International Journal of Industrial Engineering & Production Research

March 2016, Volume 27, Number 1
pp. 41 - 55

http://IJIEPR.iust.ac.ir/

pISSN: 2008-4889

Optimal Pricing and Advertising Decisions in a Three-level Supply Chain with Nash, Stacked berg and Cooperative Games

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KEYWORDS

Three-level supply chain,
Advertising,
Pricing,
Game theory.

ABSTRACT

Pricing and advertising are two important marketing strategies in the supply chain management which lead to increase customer demand, therefore higher profit for members of supply chains. This paper considers advertising, and pricing decisions simultaneously for a three-level supply chain with one supplier, one manufacturer and one retailer. The amount of market demand is influenced by pricing and advertising. In this paper, three well-known approaches in the game theory including the Nash, Stackelberg and Cooperative games are exploited to study the effects of pricing and advertising decisions on the supply chain. Using these approaches, we identify optimal decisions in each case for the supplier, the manufacturer and the retailer. Also, we compare the outcomes of decisions among the mentioned games. The results show that, the Cooperative and the Nash games have the highest and lowest advertising expenditure, respectively. The price level in the Nash game is more than the Stackelberg game for all three levels, and the retailer price in the Stackelberg and Cooperative games are equal. The system has the highest profit in the Cooperative game. Finally, the Nash bargaining model will be presented and explored to investigate the possibilities for profit sharing.

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1. Introduction[1]

Today's business has rapidly changed and has become more competitive, so winning customer satisfaction is one of the primary elements of survival in the market [1]. Supply chain coordination is an important issue in SCM in order to increase sales and profit and help to be more competitive. In this paper we coordinate the supply chain through the advertising and pricing strategies. The appropriate pricing helps the firms to gain an income with respect to the good's and service's value and keeps the firm's position among its competitors. The advertising can persuade customers to choose and buy a good with special brand among many brands that exist in the competitive markets.

The advertising may be divided into two main categories: static and dynamic. In the first category, the advertising is studied over a single period and in the second category, the customer's goodwill function is considered for investigating the carryover effect of advertising. Sometime the manufacturer agrees to pay part of the retailer's local advertising costs in order to make more

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Paper first received Apr. 08, 2015, is accepted in Feb. 06, 2016

promotional initiatives aimed at increasing immediate sales. This type of advertising is called cooperative (co-op) advertising.

A common approach adopted for investigating the role of advertising and pricing models in the supply chain is the game theory. Berger was the first to investigate the vertical co-op advertising from a mathematical viewpoint [2]. Jørgensen et al. studied dynamic Cooperative advertising in a channel [3]. They survived the cooperative advertising in a marketing channel and resolved the first step to the problem that if any channel member potentially can leads the channel and if there is a way to fully endogenize the choice of the leader [4]. Huang and Li and Li et al. investigated co-op advertising models of manufacturer-retailer supply chains [5, 6]. Prasad and Sethi studied the competitive advertising under uncertainty with a stochastic differential game approach. In another paper they applied advertising and pricing in a new-product adoption model [7, 8]. Karray and Zaccour survived if co-op advertising could be a manufacturer's counter strategy to store brands [9]. Yue et al. investigated the Cooperative advertising in a two-level supply chain in which the manufacturer offers discount in order to coordinate the channel[10].

Many researchers have also devoted their efforts to investigating methods of advertising and pricing. Szmerekovsky and Zhang and Xie and Wei investigated pricing and advertising with one manufacturer and one retailer [11, 12]. For the first time, Xie and Neyret applied the Stackelberg-retailer game in order to investigate these models [13]. In the Stackelberg-retailer game, the retailer is the manufacturer’s leader. Jørgensen et al. studied optimal pricing and advertising policies for an entertainment event [14]. In their model there are two periods, an initial period of regular price sales and a terminal period of last-minute sales at a (possibly) reduced price. Kumar and Sethi investigated the dynamic pricing and advertising for web content providers [15]. Krishnamoorthy et al. investigated the optimal pricing and advertising in a durable-goods duopoly [16]. In their model when sale increases the price doesn’t change but the advertising level decreases. Yan studied the Cooperative advertising, pricing strategy and firm performance in the e-marketing age [17]. His local and national advertising model is similar to the model of Xie and Wie [12]. SeyedEsfahani et al. developed the pricing and advertising models by incorporating concave, convex, and linear price demand curves [18]. Wang et al. studied the coordination of Cooperative advertising models in a one-manufacturer two-retailer supply chain system with the Nash-Cournot, Stackelberg-Cournot, Stackelberg-Coalition and Nash-Coalition games models [19]. Ahmadi-Javid and Hoseinpour applied a game-theoretic approach to analyze coordinating cooperative advertising in a supply chain. In their model the manufacturer offers no advertising support to the retailer when there is no channel leader [20]. Chutani and Sethi Studied the role of advertising and pricing in a dynamic durable goods supply chain [21].

Dietl et al. worked on the advertising and pricing models in media markets [22]. In their model the paid media platform generates revenues from media consumers through subscription fees, while the free media platform generates revenues from charging advertisers either on a lump-sum basis or on a per-consumer basis. Helmes et al. studied the dynamic advertising and pricing with constant demand elasticity [23]. Helmes and Schlosser analyzed a stochastic dynamic advertising and pricing model with isoelastic demand in a class of general new-product adoption models [24]. Aust and Buscher extended the model of SeyedEsfahani et al. by relaxing the restrictive assumption in equal margin profit [25]. In their model the state space is discrete, time is continuous and the planning horizon is allowed to be finite or infinite. They used the dynamic version of the Dorfman–Steiner identity in order to solve the problem. Liu et al. investigated an inventory decision problem under the pricing and advertising dependent stochastic demand [26]. They considered a joint decision on the pricing and advertising for competing retailers who operate short-life-cycle products under emergency purchasing. Giri and Sharma studied two-level supply chain consist of one manufacturer and two competing retailers with advertising cost dependent demand. The manufacturer acts as the leader who specifies wholesale price for retailers and two retailers compete with each other in advertising level [22]. Jørgensen and Zaccour studied the game-theoretic models of the Cooperative advertising [27].

