Overview and Comparison of Short-term Interval Models for Financial Time Series Forecasting

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ABSTRACT

In recent years, various time series models have been proposed for financial markets forecasting. In each case, the accuracy of time series forecasting models are fundamental to make decision and hence the research for improving the effectiveness of forecasting models have been curried on. Many researchers have compared different time series models together in order to determine more efficient once in financial markets. In this paper, the performance of four interval time series models including autoregressive integrated moving average (ARIMA), fuzzy autoregressive integrated moving average (FARIMA), hybrid ANNs and fuzzy (FANN) and Improved FARIMA models are compared together. Empirical results of exchange rate forecasting indicate that the FANN model is more satisfactory than other those models. Therefore, it can be a suitable alternative model for interval forecasting of financial time series.

1. Introduction

Exchange rate is one of the most effective variables in financial environments and its changes can be very important for economic decision makers [1]. Several investigations have been accomplished in the field of exchange rate forecasting [2-5] that number of these investigations represents the mentioned issue importance. Nowadays, despite the numerous financial time series models available, accurate forecasts of exchange rate are not easy task [6-11].

Several different models have been suggested for time series forecasting, which are generally categorized in to linear and nonlinear models. One of the most important and widely used linear time series models are autoregressive integrated moving average (ARIMA) models that have enjoyed fruitful applications in forecasting social, economic, engineering, foreign exchange, and stock problems. Second class of time series models are nonlinear models. Artificial neural networks are one of these models that are able to approximate various nonlinearities in the data and are flexible computing frameworks for modelling a broad range of nonlinear problems.

One significant advantage of ANNs over than other nonlinear classes is that ANNs are universal approximators which can approximate a large class of functions with a high degree of accuracy. No prior assumption of the model form is required in the model building process.

Instead, the network model is largely determined by the characteristics of the data. Commonly used neural networks include multi-layer perceptrons (MLPs), radial basis functions (RBFs), probabilistic neural networks (PNNs), and general regression neural networks (GRNNs) [12]. Single hidden layer feedforward network is the most widely used model form for time series forecasting [13].

Forecasting accuracy is one of the most important factors to choose the forecasting model; therefore,

several researchers have compared different time series models together in order to determine more accurate once. Ture compared the performance of four different time series models to forecast the hepatitis *A* virus infection [14]. Taylor *et al.* compared the univariate models for forecasting electricity demand [15]. Kima [16] forecasted the international tourist flows to Australia for comparison between the direct and indirect models. Cho also compared the three different approaches to tourist arrival forecasting [17]. Some other researches in this field are as follows: Weatherforda [18] to hotel revenue management forecasting, Smith [19] to traffic flow forecasting, and Sfetsos [20] to mean hourly wind speed time series forecasting.

As similar, various researches have been also done in the financial fields. Alon compared the performance of artificial neural networks and traditional models to aggregate retail sales forecasting [21]. Meade [22] compared the accuracy of short term foreign exchange forecasting models. Leunga *et al.* [23] compared the classification and level estimation models to forecasting the stock indices. Lisi also compared the neural networks and chaotic models for exchange rate prediction [24]. In this paper, the performance of four different interval time series models is compared for financial markets forecasting. The rest of the paper is organized as follows. In the next section, concepts of four used time-series models are briefly reviewed. Empirical results from forecasting the exchange rate (US dollar/ Iran rial) are reported in Section 3. The performance of each model is compared together in section 4, and finally the conclusions are discussed.

2. Time Series Forecasting Models

There are several different approaches for time series modelling. Interval models are a special class of the quantitative forecasting models. In interval models, an interval is calculated as optimum forecast of independent variable. In this section, four interval models are briefly reviewed.

2-1. The Auto-Regressive Integrated Moving Average Model

In an autoregressive integrated moving average model, the future value of a variable is assumed to be a linear function of several past observations and random errors. That is, the underlying process, generating the time series has the following form:

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + ... + \phi_p y_{t-p} + \varepsilon_t$$
$$- \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - ... - \theta_q \varepsilon_{t-q} \tag{1}$$

where y_t and ε_t are the actual value and random error at time period t, respectively; ϕ_i $(i = 1,2,...,p)$ and θ_j $(j = 1,2,...,q)$ are model parameters. p and q are integers and often referred to as orders of the model.

Random errors, ε_t, are assumed to be independently and identically distributed with a mean of zero and a constant variance of σ^2.

