A Robust Desirability-based Approach to Optimizing Multiple Correlated Responses

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ABSTRACT

There are many real problems in which multiple responses should be optimized simultaneously by setting of process variables. One of the most common approaches for optimization of multi-response problems is the desirability function. In most real cases, there is a correlation structure between responses; therefore, ignoring the correlation may lead to incorrect results. Hence, in the present paper a robust approach, based on desirability function is provided which can consider the correlation structure in the optimization of the responses. The current study mainly aims to synthesize the ideas considering correlation structure in robust optimization through defining joint confidence interval and desirability function methods. A genetic algorithm is employed to solve the introduced problem. We have tried to enhance the effectiveness of the proposed method through some computational examples and comparisons with previous methods which are incorporated to show the applicability of the proposed approach. Also, a sensitivity analysis is provided to show the relationship between correlation and robustness in these approaches.

1- Introduction

There are usually multiple responses in the real experiments which can make optimization complicated considerably. Investigating the literature shows that many works have been done around multiple response optimization (MRO). Some of researchers tried to convert multiple response variables into a single measure through aggregating them into a composite function. Desirability function and loss function are two popular approaches among practitioners. The desirability function method proposed by Harrington [1] transforms each response value into a scale-free value called desirability. On the other hand, the loss function approach considers the distance of the response value from its predefined target. Moreover, some other approaches have been used for optimizing multiple responses. Some of these approaches are based on compromise programming, goal programming, inspection of contour plots, probability-based performance indices, neural networks, and vectorial optimization.

Among aforementioned approaches, desirability-based methods have less sophistication, more applicability and flexibility than other methods in weighting individual responses. The desirability-based method proposed by Derringer and Suich's method [2] for optimization of multi-response

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problems, and its modifications [3] became very popular such that they are available in many data analysis software packages.

In contrast, there have been few works on multi-response problems that consider both the optimality and robustness of the solution. As some examples; He et al. [4] proposed a method considering the optimality as well as robustness using desirability function. A hybrid quality function-based multi-objective model is proposed with combining reliability-based design optimization (RBDO) and robust design optimization by Yadav et al. [5]. Wang et al. [6] proposed an integrated grey relational grade (IGRD) index and used TOPSIS to find optimal robust parameter design for dynamic multi-response system.

Note that in these works, researchers did not consider uncertainty in the models. On the other hand, Xu and Albin [7] proposed a robust optimization method for experimentally obtained objective functions where uncertainties related to model coefficients were considered. In reality, they proposed worst case strategy that is closely related to Watson's robust counterpart formulation [8]. Peterson [9] presented a Bayesian posterior predictive approach to deal with the model parameter uncertainty for MRO problems. However, this approach does not seem to be favorable for researchers because they need a statistical background.

Stinstra and Hertog [10] presented the robust counterpart for constrained optimization method in which the two error types, namely model error and implementation error, have been considered apart from each other.

Because the desirability function method is highly popular, interpolating the robustness concepts can equip it to find better results. For more information about how to account for design uncertainties and measuring robustness, Beyer and Sendhoff [11] can be used. Datta et al. [12] proposed a combination of the Taguchi robust optimization approach and principal component analysis to solve correlated multi-response problems. Ribeiro et al. [13] developed another approach for the optimization of correlated multi-response problems. Salmasnia et al. [14] proposed a combined approach in that correlated covariates are considered as well as correlated responses. Salmasnia et al. [15] proposed a novel approach for optimization of

correlated multi-response problems by using of ANFIS as predicting tool. They used principal component analysis to remove correlation structure and employed desirability function for optimization. Salmasnia et al. [16] introduced a method to identify process variables to consider correlation among quality characteristics and minimize the variation in deviation of responses from their targets.

It also accommodates dispersion effects and specification limits as well as location effects in a unified framework based on desirability functions. Bashiri and Bakhtiarifar [17] proposed a new method which used multivariate normal probability to find the optimal treatment in an experimental design. Also, they developed a heuristic method to find better factors' level in all possible combinations in the designs with large number of controllable factors and levels.

