and modifications on the tooth contact of the mating gears under light load. Apart from contact ones, no other deflections were considered. Such an analysis was practically used throughout the history to evaluate the gears and predict their performance at the higher loads. Gear teeth were painted, gears put in mesh at the proper positions (mounting distances) and *rolled* on the rolling machines under low load or, when possible in the actual assembly. The thin layer of paint is transferred from one gear to the other (if only one member is painted), or it is removed from the gear teeth surfaces (if both members are painted) in the areas where the

gears are contacting each other, indicating the contact pattern and its position. Such a process can be modeled by unloaded tooth contact analysis.
The first step in the contact analysis is determination of the position where the surfaces of the teeth would contact each other. Then, assuming the maximum amount of contact deflection and considering the shape of the gear surfaces, the contact area (patch) is found out at each position (rotation angle) of the gears. The complete area of the contact is determined by rolling the gears through the complete mesh cycle (motion to ensure the contact travel from fillet to tip). Contact between modified tooth surfaces is found by solving the system of five nonlinear equations formed by equating coordinates (three equations) and normals (two equations) of the pinion and sidegear surfaces. There are 6 unknows (R_i , β_i from equation (1) and rotation angle of each gear, ε_i) and one unknown has to be the input into the solution algorithm (usually angle of one gear). In this work the nonlinear equations are first linearized by using Newton’s iteration method [8], and the system of five linear equations is then solved by Gauss-Jordan method. The solution of the Gauss-Jordan method is used to obtain update for the Newton’s iteration method, and the algorithm is followed until all of the update values are lower that the predetermined small values. As the algorithm has to find the unique solutions, it is obvious that at least one surface has to have lead crown modification.
Gear teeth without such a modification would have contact along line, and unique set of solutions (contact line) can not be obtained in a simple and fast manner by using the proposed procedure. This is not considered to be a drawback, because all gear pairs have to have at least one member modified for their proper operation in actual applications. In addition, gears produced by hot (or warm) forging experience more shrinking of the material in the center of their teeth (due to non-uniform shrinking during cooling). There is a danger that the teeth might experience negative lead crown (‘hollow’) which is highly undesirable effect. For that reason forged straight bevel gears have at least a small lead crowning. The solutions of the above algorithm for a series of gear rotation positions yield *contact path*, shown in Figure 7. It is apparent that the contact path in the system with lead and profile modifications does not start at the fillet (root) of the teeth and does not continue all the way to the tip. Under light load and perfect alignment of the gears, the contact paths shown in figures 7(a1) and 7(b1) would be desirable, because the gears would transfer the motion in uniform manner and have the highest contact ratio (longest contact path).

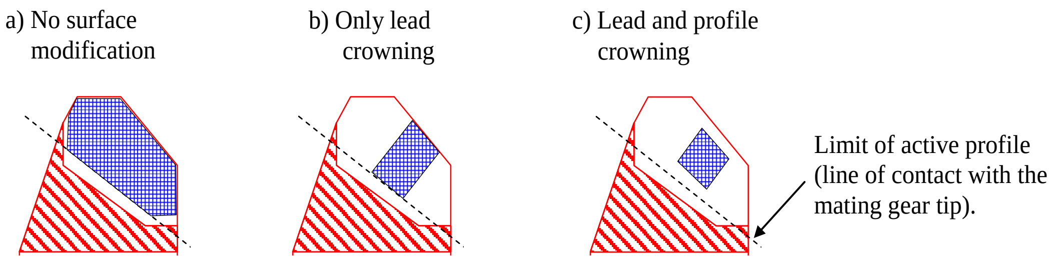


Fig. 6. Contact pattern of lightly loaded gears with a) no surface modification, b) only lead crowning and c) lead and profile crowning

