Implementation of Distributed Model Predictive Controllers Based on Orthonormal Basis Functions to Increase Supply Chain Robustness

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ABSTRACT

Today, companies need to make use of appropriate patterns such as supply chain management system to gain and preserve their positions in competitive world-wide market. Supply chain is a large scale network consists of suppliers, manufacturers, warehouses, wholesalers, retailers and final customers which are in coordination with each other in order to transform products from raw materials into finished goods with optimal placement of inventory within the supply chain and minimizing operating costs in the face of demand fluctuations. Model Predictive Control (MPC) is a widely used means on supply chains, but in presence of long delays and sudden disturbance changes (customer demand changes) , tuning of MPCs requires a time consuming trial-and-error procedure and system robustness will be highly decreased. In this paper, a control scheme is proposed to increase supply chain robustness. Due to the large scale characteristic of supply chain, the model is divided into different subsystems and is controlled by distributed model predictive controllers. Each subsystem model has been changed in which an integrated is imbedded, input and output changes are highly penalized in cost functions and Laguerre orthonormal basis functions are added in MPC's structures and it will be shown that the supply chain robustness will be increased toward high changes in customer demand and toward long constant delays in distribution centres, also Results will be compared to previous conventional MPCs applied on supply chains by other authors.

1- Introduction

Historically companies leveraged a variety of factors to differentiate themselves from their competition, including product features, price, quality, product availability and customer service. In today's dynamic market, companies can no longer exploit the same drivers, or must exploit them differently, in order to remain competitive. The supply chain is the best answer to above problems. A supply chain consists of all parties involved, directly or indirectly in fulfilling a customer request. The supply chain not only includes the manufacturer and suppliers, but also contains transporters, warehouses, wholesalers, retailers and

customers to transform products from raw materials into finished goods and deliver those goods into the hands of the end customers as soon as possible. So the supply chain is a large scale system with different subsystems that are highly coupled with each other.

There are some important issues that should be considered in the control of supply chains, such as safety stock control, inventory control, and bullwhip effect control. Safety stock is a term used by supply chain managers to describe a level of extra stock or inventory which should be maintained in warehouses to prevent risk of stock-outs due to uncertainties in supply and demand. Each stage or subsystem in a supply chain has specific safety storage level.

Companies today must be fast and nimble enough to react quickly to changes in customer demand and do it with little inventory. Inventory control is concerned with minimizing the total cost of inventory. The purpose of inventory control is full filling any customer demand with lowest level of inventory (safety stock levels) and reducing stocks-out without excessive inventories.Another term in the supply chains is bullwhip effect. The bullwhip effect is an observed phenomenon in forecast-driven distribution channels. It refers to a trend of larger and larger swings in inventory in response to changes in customer demand (Fig .1) .Our goal is to keep it as minimum as possible.

Fig.1. Bullwhip effect rate

According to [1], the bullwhip effect in each subsystem is obtained by following formula

$$BE(t) = \frac{Var(inventory\ level)}{Var(customer\ demand)} \tag{1}$$

Where Var(.) is the variance in time t.

Different methods and control approaches are applies on supply chains. For example, reference number [2] has proposed a mathematical method for managing inventories in a dual channel supply chain, a fuzzy approach is proposed for multi-objective supplier selection in [3]. Model Predictive controller (MPC) is a widely used means to deal with large multivariable constraint control issues in industry. In supply chain studies there are a lot of papers using MPC to improve supply chain performance.Kapsiotis was the first to apply MPC to an inventory management problem [4]; Tzafestas et al., considered a generalized production planning problem that includes both production/inventory and marketing decisions [5]; Perea-Lopez et al. employed MPC to manage a multi-

product, multi-echelon production and distribution network with lead times [6]; Dunbar and Desa applied a recently developed distributed /decentralized implementation of MPC to the problem of dynamic supply chain management problem, reminiscent of the classic MIT "Beer Game" [7]; Sarmveis has written a review for dynamic modeling and control of supply chain systems[6]; Miranbeigi uses a move suppression term in supply chain cost functions in order to increase robustness toward changes in customer demands [9]; Jie Li and Mian Peng have written a paper to solve inventory fluctuation caused by uncertain time delay in a multi-level supply chain [10]; Grasso presents a MPC strategy enriched with soft control techniques as neural networks and fuzzy logic to MPC self-tuning capability in a water drinks supply chain [11].

As it is clear, MPC is a widely used means on supply chains, but in presence of long delays and sudden disturbance changes (customer demand changes), tuning of MPCs requires a time consuming trial-and-error procedure and system robustness will be highly decreased. In this paper, a control scheme is proposed to increase supply chain robustness. Due to the large scale characteristic of supply chain, the model is divided into different subsystems and is controlled by distributed model predictive controllers. Each subsystem model has been changed in which an integrated is imbedded, input and output changes are highly penalized in cost functions and Leaguered orthonormal basis functions are added in MPC's structures and it will be shown that the supply chain robustness will be increased toward high changes in customer demand and toward long constant delays in distribution centers, also results will be compared to previous conventional MPCs applied on supply chains by other authors.

