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The Cost Function for a Two-warehouse and Three-Echelon Inventory System

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KEYWORDS

Three-echelon inventory system,
One-for-one Replenishments,
Poisson Demands

ABSTRACT

In this paper, the cost function for a three-echelon inventory system with two warehouses is derived. Transportation times are constant and retailers face independent Poisson demand. Replenishments are one-for-one. The lead time of a retailer is determined not only by the constant transportation time but also by the random delay incurred due to the availability of stock at the warehouses. We consider two warehouses in the second echelon which may leads to having more delays which were incurred in the warehouses and facing different behaviors of independent Poisson demands. Because the replenishment policy is base stock, the obtained function can also be used in different ordering policies to compute the inventory holding and shortage costs.

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1. Introduction

Supply chains are generally complex and are characterized by numerous activities spread over multiple functions and organizations (Arshinder & Deshmukh, 2008). Supply chain has evolved very rapidly since 1990s showing an exponential growth in papers in different journals of interest to academics and practitioners (Burgess, et. al. 2006). Researchers were able to enhance some of the previously developed models, methods and optimization techniques for modeling and improving the performance of the supply chain.

Multi-echelon systems have received considerable attention during the last decades. Competitive forces and the high costs of unsatisfied demands have resulted in the use of multi-echelon inventory systems in providing service support for products with customers distributed over an extensive geographical region. These systems usually consist of a number of local stocking sites that serve as a first level of product support for customer demands. These sites will act, in turn, as customers to a higher level stocking site, or a warehouse, for replenishment of their stock after they are depleted by customer demands.

Since most of the repairable products are of high value with infrequent failures, most of the past studies have employed a one-for-one, (S-1, S), inventory ordering policy (Moinzadeh et al., 1986). Examples of these studies include Simon [16], Graves [5], Higa et, al [10], and Hajiaghaei-Keshteli and Sajadifar [7]

Poisson models with one-for-one ordering policies can be solved very efficiently. Karush [13] presents the lost-sales model with base-stock S, and also shows that mutually independent lead times can be solved by Erlang's loss formula, and the formula is convex. Convexity has later been proved differently by Jagers and Van Doorn [12]. Simon [16] derives the steady-state distribution for inventory levels at each site. Higa et al. [10] show Steady-state distribution functions those are derived for waiting time in an (S - 1, S) inventory system.

Axsäter [2] provides a simple recursive procedure to determine holding and shortage costs. He considers an inventory system with one warehouse and N retailers.

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Lead times are constant and the retailers face independent Poisson demand. He limits this inventory system to one-for-one Replenishment policies. Axsäter [3] first expressed costs as a weighted mean of costs for one-for-one ordering polices.

Svoronos and Zipkin [17] study base-stock policies for similar multi-echelon inventory systems. They show that lead time variances play an important role in system performance. Hausman [9] considers low demand, high cost items controlled on an (S - 1, S) basis, with all warehouse stock outs met on an emergency ordering basis.

Hill [11] considers continuous-review lost-sales inventory models with no fixed order cost and a Poisson demand process, holding cost per unit per unit time and a lost sales cost per unit.

Hajiaghaei-Keshteli and Sajadifar [7] considered a three-echelon inventory system with two warehouses and N retailers. Transportation times are constant and retailers face independent Poisson demand. Replenishments are one-for-one. The lead time of the retailer is determined not only by the constant transportation time but also by the random delay incurred due to the availability of stock at the warehouses.

They extend Axsater [2] work and add a warehouse as third echelon and consider having one more delay in shipment which may incurred in the new warehouse. They obtained the cost function for this inventory system and tested it by several examples.

In this paper, we are to derive the expected total holding and shortage costs for a unit demand in three-echelon inventory system with a one-for-one ordering policy for a three-echelon inventory system, using the idea of Hajiaghaei-Keshteli and Sajadifar [7]. We have two warehouses in second echelon. Therefore, this may leads to having more delays which were incurred in the warehouses and facing different behaviors of independent Poisson demand.