A supply chain is consists of different members such as the supplier, the manufacturer, the distributer and the retailer. The comprehensive view to supply chain helps the better coordination of member’s decision. With regarding the channel member’s policies and decisions, the supply chain management could be better. In above mentioned studies all the investigations were about the role of advertising and pricing in a two-level supply chain, consist of one retailer and one manufacturer. The aim of the present study is to investigate the optimal decisions of channel members in a three-level supply chain consist of one supplier, one manufacturer and one retailer with one Cooperative and two non-cooperative games including the Stackelberg and the Nash games. So we can be one step closer to the better management and optimization of the channel.

In the non-cooperative games each member is a separate economic entity that makes its operational decisions independently [28]. In this study, in the Stackelberg game, the manufacturer is the retailer's follower and the supplier's leader. One example in which the manufacturer is retailer's follower is Wal-Mart[18]. The Wal-Mart is a powerful retailer who is able to limit manufacturer's margin. In many cases, the manufacturer is powerful than the suppliers. In these situations, the manufacturer is the supplier's leader. The Nash game is an equal power game. The customer's demand is influenced by the advertising and pricing. Tab. 1 presents a number of relevant key papers and the contributions made by the present study to the field.

The rest of the paper is organized as follows: In Section 2, a description of the model is presented. In Section 3, the non-Cooperative games and the Cooperative one are presented. Illustrative results of the models are presented in Section 4. Section 5 deals with the use of the Nash bargaining problem for profit sharing. Conclusions, trends for future research, and the summary of results are presented in Section 6.

Tab. 1. The relevant studies and the proposed model

paper	Equality of margins	Price demand	Advertising demand	Channel members	Game structures
[6]	----	----	$\alpha - \beta a^{-\gamma} A^{-\delta}$	manufacturer and retailer	N, SM and Co
[12]	----	$1 - \beta p$	$k_r\sqrt{a} + k_m\sqrt{A}$	manufacturer and retailer	SM and Co
[13]	*Assumed*	$\alpha - \beta p$	$A - Ba^{-\gamma} A^{-\delta}$	manufacturer and retailer	N, SR, SM and Co
[18]	*Assumed*	$(\alpha - \beta p)^{\frac{1}{\nu}}$	$k_r\sqrt{a} + k_m\sqrt{A}$	manufacturer and retailer	N, SR, SM and Co
[29]	*Relaxed*	$(\alpha - \beta p)^{\frac{1}{\nu}}$	$k_r\sqrt{a} + k_m\sqrt{A}$	manufacturer and retailer	N, SR, SM and Co
Proposed Model	*Assumed*	$(\alpha - \beta p)^{\frac{1}{\nu}}$	$k_r a^{\frac{1}{\tau}} + k_m A^{\frac{1}{\tau}}$	supplier, manufacturer and retailer	N, SR and Co

N: Nash game
SM: Stackelberg Manufacturer game
SR: Stackelberg Retailer game
Co: Cooperative game
$\alpha, \beta, a, A, k_r, k_m, \nu,$ and τ are defined in Table 2

2. The Model and Notations

In this paper, we investigate a three-level supply chain consist of one supplier, one manufacturer and one retailer in which the supplier sells raw material to the manufacturer who sells her/his products via the retailer channel. The supplier determines the supplier price. The manufacturer specifies the wholesale price and advertising budget. The retail price and the local advertising budget are determined by the retailer. The structure of the considered dual-channel supply chain is shown in Fig. 1. Tab. **2** presents the decision variables and the parameters used in this paper.

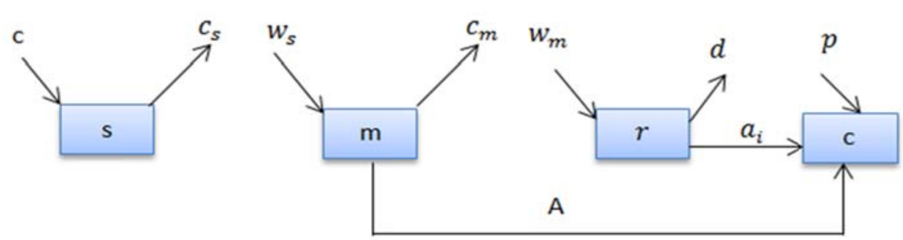

Fig. 1. The structure of the considered three-level supply chain

Tab. 2. Symbols Used

Variable		Parameters	
w_s	supply price	α	Price demand potential
w_m	Whole sale price	β	Price sensitivity
p	Retail price	ν	Price-demand curve's shape parameter
A	National advertising	k_m	Effectiveness of national advertising
a	Local advertising	k_r	Effectiveness of local advertising
Π_s	Supplier's profit	D_0	Customer's basic demand
Π_m	Manufacturer's profit	c	Supplier's procurement cost of raw material for unit product
Π_r	Retailer's profit	c_s	Supplier's processing cost of unit basic module
Π_{s+m+r}	System's profit	c_m	Manufacturer's value-added cost of unit product
		d	Retailer's unit handling cost
		τ	Advertisement-demand curve's shape parameter

The customer demand function can be assumed as follows similar to the relevant models in pricing and advertising [12, 13, 18].

$$D(p, a_{r,} A) = D_0 g(p) h(a, A) \tag{1}$$

The effects of the retail price and advertising on the demand are shown by $g(p)$ and $h(a, A)$, respectively. The effect of retail price is similar to the model of SeyedEsfahani et al. [18]. The demand changes when the price changes within an inverse relationship. One of our contributions in this paper is using a more general advertising function than the existing relevant research because it can show any shapes of advertising and demand relationship (see Tab. 1). These functions are shown as follows:

$$g(p) = (\alpha - \beta p)^{\frac{1}{\nu}} \tag{2}$$

$$h(a, A) = k_r a^{\frac{1}{\tau}} + k_m A^{\frac{1}{\tau}} \tag{3}$$

Based on Equations (2) and (3), the demand function is written as follows:

$$D(p, a_{r,} A) = D_0 (\alpha - \beta p)^{\frac{1}{\nu}} (k_r a^{\frac{1}{\tau}} + k_m A^{\frac{1}{\tau}}) \tag{4}$$