2. Supply Chain Modeling and Control Strategy

2-1. Modeling

Assume a supply chain with suppliers(S), manufactures (M), Wholesalers (W), Retailers (R) and final customers(C) as the nodes of the system. For each node k, there is an upstream node denoted by k' which can supply node k and a downstream node denoted by k" which can be supplied by k. Inventory control strategies are made to avoid uncertain fluctuations. Each node has its own desired inventory level and inventory fluctuations should perform in certain small domain around their desired levels even though demands don't obey any specific structure. Each subsystem model is as follows.

$$\frac{d(inv_k(t))}{dt} = p_{k',k}(t - \tau_{k',k}) - d_{k,k''}(t) \tag{2}$$

$$\frac{d(BO_k(t))}{dt} = p_{k,k''}(t) - d_{k,k''}(t)$$

$$\forall k \in \{R, W, M\}$$

Where $P_{k',k}(t)$ is the transferred product through (k',k) channel and is considered as manipulated variable, $\tau_{k',k}$ is transportation delay time through channel k' and k , $d_{k,k''}$ is actual number of delivered products from node k to node k" and is considered as a manipulated variable , $p_{k,k''}$ is demand rate sent from node k" to node k and is considered as disturbance , $inv_k(t)$ and $BO_k(t)$ are inventory level and backorder rate in node k which are considered as system outputs [9]." $p_{k,k''}$ and $\tau_{k',k}$ are the arbitrary parameters in our model . The control goal in our paper is to increase supply chain robustness through any kind of delay ($\tau_{k',k}$) and any kind of demand ($p_{k,k''}$). In order to use MPC, the above continues model should be discretized as follows:

$$inv_k(t+1) = inv_k(t) + p_{k',k}(t-1-L_{k',k}) - d_{k,k''}(t)$$
$$BO_k(t+1) = BO_k(t) + p_{k,k''}(t) - d_{k,k''}(t) \qquad (3)$$

2-2. Control Strategy

In large scale applications, such as power systems, water distribution systems, traffic systems, manufacturing systems, and economic systems, such a centralized control scheme may not suitable or even possible for technical or commercial reasons, it is useful to have distributed or decentralized control schemes. Each controller will be optimized by its own policy and its own cost function and will sent its optimal signal controls as disturbances (demands) to the next coupled controllers. The structure of distributed controllers is illustrated in Fig.2.
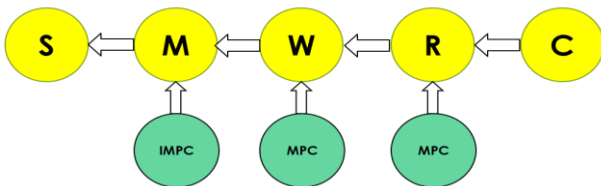

Fig.2.Method 1, Decentralized conventional MPCs.

In this paper, conventional MPCs in Fig.2 are replaced with orthonormal based MPCs in order to increase supply chain robustness Fig.3.
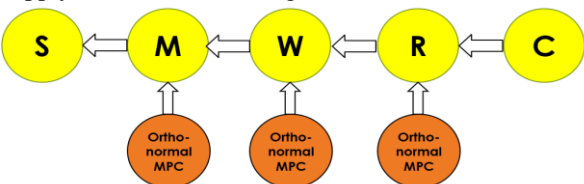

Fig.3. Method 2, Decentralized orthonormal based MPCs.

Some advantages of method 2 than method 1 are:
1-Simple tuning.
2- Using orthonormal functions in method 2 reduces

number of parameters used for description of future control trajectory, so computational volume decreases.
3- Using augmented model in method 2 and penalizing input and output changes causes smoother responses.

2-3. Cost Functions

In this paper decentralized formulation will be used. So instead of a centralized cost function, three individual cost functions will be used. Each cost function will do its own optimization in order to achieve optimal orders for its coupled controllers. The cost functions are in quadratic forms. Each of them includes two least square parts. First part is to penalize outputs and the second part is to penalize inputs. $\forall k \in \{R, W, M\}$ The cost functions and constraints are obtained by

$$\min_{P_{k',k}, d_{k',k}} W_{out}\left(\begin{bmatrix} inv_k(t) - Tinv_k \\ BO_k(t) - 0 \end{bmatrix} \right)^2 + W_{in}\left(\begin{bmatrix} P_{k',k}(t) \\ d_{k,k''}(t) \end{bmatrix} \right)^2 \qquad (4)$$

Subject to
$$0 \le inv_k(t) \le inv^{max} , \quad t \ge 0$$

Where Win>0 and W$_{out}$> 0 are weighting matrices.