In the presence of misalignments, the rotation of the gears is not uniform and contact path can shift towards the tip and root portion of the gears. It is possible that the next tooth pair is not yet in the position to take over the transmission of load and motion when the contact on the current tooth pair is finishing (when the contact reaches tip of one of the gears).
In such case the motion and load are transferred through a sudden rotation of the gears and impact of the next tooth pair, leading to the vibration and high (impact) loads. Profile modification, which in itself causes transmission error under low load and perfect alignment of gears, makes the transfer of power smoother when gears operate under heavier load and common assembly errors. The contact paths shown in Figure 7(a2) and 7(b2) are more desirable in actual applications. Due to the elastic deformation of the material, contact between two gears in mesh spreads over the area (does not stay confined to the contact path). Finding that area, its position and shape is an important part of the design and analysis of any type of gears. As already mentioned earlier, there are several ways to perform contact analysis, and the classical approach, applied by Litvin [9, 11] is used in this work. The obtained contact area on a sidegear is shown in the Figure 8. Apparently, the analyzed gears were modified in both, lead and profile direction. The area is formed by adding contact ellipses (formed by elastic contact of the surfaces) at each point of the contact path. Figure 8 also shows the boundaries of the actual sidegear tooth with truncated teeth and front and back webbing. The theoretical surfaces (shown in red in Figure 8), together with the root region are exported to the CAD application and trimmed to provide the surfaces for the actual near-net formed straight bevel gears.

In order to verify the proposed procedure, predicted and actual contact patterns are compared in Figure 9. The actual gears were placed at their proper positions on a rolling stand (specialized machine which rotates the gears). Marking compound (yellow paint) was applied to both gears and they were rotated in mesh under light load.

The contact pattern can be seen in Figure 9 as the darker region of the teeth from which the paint was removed during gear meshing (rotation). The shape and position of the predicted (blue regions in the Figure 9) and actual contact patterns are in good agreement, and can be used as the verification of the analysis procedure. The absence of the paint in the tip, front and back portions of the teeth indicates that the applied modifications are adequate for the practical applications.

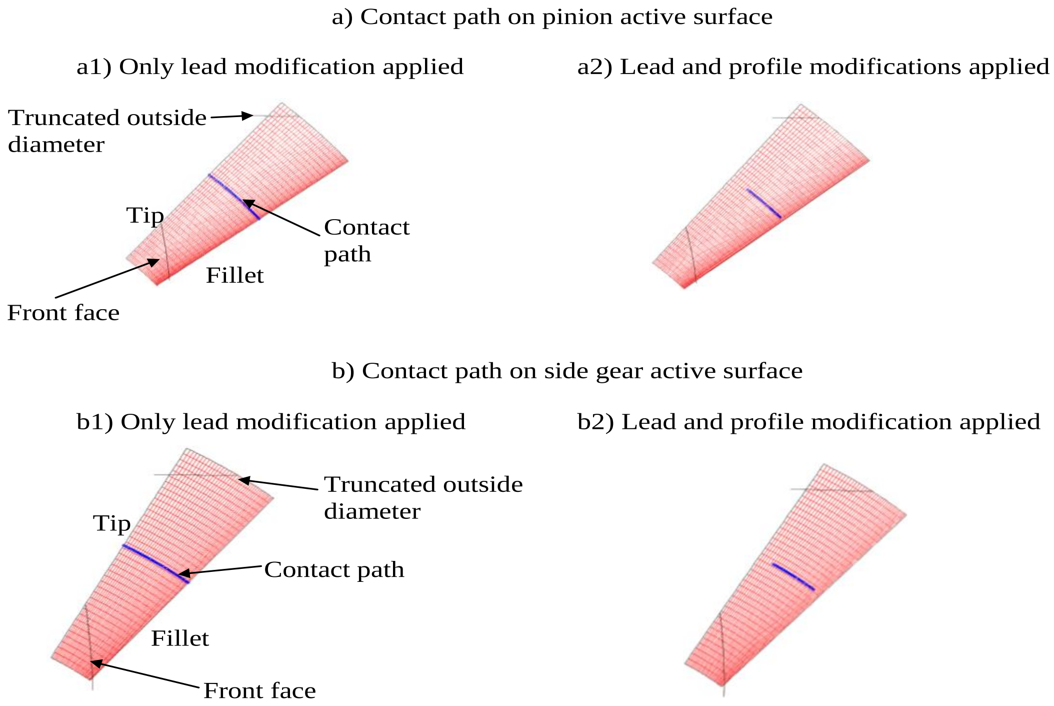


Fig. 7. Contact path on a) pinion and b) sidegear with only lead (a1, b1) and lead and profile (a2, b2) modifications