Considering of previous studies shows that lots of works have concentrated on the desirability function approach, however considering of correlation structure usually is ignored in some of desirability-based studies.

A comparison among some new studies about using desirability function in MRO and the current work is provided in Table 1 to show the novelty of the proposed method. As can be seen, there is not any other method which can consider correlation structure, weight of the responses, location and dispersion effects, and robustness, simultaneously.

In this paper, a robust approach based on desirability function has been developed to optimizing multiple correlated responses by considering both location and dispersion effects. To do this, we have considered simultaneous confidence intervals for correlated responses. To deal with the problem we try to combine robust counterpart approach with desirability function method and then employ a well-known genetic algorithm (GA) to optimize the problem.

The rest of the current paper is structured as follows. The next section includes problem statement. Thereafter proposed method including robust desirability function and optimization algorithm is discussed in detail in section 3. In the section 4, the performance of the proposed approach is evaluated through some numerical examples. Also this section includes sensitivity analysis on the correlation coefficient to compare the proposed method with previous ones. Finally,

concluding remarks and some future study suggestions are expressed in section 5.

Tab.1. a comparison on new desirability-based methods in MRO

Authors	Correlation	Weights	location effect	dispersion effect	Robustness
Costa, et al.[18]	✓	✓	✓	✓	
Mostafa, et al.[19]			✓		
Wan and Birch[20]			✓		
Salmasnia, et al.[15]	✓		✓	✓	
He, et al.[21]		✓	✓		✓
Salmasnia, et al.[16]	✓		✓	✓	
Chen, et al.[22]			✓	✓	
Zhang, et al.[23]		✓	✓	✓	
Proposed Method	✓	✓	✓	✓	✓

2-   Problem Statement

In this paper an experiment with multiple correlated response variables has been addressed. It is supposed that response variables have a multivariate normal distribution. For each of the responses a specification limit including upper and lower bounds as well as target value are defined. The aim of this study is setting of controllable variables of the experiment such that responses get nearest value to their corresponding targets. In the proposed method a robust desirability approach is considered to select best setting of controllable factors with considering of worst case strategy.

3-   Proposed Method

Using the desirability function method in experimental design was proposed by Derringer and Suich [2]. In this method any response value is converted to a desirability value between 0 and 1 based on its distance from the corresponding target. One strength of the desirability function is its ability to aggregating multiple responses by considering their importance into a value between 0 and 1. However it cannot consider the correlation structure of the responses. Also it considers the location effect of responses while the dispersion effect is ignored. The formulation of desirability functions for different type of

responses based on Derringer and Suich [2] suggestion are according to Equations (1-3) which are for nominal-the-best (NTB), larger-the-best (LTB) and smaller-the-best (STB) type of responses, respectively.

$$d_i = \begin{cases} \left[\dfrac{\hat{y}_l - LSL_i}{t_i - LSL_i} \right] & LSL_i \leq \hat{y}_l \leq t_i \\[2em] \left[\dfrac{\hat{y}_l - USL_i}{t_i - USL_i} \right] & t_i \leq \hat{y}_l \leq USL_i \\[2em] 0 & \hat{y}_l \geq USL_i \; or \; \hat{y}_l \leq LSL_i \end{cases} \quad (1)$$

$$d_i = \begin{cases} 0 & \hat{y}_l \leq LSL_i \\[1.5em] \left[\dfrac{\hat{y}_l - USL_i}{LSL_i - USL_i} \right] & LSL_i \leq \hat{y}_l \leq USL_i \\[1.5em] 1 & \hat{y}_l \geq USL_i \end{cases} \quad (2)$$

$$d_i = \begin{cases} 0 & \hat{y}_l \leq LSL_i \\[1.5em] \left[\dfrac{\hat{y}_l - LSL_i}{USL_i - LSL_i} \right] & LSL_i \leq \hat{y}_l \leq USL_i \\[1.5em] 1 & \hat{y}_l \geq USL_i \end{cases} \quad (3)$$

which LSL_i and USL_i are lower and upper limits of the ith response variable, respectively, $\hat{y}_l$ is the estimation of ith response (y_i). As Harrington [1] proposed the total desirability (D) index computed as following:

$$D(d_1(y_1), d_2(y_2), ..., d_l(y_l)) = \left(\prod_{i=1}^{n} d_i \right)^{\frac{1}{l}} \quad (4)$$