The Box–Jenkins [25] methodology includes three iterative steps of model identification, parameter estimation and diagnostic checking. The basic idea of model identification is that if a time series is generated from an ARIMA process, it should have some theoretical autocorrelation properties. By matching the empirical autocorrelation patterns with the theoretical ones, it is often possible to identify one or several potential models for the given time series. Box and Jenkins [25] proposed to use the autocorrelation function (ACF) and the partial autocorrelation function (PACF) of the sample data as the basic tools to identify the order of the ARIMA model.

Once a tentative model is specified, estimation of the model parameters is straightforward. The parameters are estimated such that an overall measure of errors is minimized. This can be done with a nonlinear optimization procedure. The last step of model building is the diagnostic checking of model adequacy. This is basically to check if the model assumptions about the errors, ε_t, are satisfied.

Several diagnostic statistics and plots of the residuals can be used to examine the goodness of fit of the tentatively entertained model to the historical data. If the model is not adequate, a new tentative model should be identified, which is again followed by the steps of parameter estimation and model verification. Diagnostic information may help suggest alternative model(s). This three-step model building process is typically repeated several times until a satisfactory model is finally selected.

2-2. The Fuzzy Auto-Regressive Integrated Moving Average

The parameter of ARIMA(p,d,q), $\varphi_1, \varphi_2,....,\varphi_p$ and $\theta_1, \theta_2,....,\theta_q$ are crisp. Instead of using crisp, fuzzy parameters, $\tilde{\varphi}_1, \tilde{\varphi}_2,....,\tilde{\varphi}_p$ and $\tilde{\theta}_1, \tilde{\theta}_2,....,\tilde{\theta}_q$, in the form of triangular fuzzy numbers are used in fuzzy autoregressive integrated moving average models [26]. A fuzzy ARIMA(p,d,q) model is described by a fuzzy function with a fuzzy parameter:

$$\tilde{\Phi}_p(B)W_t = \tilde{\theta}_q(B)a_t \tag{2}$$

$$W_t = (1-B)^d (Z_t - \mu) \tag{3}$$

$$\tilde{W}_t = \tilde{\varphi}_1 W_{t-1} + \tilde{\varphi}_2 W_{t-2} + ... + \tilde{\varphi}_p W_{t-p} + a_t$$
$$- \tilde{\theta}_{p+1} a_{t-1} - \tilde{\theta}_{p+2} a_{t-2} - ... - \tilde{\theta}_{p+q} a_{t-q} \tag{4}$$

where $\{Z_t\}$ are observations, $\tilde{\varphi}_1, \tilde{\varphi}_2,....,\tilde{\varphi}_p$ and $\tilde{\theta}_1, \tilde{\theta}_2,....,\tilde{\theta}_q$, are fuzzy numbers. Equation (4) is modified as:

$$\tilde{W}_t = \tilde{\beta}_1 W_{t-1} + \tilde{\beta}_2 W_{t-2} + …. + \tilde{\beta}_p W_{t-p} + a_t$$
$$- \tilde{\beta}_{p+1} a_{t-1} - \tilde{\beta}_{p+2} a_{t-2} - … - \tilde{\beta}_{p+q} a_{t-q} \tag{5}$$

Fuzzy parameters in form of triangular fuzzy numbers are used:

$$\mu_{\beta_i}(\beta_i) = \begin{cases} 1 - \dfrac{|\alpha_i - \beta_i|}{c_i} & if \quad \alpha_i - c_i \le \beta_i \le \alpha_i + c_i, \\ 0 & otherwise, \end{cases} \tag{6}$$

where $\mu_{\tilde{\beta}}(\beta_i)$ is the membership function of the fuzzy set that represents parameter β_i, α_i is the centre of the fuzzy number, and c_i is the width or spread around the centre of the fuzzy number. Using fuzzy parameters β_i in the form of triangular fuzzy numbers and applying the extension principle, it becomes clear [27] that the membership of W in (5) is given as (7).

$$\mu_{\tilde{w}}(W_t) = \begin{cases} 1 - \dfrac{\left| W_t - \sum\limits_{i=1}^{p} \alpha_i W_{t-i} - a_t + \sum\limits_{i=1}^{p} \alpha_i a_{t+p-i} \right|}{\sum\limits_{i=1}^{p} c_i |W_{t-i}| + \sum\limits_{i=1}^{p} c_i |a_{t+p-i}|} \\ \qquad for \quad W_t \ne 0, \quad a_t \ne 0 \\ 0 \qquad\qquad otherwise \end{cases} \tag{7}$$

