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### **Economic-Statistical Design of a Control Chart for High Yield Processes When the Inspection is Imperfect**

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#### **KEYWORDS**

CCC-r control chart, high-yield process, inspection error.

#### ABSTRACT

CCC-r control chart is a monitoring technique for high yield processes. It is based on the analysis of the number of inspected items until observing a specific number of defective items. One of the assumptions in implementing CCC-r chart that has a significant effect on the design of the control chart is that the inspection is perfect. However, in reality, due to the multiple reasons, the inspection is exposed to errors. In this paper, we study the economic-statistical design of CCC-r charts when the inspection is imperfect. Minimization of the average cost per produced item is considered as the objective function. The economic objective function, modified consumer risk, and modified producer risk are simultaneously considered, and then the optimal value of r parameter is selected.

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#### 1. Introduction

Nowadays, we are mostly dealing with high-yield processes in which the proportion of defective items is very small. While an attribute control chart is a very effective tool for statistical Proses Control (SPC), the Shewhart control charts are ineffective in high-yield processes. Timebetween-events (TBE) control charts, which consider the number of successes between failures, have been shown to be useful for monitoring the high-yield processes. The cumulative count of conforming (CCC) control chart is based on the cumulative number of conforming items between two consecutive defective items. The idea of CCC chart was firstly proposed by Calvin[1] and further developed by Goh [2]. Since the geometric probability distribution function is highly asymmetric, the CCC control chart is not very sensitive to small incremental changes in the defective proportion[3]. For this purpose, several authors have proposed methods for solving this Acosta-Mejia[4] problem. suggested two geometric charts with runs rules. They observed that runs rules are appropriate for monitoring a distribution that is approximately unimodal and symmetric. Khilare and Shirka[5] studied the performance of a m-of-m control chart based on the cumulative count of conforming items.

Xie et al. [6] introduced the extended state of CCC control chart based on the cumulative count of conforming items until observing r defective

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ones called CCC-r control chart. CCC-r charts negative binomial probability follow a distribution and have greater efficiency in comparison with CCC chart in the high-yield In the CCC-r charts, too many processes. samples are required to be inspected in order to plot a point on the chart, and the related cost is fairly high[7]. In order to minimize the costs, the selection of parameter r has a significant influence on the charts' effectiveness. Ohta et al. [3] suggested a method to obtain parameter r and sampling interval using a simplified optimal design method.

Di Bucchianico et al. [8] presented a case study for monitoring a high-volume production process with a high yield. They discussed several implementation aspects of CCC-r charts in their case study and proposed a method to obtain the value of parameter r. Albers [9] studied one-sided CCC-r charts that determine the optimal value of parameter r by computing approximate results on the Lower Control Limit and Average Run Length (ARL) values.

Inspection errors have important effects on the results of control charts. Johnson et al.[10] and Ryan[11] discussed inspection errors and their impact on control charts. Burke et al. [12] denoted that the estimated value of fraction proportion of the sample might deviate from the true value due to the presence of inspection errors.

Lu et al. [13] studied the CCC chart in the presence of inspection errors. They computed the adjusted control limits based on the relationship between the true and observed values of the fraction proportion. In order to obtain the maximum ARL, Ranjan et al. [14] suggested a procedure to set control limits for CCC charts in the presence of inspection errors. Some other researchers studied the effects of inspection errors, such as Case[15], Lindsay[16], Suich[17], Suich[18], Chen and Chung[19], Wang and Chen[20], Nezhad and Nasab[21], and M. Fallahnezhad and A. Y. Babadi[21].

In the global market competition, we should pay more attention to cost optimization by determining the essential parameters of the control charts. For this reason, many economic studies have been performed. First, Duncan[22] investigated the economic design of control charts. Lorenzen and Vance[22] proposed the economic model applicable to many types of control charts. Tang et al.[23] studied economicstatistical design of CCC chart with conditional control limits. Ohta et al.[3] investigated the economic model of CCC-r chart, determined parameter r, and obtained sampling interval. Xie et al.[24] studied the economic design of CCC chart based on Duncan model[22]. Chan et al.[25] investigated the economic model of CCC charts based on sampling plans and the approach of acceptance risks. Zhang et al.[26] studied the economic design of the time-between-events (TBE) control chart. Yilmaz and Burnak[27] discussed the importance of cost consideration and sensitivity analysis of statistical risks in CCC chart. Fallahnezhad and Golbafian[28] proposed a mathematical model for CCC-r control charts based on the average number of inspected items. The optimal control limits and r values were cumputed for different defective fractions and different parameters in each iteration.