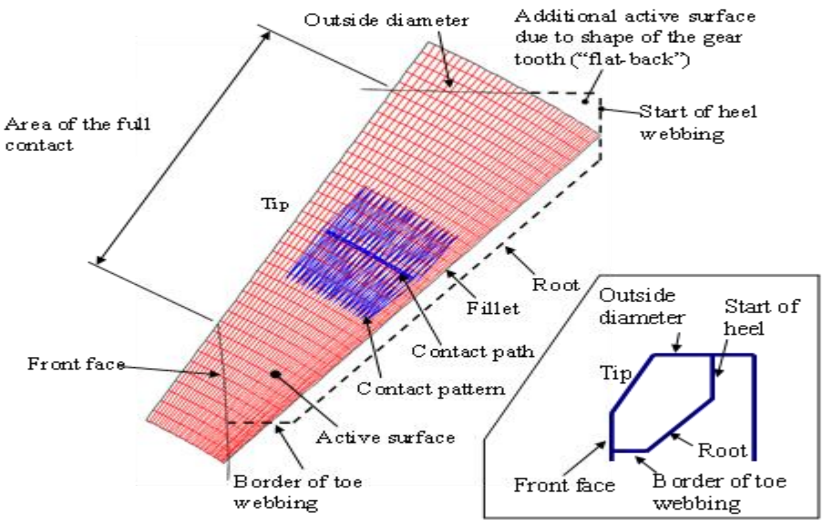
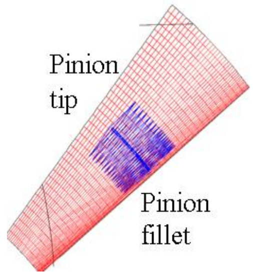
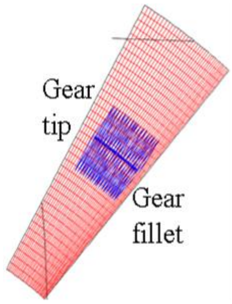
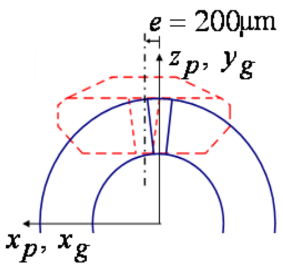
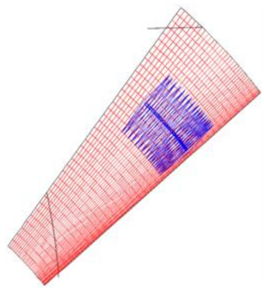
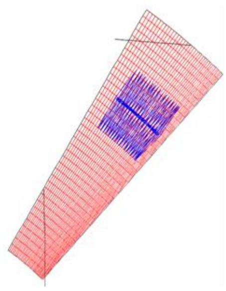
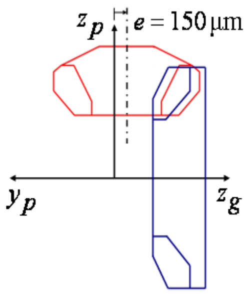
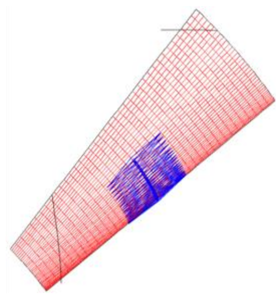
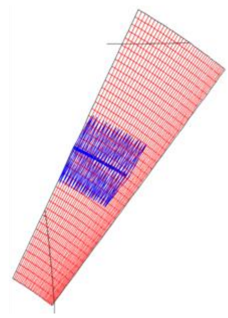
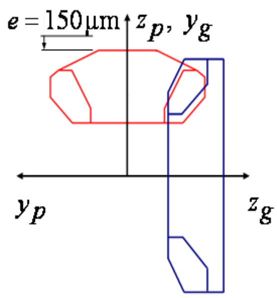
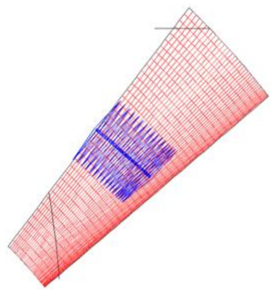
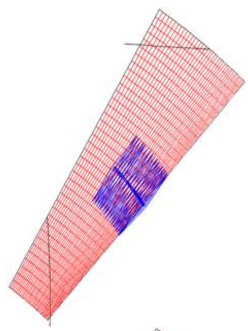


Fig. 8. Actual boundaries and obtained contact pattern on the sidegear in a system with lead and profile modifications.