Simultaneously, Z_t represents the tth observation, and h-level is the threshold value representing the degree to which the model should be satisfied by all the data points $y_1, y_2, …., y_k$ to a certain h-level.

$$\mu_y(y_t) \ge h \qquad for \quad t = 1,2,…, k \tag{8}$$

The index t refers to the number of nonfuzzy data used for constructing the model. On the other hand, the fuzziness S included in the model is defined by:

$$S = \sum_{i=1}^{p} \sum_{t=1}^{k} c_i |\varphi_{ii}| \|W_{t-i}\| + \sum_{i=p+1}^{p+q} \sum_{t=1}^{k} c_i |\rho_{i-p}| \|a_{t+p-i}\| \tag{9}$$

Where ρ_{i-p} is the autocorrelation coefficient of time lag i-p, φ_{ii} is the partial autocorrelation coefficient of time lag i. The weight of c_i depends on the relation of time lag i and the present observation, where the p of AR (p) is derived by PACF and the q of MA (q) is derived by ACF.

Next, the problem of finding the fuzzy ARIMA parameters was formulated as a linear programming problem as (10). At last, according to the Ishibuchi and Tanaka [28] opinion, the data around the model's upper bound and lower bound is deleted when the fuzzy ARIMA model has outliers with wide spread, and then reformulating the fuzzy regression model.

$$Minimize \quad S = \sum_{i=1}^{p} \sum_{t=1}^{k} c_i |\varphi_{ii}| \|W_{t-i}\| + \sum_{i=p+1}^{p+q} \sum_{t=1}^{k} c_i |\rho_{i-p}| \|a_{t+p-i}\|$$

$$\sum_{i=1}^{p} \alpha_i W_{t-i} + a_t - \sum_{i=p+1}^{p+q} \alpha_i a_{t+p-i} + (1+h)\left(\sum_{i+1}^{p} c_i |W_{t-i}| + \sum_{i=p+1}^{p+q} c_i |a_{t+p-i}| \right) \ge W_t \quad t = 1,2,.., k$$

$$subject.to \quad \sum_{i=1}^{p} \alpha_i W_{t-i} + a_t - \sum_{i=p+1}^{p+q} \alpha_i a_{t+p-i} + (1+h)\left(\sum_{i+1}^{p} c_i |W_{t-i}| + \sum_{i=p+1}^{p+q} c_i |a_{t+p-i}| \right) \le W_t \quad t = 1,2,.., k$$

$$c_i \ge 0 \quad for\ i = 1,2,…, p + q \tag{10}$$

2-3. The Improved FARIMA Model with Probabilistic Neural Networks (PNNs)

Forecasting interval of the fuzzy autoregressive integrated moving average models is extended in some specific data conditions. According to the Ishibuchi and Tanaka opinion, forecasting interval can be too wide, when training data set includes the significant difference or outlying case.

In improved model, the abilities of the probabilistic neural networks (PNNs) [29] is used in order to recognize more probability spaces in forecasting interval of FARIMA model. Technically, PNN is a classifier and is able to deduce the class/group of a

given input vector after the training process is completed. PNN is conceptually built on the Bayesian model of classification which, given enough data, is capable of classifying a sample with the maximum probability of success [30]. The procedure of improved model is as follows:

Phase I: Fitting the FARIMA model using the available observations. The result of phase I is:

$$\tilde{W}_t = \langle \alpha_1, c_1 \rangle W_{t-1} + … + \langle \alpha_p, c_p \rangle W_{t-p} + a_t$$
$$- \langle \alpha_{p+1}, c_{p+1} \rangle a_{t-1} - …. - \langle \alpha_{p+q}, c_{p+q} \rangle a_{t-q}, \tag{11}$$

where $W_t = (1 - B)^d (Z_t - \mu)$, α_i is the centre of the fuzzy number, and c_i is the width or spread around the centre of the fuzzy number. Then, the obtained interval of FARIMA model is divided to n equal sections for using in probabilistic neural network. The subinterval which includes the real value or *n-1* other subintervals are considered as target data. Other information -results of FARIMA and time series data- is considered as training data.

Phase II: Designing and training a network to recognize more probability spaces in forecasted interval of FARIMA model. The result of this phase is a interval with *1/n* width and confidence coefficient α. (α is the performance of PNN in the test data).