In this paper, we study the Economic-Statistical Design of CCC-r chart when inspection is imperfect. The paper is organized as follows: In section 2, the problem will be formulated. In section 3, the economic model will be denoted. In section 4, the implementation procedure and some comparative studies will be discussed, and sensitivity analysis is conducted. Finally, the paper is concluded in section 5.

#### 2. Inspection Errors and Modification of CCC-r Chart

#### 2-1. Review of *CCC-r* chart

CCC-r chart is based on the analysis of number of inspected items until observing a specific number of defective items. Suppose that r is a specific positive integer number. Given that the probability of defection of each defective item is a fixed value, the number of inspected items, x, until observing r defective items is a random variable that follows a Pascal distribution (also known as a negative binomial distribution). The probability distribution function (p.d.f) and the cumulative distribution function (c.d.f) of the distribution are as follows:

$$\begin{split} f_{r,p}(x) &= \binom{x-1}{r-1}(1-p)^{x-r}p^r; \ for \ x = r, r + \\ 1, \dots & (1) \\ F_{r,p}(n) &= \sum_{i=r}^x \binom{i-1}{r-1} \ (1-p)^{i-r}p^r; \ for \ x = \\ r, r+1, \dots & (2) \end{split}$$

Given that the probability of type I error is  $\alpha$ , the lower control limit, LCL, for the CCC-r chart is computed using the equation below:

$$F(LCL, r, p) = \sum_{i=r}^{LCL} {i-1 \choose r-1} p^r (1-p)^{i-r} = \alpha$$
(3)

Average run length (ARL) and average number of inspected items (ANI) are two important criteria to analyze the performance of control charts. While ARL is defined based on the average number of inspected samples until releasing a signal from the control chart, ANI is based on the average number of inspected items until the control chart issues a signal. To analyze the performance of CCC-r charts, ANI is a better criterion in comparison with ARL. In a *CCC-r* chart, ANI can be approximately computed as follows:

$$ANI = \frac{r}{p}ARL = \frac{r}{p} * \frac{1}{1 - p(LCL \le x)}$$
(4)

#### 2-2. Inspection errors

As stated by Xie *et al.*[29], due to the existance of errors in items inspection, the observed value of defective propoation may be different from the actual value of defective propotion. There may be two types of errors in item inspection: (1) classification of a defective item as conforming; (2) classification of a conforming item as defective. Let's define p' and p as the true and observed values of defective propoation, respectivley. Also, let  $e_1$  and  $e_2$  denote the probablity of classification of a conforming item as a defective item and the probability of classification of defective item as a confroming item, respectively. Given these definitions, the following equation holds true:

$$\dot{p} = p(1 - e_2) + (1 - p)e_1 \tag{5}$$

Burke et al. [12] proposed the following procedures to estimate the values of  $e_1$  and  $e_2$  errors:

Step1. Create k lots, while each lot contains N items.

Step2. For each lot, according to the experts' opinion, the conforming and nonconforming items are distinguished.

step3. The inspectors inspect a random assignment of lots, so that each lot is inspected m times.

Now, the following values are defined:

 $\bar{x}_j$ : the average value of type I error for the j<sub>th</sub> lot  $\bar{y}_j$ : the average value of type II error for the j<sub>th</sub> lot

Then, using these values, we have the following equations:

$$\hat{e}_1 = \frac{1}{k} \sum_{j=1}^k \bar{x}_j \tag{6}$$

$$\hat{e}_2 = \frac{1}{k} \sum_{j=1}^k y$$
 (7)

the adjusted acceptable risk of false  $alarm(\alpha^*)$  in the presence of inspection errors can be computed as follows[13]:

$$\alpha^* = \frac{\alpha \, p_0}{p_0'} \tag{8}$$

Then, the adjusted lower control limits in the presence of inspection errors can be obtained as follows:

$$\sum_{i=r}^{LCL_a} {i-1 \choose r-1} p'_0{}^r (1-p'_0)^{i-r} = \alpha^*$$
(9)