In order to show saturation effect of the advertising on the customer's demand we assume $\tau > 1$. To avoid the negative effect of the pricing and advertising on the demand when they are committed together, the following condition should be verified:

$$p < \frac{\alpha}{\beta} \tag{5}$$

The profit function of the channel members and the system are as follows:

$$\Pi_s(w_s) = D_0 (w_s - c_s - c)(\alpha - \beta p)^{\frac{1}{\nu}} \left(k_r a^{\frac{1}{\tau}} + k_m A^{\frac{1}{\tau}}\right) \tag{6}$$

$$\Pi_m(w_m, A) = D_0 (w_m - w_s - c_m)(\alpha - \beta p)^{\frac{1}{\nu}} \left(k_r a^{\frac{1}{\tau}} + k_m A^{\frac{1}{\tau}}\right) - A \tag{7}$$

$$\Pi_r(p, a) = D_0 (p - w_m - d)(\alpha - \beta p)^{\frac{1}{\nu}} \left(k_r a^{\frac{1}{\tau}} + k_m A^{\frac{1}{\tau}}\right) - a \tag{8}$$

$$\Pi_{s+m+r}(p, A, a) = D_0 (p - c_s - c - c_m - d)(\alpha - \beta p)^{\frac{1}{\nu}} \left(k_r a^{\frac{1}{\tau}} + k_m A^{\frac{1}{\tau}}\right) - A - a \tag{9}$$

In this paper, s, m, r, and $s + m + r$ represent the supplier, the manufacture, the retailer, and the system, respectively. Equations (6)-(8) should satisfy the following conditions in order to avoid backwash effect [18]:

$$\Pi_s > 0 \rightarrow w_s > c_s + c; \tag{10}$$

$$\Pi_m > 0 \rightarrow w_m > w_s + c_m; \tag{11}$$
$$\Pi_r > 0 \rightarrow p > w_m + d \tag{12}$$

The variables are changed for ease of analysis as follows:

$$w'_m = \frac{\beta}{\alpha'}(w_m - (c_s + c + c_m)) \tag{13}$$
$$w'_s = \frac{\beta}{\alpha'}(w_s - (c_s + c)) \tag{14}$$
$$p' = \frac{\beta}{\alpha'}(p - (c_s + c + c_m + d)) \tag{15}$$
$$k'_r = D_0 \frac{\alpha'^{1+\frac{1}{\nu}}}{\beta} k_r \tag{16}$$
$$k'_m = D_0 \frac{\alpha'^{1+\frac{1}{\nu}}}{\beta} k_m \tag{17}$$
$$\alpha' = \alpha - \beta(c_s + c + c_m + d) \tag{18}$$

Based on the above changes, Equations (6)-(9) can be rewritten as follows:

$$\Pi_s(w'_s) = w'_s(1 - p')^{\frac{1}{\nu}}\left(k'_r a^{\frac{1}{\tau}} + k'_m A^{\frac{1}{\tau}}\right) \tag{19}$$
$$\Pi_m(w'_m, A) = (w'_m - w'_s)(1 - p')^{\frac{1}{\nu}}\left(k'_r a^{\frac{1}{\tau}} + k'_m A^{\frac{1}{\tau}}\right) - A \tag{20}$$
$$\Pi_r(p', a) = (p' - w'_m)(1 - p')^{\frac{1}{\nu}}\left(k'_r a^{\frac{1}{\tau}} + k'_m A^{\frac{1}{\tau}}\right) - a \tag{21}$$
$$\Pi_{s+m+r}(p', A, a) = p'(1 - p')^{\frac{1}{\nu}}\left(k'_r a^{\frac{1}{\tau}} + k'_m A^{\frac{1}{\tau}}\right) - A - a \tag{22}$$

For simplicity in the sequence of equations, the superscript ($'$) is removed.

3. Three Game Models

In this section three games, consisting of one Cooperative and two non-cooperative games, are described.

3.1- The Nash Game

The Nash game is applied where the members have equal power and their decisions are made simultaneously and independently. The solution of this game is called the 'Nash equilibrium' which is obtained by solving the following three models:

$$\begin{aligned}&\max \Pi_s(w_s) = w_s(1-p)^{\frac{1}{\nu}}\left(k_r a^{\frac{1}{\tau}} + k_m A^{\frac{1}{\tau}}\right);\\ &st: \quad 0 \le w_s \le 1\end{aligned} \tag{23}$$
$$\begin{aligned}&max\, \Pi_m(w_m, A) = (w_m - w_s)(1-p)^{\frac{1}{\nu}}\left(k_r a^{\frac{1}{\tau}} + k_m A^{\frac{1}{\tau}}\right) - A;\\ &s.t: \quad w_s \le w_m \le 1 \quad A \ge 0\end{aligned} \tag{24}$$
$$\begin{aligned}&\max \Pi_r\,(p, a) = (p - w_m)(1-p)^{\frac{1}{\nu}}\left(k_r a^{\frac{1}{\tau}} + k_m A^{\frac{1}{\tau}}\right) - a;\\ &s.t: \quad w_m \le p \le 1 \quad a \ge 0\end{aligned} \tag{25}$$

Proposition 1. The Nash equilibrium is obtained as follows:

$$p^N = \frac{3\nu}{1 + 3\nu} \tag{26}$$
$$w_s^N = \frac{\nu}{1 + 3\nu} \tag{27}$$
$$w_m^N = \frac{2\nu}{1 + 3\nu} \tag{28}$$
$$A^N = \left(\frac{\frac{\nu}{1+3\nu}\left(\frac{1}{1+3\nu}\right)^{\frac{1}{\nu}}}{\tau} k_m\right)^{\frac{\tau}{\tau-1}} \tag{29}$$
$$a^N = \left(\frac{\frac{\nu}{1+3\nu}\left(\frac{1}{1+3\nu}\right)^{\frac{1}{\nu}}}{\tau} k_r\right)^{\frac{\tau}{\tau-1}} \tag{30}$$