The retailers face independent Poisson demands. Unfilled demand is backordered and the shortage cost is a linear function of time until delivery, or equivalently, a time average of the net inventory when it is negative. Transportation times are constant and each echelon follows a base stock, or (S - 1, S), or one-for-one replenishment policies. Orders that cannot be delivered instantaneously from the warehouse are ultimately delivered on a first come, first serve basis. We obtained the cost function for the presented inventory system.

Also, because the replenishment policy is base stock, the obtained function can also be used in different inventory policy to compute the inventory holding and shortage costs.

For instance, Hajiaghaei-Keshteli et al. (2010), used the cost function which obtained from base stock policy, to compute (R,Q) ordering policy in three-echelon inventory system. So this means that the obtained function in this paper is very useful for future researches and researchers who want to determine the cost of other ordering policies in multi-echelon inventory systems or who want to present new ordering policy. For future investigations in this area one can see how Haji et al. (2008) and Moinzadeh (2002) presented new ordering policies and derived cost function.

2. Deriving the Cost Function

In this section, we consider a three-echelon inventory system with two warehouses (suppliers) in second echelon and two retailers belong to each warehouse as shown in Fig. 1.

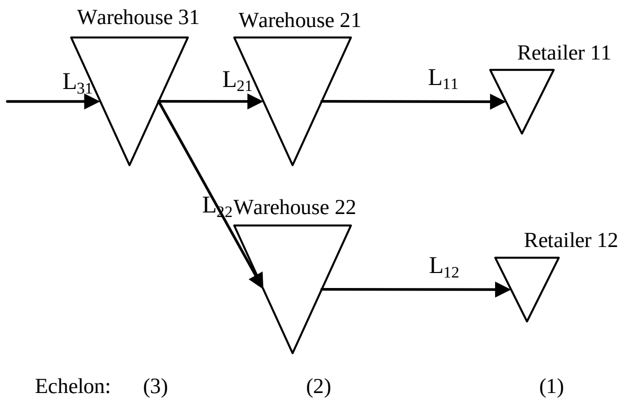

Fig. 1.Three-echelon Inventory System with Two Warehouses

Axsäter [2] found the expected total holding and shortage costs for a unit demand, that is, $c(S_0, S_1)$ for the one-for-one ordering policy in two-echelon inventory system.

Hajiaghaei-Keshteli and Sajadifar [7] found the expected total holding and shortage costs for a unit demand in three-echelon inventory system with two warehouses (suppliers) and one retailer. In their presented inventory system, with two warehouses (suppliers) and one retailer, as shown in Fig. 2, transportation times from an outside source to the warehouse 3, between warehouses, and also from the warehouse 2 to the retailers are constant. They assume that the retailer faces Poisson demand. Unfilled demand is backordered and the shortage cost is a linear function of time until delivery, or equivalently, a time average of the net inventory when it is negative. Each echelon follows a base stock, or (S-1, S), or one-for-one replenishment policies. This means essentially that they assume that ordering costs are low and can be disregarded.

When a demand occurs at a retailer, a new unit is immediately ordered from the warehouse 2 to warehouse 3 and also warehouse 3 immediately orders a new unit at the same time, that is, each echelon faces

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the same demand intensity. If demands occur while the warehouses are empty, shipments are delayed. When units are again available at the warehouses, delivered according to a first come, first served policy. In such situation the individual unit is, in fact, already virtually assigned to a demand when it occurs, that is, before it arrives at the warehouse.

They found $C(S_3,S_2,S_1)$ as the cost function which relates to S_3, S_2 , and S_1 that indicate the warehouse 3, warehouse 2, and the retailer inventory positions respectively in their system. For the one-for-one ordering policy, an arbitrary customer consumes $(S_1+S_2+S_3)^{th}$, $(S_1+S_2)^{th}$ and S_1^{th}, order placed by the warehouse 3, warehouse 2, and the retailer, respectively, just before his arrival to the retailer.