The presence of inspection errors leads to a change in the values of type I error, ARL and ANI. The adjusted value of ARL in the in-control state can be obtained as follows:

$$ARL_a = \frac{1}{\alpha^*_{actual}} = \frac{1}{1 - p(LCL_a \le x)}$$
(10)

Also, the adjusted ANI is computed as follows:  $ANI_{a_{\ell}}\dot{p}, e_1, e_2) = \frac{r}{\dot{p}} * \frac{1}{1 - p(LCL_a \le x)}$  (11)

#### 3. Economic Model Introduction

The proposed economic model is developed based on the model of Chan *et al.*[25]. To minimize the average cost per produced item, the value of r is considered as a decision variable. In the following subsection, the other assumptions of the model are explained.

#### **3-1.** Assumptions

(1) One or more assignable causes may affect the process so that the defective proportion changes from  $p_0$  to  $p_1(p_1 > p_0)$ . Before producing the first item or between producing two successive items, the probability of occurrence of the assignable causes is assumed to be  $\pi$ . The shift does not occur during the period of investigation. Hence, the probability of the occurrence of assignable cause before producing the  $i_{th}$  item follows a geometric distribution as follows:

$$p(i) = \pi (1 - \pi)^{i-1} \tag{12}$$

(2) After production, all items are inspected so that the defective items are reworked with the cost of  $C_{rw}$  per item.

(3) After releasing an out-of-control signal, an investigation is performed which incurs  $C_{inv}$ .

(4) The process is not stopped during the investigation. It is assumed that N items are produced during the investigation period.

(5) If the investigation concludes that the chart signal is a false alarm, then the production continues without any further interruption.

(6) After investigation, if one or more assignable causes are identified,  $C_{rec}$  is incurred as a rectification cost.

Fig. 1 illustrates the evolution of a production cycle. A target defective proportion, p, changes from  $p_0$  to  $p_1$  after the production of  $(i-1)_{th}$ item. Totally, j items are produced until an outof-control alarm is released, including the  $i_{\rm th}$ item. According to Fig.1, the production cycle can be determined. Target defective proportion pchanges from  $p_0$  to  $p_1$  after the production of  $(i-1)_{th}$  item. Including the  $i_{th}$  item, j items are produced altogether until an out-of-control alarm is observed. The small dots "..." denote production of items, the circle "o" denotes the production of a defective item, which does not give an out-of-control alarm, the heavy dots "•" denote the production of a defective item which gives an out-of-control alarm, and the star "\*" denotes the start of an investigation (Chan et al., 2003). Now, we proceed to develop the economic model based on a production cycle.

#### **3-2.** Notations

The parameters of the economic model can be classified into the following three groups:

1. Design parameter (decision variable): r

2. Fixed parameters:  $\pi$ , N,  $C_{rw}$ ,  $C_{inv}$ ,  $C_{rec}$ ,  $C_{l}$ ,  $C_{2}$ ,  $C_3, e_1, e_2$ 

3. Process parameters:  $p_0, p_1, LCL, UCL, \not{p}_0, \not{p}_1$ Notations

r: required number of defective items

 $p_0$ : in-control defective proportion

 $p_1$ : out-of-control defective proportion

 $p'_0$ : observed in control defective proportion in the presence of inspection errors

 $p'_1$ : observed out of control defective proportion in the presence of inspection errors

 $\pi$ : probability of change of defective proportion from  $p_0$  to a larger value as  $p_1$ 

N: the number of items produced during a period of investigation

 $C_{rw}$ : cost incurred to rework a defective item

 $C_{rw}$ : cost incurred to rework a conforming item  $C_{inv}$ : investigation cost

 $C_{rec}$ : process rectification cost

 $C_{l}$ : the cost of one identified defective item

 $C_2$ : the cost of classifying an item as conforming when it is defective

 $C_3$ : the cost of classifying an item as defective when it is conforming

 $e_i$ : the probability of classifying a conforming item as defective

 $e_2$ : the probability of classifying a defective item as conforming

LCL: Lower control limit

The values of fixed parameters are defined in Table 1.