Proof

Π_s and Π_m is increasing in line with w_s and w_m respectively, which means the optimal value for w_s and w_m is the maximum possible value for w_s and w_m.
To find the maximum possible value for w_s and w_m, we apply the similar approach as proposed by [18]; we assume that the manufacturer will not buy any raw material from the supplier and the retailer will not sell the product if they do not get a minimum unit margin. The supplier's and the manufacturer's unit margin are as such minimum level for the manufacturer and the retailer, respectively. So the below constraints should be verified:

$$\left.\begin{array}{l}\mu_m \geq \mu_s \rightarrow w_m - w_s \geq w_s \rightarrow w_m \geq 2w_s \\ \mu_r \geq \mu_m \rightarrow p - w_m \geq w_m - w_s \rightarrow p \geq 2w_m - w_s\end{array}\right\} \begin{array}{l} w_s^N = \dfrac{p}{3} \\ w_m^N = \dfrac{2p}{3}\end{array}$$

$$\frac{\partial \Pi_m(w_m, A)}{\partial A} = 0 \rightarrow \frac{A^{\frac{1}{\tau}} k_m (w_m - w_s)(1-p)^{\frac{1}{\nu}}}{\tau A} - 1 = 0 \rightarrow A^N = \left(\frac{k_m (w_m - w_s)(1-p)^{\frac{1}{\nu}}}{\tau}\right)^{\frac{\tau}{\tau-1}} \quad (31)$$

The second partial derivative of Π_m w.r.t. A is negative, hence, Π_m is concave w.r.t. A and the optimal value is achieved by solving the first order condition which shown above.

$$\frac{\partial \Pi_r(p, a)}{\partial a} = 0 \rightarrow \frac{a^{\frac{1}{\tau}} k_r (p - w_m)(1-p)^{\frac{1}{\nu}}}{\tau a} - 1 = 0 \rightarrow a^N = \left(\frac{k_r (p - w_m)(1-p)^{\frac{1}{\nu}}}{\tau}\right)^{\frac{\tau}{\tau-1}} \quad (32)$$

By the same token, $\partial \Pi_r$ is concave w.r.t. a so the optimal value is achieved by solving the first order condition which shown above.

$$\frac{\partial \Pi_r(p, a)}{\partial p} = 0 \rightarrow (1-p)^{\frac{1}{\nu}}\left(k_r a^{\frac{1}{\tau}} + k_m A^{\frac{1}{\tau}}\right) - \frac{(p - w_m)(1-p)^{\frac{1}{\nu}}\left(k_r a^{\frac{1}{\tau}} + k_m A^{\frac{1}{\tau}}\right)}{\nu(1-p)} = 0 \rightarrow p^N = \frac{w_m + \nu}{1+\nu} \quad (33)$$

In order to prove that p^N which is obtained above is maximum value of retailer's price, we define the retailer's income as x $\left(x = (p - w_m)(1-p)^{\frac{1}{\nu}}\left(k_r a^{\frac{1}{\tau}} + k_m A^{\frac{1}{\tau}}\right)\right)$. We compare the value of retailer's income between $p = \frac{w_m+\nu}{1+\nu}$, p $= w_m$ and p = 1:

$x(p = w_m) = 0$

$x(p = 1) = 0$

$$x\left(p = \frac{w_m + \nu}{1+\nu}\right) = \left(k_r^{\frac{\tau}{\tau-1}} + k_m^{\frac{\tau}{\tau-1}}\right)\left(\frac{\nu}{1+3\nu}\right)\left(\frac{1}{1+3\nu}\right)^{\frac{1}{\nu}}\left(\frac{\left(\frac{\nu}{1+3\nu}\right)\left(\frac{1}{1+3\nu}\right)^{\frac{1}{\nu}}}{\tau}\right)^{\frac{1}{\tau-1}} > 0$$

Thus, the maximum of retailer's profit and Π_r is obtained while $p = \frac{w_m+\nu}{1+\nu}$.

The above optimal points are the functions of each other. After solving them simultaneously, the Nash equilibrium is achieved as shown in proposition 1.

.

3-2. The Stackelberg game

The players of this game are the leader or the follower. In this paper, we assume the retailer is the manufacturer's leader and the supplier is manufacturer's follower. The solution of this game is called the Stackelberg equilibrium. In order to determine the equilibrium by backward induction, at first the follower's optimal problem should be solved. The leader's decision problem is solved based on the follower's response. The best response of the supplier is as follows:

$$w_s^* = \frac{w_m}{2} \quad (34)$$

Now this value should be substituted in the profit function of the manufacturer. Regarding the above value of w_s^*, the manufacturer's response is as below:

$$w_m^* = \frac{2p}{3} \tag{35}$$

$$A^* = (\frac{w_m(1-p)^{\frac{1}{\nu}}}{2\tau}k_m)^{\frac{\tau}{\tau-1}} \tag{36}$$

In order to gain the best response of retailer, the above values are substituted his profit function. The retailer's response is as below:

$$p^* = \frac{\nu}{\nu+1} \tag{37}$$

$$a^* = (\frac{p(1-p)^{\frac{1}{\nu}}}{3\tau}k_r)^{\frac{\tau}{\tau-1}} \tag{38}$$

Proposition 2. The equilibrium of the Stackelberg game is obtained as follows by solving the above equations simultaneously:

$$p^{St} = \frac{\nu}{1+\nu} \tag{39}$$

$$A^{St} = (\frac{\frac{\nu}{1+\nu}(\frac{1}{1+\nu})^{\frac{1}{\nu}}}{3\tau}k_m)^{\frac{\tau}{\tau-1}} \tag{40}$$

$$w_s^{St} = \frac{\nu}{3+3\nu} \tag{41}$$

$$a^{St} = (\frac{\frac{\nu}{1+\nu}(\frac{1}{1+\nu})^{\frac{1}{\nu}}}{3\tau}k_r)^{\frac{\tau}{\tau-1}} \tag{42}$$

$$w_m^{St} = \frac{2\nu}{3+3\nu} \tag{43}$$

The proof of this proposition is similar to the proof of proposition 1.