2-4. The Hybrid Artificial Neural Networks and Fuzzy Logic

A hybrid FANN model can be described by a fuzzy function as follows [31]:

$$\tilde{y}_t = f(\tilde{b}_0 + \sum_{j=1}^{q} \tilde{w}_j \cdot g(\tilde{b}_{0j} + \sum_{i=1}^{p} \tilde{w}_{i,j} \cdot y_{t-i})) \quad (12)$$

where y_t are observations, $\tilde{w}_j, \tilde{w}_{i,j}, \tilde{b}_0, \tilde{b}_{0,j}$ are fuzzy numbers. Equation (12) is modified as:

$$\tilde{y}_t = f(\tilde{b}_0 + \sum_{j=1}^{q} \tilde{w}_j \cdot \tilde{X}_{t,j}) = f(\sum_{j=0}^{q} \tilde{w}_j \cdot \tilde{X}_{t,j}) \quad (13)$$

where $\tilde{X}_{t,j} = g(\tilde{b}_{0j} + \sum_{i=1}^{p} \tilde{w}_{i,j} \cdot y_{t-i})$. Fuzzy parameters in the form of triangular fuzzy numbers are used as (14),

$$\mu_{\tilde{w}_{i,j}}(w_{i,j}) = \begin{cases} \dfrac{1}{b_{i,j} - a_{i,j}} (w_{i,j} - a_{i,j}) \\ \qquad if \quad a_{i,j} \le w_{i,j} \le b_{i,j}, \\ \dfrac{1}{b_{i,j} - c_{i,j}} (w_{i,j} - c_{i,j}) \\ \qquad if \quad b_{i,j} \le w_{i,j} \le c_{i,j}, \\ \\ 0 \qquad\qquad otherwise, \end{cases} \quad (14)$$

where $\mu_{\tilde{w}}(w_{i,j})$ is the membership function of the fuzzy set that represents parameter $w_{i,j}$. Applying the extension principle, it becomes clear that the membership of $\tilde{X}_{t,j} = g(\sum_{i=0}^{p} \tilde{w}_{i,j} \cdot y_{t-i})$ in (13) is given as (15) [31].

$$\mu_{\tilde{w}_{i,j}} = \begin{cases} \dfrac{\left(x_{t,j} - g\left(\sum_{i=0}^{p} a_{i,j} \cdot y_{t-i}\right)\right)}{g\left(\sum_{i=0}^{p} b_{i,j} \cdot y_{t-i}\right) - g\left(\sum_{i=0}^{p} a_{i,j} \cdot y_{t-i}\right)} \\ if \quad g\left(\sum_{i=0}^{p} a_{i,j} y_{t-i}\right) \le x_{t,j} \le g\left(\sum_{i=0}^{p} b_{i,j} y_{t-i}\right), \\ \dfrac{\left(x_{t,j} - g\left(\sum_{i=0}^{p} c_{i,j} \cdot y_{t-i}\right)\right)}{g\left(\sum_{i=0}^{p} b_{i,j} \cdot y_{t-i}\right) - g\left(\sum_{i=0}^{p} c_{i,j} \cdot y_{t-i}\right)} \\ if \quad g\left(\sum_{i=0}^{p} b_{i,j} y_{t-i}\right) \le x_{t,j} \le g\left(\sum_{i=0}^{p} c_{i,j} y_{t-i}\right), \\ \\ 0 \qquad\qquad\qquad otherwise \end{cases} \quad (15)$$

Now, considering the fuzzy parameters w_j as (16)

$$\mu_{\tilde{w}_j}(w_j) = \begin{cases} \dfrac{1}{e_j - d_j}(w_j - d_j) \\ \qquad if \quad d_j \le w_j \le e_j, \\ \dfrac{1}{e_j - f_j}(w_j - f_j) \\ \qquad if \quad e_j \le w_j \le f_j, \\ 0 \qquad\qquad otherwise, \end{cases} \quad (16)$$

the membership function of $\tilde{y}_t = f(\tilde{b}_0 + \sum_{j=1}^{q} \tilde{w}_j \cdot \tilde{X}_{t,j}) = f(\sum_{j=0}^{q} \tilde{w}_j \cdot \tilde{X}_{t,j})$ is given as (17).