If the ordered unit arrives prior to its (assigned) demand, it is kept in stock and incurs carrying cost; if it arrives after its assigned demand, this customer demand is backlogged and shortage costs are incurred until the order arrives. This is an immediate consequence of the ordering policy and of our assumption that delayed demands and orders are filled on a first come, first served basis.

In this paper, based on the one-for-one ordering policy as described above, we want to obtain the exact value of $C(S_{31},S_{21},S_{22},S_{11},S_{12})$, the expected total holding and shortage costs per time unit for the inventory system as shown in Fig. 1.

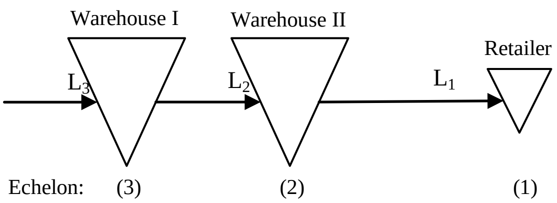

Fig. 2.Three-echelon Serial Inventory System

We fix the two retailers (11 and 12), the two warehouses (21 and 22), and the warehouse 31, to echelon one, two and three respectively as shown in Fig. 2. In order to derive the cost function, the following notations are used for this inventory system:

- S_{31} Inventory position at warehouse 31
- S_{21} Inventory position at warehouse 21
- S_{22} Inventory position at warehouse 22
- S_{11} Inventory position at retailer 11
- S_{12} Inventory position at retailer 12
- L_{11} Transportation time from the Warehouse 21 to the retailer 11
- L_{12} Transportation time from the Warehouse 22 to the retailer 12
- L_{21} Transportation time from the Warehouse 31 to the Warehouse 21
- L_{22} Transportation time from the Warehouse 31 to the Warehouse 22
- L_{31} Transportation time from the outside source to the Warehouse 31(Lead time of the Warehouse 31)
- T_{31} Random delay incurred due to the shortage of stock at the Warehouse 31
- T_{21} Random delay incurred due to the shortage of stock at the Warehouse 21
- T_{22} Random delay incurred due to the shortage of stock at the Warehouse 22
- λ_{11} Demand intensity at retailer 11
- λ_{12} Demand intensity at retailer 12
- λ_{31} Demand intensity at the warehouse 31
- h_{11} Holding cost per unit per unit time at retailer 11
- h_{12} Holding cost per unit per unit time at retailer 12
- h_{21} Holding cost per unit per unit time at warehouse 21
- h_{22} Holding cost per unit per unit time at warehouse 22
- h_{31} Holding cost per unit per unit time at warehouse 31
- β_{11} Shortage cost per unit per unit time at the retailer 11
- β_{12} Shortage cost per unit per unit time at the retailer 12

We characterizes our one-for-one replenishment policy by the $(S_{31},S_{21},S_{22},S_{11},S_{12})$ of order-up-to inventory positions which S_{31}, S_{21}, S_{22}, S_{11}, S_{12} are the inventory position at warehouse 31 (echelon 3), the inventory position at warehouse 21 and 22 (echelon 2), and the inventory position at retailer 11 and 12 (echelon 1), respectively. So we consider a one-for-one replenishment rule with $(S_{30},S_{21},S_{22},S_{11},S_{12})$ as the vector of order-up-to levels.

When a demand occurs at retailer 11 with a demand density, λ_{11}, a new unit is immediately ordered from the warehouse 21 to warehouse 31 and also warehouse 31 immediately orders a new unit at the same time. This action also occurs for retailer 12, warehouse 22 and warehouse 31 with demand density λ_{11} respectively.