3. Discrete-time MPC using Laguerre functions
3-1. Control Signal Trajectory and Prediction

The z-transfer function of Leaguers function is given as [12]

$$\Gamma_k(z) = \Gamma_{k-1}(z)\frac{z^{-1} - \alpha}{1 - az^{-1}} \qquad (5)$$

With $\Gamma_1 = \frac{\sqrt{1-\alpha^2}}{1-\alpha z^{-1}}$ .where $0 \le \alpha < 1$ is called the scaling factor and is selected by the user. Letting $l_1(k)$ to $l_N(k)$ denote the inverse z-transforms of $\Gamma_1(z)$ to $\Gamma_2(z)$. This set of discrete-time Laguerre functions are expressed in a vector form as

$$L(k) = \begin{bmatrix} l_1(k) & l_2(k) & \cdots l_N(k) \end{bmatrix}^T \qquad (6)$$

According to (5), the set of discrete-time Laguerre functions in vector (6) satisfies the following equation
$$L(k+1) = A_l L(k) \qquad (7)$$

Where A_l is ($N \times N$) and can be expressed as

$$\begin{bmatrix} \alpha & 0 & 0 & \cdots & 0 \\ \beta & \alpha & 0 & \cdots & 0 \\ -\alpha\beta & \beta & \alpha & \cdots & 0 \\ \alpha^2\beta & -\alpha\beta & \beta & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ (-1)^{-N-2}\alpha^{N-2}\beta & (-1)^{N-3}\alpha^{N-3}\beta & \cdots & \beta & \alpha \end{bmatrix} \qquad (8)$$

Where $\beta = 1 - \alpha^2$ and the initial condition is given by
$$L(0)^T = \sqrt{\beta}\begin{bmatrix} 1-\alpha^2 & \alpha^2 - \alpha^3 & \cdots & (-1)^{N-1}\alpha^{N-1} \end{bmatrix} \qquad (9)$$

Assume that a discrete time model has the form as
$$x_m(k+1) = A_m x_m(k) + B_m u(k)$$
$$y(k) = C_m x_m(k) \qquad (10)$$

We need to change the model to suit our design

purpose in which an integrator is embedded. The augmented model can be expressed as follows according to [13]

$$\begin{bmatrix} \Delta x_m(k+1) \\ y(k+1) \end{bmatrix} = \begin{bmatrix} A_m & 0_m^T \\ C_m A_m & I \end{bmatrix} \begin{bmatrix} \Delta x_m(k) \\ y(k) \end{bmatrix}$$

$$+ \begin{bmatrix} B_m \\ C_m B_m \end{bmatrix} \Delta u(k) \qquad (11)$$

$$y(k) = \begin{bmatrix} 0_m & 1 \end{bmatrix} \begin{bmatrix} \Delta x_m(k) \\ y(k) \end{bmatrix}$$

Where $\Delta u = u(k) - u(k-1)$ and $\Delta x_m(k) = x_m(k) - x_m(k-1)$ denote the difference of the control input and the state variable respectively. By using discrete-time Laguerre functions, $\Delta u(k_i + k)$ can be approximated by following equation

$$\Delta u(k_i + k) = \sum_{j=1}^{N} c_j(k) l_j(k) = L(k)^T \eta \qquad (12)$$

Where $\eta = \begin{bmatrix} c_1 & c_2 & \cdots & c_N \end{bmatrix}$. The state prediction has the form as follows:

$$x(k_i + m | k_i) = A^m x(k_i) +$$

$$\sum_{i=0}^{m-1} A^{m-i-1} BL(i)^T \eta \qquad (13)$$

Where (13) is obtained by replacing $L(i)^T \eta$ instead of $\Delta u(k_i + k)$. Similarly the prediction for the plant output can be written as

$$y(k_i + m | k_i) = CA^m x(k_i) +$$

$$\sum_{i=0}^{m-1} CA^{m-i-1} BL(i)^T \qquad (14)$$

The cost function is in the following quadratic form

$$J = \sum_{m=1}^{N_p} (r(k_i + m | k_i) - y(k_i + m | k_i))^T \times$$

$$Q \times (r(k_i + m | k_i) - y(k_i + m | k_i)) + \qquad (15)$$

$$\sum_{m=0}^{N_p-1} \Delta U(k_i + m | k_i)^T \times R \times \Delta U(k_i + m | k_i)$$

The weighting matrices are Q >0 and R> 0. By substituting (12) into the cost function (15) we obtain

$$J = \sum_{m=1}^{N_p} (r(k_i + m | k_i) - y(k_i + m | k_i))^T \times$$

$$Q \times (r(k_i + m | k_i) - y(k_i + m | k_i)) + \qquad (16)$$

$$\eta^T R \eta$$

The objective is to minimize the cost function (16) to find the optimal coefficient vector η in the presence of input and states constraints

3-2. The Unconstrained Solution

Without constraints, by substituting (14) into (16), according to [12] the optimal solution of the cost function (16), is found as following equation

$$\eta = -\Omega^{-1} \psi x(k_i) \qquad (17)$$

Where $\phi(m)^T = \sum_{i=0}^{m-1} A^{m-i-1} BL(i)^T$,

$\Omega = (\sum_{m=1}^{N_p} \phi(m) Q \phi(m)^T + R)$ and $\Psi = (\sum_{m=1}^{N_p} \phi(m) QA^m)$

3-3. The Inequality Constraints

Model predictive control has the ability to handle hard constraints in the design. With parameterization of the control signal trajectory, we can choose the locations of the future constraints. This could potentially reduce the number of constraints within the prediction horizon [12].