By performing the experiment a value is obtained for each response in each treatment. Using these values response models can be estimated for the problem. Note that we estimate the relationship of the responses and controllable factors by using least squares method. In such situation total desirability can be calculated for unperformed

trials based on response estimations as shown in equation (5).

$$D(d_1[\hat{y}_1(\eta_1)], d_2[\hat{y}_2(\eta_2)], ..., d_l[\hat{y}_l(\eta_l)]) = \left(\prod_{i=1}^{n} d_i(\eta) \right)^{\frac{1}{l}} \quad (5)$$

where η_l refers to the selected l^{th} point from the joint confidence interval (JCI).

Afterwards, for each control setting, we define a confidence interval for correlated responses and try to select an enough large number of points through the simulation from the mentioned area. Then we choose the minimum desirability value as representative score of the selected control setting.

$$D_m(x) = \min_g \{D[d_1(\eta_1), d_2(\eta_2), ..., d_l(\eta_l)]|$$
$$(\eta_1, \eta_2, ..., \eta_l) \in JCI\} \quad (6)$$

where $D_m(x)$ is the robust overall desirability function that called the worst case desirability. Mentioned measure considers the location effect only.

To interpolating the dispersion effect in the proposed approach, we estimate variance model for each response as well. Then, desirability of the variances are calculated by considering their maximum and minimum possible values as upper and lower limits. In the following equation $D_v(x)$ shows total variances desirability.

$$D_v(x) = d_1(var_1(y_1)) \times d_2(var_2(y_2)) \times ... \times d_l(var_l(y_l)) \quad (7)$$

Thereafter, first integrated desirability is calculated by multiplying mentioned desirability values where is shown in equation (8) and called as $D_{mv}(x)$.

$$D_{mv}(x) = \left(\left(D_m(x) \right)^{w_1} \times \left(D_v(x) \right)^{w_2} \right)^{\left(\frac{1}{w_1+w_2} \right)} \quad (8)$$

where w_1 and w_2 are geometric weights of location and dispersion effects respectively and are defined by the designer.

As a solution approach, an iterative heuristic algorithm is proposed to find the best controllable factors setting. The proposed solution procedure has been depicted in Fig 1

4-   Numerical examples

The proposed method is shown with two experiments extracted from He et al. [21] and Harper et al. [26], respectively.

3-1.   Example 1: chemical process

In this section two types of examples are illustrated. In the first type which is based on the example reported by Montgomery [24] the analysis is performed using the proposed method and the results are compared with results of previous approaches. In the second type of examples some data are simulated by changing data generation parameters such as correlation between responses in data. Then a sensitivity analysis is performed.

In the first example two responses, y_1 and y_2 are considered that are yield and viscosity of a chemical action, respectively. These responses are related to reaction time (x_1) and reaction temperature (x_2). The first controllable variable is considered between 80 and 90 minutes and the second, between 170 and 180 degrees Fahrenheit. Target values of the responses are assumed to be 80 and 65, respectively. To analyze the experiment a central composite design (CCD) experiment with 13 treatments has been considered. The results of experimentation have been reported in Table 2. The existence of the multivariate normal distribution for responses was checked by Royston multivariate normality test [27] and a p-value of 0.08 confirmed it. Equations (9-13) show the first and second response means, variances and the correlation coefficient models, respectively, estimated using least squares method.