#### **3-3.** Cost equations in the economic model

Since  $p(i) = \pi (1 - \pi)^{i-1}$ , thus  $\sum_{i=1}^{\infty} p(i) = 1$ 1 and  $\sum_{i=1}^{\infty} (i-1) p(i) = \frac{1-\pi}{\pi}$ . The different cost functions are obtained as

follows:

(1) When the control chart signals, cost  $C_I$  is incurred to investigate the process. During the production of the first (i - 1) items, the expected number of the out-of-control signals is equal to:  $\frac{i-1}{ANI_a(p'_0)}$ . However, only the fraction  $\left(\frac{ANI_a(p'_0)}{ANI_a(p'_0)+N}\right)$  of the alarms needs the investigation.

That is because, during the investigation of an out-of-control signal, no other investigations are performed, although some signals in this period may be observed. Thus, during the production of the first (i-1) items, the expected number of out-of-control signals that needs investigation is equal to:  $\frac{i-1}{ANI_a(p'_0)+N}$ . When p jumps from  $p'_0$  to an out-of-control signal, the alarm would appear which needs an investigation with cost  $C_{inv}$ :

$$E[C_{I}] = C_{inv} \left( \sum_{i=1}^{\infty} \frac{(i-1)p(i)}{(ANI_{a}(\dot{p_{0}})+N)} \right) + C_{inv} = C_{inv} \frac{(1-\pi)}{(\pi(ANI_{a}(\dot{p_{0}})+N))} + C_{inv}$$
(13)

(2) The cost incurred to rework the defective items found in the inspection is denoted by  $C_R$ . This cost can be divided into two terms: the first is related to the reworking cost during the period when the defective proportion of the process is  $p'_0$  and the second term is associated with the reworking cost during the period when defective proportion of the process is  $p'_1$ .

Thefollowing equations hold true for the first term of reworking  $\cot(C_{rw} < C'_{rw})$ :

$$C_{R1} = C_{rw} \sum_{i=1}^{\infty} (i-1) p(i) p_0(1-e_2) = C_{rw} \frac{(1-\pi)}{\pi} p_0(1-e_2)$$
(14)  

$$C'_{R1} = C'_{rw} \sum_{i=1}^{\infty} (i-1) p(i)(1-p_0) e_1 = C'_{rw} \frac{(1-\pi)}{\pi} (1-p_0) e_1$$
(15)

In addition, the equation below expresses the second term of the reworking cost:

$$C_{R2} = C_{rw}(ANI(p_1) + N)p_1(1 - e_2)$$
(16)

$$C'_{R2} = C'_{rw}(ANI(p_1) + N)(1 - p_1)e_1$$
 (17)  
Finally, the expected reworking cost is as follows:

$$E[C_R] = C_{R1} + C_{R2} + C'_{R1} + C'_{R2}$$
(18)

(3) cost  $C_{rec}$  is incurred to rectify the process when one or more assignable causes are detected.

(4) The cost associated with the inspection errors in a cycle is denoted by  $C_{IEr}$ . The inspection error costs can be classified into the following terms:

1- The first term is related to the cost of defective items identified when the process is incontrol state. This term is equal to  $\left(\frac{1-\pi}{\pi}\right)p_0(1-e_2)C_1$ .

2- The second term of the inspection error costs is related to the cost of classifying a defective item as conforming when the process is in-control state. This term is equal to:  $\left(\frac{1-\pi}{\pi}\right)p_0(e_2)C_2$ .

3- The third term of the inspection error costs is related to the cost of conforming items classified as a defective item when the process is in control. This term is equal to  $\left(\frac{1-\pi}{\pi}\right)(1-p_1)(e_1)C_3$ .

4- The fourth term of the inspection error costs is related to the cost of detected defective items when the process is out of control. This term is equal to  $((ANI_a(p_1) + N)p_1(1 - e_2)C_1)$ .

5- The fifth term of the inspection error costs is related to the cost of classifying a defective item as conforming when the process is out of control. This term is equal to  $((ANI_a(p_1) + N)p_1(e_2)C_2)$ 

6- The last term of the inspection error costs is related to the cost of conforming items classified as defective items when the process is out of control. This term is equal to:  $((ANI_a(p_1) + N)(1 - p_1)(e_1)C_3)$ 