3-3-1. Constraints on the Amplitudes of the Control Signal

Suppose that the limits on the control signals are u_{low} and u_{high}. Noting that the increment of the control signal is $u(k) = \sum_{i=0}^{k-1} \Delta u(i)$, then the inequality constraint for the future time k, k=1, 2. . . is expressed in following inequality as [12]

$$u_{low} \leq (\sum_{i=0}^{k-1} L(i)) \eta + u(k_i - 1) \leq u_{high} \qquad (18)$$

3-3-2. Constraints on the Output Variables

Suppose that the constraints on the process output variables are given by y_{low} and y_{high}. Then the inequality constraint for the future time instant m is in following inequality as [12]

$$y_{low} \leq CA^m x(k_i) + \phi(m)^T \eta \leq y_{high} \qquad (19)$$

4. Simulation

A multi echelon supply chain is used in simulation examples. The supply chain network consists of supplier, manufacturer, wholesaler, retailer and final customer. Only a single type of product will be distributed along the chain and the purpose of our inventory control isn't zero safety stock; safety storage is to keep customer demand satisfaction, set as 100 units. Whole simulation time is 80 time unit (day). Prediction horizon and control horizon of 30 time periods are selected. Inventory set points, initial inventory levels, maximum storage capacity at every node are reported in Table 1.

Tab.1. Supply Chain Data

Data	Retailer	Warehouse	Manufacture
Initial Inventory level	20	30	40
Desired Inventory	80	100	150
Maximum storage capacity	300	300	300

As it said before , customer demand and transportation delays are the arbitrary parameters in our supply chain model and the control goal is to increase supply chain robustness through a sort of delay and any kind of customer demand, so two kind of customer demand is considered, first type of demand is a random pulsatory demand with small domain changes between 0 and 50

units and the second demand is a random pulsatory demand with high and sudden domain changes between 0 and 600 units and Also two kind of short and long transportation time delays are considered.

In this part two control methods is applied. In first method, distributed regular model predictive controllers are applied and in second method the supply chain model is changed in which an integrator is embedded , input and output changes are highly penalized in cost functions and also Orthonormal basis functions are used in MPCs' structure. Laguerre parameters are chosen as a= [0.9 0.9], N= [40 40].

It will be shown that if suddenly demand changed, the first method cannot predict this changes and has poor efficiency, Also in presence of long delays, tuning of conventional MPCs (first method) requires a time consuming trial-and-error procedure and retuning is needed for any changes, but our proposed method (method 2) is robust through sudden demand changes and toward any short or long transportation delays and there is no need for retuning. The input and output tuning weight matrices in each echelon in both methods are given in Table 2 and table 3.