$$\hat{y}_1(x) = 79.940 + 0.995x_1 + 0.515x_2 - 1.376x_1^2$$
$$-1.001x_2^2 + 0.250x_1x_2 (R^2 = 98.28) \quad (9)$$

$$\hat{y}_2(x) = 70.000 - 0.155x_1 - 0.948x_2 - 0.688x_1^2$$
$$-6.688x_2^2 - 1.250x_1x_2 (R^2 = 89.97) \quad (10)$$

$$var_1 = 0.538 - 0.030x_1 + 0.042x_2 + 0.239x_1^2$$
$$-0.087x_2^2 - 0.063x_1x_2 (R^2 = 79.82) \quad (11)$$

$$var_2 = 4.656 + 0.366x_1 + 0.602x_2 + 0.349x_1^2$$
$$+1.350x_2^2 - 0.904x_1x_2 (R^2 = 15.53) \quad (12)$$

$$cor = -0.116 - 0.156x_1 - 0.001x_2 - 0.095x_2^2$$
$$-0.189x_1x_2 \ (R^2 = 52.78) \qquad (13)$$

Tab. 2. Results of experimentation for the chemical process example

Order N	Natural variable		Coded variable		Responses		Desirability		
	A	B	x_1	x_2	y_1	y_2	D_m	D_v	D_{mv}
1	80.00	170.00	-1.000	-1.000	76.5	62	0.763	0.2461	0.749
2	90.00	170.00	1.000	-1.000	78.0	66	0.902	0.115	0.871
3	80.00	180.00	-1.000	1.000	77.0	60	0.671	0.064	0.646
4	90.00	180.00	1.000	1.000	79.5	59	0.664	0.152	0.648
5	77.93	175.00	-1.414	0.000	75.6	71	0.755	0.023	0.713
6	92.07	175.00	1.414	0.000	78.4	68	0.863	0.050	0.824
7	85.00	167.93	0.000	-1.414	77.0	57	0.497	0.255	0.492
8	85.00	182.07	0.000	1.414	78.5	58	0.573	0.112	0.557
9	85.00	175.00	0.000	0.000	79.9	72	0.791	0.276	0.777
10	85.00	175.00	0.000	0.000	80.3	69	0.874	0.276	0.857
11	85.00	175.00	0.000	0.000	80.0	68	0.893	0.276	0.876
12	85.00	175.00	0.000	0.000	79.7	70	0.840	0.276	0.825
13	85.00	175.00	0.000	0.000	79.8	71	0.812	0.276	0.798

After estimating responses, variances and correlation coefficient, we calculate the confidence interval by considering correlation between the responses. Then, we select η points from this confidence interval and find a robust solution for them by employing max-min approach which expressed in details in the previous section. Finally we should find a control setting to maximize the integrated desirability (D_{mv}).

So, the desirability of the response means, and variances were calculated, and then corresponding total desirability was computed based on Equation 8. As shown in Table 2, the second treatment has maximum total desirability among others. Also, treatments (9-13) have best variance desirability values. Finally, best total desirability belongs to the 13th treatment.

As illustrated before, the proposed solution approach is a heuristic algorithm while some computations is needed to perform in each iteration. Moreover metaheuristic algorithms have been proposed before by previous researchers to find optimal settings of controllable factors in the MRO problems. For example He et al. [21] used hybrid genetic algorithm and etc. So in this study the genetic algorithm (GA) is used to search in the problem space for finding optimal factor setting of the robust MRO problem. We used Matlab R2012b 64bit under Windows 7 64bit to code the

proposed algorithm. In the genetic algorithm, at first a pool of chromosomes is generated. The chromosome is considered as a set of genes which are generated randomly from [-1 1] interval. These genes are denoted as x variable. Then a rate of crossover and mutation are selected randomly from mentioned pool. In this paper we used single point crossover. In mutation section, firstly, a random integer number from mentioned interval is selected as a point. Then, a new x variable is replaced in this point. Thereafter by considering new and old chromosomes we select the solutions with best fitness values according to predetermined population size (elitist selection). In this regard, there are some parameters such as crossover rate, mutation rate and population size that should be tuned to increase the efficiency of the GA. To do this, we applied the Taguchi method and the results have been reported in Table 3. As can be seen, the eighth treatment has better result and is selected as optimal GA parameters setting.