Thus, the following equation holds true for the inspection error costs in a cycle:

$$\begin{split} C_{IEr} &= \left(\frac{1-\pi}{\pi}\right) p_0 (1-e_2) C_1 + \left(\frac{1-\pi}{\pi}\right) p_0(e_2) C_2 + \\ &\left(\frac{1-\pi}{\pi}\right) (1-p_1) (e_1) C_3 + ((ANI_a(\not{p}_1) + N)p_1(1-e_2) C_1 + ((ANI_a(\not{p}_1) + N)p_1(e_2) C_2 + \\ &\left((ANI_a(\not{p}_1) + N)(1-p_1) (e_1) C_3 \right) \end{split}$$

#### **3-4.** Objective function

The objective function in designing the one-sided CCC-r chart is the expected cost per producing one item. Thus, the objective function is expressed as the expected total cost in a production cycle divided into the expected number of the items produced in a production cycle. Expected cost in a production cycle is equal to  $[C_I] + E[C_R] + C_{rec} + C_{IEr}$ . In addition, the expected number of items produced in a produced in a production cycle is equal to:

$$E[(i-1) + j + N] = E(i-1) + ANI_a(\not p_1) + N = \frac{1-\pi}{\pi} + ANI_a(\not p_1) + N$$
(20)

Finally, the objective function is as follows:

$$Min C_{avg} = \frac{C_{inv} \left( 1 + \frac{1 - \pi}{\pi} \left( \frac{1}{ANI_a(\dot{p}_0) + N} \right) \right) + C_{rw} \left( \frac{1 - \pi}{\pi} (\dot{p}_0) + \dot{p}_1 (ANI_a(\dot{p}_1) + N) \right) + C_{rec} + C_{IEr}}{\frac{1 - \pi}{\pi} + ANI_a(\dot{p}_1) + N}$$
(21)

In the objective function, the only decision variable is r. By defining a search interval for the decision variable of the model, r, the optimal solution is determined so that the objective function is minimized.

#### 4. **Performance Study of the Model**

To analyze the performance of the proposed model and select the optimal value of r parameter in the *CCC-r* chart, considering the inspection errors and reworking of defective items, we simulate the model in an example. The constant values in the model are specified in Table 1. Accordingly, taking into account r parameter value in the range of integers between 2 and 7 and the values of  $p_0$  and  $p_1$ , the low control limit is determined. In order to choose *r* parameter by considering other important aspects of the control chart, the two criteria of modified producer and consumer risks are calculated with regard to the inspection errors as in the following equations. These amounts are given in Table 2.

Producer Risk = 
$$\frac{1}{ANIp'_0}$$
  
Consumer Risk =  $1 - \frac{1}{ANIp'_1}$  (22)

	Tab. 1. Fixed parameters in the example with $p_0 = 0.01$ and $p_1 = 0.03$							
		ters						
	$e_1 e_2$	$C_{inv}$ $C_{rw}$ $C'_{rw}$	$C_{rec}$ $\pi$ N					
	0.001 0.005	0.5 50 70	400 0.2 10					
	1ab. 2. The o	btained values of the de	termined criteria					
r	Producer Risk	Consumer Risk	Objective Function					
2	0.005452455	0.98495215	5.022979761					
3	0.003637496	0.990143887	4.965436122					
4	0.0027285	0.992849392	4.933389838					
5	0.002182551	0.994589617	4.912012279					
6	0.001818862	0.995789354	4.896896724					
7	0.001559058	0.996690089	4.885350636					

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According to Table 2, the optimal r value based on the objective function is a time when it is at the lowest value. However, the minimum amount of the objective function is obtained when the modified consumer risk takes a high amount. Therefore, multi-criteria decision-making methods can be used to select optimal r. Among the multi-criteria decision-making methods, one can mention AHP and TOPSIS. In this article, both methods will be used.

### 4-1. Choosing *r* parameter based on AHP method

In the AHP method, to determine the importance level of each of the three specified criteria, paired comparisons based on expert judgments are used. The paired comparisons and importance of each criterion in this problem are presented in Table 3. The paired comparisons between criteria and showing compatibility and incompatibility of decision are the most important advantages of this method. Pair comparisons are used to determine the importance of each alternative as compared with other alternatives. For example, the scale used to compare consumer risks in different scenarios is shown in the following table.

rubi et i un eu computibilit et e effectua						
	Objective Function	Modified Producer Risk	Modified Consumer Risk	Weight		
Objective Function	1	4	2	0.5643		
Modified Producer Risk	0.25	1	0.4	0.1312		
Modified Consumer Risk	0.5	2.5	1	0.3043		