Tab.2. Input and output weights in first method

Delay type / Demand type	Delay type 1: Short transportation delays	Delay type 2: Long transportation delays
Demand type	$\begin{bmatrix} L_{S,W} & L_{M,W} & L_{W,R} & L_{R,C} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 2 & 1 \end{bmatrix}$	$\begin{bmatrix} L_{S,W} & L_{M,W} & L_{W,R} & L_{R,C} \end{bmatrix} = \begin{bmatrix} 6 & 5 & 8 & 7 \end{bmatrix}$
Demand type 1:	$\begin{bmatrix} W_{in_R} \end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix}, \begin{bmatrix} W_{out_R} \end{bmatrix} = \begin{bmatrix} 0.3 & 0.09 \end{bmatrix}$ $\begin{bmatrix} W_{in_W} \end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix}, \begin{bmatrix} W_{out_W} \end{bmatrix} = \begin{bmatrix} 0.3 & 0.09 \end{bmatrix}$ $\begin{bmatrix} W_{in_M} \end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix}, \begin{bmatrix} W_{out_M} \end{bmatrix} = \begin{bmatrix} 0.3 & 0.09 \end{bmatrix}$ $\begin{bmatrix} W_{in_S} \end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix}, \begin{bmatrix} W_{out_S} \end{bmatrix} = \begin{bmatrix} 0.3 & 1.5 \end{bmatrix}$	$\begin{bmatrix} W_{in_R} \end{bmatrix} = \begin{bmatrix} 30 & 20 \end{bmatrix}, \begin{bmatrix} W_{out_R} \end{bmatrix} = \begin{bmatrix} 10 & 5 \end{bmatrix}$ $\begin{bmatrix} W_{in_W} \end{bmatrix} = \begin{bmatrix} 15 & 10 \end{bmatrix}, \begin{bmatrix} W_{out_W} \end{bmatrix} = \begin{bmatrix} 20 & 15 \end{bmatrix}$ $\begin{bmatrix} W_{in_M} \end{bmatrix} = \begin{bmatrix} 15 & 25 \end{bmatrix}, \begin{bmatrix} W_{out_M} \end{bmatrix} = \begin{bmatrix} 20 & 15 \end{bmatrix}$ $\begin{bmatrix} W_{in_S} \end{bmatrix} = \begin{bmatrix} 20 & 20 \end{bmatrix}, \begin{bmatrix} W_{out_S} \end{bmatrix} = \begin{bmatrix} 30 & 20 \end{bmatrix}$
Demand type 2:	$\begin{bmatrix} W_{in_R} \end{bmatrix} = \begin{bmatrix} 0.3 & 0.5 \end{bmatrix}, \begin{bmatrix} W_{out_R} \end{bmatrix} = \begin{bmatrix} 1.5 & 1.2 \end{bmatrix}$ $\begin{bmatrix} W_{in_W} \end{bmatrix} = \begin{bmatrix} 0.4 & 0.5 \end{bmatrix}, \begin{bmatrix} W_{out_W} \end{bmatrix} = \begin{bmatrix} 1.1 & 0.8 \end{bmatrix}$ $\begin{bmatrix} W_{in_M} \end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix}, \begin{bmatrix} W_{out_M} \end{bmatrix} = \begin{bmatrix} 1.3 & 0.9 \end{bmatrix}$ $\begin{bmatrix} W_{in_S} \end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix}, \begin{bmatrix} W_{out_S} \end{bmatrix} = \begin{bmatrix} 1.3 & 1.5 \end{bmatrix}$	$\begin{bmatrix} W_{in_R} \end{bmatrix} = \begin{bmatrix} 35 & 30 \end{bmatrix}, \begin{bmatrix} W_{out_R} \end{bmatrix} = \begin{bmatrix} 15 & 10 \end{bmatrix}$ $\begin{bmatrix} W_{in_W} \end{bmatrix} = \begin{bmatrix} 25 & 20 \end{bmatrix}, \begin{bmatrix} W_{out_W} \end{bmatrix} = \begin{bmatrix} 20 & 20 \end{bmatrix}$ $\begin{bmatrix} W_{in_M} \end{bmatrix} = \begin{bmatrix} 35 & 30 \end{bmatrix}, \begin{bmatrix} W_{out_M} \end{bmatrix} = \begin{bmatrix} 25 & 20 \end{bmatrix}$ $\begin{bmatrix} W_{in_S} \end{bmatrix} = \begin{bmatrix} 25 & 20 \end{bmatrix}, \begin{bmatrix} W_{out_S} \end{bmatrix} = \begin{bmatrix} 30 & 25 \end{bmatrix}$

Tab.3. Input and output weights in second method

Delay type / Demand type	Short and Long delay (Delay type 1 and Delay type 2)
Demand type 1 and 2	$\begin{bmatrix} W_{in_R} \end{bmatrix} = \begin{bmatrix} 0.08 & 5 \end{bmatrix}, \begin{bmatrix} W_{out_R} \end{bmatrix} = \begin{bmatrix} 0.03 & 0.03 \end{bmatrix}, \begin{bmatrix} W_{in_W} \end{bmatrix} = \begin{bmatrix} 0.1 & 0.61 \end{bmatrix}, \begin{bmatrix} W_{out_W} \end{bmatrix} = \begin{bmatrix} 2 & 2 \end{bmatrix},$ $\begin{bmatrix} W_{in_M} \end{bmatrix} = \begin{bmatrix} 0.1 & 0.61 \end{bmatrix}, \begin{bmatrix} W_{out_M} \end{bmatrix} = \begin{bmatrix} 1.3 & 0.9 \end{bmatrix}, \begin{bmatrix} W_{in_S} \end{bmatrix} = \begin{bmatrix} 0.1 & 0.5 \end{bmatrix}, \begin{bmatrix} W_{out_S} \end{bmatrix} = \begin{bmatrix} 2 & 2 \end{bmatrix}$

In the following, proposed methods should be implemented to check whether they have an acceptable performance in critical issues such as safety stock control, inventory control and bullwhip effect control. Also the approaches must provide optimal orders for upstream nodes and optimal number of delivered products (satisfied orders) to downstream nodes.

Simulation results for each combination of customer demand and delays {(demand_1 ,delay _short), (demand_1,delay_Long), (demand_2 ,delay _short), (demand_2,delay_Long) } are illustrated in figures 4-11 and both methods are compared with each other.