3.3- The Cooperative game

In this game, the channel members cooperate together to maximize the profits of the whole system, and then they bargain to share the profit.

$$\max \Pi_{s+m+r}(p,A,a) = p(1-p)^{\frac{1}{\nu}}\left(k_r a^{\frac{1}{\tau}} + k_m A^{\frac{1}{\tau}}\right) - A - a$$
$$s.t: \quad 0 \le p \le 1 \qquad A, a \ge 0 \tag{44}$$

Proposition 3. The equilibrium of the Cooperative game is obtained as follows:

$$p^{co} = \frac{\nu}{1+\nu} \tag{45}$$

$$A^{co} = (\frac{\frac{\nu}{1+\nu}(\frac{1}{1+\nu})^{\frac{1}{\nu}}}{\tau}k_m)^{\frac{\tau}{\tau-1}} \tag{46}$$

$$a^{co} = (\frac{\frac{\nu}{1+\nu}(\frac{1}{1+\nu})^{\frac{1}{\nu}}}{\tau}k_r)^{\frac{\tau}{\tau-1}} \tag{47}$$

Proof

$$\frac{\partial \Pi_{s+m+r}(p,A,a)}{\partial p} = 0 \rightarrow (1-p)^{\frac{1}{\nu}}\left(k_r a^{\frac{1}{\tau}} + k_m A^{\frac{1}{\tau}}\right) - \frac{p(1-p)^{\frac{1}{\nu}}\left(k_r a^{\frac{1}{\tau}} + k_m A^{\frac{1}{\tau}}\right)}{\nu(1-p)} = 0 \rightarrow p^{co} = \frac{\nu}{1+\nu} \tag{48}$$

With the same token in the proof of proposition 1 in order to find the optimal value of p, we define the value of x as $p(1-p)^{\frac{1}{\nu}}\left(k_r a^{\frac{1}{\tau}} + k_m A^{\frac{1}{\tau}}\right)$. To define the domain of x we compared its value in $p = \frac{\nu}{1+\nu}$ with p =0 and p = 1:

$x(p=0) = 0$
$x(p=1) = 0$

$$x\left(p = \frac{\nu}{1+\nu}\right) = (k_r^{\frac{\tau}{\tau-1}} + k_m^{\frac{\tau}{\tau-1}})(\frac{\nu}{1+\nu})\left(\frac{1}{1+\nu}\right)^{\frac{1}{\nu}}(\frac{(\frac{\nu}{1+\nu})\left(\frac{1}{1+\nu}\right)^{\frac{1}{\nu}}}{\tau})^{\frac{1}{\tau-1}} > 0$$

Thus, the maximum of Π_r is obtained while $p = \frac{\nu}{1+\nu}$.

$$\frac{\partial \Pi_{s+m+r}(p,A,a)}{\partial A} = 0 \rightarrow \frac{A^{\frac{1}{\tau}} k_m p(1-p)^{\frac{1}{\nu}}}{\tau A} - 1 = 0 \rightarrow A^{co} = \left(\frac{k_m p(1-p)^{\frac{1}{\nu}}}{\tau}\right)^{\frac{\tau}{\tau-1}} \tag{49}$$

$$\frac{\partial \Pi_{s+m+r}(p,A,a)}{\partial a} = 0 \rightarrow \frac{a^{\frac{1}{\tau}} k_r p(1-p)^{\frac{1}{\nu}}}{\tau a} - 1 = 0 \rightarrow a^{co} = \left(\frac{k_r p(1-p)^{\frac{1}{\nu}}}{\tau}\right)^{\frac{\tau}{\tau-1}} \tag{50}$$

In order to prove that the above A^{co} and a^{co} which derived from the first order condition, show the maximum values we used Hessian matrix.

$$\mathrm{H}(\Pi_{s+m+r}) = \begin{bmatrix} \frac{\partial^2 \Pi_{s+m+r}(p,A,a)}{\partial \mathrm{A}^2} & \frac{\partial^2 \Pi_{s+m+r}(p,A,a)}{\partial \mathrm{A}\, \partial a} \\ \frac{\partial^2 \Pi_{s+m+r}(p,A,a)}{\partial a\, \partial \mathrm{A}} & \frac{\partial^2 \Pi_{s+m+r}(p,A,a)}{\partial a^2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\mathrm{p}(1-\mathrm{p})^{\frac{1}{\nu}} k_m A^{\frac{1}{\tau}}}{\tau^2 A^2}(1-\tau) & 0 \\ 0 & \frac{\mathrm{p}(1-\mathrm{p})^{\frac{1}{\nu}} k_r a^{\frac{1}{\tau}}}{\tau^2 a^2}(1-\tau) \end{bmatrix}$$

The odd minor is negative and the even minor is positive so the Hessian matrix is a concave one so the extreme points are maximum points.

The above optimal points are the functions of each other. After solving them simultaneously, the Cooperative game's equilibrium is achieved as shown in proposition 3.

In Table 3 the optimal solutions in three game models are shown.

In Table 3. Summary of the optimal solutions in three game models

	Nash game	Stackelberg game	Cooperative game
supply price	$\frac{\nu}{1+3\nu}$	$\frac{\nu}{3+3\nu}$	-
Whole sale price	$\frac{2\nu}{1+3\nu}$	$\frac{2\nu}{3+3\nu}$	-
Retail price	$\frac{3\nu}{1+3\nu}$	$\frac{\nu}{1+\nu}$	$\frac{\nu}{1+\nu}$
National advertising	$\left(\frac{\frac{\nu}{1+3\nu}\left(\frac{1}{1+3\nu}\right)^{\frac{1}{\nu}}}{\tau}\mathrm{k_m}\right)^{\frac{\tau}{\tau-1}}$	$\left(\frac{\frac{\nu}{1+\nu}\left(\frac{1}{1+\nu}\right)^{\frac{1}{\nu}}}{3\tau}\mathrm{k_m}\right)^{\frac{\tau}{\tau-1}}$	$\left(\frac{\frac{\nu}{1+\nu}\left(\frac{1}{1+\nu}\right)^{\frac{1}{\nu}}}{\tau}\mathrm{k_m}\right)^{\frac{\tau}{\tau-1}}$
Local advertising	$\left(\frac{\frac{\nu}{1+3\nu}\left(\frac{1}{1+3\nu}\right)^{\frac{1}{\nu}}}{\tau}\mathrm{k_r}\right)^{\frac{\tau}{\tau-1}}$	$\left(\frac{\frac{\nu}{1+\nu}\left(\frac{1}{1+\nu}\right)^{\frac{1}{\nu}}}{3\tau}\mathrm{k_r}\right)^{\frac{\tau}{\tau-1}}$	$\left(\frac{\frac{\nu}{1+\nu}\left(\frac{1}{1+\nu}\right)^{\frac{1}{\nu}}}{\tau}\mathrm{k_r}\right)^{\frac{\tau}{\tau-1}}$

4. Discussion of The Results

In this section, the optimal solutions of the two mentioned non-cooperative games and the Cooperative game will be compared together. We make a comparison among the price, the advertising expenditures and the profit function of the members and whole system in above-mentioned games. The comparison results of advertising decision among games are the similar to the comparison results of the national advertising.