Tab. 3. Results of tuned Taguchi method for presented genetic algorithm

N	Max iteration	Number of population	Crossing probability	Number of η	D_{mv}
1	10	50	0.7	70	0.9325
2	10	100	0.3	100	0.9235
3	20	50	0.7	100	0.9272
4	20	100	0.3	70	0.9316
5	30	50	0.3	70	0.9323
6	30	100	0.7	100	0.9309
7	60	50	0.3	100	0.9296
8	60	100	0.7	70	0.9350

The proposed approach is illustrated in Fig 1 To compare the proposed method with previous approaches, we try to calculate our robust desirability based on best results reported by He et al. [21], and solutions 1 and 2 of Derringer's method [2]. It can be seen that our result are meaningful better than other approaches from the results reported in Table 4.
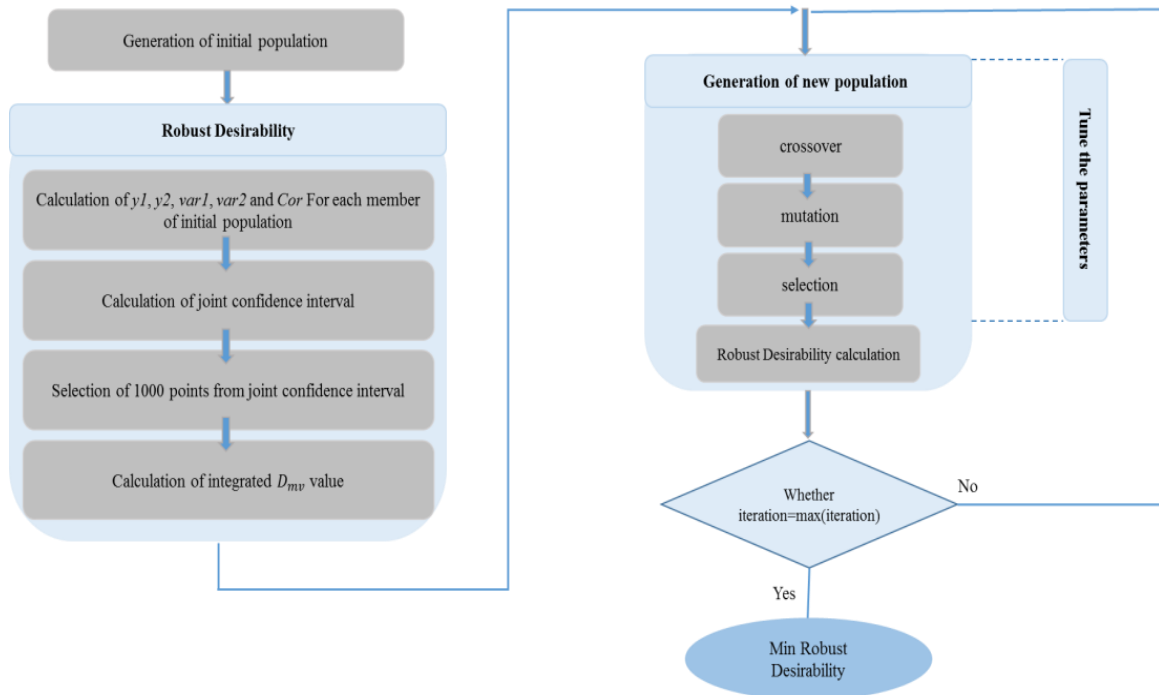
Fig. 1. Flowchart of the proposed iterative solution algorithm for the robust multiple response optimization

Tab. 4. Comparison of results by different approaches

Methods	x_1	x_2	D_{mv}
Proposed method	0.2375	0.7338	0.9361
He et al.[21] method	0.1690	-0.8513	0.9009
Solution 1. Derringer's method [2]	-0.2852	0.6157	0.8861
Solution 2. Derringer's method [2]	0.2117	-0.7366	0.8819