When demand is type 1 and distribution delay is type 1 (delay _short), results are shown in Figs 4.a, 5.a and 6.a. However Method 2 performs better than method 1, but both applied methods have good performances; It means that, inventory levels in all echelons are almost

near safety stock level (desired reference), (Fig4.a), Each upstream node satisfies its downstream node demand perfectly (fig6.a), and Also Bullwhip effect rate which is calculated according to formula (1), is under control , Fig (5.a). But the question is how is the performance of method 1 in face of sudden demand fluctuations and in the presence of long distribution delays?

However much effort has been done to tune MPCs in method 1, but as illustrated in Fig (4.b), fig (5.b) and Fig (7), in the presence of long delays, method 1 loses its efficiency; It means that there are lots of overshoots in output responses, each node cannot satisfy its downstream node's demand precisely and bullwhip effect rates are increased.

Also when there is sudden changes in customer demand (demand type 3) , method 1 cannot control the

supply chain, Fig (8a , 9a , 10) and method 1 performance becomes poorer when distribution delay time increases , Fig (8b , 9b, 11)

As it is clear, in all figures, method 2 plays much better performance than method 1 and method 2 is always robust toward any demand changes and any delay. All the results are summarized in Table 4.

Tab.4. Results summarization

Delay type Demand type	Delay type 1 (Short Delay)	Delay type 2 (Long Delay)
Demand type 1 (pulsatory demand with Small domain changes)	Method 1 √ Method 2 √	Method 1 × Method 2 √
Demand type 2 (pulsatory demand with high and sudden domain changes)	Method 1 × Method 2 √	Method 1 × Method 2 √
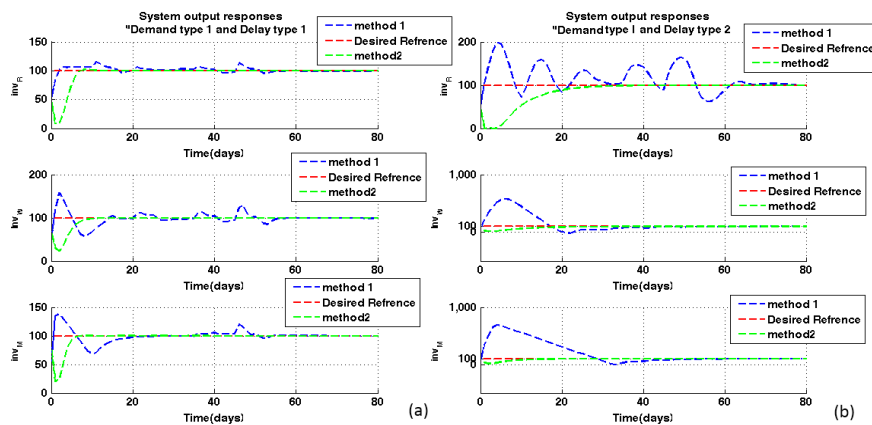
Fig.4. Method 2 plays better performance than method 1 on inventory control and safety stock control, Fig (a), especially when there is delay type 2, Fig (b).
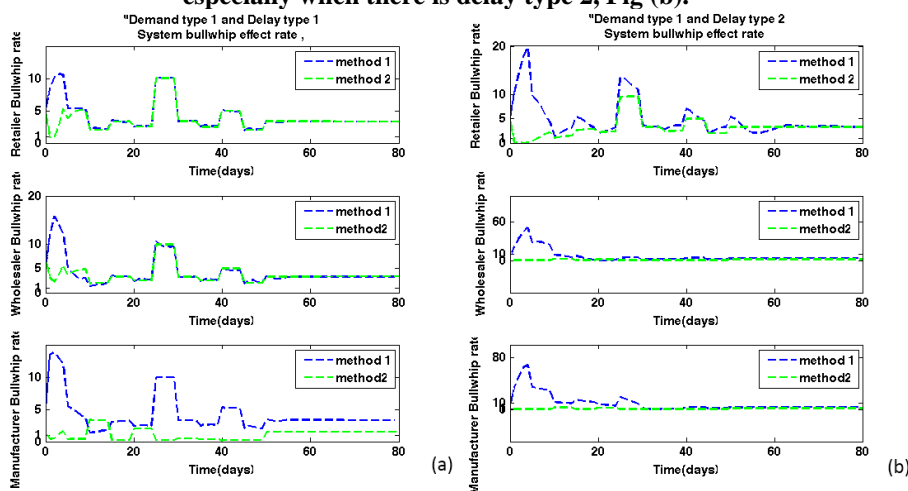
Fig.5. Method 2 plays better performance than method 1 on bullwhip effect rate reduction, Fig (a), especially when there is delay type 2, Fig (b)
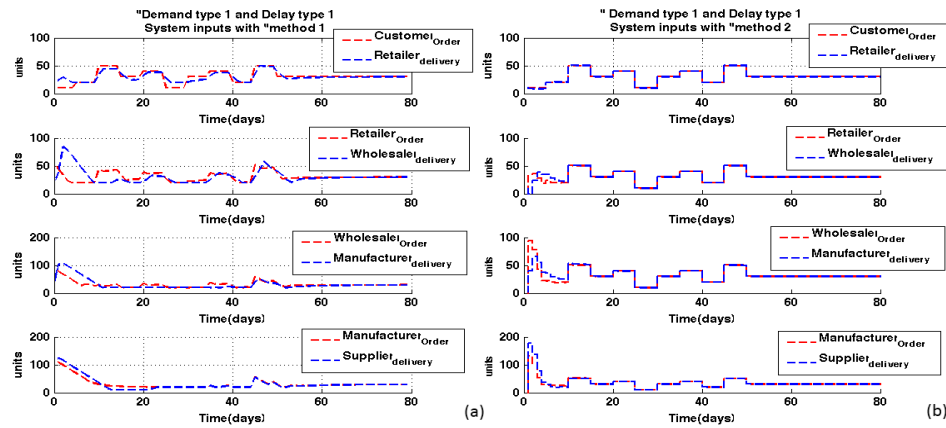
Fig.6. When demand is type 1 and delay is type 1, both methods have good performances in demand satisfaction, Fig (a), Fig (b); however method 2 has better performance than method 1, Fig (b).
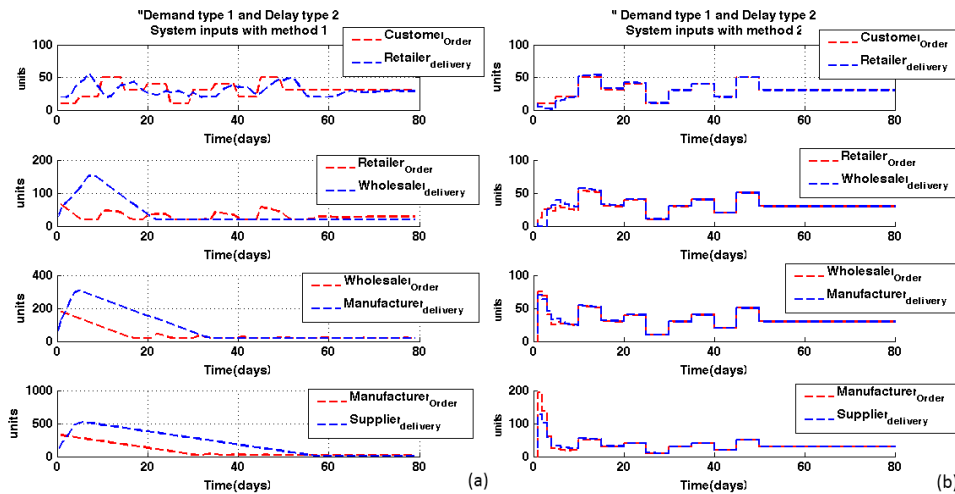