For the one-for-one ordering policy as described above, any unit ordered by the retailer 11 is used to fill the S_{11}^{th} demand following this order, hereafter, referred to as its demand. It means that, an arbitrary customer consumes S_{11}^{th} order placed by the retailer 11 just before his arrival to the retailer and we can also say that the customer consumes S_{21}^{th} (S_{31}^{th}) order placed by the warehouse 21 (warehouse 31) just before his arrival to the retailer. If the ordered unit arrives prior to its (assigned) demand, it is kept in stock and incurs carrying cost; if it arrives after its assigned demand, this customer demand is backlogged and shortage costs are incurred until the order arrives. This is an immediate consequence of the ordering policy and of

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our assumption that delayed demands and orders are filled on a first come, first served basis.

We confine ourselves to the case where inventory position in all warehouses and retailers is equal or greater than zero.

To find the total cost, following the Axsäter [3], Hajiaghaei-Keshteli and sajadifar [7]'s idea, let $g_{ij}^{S_{ij}}(.)$ (ij= 11,12,21,22,31) denote the density function of Erlang (λ_{xy}, S_{ij}) distribution of the time elapsed between the placement of an order and the occurrence of its assigned demand unit:

$$g_{ij}^{S_{ij}}(t)=\frac{\lambda_{xy}{}^{S_{ij}}t^{S_{ij}-1}}{(S_{ij}-1)!}e^{-\lambda t} \qquad \begin{matrix} xy=11,12 \\ ij=11,12,21,22,31 \end{matrix} \tag{1}$$

The corresponding cumulative distribution function:

$G_{ij}^{S_{ij}}(t)$ is:

$$G_{ij}^{S_{ij}}(t)=\sum_{k=S_{ij}}^{\infty}\frac{(\lambda_{xy}t)^k}{k!}e^{-\lambda_{xy}t} \tag{2}$$

An order placed by the retailer 11, arrives after $L_{11}+T_{21}$ time units, and an order placed by warehouse 21, arrives after $L_{21}+T_{31}$ time units, where T_{ij} (ij=31,21,22) is the random delay encountered at echelon 2 and 3 in case a warehouse in the echelon 2 or 3 is out of stock.

As we mentioned earlier, we want to show deriving the cost function clearly in this section.

So, let trace one path; 11, 21, and 31. Let $\pi_{11}^{S_{11}}(t_{21})$ denotes the expected retailer carrying and shortage costs incurred to fill a unit of demand at retailer when inventory position at retailer is S_{11}. We evaluate this quantity by conditioning on $T_{21}=t_{21}$. Note that the conditional expected cost is independent of S_{21} and S_{31}, and is given by:

$$\pi_{11}^{S_{11}}(t_{21})=\beta_{11}\int_0^{L_{11}+t_{21}}(L_{11}+t_{21}-s)g_{11}^{S_{11}}(s)ds + h_{11}\int_{L_{11}+t_{21}}^{\infty}(s-L_{11}-t_{21})g_{11}^{S_{11}}(s)ds, S_{11}>0; \tag{3}$$

$$\pi_{11}^{0}(t_{21})=\beta_{11}(L_{11}+t_{21}); \tag{4}$$

The conditional distribution of T_{21}, on condition that $T_{31}=t_{31}$, obtained from:

$$P\langle T_{21}=0 \mid T_{31}=t_{31}\rangle=\sum_{k=0}^{S_{21}-1}\frac{\lambda_{11}^k(L_{21}+t_{31})^k}{k!}e^{-\lambda_{11}(L_{21}+t_{31})} = 1-G_{21}^{S_{21}}(L_{21}+t_{31}) \tag{5}$$

Also the conditional density function $f(T_{21})$ for $0\leq T_{21}\leq L_{21}+t_{31}$ is given by:

$$f\langle T_{21}=t_{21} \mid T_{31}=t_{31}\rangle=g_{21}^{S_{21}}(L_{21}+t_{31}-t_{21}) = \frac{\lambda_{11}^{S_{21}}(L_{21}+t_{31}-t_{21})^{S_{21}-1}}{(S_{21}-1)!}e^{-\lambda_{11}(L_{21}+t_{31}-t_{21})} \tag{6}$$