Fig. 2. Joint interval of a bi-variate problem with less correlation between responses

To appraise performance of the proposed method, Fig 2 and 3 show the effect of correlation value on the joint confidence interval. As can be seen, correlation and robustness have direct relation together. In fact, increasing of the correlation between response variables leads to increasing of robustness.
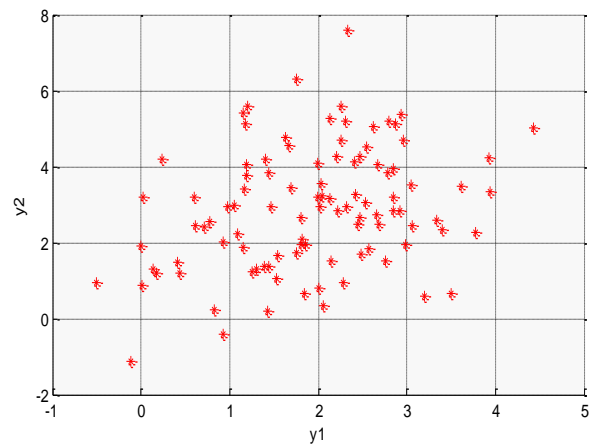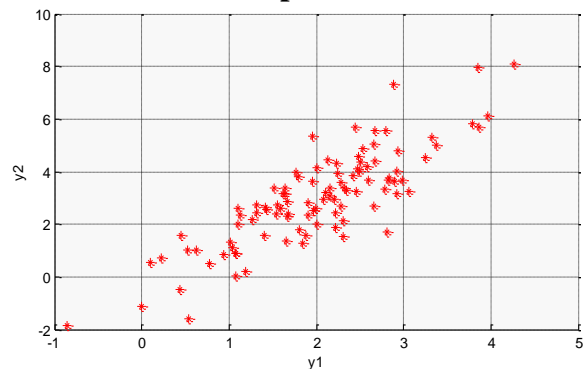

Fig. 3. Joint interval of a bi-variate problem with more correlation between responses

To investigate the effect of correlation coefficient on the results of this study, we try to solve the previous example with different correlation coefficient values. So by simulated data all previous activities are replicated and the final integrated desirability is calculated. Table 5 shows the best results and it can be seen that by decreasing the correlation coefficient value, the best D_{mv} is decreased as well.

Tab. 5. Result of the proposed method with different examples

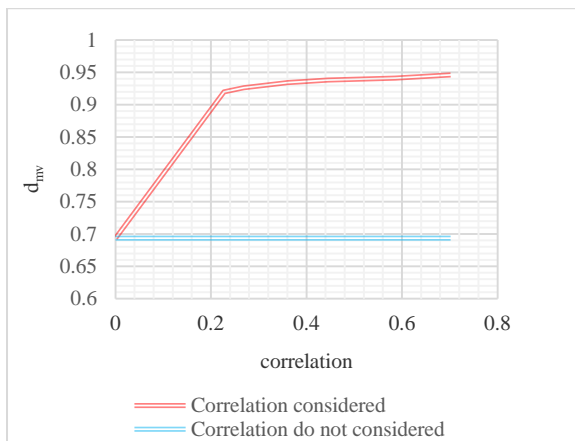| n	Correlation	x_1	x_2	D_{mv}
1	0.7018	0.0593	0.3302	0.9460
2	0.5858	0.5294	0.3294	0.9412
3	0.4449	0.8364	0.2200	0.9383
4	0.3605	0.4537	0.7094	0.9343
5	0.2691	0.4030	0.6910	0.9263
6	0.2276	1.1458	0.4616	0.9199

Fig. 4. The effect considering correlation between responses

As illustrated in Fig 4, increasing in the correlation leads to increasing of the proposed D_{mv} (red line). However the results D_{mv} which is calculated based on the He et al. [21] is constant because of ignoring the correlation structure.