Fig.7. When there is delay type 2, method 2 has poor performance in demand satisfaction , Fig(a) , but method 2 has good performance and keeps its robustness through delay type 2 , Fig(b).
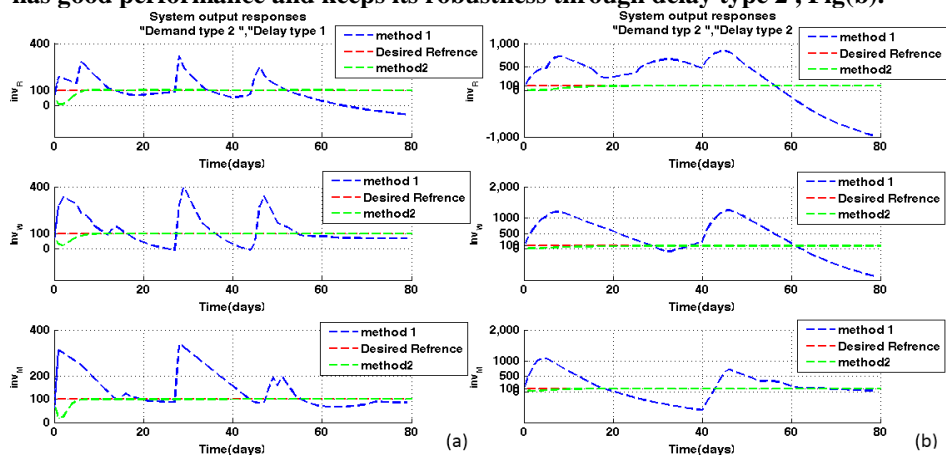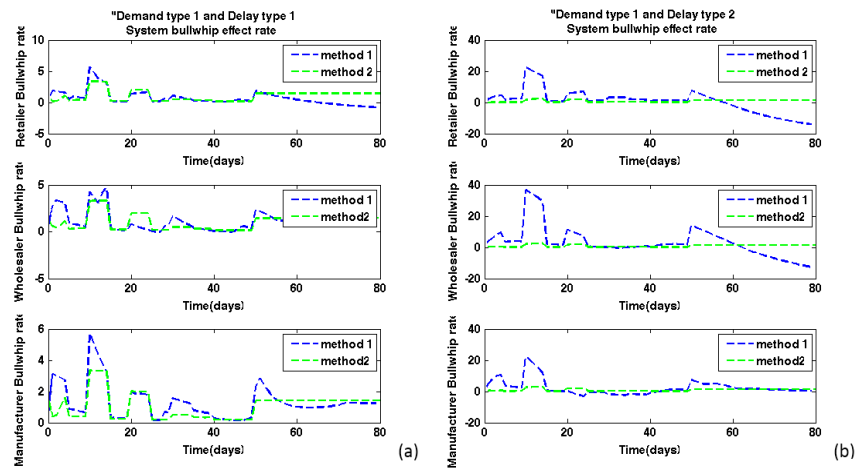