4-1. Comparisons on prices

The summary of the results provided in Table 3 show that the optimal retail prices in the Stackelberg and the Cooperative games are similar and the Nash game has the highest optimal retail price. The supply and whole sale price in Nash game is higher than the Stackelberg game. In the Nash game each member tries to increase its profit through increasing price without attention to other members. In the Stackelberg game, the optimal value of price is obtained with respect to the leader's decision and in the

Cooperative game the optimal price is one which maximizes the whole system's profit, so it is lower than the price in the Nash game.

4-2. Comparisons on advertising expenditures

Fig. 2 shows the comparisons of the advertising expenditures. The result of comparison among the national and local advertising is same. As it is obvious the advertising expenditures has the most value in the Cooperative game and the least value in the Nash game. The increasing in the parameters of the advertisement-demand curves shape, leads to increase in the advertising expenditures, because when τ increases, the saturation effects increases. So in order to attract one customer, the more advertising expenditure is needed.

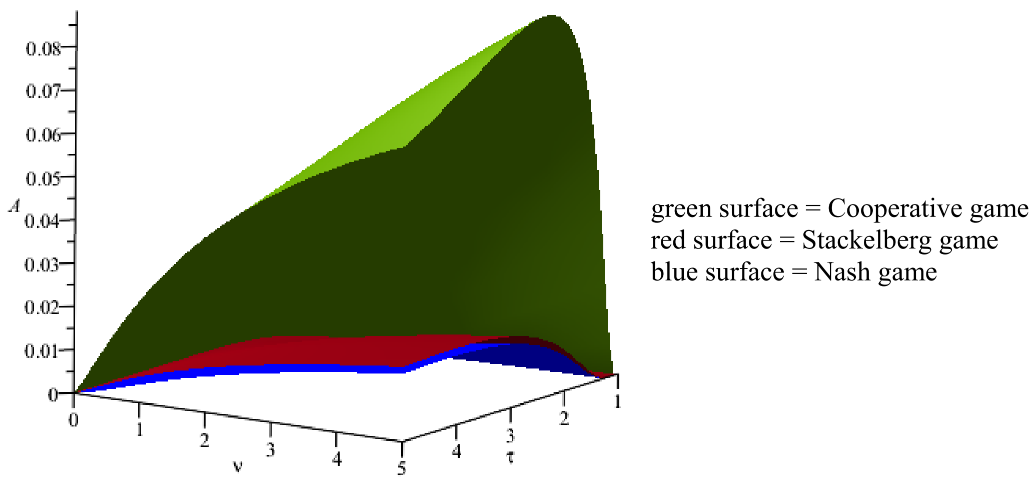

Fig. 2. The national advertising expenditures

4.3- Comparisons on Profits

The comparison on the profit functions among the mentioned games is done after substitution of the optimal values in the member's profit function. The results of the supplier's, the manufacturer's and retailer's profits is provided in Fig. 3, 4 and 5 respectively. This figures show that in some regions the member's profit is more in the Nash game and in some other regions it is vice versa. After determining the τ and ν according to situation of a real problem, the decision makers can use these figures in order to choose to participate in same power game or be the other's follower to gain more profit. As it is shown below in the figures, when ν increases, the profit of members increases. Because the higher value of ν means the lower sensitivity of customer's demand to price, so the supply chain members can increase price without missing important percent of customer's demand. The change in τ has no important effect in the profit. Because when τ changes, it effects both in revenue and costs.