$$\mu_{\tilde{Y}}(y_t) \cong \begin{cases} \dfrac{-B_1}{2A_1} + \left[\left(\dfrac{B_1}{2A_1}\right)^2 - \dfrac{C_1 - f^{-1}(y_t)}{A_1}\right]^{1/2} \\ \qquad if \quad C_1 \le f^{-1}(y_t) \le C_3, \\ \dfrac{B_2}{2A_2} + \left[\left(\dfrac{B_2}{2A_2}\right)^2 - \dfrac{C_2 - f^{-1}(y_t)}{A_2}\right]^{1/2} \\ \qquad if \quad C_3 \le f^{-1}(y_t) \le C_2, \\ \\ 0 \qquad\qquad\qquad otherwise, \end{cases} \quad (17)$$

Where

$$A_1 = \sum_{j=0}^{q}\left((e_j - d_j) \cdot g\left(\sum_{i=0}^{p} b_{i,j} y_{t-i}\right) - g\left(\sum_{i=0}^{p} a_{i,j} y_{t-i}\right)\right),$$

$$A_2 = \sum_{j=0}^{q} (f_j - e_j) \cdot \left(g\left(\sum_{i=0}^{p} c_{i,j} y_{t-i} \right) - g\left(\sum_{i=0}^{p} b_{i,j} y_{t-i} \right) \right),$$

$$B_1 = \sum_{j=0}^{q} \left(d_j \cdot \left(g\left(\sum_{i=0}^{p} b_{i,j} y_{t-i} \right) - g\left(\sum_{i=0}^{p} a_{i,j} y_{t-i} \right) \right) \right. $$
$$\left. + g\left(\sum_{i=0}^{p} a_{i,j} y_{t-i} \right) \cdot (e_j - d_j) \right),$$

$$B_2 = \sum_{j=0}^{q} \left(f_j \cdot \left(g\left(\sum_{i=0}^{p} c_{i,j} y_{t-i} \right) - g\left(\sum_{i=0}^{p} b_{i,j} y_{t-i} \right) \right) \right.$$
$$\left. + g\left(\sum_{i=0}^{p} c_{i,j} y_{t-i} \right) \cdot (f_j - e_j) \right),$$

$$C_1 = \sum_{j=0}^{q} \left(d_j \cdot g\left(\sum_{i=0}^{p} a_{i,j} \cdot y_{t-i} \right) \right),$$

$$C_2 = \sum_{j=0}^{q} \left(f_j \cdot g\left(\sum_{i=0}^{p} c_{i,j} \cdot y_{t-i} \right) \right),$$

$$C_3 = \sum_{j=0}^{q} \left(e_j \cdot g\left(\sum_{i=0}^{p} b_{i,j} \cdot y_{t-i} \right) \right),$$

Now, considering threshold level h for all membership function value of observations according to (8) the nonlinear programming is given as (18).

$$Min \quad \sum_{t=1}^{k} \sum_{j=0}^{q} \left(f_j \cdot g\left(\sum_{i=0}^{p} c_{i,j} \cdot y_{t-i} \right) \right)$$
$$- \left(d_j \cdot g\left(\sum_{i=0}^{p} a_{i,j} \cdot y_{t-i} \right) \right)$$

$$S.T \begin{cases} \dfrac{-B_1}{2A_1} + \left[\left(\dfrac{B_1}{2A_1} \right)^2 - \dfrac{C_1 - f^{-1}(y_t)}{A_1} \right]^{\frac{1}{2}} \le h \\ \quad if \quad C_1 \le f^{-1}(y_t) \le C_3, \\ \dfrac{B_2}{2A_2} + \left[\left(\dfrac{B_2}{2A_2} \right)^2 - \dfrac{C_2 - f^{-1}(y_t)}{A_2} \right]^{\frac{1}{2}} \le h \\ \quad if \quad C_3 \le f^{-1}(y_t) \le C_2, \end{cases} \quad (18)$$

3. Application of Hybrid Models for Forecasting

In this section, the appropriateness and effectiveness of four aforementioned models are compared together in application of exchange rate (US dollar/Iran rial) forecasting. The information of this investigation consists of 42 daily observations from *5 Nov to 16 Dec 2005*. In all models, the first 35 observations are used to formulate the model and the next 7 observations in order to evaluate the performance of the models.

3-1. Autoregressive Integrated Moving Average Model

Using the *Eviews* package software, the best-fitted model is ARIMA(2,1,0). The actual values and 95% confidence interval of the ARIMA model are given in Table 1.