4-2.   Example 2: Wheel cover component experiment

In the second example two responses y_1 and y_2 are considered which are weight and balance of the wheel cover component experiment, respectively. These responses are related to seven factors that defined as controllable variables considered between -1 and 1 interval. In this regard, target values of the responses are assumed to be 717.5 and 2, respectively. To analyze the experiment, we designed aorthogonal array with 8 treatments. The results have been reported in Table 6. Equations (14-18) show the first and second response means, variances and the correlation coefficient models, respectively. Note that, the corresponding coefficient of determination (R^2) is provided in parenthesis.

$$\hat{y}_1(x) = 720.763 + 1.873x_1 + 5.318x_5 - 3.408x_7$$
$$(R^2 = 88.03) \tag{14}$$

$$\hat{y}_2(x) = 0.967 + 0.113x_1 + 0.328x_5 - 0.174x_7$$
$$(R^2 = 93.86) \tag{15}$$

$$var_1 = 7.905 + 6.008x_4 \, (R^2 = 89.19) \tag{16}$$

$$var_2 = 0.011 - 0.007x_2 + 0.003x_4 + 0.004x_6$$
$$(R^2 = 98.30) \tag{17}$$

$$cor = 0.269x_1 - 0.289x_5 \ (R^2 = 81.65) \tag{18}$$

Tab. 6. Results of experimentation for the wheel cover component example

Order N	Coded variable							Responses		Desirability		
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	y_1	y_2	D_m	D_v	D_{mv}
1	-1	-1	-1	-1	-1	-1	-1	712	0.59	0.33	0.51	0.34
2	-1	-1	-1	1	1	1	1	720	0.86	0.42	0.02	0.40
3	-1	1	1	-1	-1	1	1	711	0.49	0.26	0.70	0.27
4	-1	1	1	1	1	-1	-1	731	1.46	0.26	0.34	0.26
5	1	-1	1	-1	1	-1	1	722	1.31	0.50	0.51	0.50
6	1	-1	1	1	-1	1	-1	723	0.96	0.43	0.02	0.41
7	1	1	-1	-1	1	1	-1	730	1.53	0.33	0.70	0.34
8	1	1	-1	1	-1	-1	1	714	0.50	0.36	0.34	0.36

As shown in Table 6, the fifth treatment has maximum D_m value among others. However, treatments 3 and 7 have best D_v in comparison to others. Finally, best total desirability belongs to the fifth treatment.

To do a comparison between the proposed method and previous approach, we try to calculate the robust desirability index for the best result reported by Chiao and Hamada [25]. As can be seen in Table 7, our result is

meaningfully better than mentioned method. And it means that the proposed approach is more efficient in finding the best treatment when all aspects of mean, variance, correlation and robustness are important in the decision making.

Tab. 7. Comparison of results by other approach

Methods	x_1	x_2	x_3	x_4	x_5	x_6	x_7	D_{mv}
Proposed method	-0.154	0.310	0.740	-0.233	0.475	0.748	-0.762	0.522
Chiao and Hamada [25] method	1	1	1	-1	-1	1	1	0.254

5-   Conclusion

In this paper a robust optimization approach based on desirability function was proposed for multiple correlated responses. To do this, we generated several points from the joint confidence interval area. Then, the minimum desirability value is calculated as a robust representative value of joint interval desirability. To decrease the uncertainty of the results the process is iterated many times and the setting of factors with maximum desirability is selected. Thereafter a tuned genetic algorithm employed to find the best factor setting. Two numerical examples from previous works was used to compare our results with some older approaches. The results show the efficiency of our proposed method. Finally a sensitivity analysis showed that by decreasing the correlation coefficient value, the best D_{mv} was decreased as well.

The comparisons confirm that the proposed method is more reliable than previous works when we pay attention to the robustness. As future studies, other multivariate distributions for responses rather than normal as well as qualitative responses can be considered. Besides considering replications for responses and incorporating the effect of in-treatment variance can be another fruitful subject. Moreover consideration of three or more responses for the estimated response model, and also using another metaheuristic algorithm to solve the presented model can be considered

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