Tab. 2. Example of contact analysis of gears under common misalignments

<table>
<tr><th>Misalignment explanation</th><th>Illustration of misalignment</th><th>Contact pattern on pinion</th><th>Contact pattern on sidegear</th></tr>
<tr><td>10A
Perfect alignment
(no misalignment)</td><td>Perfect alignment
(no misalignment)</td><td>
</td><td>
</td></tr>
<tr><td>10B
Pinion pushed 200 µm along x_p axis</td><td>
</td><td></td><td></td></tr>
<tr><td>10C
Pinion pushed 150 µm along z_g axis</td><td>
</td><td></td><td></td></tr>
<tr><td>10D
Pinion pushed -150 µm along z_p axis</td><td>
</td><td></td><td></td></tr>
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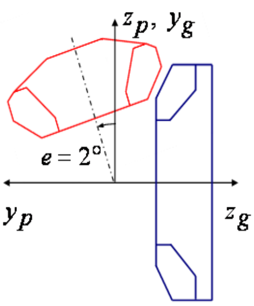
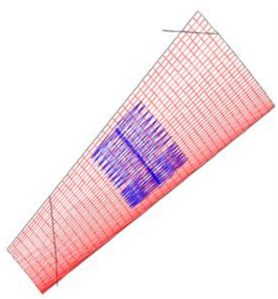
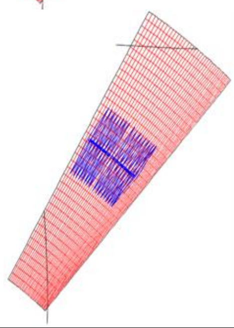
Pinion rotated -2° around x_p axis</td><td>
</td><td></td><td></td></tr>
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The influence of the common misalignments on the performance of the gears can be assessed by performing contact analysis. For such analysis the coordinates of the contacting surfaces are modified to reflect the intended misalignment. Figure 10 clarifies the rotation direction, orientation of the axes of the gears, and loaded flank (side) of the teeth, used in the contact analysis of the misaligned gears.

Table 2 shows the results of several such analyses. The case 10A shows the contact pattern of the gearset operating under perfect alignment. The contact pattern on both of the gears is contained safely within borders of the active surfaces of the teeth.

When the pinion is pushed by 200 μm into the sidegear tooth (along x_p), the contact on both members moves towards the back (heel). Under heavier load, the contact spreads towards the heel, and it is possible that in this case (10B) it will reach the heal portion of the gears and cause undesirable edge loading. In case 10C, the pinion is moved 150 μm from its design position along axis z_g (or $-y_g$), and it causes contact pattern on pinion to move towards root, and on the sidegear towards the tip. It is very likely that the gears in this case would experience high impact loading. In the next case, 10D, the pinon is moved -150 μm from its design position along its center axis (z_p), causing the contact pattern on pinion to move towards its tip, while on the sidegear it moves towards the root.

Finally, case 10E shows angular (shaft) misalignment where the pinion is rotated by 2° away from its design center axis. The contact pattern in this case moves towards the tip for both members which is expected, but it is still very close to the case 10A. Such behavior is expected for spherical involutes, because they are insensitive to angular misalignment in this direction. The cases in Table 2 represent common misalignments and the results agree well with AGMA standard [12]. The procedure used in this work can also simulate the influence of profile error (commonly occurring in forging) and several other shaft misalignments or their combinations. While not shown here, the presented procedure also yields the motion transmission error resulting from the modifications, as well as from the misalignments.