Tab. 1. Actual values and 95% confidence interval of ARIMA model

Date	Actual value	Lower bound	Upper bound
10- Dec	9082	9074	9090
11- Dec	9083	9075	9091
12- Dec	9083	9075	9091
13- Dec	9082	9074	9090
14- Dec	9081	9073	9089
15- Dec	9082	9074	9090
16- Dec	9082	9074	9090

3-2. Fuzzy Autoregressive Integrated Moving Average Model

Setting $(\alpha_0, \alpha_1, \alpha_2) = (9060.05, 0.607, 0.421)$, the fuzzy parameters are obtained by (10) (with *h=0*). The results after deleting the outlier data are given in Table 2.

Tab. 2. Actual values and forecasted interval of FARIMA model

Date	Actual value	Lower bound	Upper bound
10- Dec	9082	9081	9085
11- Dec	9083	9080	9084
12- Dec	9083	9081	9085
13- Dec	9082	9081	9085
14- Dec	9081	9080	9084
15- Dec	9082	9079	9083
16- Dec	9082	9080	9084

3-3. The Improved FARIMA Model with PNNs

In improved model, the probabilistic neural network is used after the FARIMA model. The best fitted network is a network with five input neurons and one output neuron. The structure of designed network is given in Fig. 2.
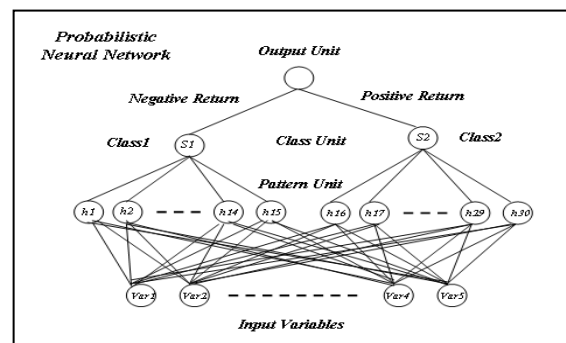

Fig. 1. The structure of designed network

where

Var1::Forecasting lower bond of time series in time t (L_t)

Var2: Forecasting upper bond of time series in time t (U_t)

Var3: Forecasting value of time series in time t ($\hat{Z}_t$)

Var4: Difference between forecasting value of time series in time t & time t-1 ($\hat{Z}_t - \hat{Z}_{t-1}$)

Var5: Difference between forecasting upper bond (lower bond) of time series in time t & time t-1 ($U_t - U_{t-1}$)

Obtained results of upper and lower bound with improved model with *100%* confidence coefficient are given in Table 3.

Tab. 3. Actual values and forecasted interval of PNN/FARIMA model

Date	Actual value	Lower bound	Upper bound
10- Dec	9082	9081	9084
11- Dec	9083	9081	9084
12- Dec	9083	9081	9084
13- Dec	9082	9081	9084
14- Dec	9081	9080	9083
15- Dec	9082	9079	9082
16- Dec	9082	9081	9084

3-4. Hybrid Artificial Neural Networks and Fuzzy Logic Model

Considering the design concepts of artificial neural networks [32-33] and using *MATLAB7* package software, the best fitted network is $N^{(3-3-1)}$.

Then, by setting the $(\alpha_0^*, \alpha_1^*, \alpha_2^*) = (9060.05, 0.607, 0.421)$ and $(\alpha_3^*, \alpha_4^*, \alpha_5^*, \alpha_6^*) = (3.37, 6.205, -1.149, -4.060)$, the fuzzy parameters are obtained using (18) (with *h=0*). It must be noted that in this case the used triangular fuzzy numbers are considered symmetric, transfer function of the output neuron is considered linear and connection weights between hidden and input layer are considered crisp. Using the revised hybrid model, the future values of the next 5 transaction days are forecasted, which are shown in Table 4.

Tab. 4. Actual values and forecasted interval of hybrid model

Date	Actual value	Lower bound	Upper bound
10- Dec	9082	9081	9084
11- Dec	9083	9082	9084
12- Dec	9083	9082	9084
13- Dec	9082	9081	9084
14- Dec	9081	9081	9084
15- Dec	9082	9081	9084
16- Dec	9082	9081	9084

4. Comparison the Performance of Models

In this section, based on the empirical results of this example, the predictive capabilities of the aforementioned models are compared together. The information of forecasted interval width and related performance of each model is given in Table 5.