The expression (6) shows the probability of time of receiving S_{21}th demand; that is after receiving $(S_{21}-1)$th demand, S_{21}th demand occurs at the time of $L_{21}+t_{31}-t_{21}$. On the other view, we can say the time distance between receiving S_{21}th demand and receiving the order from warehouse 31 ($L_{21}+t_{31}$) is t_{21} and we call it the delay time that occurred in warehouse 21. As we mentioned earlier the warehouses 21 and 22 faces a Poisson demand process with rate λ_{11} and λ_{12} respectively, like the retailers 11 and 12. But the warehouse 31, faces a Poisson demand process with rate λ_{31}, which is equal to λ_{11} plus λ_{12}. Therefore we use the expression (5) in third echelon as follows:

$$P(T_{31}=0)=\sum_{k=0}^{S_{31}-1}\frac{\lambda_{31}^k(L_{31})^k}{k!}e^{-\lambda_{31}L_{31}}=1-G_{31}^{S_{31}}(L_{31}) \tag{7}$$

The density function $f(t_{31})$ for $0\leq t_{31}\leq L_{31}$,because we assume that inventory positrons at all facilities in this system are equal or greater that zero, is given by:

$$f(t_{31})=g_{31}^{S_{31}}(L_{31}-t_{31})=\frac{\lambda_{31}^{S_{31}}(L_{31}-t_{31})^{S_{31}-1}}{(S_{31}-1)!}e^{-\lambda_{31}(L_{31}-t_{31})} \tag{8}$$

Let $\Pi_{11}^{S_{11}}(S_{31},S_{21})$ denotes the expected retailer carrying and shortage costs incurred to fill a unit of demand at retailer when S_{31}, S_{21}, and S_{11} are the inventory position at warehouse 31, warehouse 21 and the retailer 11, respectively. Considering both states that we have delay time or have not in both warehouses, we obtain the cost that incurred to fill a unit of demand at retailer, as follows:

$$\begin{aligned}\Pi_{11}^{S_{11}}(S_{31},S_{21}) &= (1-G_{31}^{S_{31}}(L_{31}))\left(\int_0^{L_{21}}g_{21}^{S_{21}}(L_{21}-t_{21})\pi_{11}^{S_{11}}(t_{21})dt_{21}+(1-G_{21}^{S_{21}}(L_{21}))\pi_{11}^{S_{11}}(0)\right) \\ &+\int_0^{L_{31}}g_{31}^{S_{31}}(L_{31}-t_{31})\left(\int_0^{L_{21}+t_{31}}g_{21}^{S_{21}}(L_{21}+t_{31}-t_{21})\pi_{11}^{S_{11}}(t_{21})dt_{21}+(1-G_{21}^{S_{21}}(L_{21}+t_{31})\pi_{11}^{S_{11}}(0))\right)dt_{31}.\end{aligned} \tag{9}$$

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The long-run average shortage and retailer carrying costs is clearly given by $\lambda_{11}\Pi_{11}^{S_{11}}(S_{31},S_{21})$.

The conditional expected warehouse 21 holding cost, $\pi_{21}^{S_{21}}(t_{31})$, on condition that $T_{31}=t_{31}$, is independent of S_{31} and given by:

$$\pi_{21}^{S_{21}}(t_{31})=h_{21}\int_{L_{21}+t_{31}}^{\infty}(s-L_{21}-t_{31})g_{21}^{S_{21}}(s)ds, S_{21}>0; \qquad (10)$$

Therefore we find the average warehouse holding cost per unit for warehouse 21 when the inventory position at warehouse 31 is S_{31} as follows:

$$\Pi_{21}^{S_{21}}(S_{31})=\int_{0}^{L_{31}}g_{31}^{S_{31}}(L_{31}-t_{31})\pi_{21}^{S_{21}}(t_{31})dt_{31}+(1-G_{31}^{S_{31}}(L_{31}))\pi_{21}^{S_{21}}(0). \qquad (11)$$

Also the average warehouse 31 holding costs per unit $\eta(S_{31})$, which depends only on the inventory position S_{31} is:

$$\eta(S_{31})=h_{31}\int_{L_3}^{\infty}g_{31}^{S_{31}}(s)(s-L_{31})ds \qquad (12)$$

and $\eta(0)=0$.