Tab. 3. Paired comparisons of 3 crit
--------------------------------------

Importance	Definition	Explanation	
9	i is extremely preferred to j	$\frac{Pr_i}{Pr_j} < 0.9$	
7	i is very strongly preferred to j	$0.7 \le \frac{Pr_i}{Pr_j} < 0.8$	
5	i is strongly preferred to j	$0.6 \le \frac{Pr_i}{Pr_j} < 0.7$	
3	i is moderately preferred to j	$0.5 \le \frac{Pr_i}{Pr_j} < 0.6$	
2	i is preferred to j	$0.4 \le \frac{Pr_i}{Pr_j} < 0.5$	
1	i is equally preferred to j	$0.4 \ge \frac{Pr_i}{Pr_i}$	

Finally, the final weight of each alternative is observed. As shown in Table 5, the highest weight represents the optimal value of r parameter, which is 5 in the solved example.

Tab. 5. AHP	weight for	each alternative

r	Weight	
2	0.05	
3	0.10	
4	0.13	
5	0.32	
6	0.19	
7	0.29	

## **4-2.** Choosing *r* parameter based on the TOPSIS method

One of the advantages of the TOPSIS method is that the optimal choice of the parameter is done by considering both the positive and negative criteria, as well as the ability to observe the impact of importance level of each criteria on alternatives ranking. The final weights, which are determined in Table 3, are used as weights of each criterion in this method, too. According to the nature of each criterion, the objective function and consumer risk are considered as negative criterion, and the producer risk is selected as positive criterion. The final result of choosing rparameter according to the TOPSIS method is given in Table 6.

Tab. 6. TOPSIS	closeness	index for each		
alternative				

alternative					
r Closeness Index					
2	0.9107				
3	0.5327				
4	0.3048				
5	0.1753				
6	0.1071				
7	0.0893				

As shown in Table 6, the optimal value for r parameter according to the closeness index will be equal to two. Consequently, the obtained value for r parameter by the two methods is different. Although this difference is due to the nature of these methods, changes in the values of objective function are not considerable. According to the importance of consumer risk and high value of these criteria in all ranges of the examined parameter and the amounts of  $p_0$  and  $p_1$  in this example, which are not so small, the TOPSIS method shows results that are more suitable.

## 4-3. Sensitivity analysis of nonconforming fractions

To verify the performance of the model in different scenarios, some different nonconforming fractions are studied. According to the previous section and described methods, the observed results are shown in Table 7. As noted in the equations, changing in non-conforming fractions leads to a change on the producer and consumer risks. To compare different modes, the above example is also presented in this Table.

	$p_0 = 0.0$	01	$p_0 = 0.$	.005	$p_0 = 0.$	.005	$p_0 = 0$	0.01
	$p_l = 0.015$		$p_l = 0.015$		$p_l = 0.03$		$p_l = 0.03$	
r	AHP final weight	TOPSIS index						
2	0.05	0.27	0.072	0.33	0.05	0.49	0.05	0.9107
3	0.13	0.33	0.12	0.23	0.11	0.64	0.1	0.5327
4	0.15	0.51	0.16	0.47	0.14	0.31	0.13	0.3048
5	0.26	0.41	0.21	0.36	0.31	0.16	0.32	0.1753
6	0.21	0.22	0.19	0.27	0.19	0.06	0.19	0.1071
7	0.28	0.15	0.27	0.12	0.29	0.05	0.29	0.0893

 Tab. 7. Performance study on different non-conforming fractions

According to Table 7 and the experiments carried out with the decreasing of  $p_0$  and  $p_1$  values, resulting in high-quality processes, the outcome of each decision-making method is almost the same. In the small non-conforming fraction, the control charts will have a better performance with the higher value of *r* parameter. This problem has been identified in the solved examples.

#### 5. Conclusion

In this paper, a model was proposed to determine the optimal value of r parameter in the CCC-r control charts. The three criteria were simultaneously taken into account in developing the model, including the objective function, modified producer risk, and modified consumer risk. Minimization of the average cost per produced item was considered as the objective function. Since multiple criteria are considered in the proposed model, it is necessary to apply the method of multi-criteria decision making to determine the value of r. Specifically, two approaches were employed in the paper: including AHP and TOPSIS. Finally, the sensitivity analysis and comparison study between the performances of these two approaches are conducted.

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