Tab. 5. Forecasted interval width and related performance of each model

Model	interval width	Related Performance			
		ARIMA	FARIMA	PNN / FARIMA	FANNs
ARIMA	16.2	0	-	-	-
FARIMA	4.2	74.1%	0	-	-
PNN/FARIMA	3.1	80.9%	26.2%	0	-
FANN	2.5	84.6%	40.5%	19.4%	0

According to the above results, the autoregressive integrated moving average model has the lowest performance and the hybrid artificial neural networks and fuzzy logic model has the better performance than other models in exchange rate forecasting.

5. Conclusions

The foreign exchange markets are among the most important and the largest financial markets in the world with trading taking place twenty-four hours a day around the globe and trillions of dollars of different currencies transacted each day. Being able to accurately forecast the movements of exchange rates can result in considerable improvement in the overall profitability of the multinational financial firm, especially for firms, conducting substantial currency transfers in the course of business.

However, predicting currency movements has always been a problematic task for academic researchers and despite the paramount modelling effort registered in the last three decades, it is widely recognized that exchange rates are extremely difficult to forecast. That is the reason why research on improving the effectiveness of time series models has been never witnessed a halt.

In this paper the performance of four different interval time series models (Auto-Regressive Integrated Moving Average (ARIMA), Fuzzy Auto-Regressive Integrated Moving Average (FARIMA), Hybrid ANNs and Fuzzy, Improved FARIMA) are compared together in exchange rate forecasting. Empirical results of exchange rate forecasting indicate that the hybrid ANNs and fuzzy model is more satisfactory than other those models. Therefore, it can be used as a suitable alternative model for interval forecasting in financial markets.

References

[1] Khashei, M., *"Forecasting and Analysis Esfahan Steel Company Production Price in Tehran Metals Exchange*

with Artificial Neural Networks", Master of Science Thesis, Isfahan University of Technology, 2005.

[2] Khashei, M., Bijari, M., "*Foreign Exchange Rate Forecasting using a Hybrid Fuzzy and Auto Regressive Integrated Moving Average Model* ", Esteghlal Journal, Volume 26, Issue 2, pp. 67- 75, 2008.

[3] Ince, H., Trafalis, T., "*A hybrid model for exchange rate prediction*", Decision Support Systems, Volume 42, pp. 1045- 1062, 2006.

[4] Chen, A., Leung, T., "*Regression Neural Network for Error Correction in Foreign Exchange Forecasting and Trading*", Computers & Operations Research, Volume 31, Issue 7, 2004, pp. 1049-1068.

[5] Martens, M., "*Forecasting Daily Exchange Rate Volatility using Intraday Returns*", Journal of International Money and Finance, Volume 20, Issue 1, 2001, pp. 1-23.

[6] Khashei, M., Bijari, M., Raissi, G. A., "*Improvement of Auto-Regressive Integrated Moving Average Models Using Fuzzy Logic and Artificial Neural Networks*", Neurocomputing, Volume 72, 2009, pp. 956- 967.

[7] Yu, L., Wang, S., Lai, K., "*A Novel Nonlinear Ensemble Forecasting Model Incorporating GLAR and ANN for Foreign Exchange Rates*", Computers & Operations Research, Volume 32, Issue 10, 2005, pp. 2523- 2541.

[8] Khashei, M., Bijari, M., "*Using Fuzzy Auto-Regressive Integrated Moving Average (FARIMA) Model to Exchange Rate Forecasting*", 6th Iranian Fuzzy Systems Conference, 2006. pp. 26-35,

[9] Balaban, E., "*Comparative Forecasting Performance of Symmetric and Asymmetric Conditional Volatility Models of an Exchange Rate*", Economics Letters, Volume 83, Issue 1, 2004, pp 99-105.

[10] Faust, J., Rogers, J., Wright, H., "*Exchange rate forecasting: the errors we've really made*", Journal of International Economics, Volume 60, Issue 1, 2003, pp 35- 59.

[11] Chen, A., Leung, M., "*A Bayesian Vector Error Correction Model for Forecasting Exchange Rates*", Computers & Operations Research, Volume 30, Issue 6, 2003, pp. 887- 900.

[12] Khashei, M., Bijari, M., Hejazi, S. R., "*Using General Regression Neural Networks for Steel Productions Price Forecasting in Tehran Metals Exchange (TME)* ", Steel Symposium 85, 2007, pp. 965- 974.

[13] Zhang, P., Min Qi, G., "*Neural Network Forecasting for Seasonal and Trend Time Series*", European Journal of Operational Research, Volume 160, 2005, pp. 501– 514.