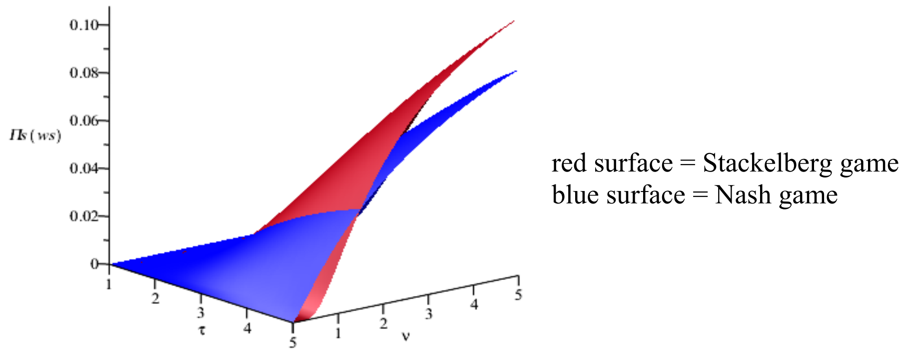

Fig. 3. The supplier's profits

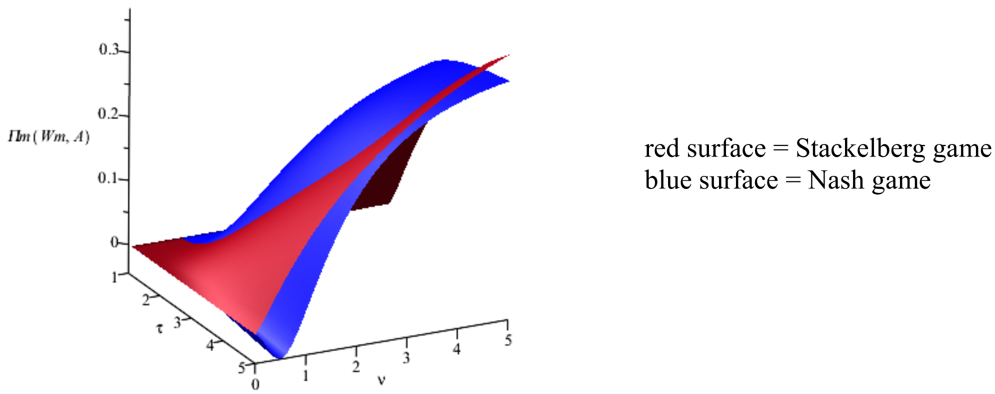

Fig. 4. The manufacturer's profits

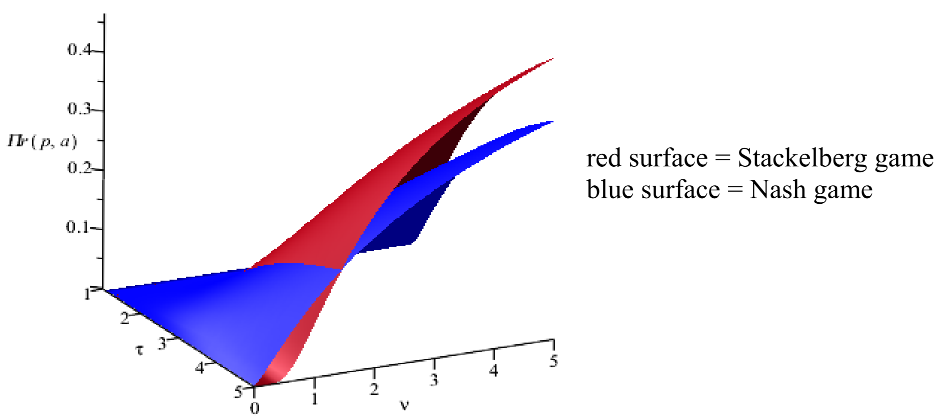

Fig. 5. The retailer's profits

The system's profit has the highest value in the Cooperative game, and the least value is achieved in some regions of each non cooperative games. These regions are shown in the Fig. 6.

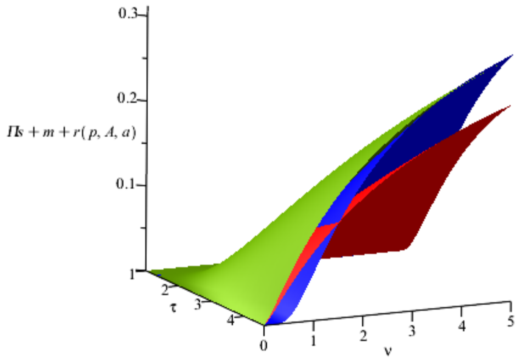

Fig. 6. The system's profits

4-4. Feasibility of the Cooperative game

For the feasibility of the Cooperative, the following conditions must hold:

$$\Pi_s^{co} = \Pi_s(p^{co}, w_s^{co}, w_m^{co}, A^{co}, a^{co}) \geq \max(\Pi_s^N, \Pi_s^{St}) \tag{51}$$

$$\Pi_m^{co} = \Pi_m(p^{co}, w_s^{co}, w_m^{co}, A^{co}, a^{co}) \geq \max(\Pi_m^N, \Pi_m^{St}) \tag{52}$$

$$\Pi_r^{co} = \Pi_r(p^{co}, w_s^{co}, w_m^{co}, A^{co}, a^{co}) \geq \max(\Pi_r^N, \Pi_r^{St}) \tag{53}$$

We integrate Equations (51)-(53):

$$\Pi_s^{co} + \Pi_m^{co} + \Pi_r^{co} = \Pi_{s+m+r}^{co} \geq \Pi_s^{max} + \Pi_m^{max} + \Pi_r^{max} \tag{54}$$

The Δ in the equation below, is the relative difference of the system's profits in the Cooperative and non-cooperative games. As shown in Fig. 6 its value is positive, because the system's profit in the Cooperative game is higher than the maximum system's profit in the non-cooperative games, so the condition in Equation (54) holds true and the feasible solution is exist.

$$\Delta = \frac{\Pi_{s+m+r}^{co} - (\Pi_s^{max} + \Pi_m^{max} + \Pi_r^{max})}{\Pi^{co}} \times 100 > 0 \tag{55}$$

The feasibility of the Cooperative game means that the channel members will cooperate. In the next section, the Nash bargaining model for sharing the extra profit gained in the cooperation is investigated.

5. Bargaining Problem

In this section, the Nash bargaining model is used to determine how to share profit between the members in the same way that it is used by Seyed Esfahani et al. [18]. First the feasible region for the variables w_s and w_m should be presented. The member's extra profit is shown below:

$$\Delta\Pi_s = \Pi_s^{co} - \Pi_s^{max} = w_s(1-p^{CO})^{\frac{1}{v}}\left(k_r a^{\frac{1}{\tau}} + k_m A^{\frac{1}{\tau}}\right) - \Pi_s^{max} = w_s B - C > 0 \tag{56}$$

$$\Delta\Pi_m = \Pi_m^{co} - \Pi_m^{max} = (w_m - w_s)(1-p^{co})^{\frac{1}{v}}\left(k_r a^{\frac{1}{\tau}} + k_m A^{\frac{1}{\tau}}\right) - A^{co} - \Pi_m^{max} = (w_m - w_s)B - D > 0 \tag{57}$$

$$\Delta\Pi_r = \Pi_r^{co} - \Pi_r^{max} = (p^{CO} - w_m)(1-p^{CO})^{\frac{1}{v}}\left(k_r a^{\frac{1}{\tau}} + k_m A^{\frac{1}{\tau}}\right) - a^{co} - \Pi_r^{max} = -w_m B + E > 0, \tag{58}$$

where

$$B = (1-p^{CO})^{\frac{1}{v}}\left(k_r a^{\frac{1}{\tau}} + k_m A^{\frac{1}{\tau}}\right) > 0$$

$\mathrm{E} = \mathrm{p}^{co}\mathrm{B} - a^{co} - \Pi_{\mathrm{r}}^{\max} > 0$
$\mathrm{C} = \Pi_{\mathrm{s}}^{\max} > 0$
$\mathrm{D} = A^{co} + \Pi_{\mathrm{m}}^{\max} > 0,$

The feasible region is beetween three lines shown in Fig. 7. with respect to $(\mathrm{w_s}, \mathrm{w_m})$. This region is made by the inequalities (56)-(58). The $\mathrm{x} = (\text{supplier, manufacturer or retailer})$ gains more from the extra profit if the solution gets nearer to $\Pi_{\mathrm{x}} = \Pi_{\mathrm{x}}^{\max}$ therefor the other member's share will be lower. All the pairs of the $(\mathrm{w_s}, \mathrm{w_m})$ on the parralel lines have equal profits for members as $\Pi_{\mathrm{s}} = \Pi_{\mathrm{s}}^{\max} + \Delta\Pi_{\mathrm{s}}$, $\Pi_{\mathrm{m}} = \Pi_{\mathrm{m}}^{\max} + \Delta\Pi_{\mathrm{m}}$ and $\Pi_{\mathrm{r}} = \Pi_{\mathrm{r}}^{\max} + \Delta\Pi_{\mathrm{r}}$.