Fig.8. When demand is type 2 , Method 2 plays much better performance than method 1 on inventory control and safety stock control (a), especially when there is delay type 2 (b).

Fig.9. When demand is type 2 , Method 2 plays much better performance than method 1 on bullwhip effect rate reduction (a) , especially when there is delay type 2 (b)
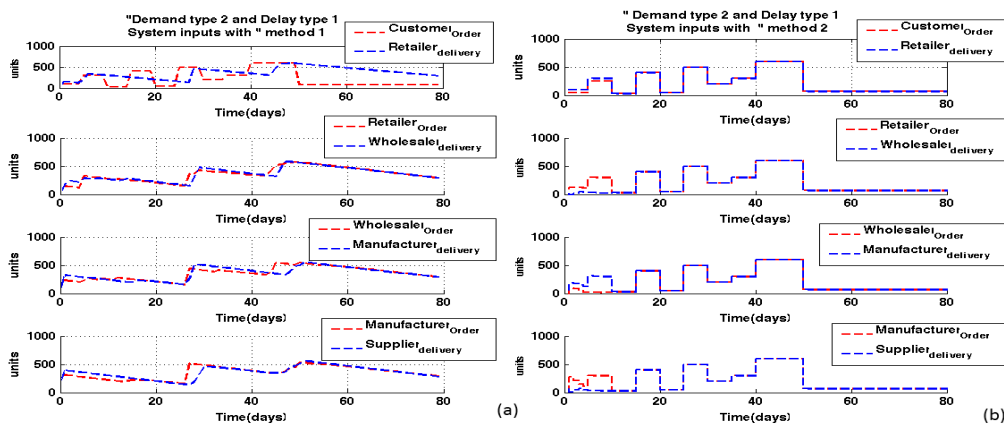

Fig.10. When demand is type 2, Method 2 in Fig (b) , plays better performance on downstream demand satisfaction than method 1 , Fig (a).
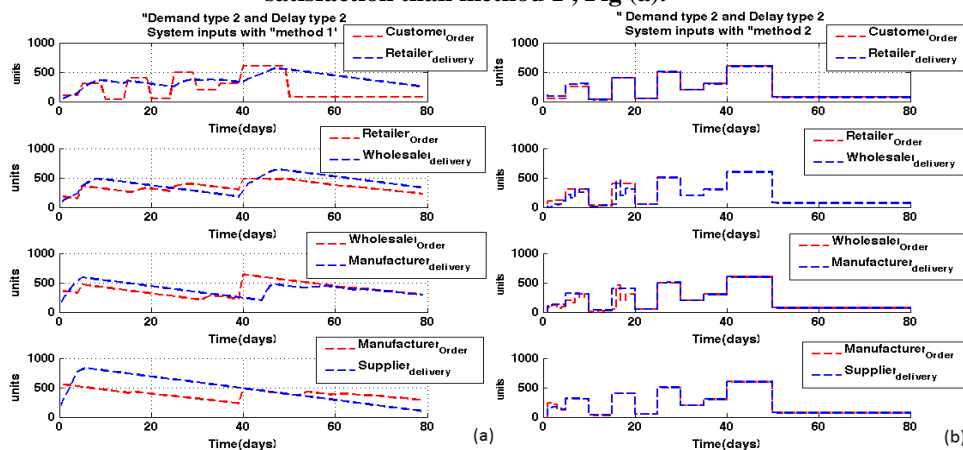

Fig.11. When demand is type 2 and delay is type 2, Method 2, Fig (b), plays much better performance on downstream demand satisfaction than method 1, Fig (a).

4. Conclusion

A framework for supply chain management based on distributed Model Predictive Controllers using orthonormal Laguerre functions was presented and also the supply chain model changed to an augmented model in which an integrator was imbedded and input and output changes were highly penalized in cost functions. The simulation results, demonstrated that the proposed control approach is robust through any customer demand and

transportation time delay and despite the demand fluctuations, in contrast to conventional MPCs without using orthonormal basis functions, the bullwhip effect rate decreased, better reference tracking and disturbance rejection achieved and smoothness of the supply chain process in all echelons and subsystems kept.

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