[14] Ture, M., Kurt, I., "*Comparison of Four Different Time Series Methods to Forecast Hepatitis A Virus Infection*", Expert Systems with Applications, Volume 31, 2006, pp. 41– 46.

[15] Taylor, J. W., de Meneze, L. M., Mc Sharry, P. E., "*A Comparison of Univariate Methods for Forecasting Eectricity Demand up to a Day Ahead*", International Journal of Forecasting, Volume 22, 2006, pp. 1– 16.

[16] Kim, J. H., Moosa, I. A., "*Forecasting International Tourist Flows to Australia: a Comparison Between the Direct and Indirect Methods*", Tourism Management, Volume 26, 2005, pp. 69– 78.

[17] Cho, V., "*A Comparison of Three Different Approaches to Tourist Arrival Forecasting*", Tourism Management, Volume 24, 2003, pp. 323– 330.

[18] Weatherford, L. R., Kimes, S. E., "*A Comparison of Forecasting Methods for Hotel Revenue Management,*" International Journal of Forecasting, Volume 19, 2003, pp. 401– 415.

[19] Smith, B., Williams, R. Keith, B. L., "*Comparison of Parametric and Nonparametric Models for Traffic Flow Forecasting*", Transportation Research Part, Volume 10, 2002, pp. 303– 321.

[20] A. Sfetsos, "*A Comparison of Various Forecasting Techniques Applied to Mean Hourly wind Speed Time series*", Renewable Energy, Volume 21, 2000, pp. 23-35.

[21] Alon, R. J., Sadowski, I., "*Forecasting Aggregate Retail Sales: a Comparison of Artificial Neural Networks and Traditional Methods*", Journal of Retailing and Consumer Services, Volume 8, 2001, pp. 147- 156.

[22] Meade, N., "*A Comparison of the Accuracy of Short Term Foreign Exchange Forecasting Methods*", International Journal of Forecasting, Volume 18, 2002, pp. 67– 83.

[23] Leung, M.T., Daouk, H.," *Forecasting Stock Indices: a Comparison of Classification and Level Estimation Models*", International Journal of Forecasting, Volume 16, 2000, pp. 173– 190.

[24] F. Lisi, R. A. Schiavo, "*A Comparison Between Neural Networks and Chaotic Models for Exchange Rate Prediction*", Computational Statistics & Data Analysis, Volume 30, 1999, pp. 87- 102.

[25] Box, P., Jenkins, G.M., "*Time Series Analysis: Forecasting and Control*", Holden-day Inc, San Francisco, CA, 1976.

[26] Tseng, F M., Tzeng, H.C., Yu, B.J.C., Yuan, G.H., "*Fuzzy ARIMA Model for Forecasting the Foreign Exchange Market*", Fuzzy Sets and Systems, Volume 118, 2001, pp. 9- 19.

[27] Tanaka, H., "*Fuzzy Data Analysis by Possibility Linear Models*", Fuzzy Sets and Systems, Volume 24, Issue 3, 1987, pp. 363- 375.

[28] Ishibuchi, H., Tanaka, H., "*Interval Regression Analysis Based on Mixed 0-1 Integer Programming Problem*", J. Japan Soc. Ind. Eng, , Volume 40, Issue 5, 1988, pp. 312- 319.

[29] Specht, D., "*Probabilistic Neural Networks for Classification, Mapping, or Associative Memory*", IEEE International Conference on Neural Networks, Volume 1; 1988, pp. 525– 532.

[30] Chen; A. S., Leung, M. T. Daouk, H.," *Application of Neural Networks to an Emerging Financial Market: Forecasting and Trading the Taiwan Stock Index*", Computers & Operations Research, Volume 30, 2003, pp. 901– 923.

[31] Khashei, M., Hejazi, S. R., Bijari, M., "*A New Hybrid Artificial Neural Networks and Fuzzy Regression Model for Time Series Forecasting* ", Fuzzy Set and Systems, Volume 159, 2008, pp. 769- 786.

[32] Khashei, M., Bijari, M., "*Forecasting and Analysis Esfahan Steel Company Production Price in Tehran Metals Exchange with Artificial Neural Networks (part I)* ", Steel Journal, Volume 133, 2006, pp. 28 -32.

[33] Khashei, M., Bijari, M., *"Forecasting and Analysis Esfahan Steel Company Production Price in Tehran Metals Exchange with Artificial Neural Networks (part II)* ", Steel Journal, Volume 134, 2006, pp. 28 -32.