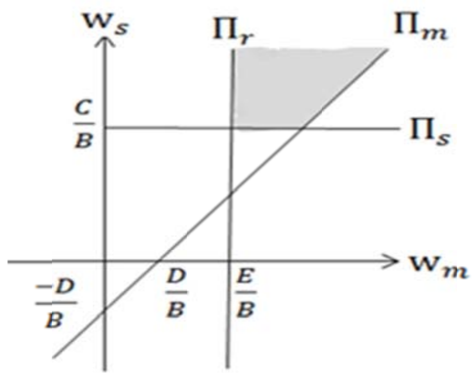

Fig. 7. Feasible region of the bargaining problem

The optimal values of (w_s, w_m) are found by maximizing the product of the members' utility function according to Nash [30]. In this paper, the utility function is assumed to be the same as the one used in SeyedEsfahani et al. [18]:

$$\mathrm{U_s}(\mathrm{w},\mathrm{t}) = \Delta\Pi_{\mathrm{s}}(w_s, w_m)^{\lambda_{\mathrm{s}}} \tag{59}$$

$$\mathrm{U_m}(\mathrm{w},\mathrm{t}) = \Delta\Pi_{\mathrm{m}}(w_s, w_m)^{\lambda_{\mathrm{m}}} \tag{60}$$

$$\mathrm{U_r}(\mathrm{w},\mathrm{t}) = \Delta\Pi_{\mathrm{r}}(w_s, w_m)^{\lambda_{\mathrm{r}}} \tag{61}$$

The parameter λ is the member's risk attitude and they gain more profit if thet seek more risk. The Nash bargaining model is solved as follows:

$$\mathrm{Max}\ \mathrm{U_s}(w_s, w_m)\mathrm{U_m}(w_s, w_m)\mathrm{U_r}(w_s, w_m) = \Delta\Pi_{\mathrm{s}}(w_s, w_m)^{\lambda_{\mathrm{s}}}\Delta\Pi_{\mathrm{m}}(w_s, w_m)^{\lambda_{\mathrm{m}}}\Delta\Pi_{\mathrm{r}}(w_s, w_m)^{\lambda_{\mathrm{r}}} \tag{62}$$

$$\Delta\Pi_{\mathrm{s}}(w_s^*, w_m^*) = \frac{\lambda_{\mathrm{s}}}{\lambda_{\mathrm{s}} + \lambda_{\mathrm{m}} + \lambda_{\mathrm{r}}}\Delta\Pi = \frac{\lambda_{\mathrm{s}}}{\lambda_{\mathrm{s}} + \lambda_{\mathrm{m}} + \lambda_{\mathrm{r}}}(\mathrm{E} - \mathrm{D} - \mathrm{c}) \tag{63}$$

$$\Delta\Pi_{\mathrm{m}}(w_s^*, w_m^*) = \frac{\lambda_{\mathrm{m}}}{\lambda_{\mathrm{s}} + \lambda_{\mathrm{m}} + \lambda_{\mathrm{r}}}\Delta\Pi = \frac{\lambda_{\mathrm{m}}}{\lambda_{\mathrm{s}} + \lambda_{\mathrm{m}} + \lambda_{\mathrm{r}}}(\mathrm{E} - \mathrm{D} - \mathrm{c}) \tag{64}$$

$$\Delta\Pi_{\mathrm{r}}(w_s^*, w_m^*) = \frac{\lambda_{\mathrm{r}}}{\lambda_{\mathrm{s}} + \lambda_{\mathrm{m}} + \lambda_{\mathrm{r}}}\Delta\Pi = \frac{\lambda_{\mathrm{r}}}{\lambda_{\mathrm{s}} + \lambda_{\mathrm{m}} + \lambda_{\mathrm{r}}}(\mathrm{E} - \mathrm{D} - \mathrm{c}) \tag{65}$$

$$\begin{cases} \mathrm{w_s^*B} = \frac{\lambda_{\mathrm{s}}(\mathrm{E}-\mathrm{D}-\mathrm{c})}{\lambda_{\mathrm{s}}+\lambda_{\mathrm{m}}+\lambda_{\mathrm{r}}} + \mathrm{c} \\ w_m^*B = E - \dfrac{\lambda_{\mathrm{r}}(\mathrm{E} - \mathrm{D} - \mathrm{c})}{\lambda_{\mathrm{s}} + \lambda_{\mathrm{m}} + \lambda_{\mathrm{r}}} \end{cases} \tag{66}$$

6. Conclusion

In this paper, a three-level supply chain consists of one supplier, one manufacturer and one retailer was studied in which the customer's demand is influenced by both pricing and advertising. Optimal decisions derived in the Nash, Stackelberg and Cooperative games show that the optimal price for all members in the Nash game is always higher than the optimal price in the Stackelberg game. The retailer price in Nash game is higher than the cooperative game and the retail price for the Cooperative and Stackelberg games are the same. The highest and the lowest advertising expenditure is in the Cooperative and the Nash games, respectivly. The supplier's profit in some regions is higher in Nash game and in some other region is higher in the Stackelberg game. The retailer's profit is same in non-cooperative games. The system's profit has the highest value in the cooperative game and the least value is achieved in some regions in the Nash game and in some other regions in the Stackelberg game, so the system gain extra profits in the Cooperative game rather than the non-cooperative games.

This problem can be solved with the multi-member in each level so the other games such as the Coalition and Bertrand can be applied in order to survey the problem for future studies, so the model would be more realistic and practical. Investigating this model with other important issues in supply chain, like inventory policy make it richer. The supply chain can be investigated in a fuzzy environment which is suitable for the imprecise or vague situations by membership functions [31]. Other types of supply chain such as agile supply chain can be considered. "Supply chain agility is a key determinant of competitiveness and is defined as the supply chain's alertness to internal and environmental changes and it's capability to use resources in responding to these changes in a timely and flexible manner.[32]" An intelligent agent supply chain can be used as an appropriate technology to coordinate and integrate different parts of the channel and makes its components relation more effective [33].

The multi-product supply chain can be investigated in order to study the effect of varying the level of substitutability coefficient of products on the profits of members [34]. A dual channel supply chain with a direct and indirect sale can and investigated in order to obtain the optimal policies of pricing, inventory and advertising can be studied [35].

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