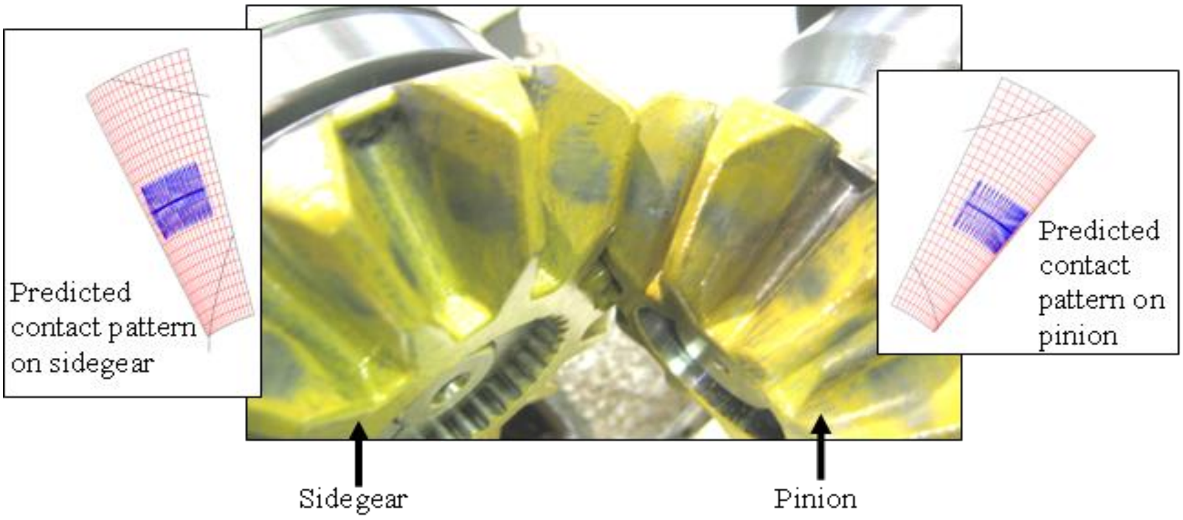

Fig. 9. Comparison of actual and predicted contact pattern on pinion and sidegear

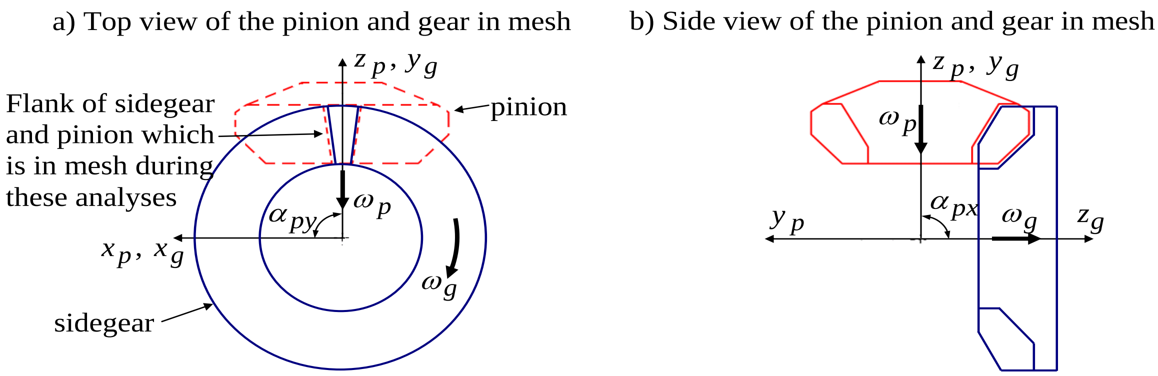

Fig. 10. Axes for pinion and sidegear in mesh, a) top view and b) side view.

5. Conclusion

Spherical involute surface is possibly the best suited tooth form for the straight bevel gears. The inability of the cutting machines to accurately (and economically) cut the spherical involute gears prevented widespread application of that tooth form. A practical application of the spherical involute tooth form to the forged (net-formed) straight bevel gears was demonstrated in this paper.

A short overview of the major modifications was given. Unloaded tooth contact analysis of the gears modified in profile and lead (lengthwise) directions was performed, and compared to the actual forged gears. The shape and position of the actual and predicted contact patterns were in good agreement, verifying the proposed procedure.

Finally, the contact analyses of gears operating under several common misalignments were performed and reported. The analysis of the gears operating under expected misalignments or tooth form errors is a very important part of the gear design and development. It was demonstrated that a rather small position error can cause large changes in the tooth contact position, and probably lead to the premature failures.

Once the amount of the contact pattern change under expected misalignments is known, the more appropriate modifications can be applied to obtain robust design which is not sensitive in the real-life applications.

Future work will concentrate on further application of the presented analysis tool. Module for the analysis of the forged spiral bevel gears with spherical involute surface will be developed. Finally, the tool for the analysis of the gears operating under higher load will be developed to better assess actual gear applications.

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