We conclude that the long-run system-wide cost for the path in this inventory system as shown in fig. 2, by adding the costs which occurred in each echelon and is given by:

$$C(S_{31},S_{21},S_{11})=\lambda_{11}(\Pi_{11}^{S_{11}}(S_{31},S_{21}) + \Pi_{21}^{S_{21}}(S_{31})+\eta(S_{31})) \qquad (13)$$

So, by considering the cost of another path (12,22,31), which can be obtained similarly, the total cost for the whole inventory system shown in fig. 2 is as follows:

$$C(S_{31},S_{21},S_{22},S_{11},S_{12})=\sum_{i=1}^{2}\lambda_{1i}(\Pi_{1i}^{S_{1i}}(S_{31},S_{2i})+\Pi_{2i}^{S_{2i}}(S_{31})+\eta(S_{31})) \qquad (14)$$

3. Numerical Examples

In this section, we present an example to reveal the convexity tendency of our formulation. We want to show that our cost function has a minimum in a specific inventory position in echelons for an inventory system. We do this for the system with two warehouses in second echelon and a retailer belongs to each warehouse. For this purpose, assuming five variables S_{11}, S_{12}, S_{21}, S_{22}, S_{31}, we fix the value of the parameters L.., h.., β.. and λ.. to 1, 1, 10, and 2, respectively, as Hajiaghai-Keshteli and Sajadifar [7] valued the same parameters in their paper. Also we use these values, because most of previous works like Haji et al. [6], and Moinzadeh [14], and some other works used the same values for the same parameters. We tested the cost function by giving some values to variables. The function has five variables, and therefore we have six dimensions.

In order to show the function's behavior we fix five variables and see the function in remained two dimensions. We give value 2 and 3 to S_{11}, S_{12}, S_{21}, and S_{22} variables and see the function's behavior with the change of S_{31} as we can see in Fig. 3. As depicted in Fig. 3, these results show the cost function's behavior and also ensure us to have a minimum cost in a specific inventory position in echelons for the inventory system.

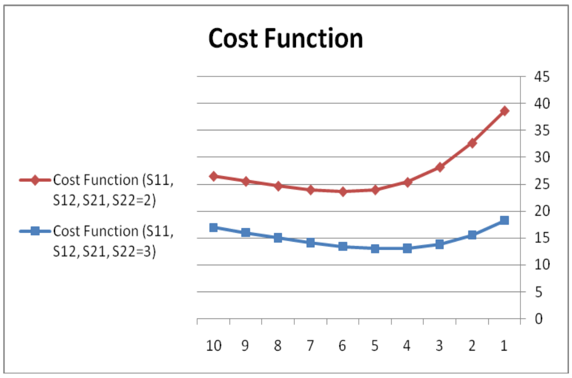

Fig. 3. The behavior of Cost Function with the change of S_{31} and fixing other variables to 2 and 3

4. Conclusion and Future Works

In this paper a three-echelon inventory system with two warehouses is considered. Transportation times are constant and retailers face independent Poisson demand. The model considered here is different from previously addressed in considering the number of facilities.

In order to find the cost function in the presented inventory system, conditional distribution for delays in shipment which may incurred in warehouses is used and behavior of different Poisson demands in retailers is considered. There are potentially unlimited opportunities for research in multi-echelon inventory systems.

For future researches, it is possible to investigate and develop the proposed model with more assumption. Other realistic aspects of the problem like ordering policies, replenishment policies and demand's behavior are proposed. Also, because the replenishment policy is base stock, the obtained function can also be used in different inventory policy to compute the inventory holding and shortage costs.

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