A Vibration Damping Optimization Algorithm for Solving the Single-item Capacitated Lot-sizing Problem with Fuzzy Parameters

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ABSTRACT

In this paper, we propose a vibration damping optimization algorithm to solve a fuzzy mathematical model for the single-item capacitated lot-sizing problem. At first, a fuzzy mathematical model for the single-item capacitated lot-sizing problem is presented. The possibility approach is chosen to convert the fuzzy mathematical model to crisp mathematical model. The obtained crisp model is in the form of mixed integer linear programming (MILP) which can be solved by the existing solver in crisp environment to find the optimal solution. Due to the complexity and NP-hardness of the problem, a vibration damping optimization (VDO) is used to solve the model for large-scale problems. To verify the performance of the proposed algorithm, we computationally compared the results obtained by the VDO algorithm with the results of the branch-and-bound method and two other well-known meta-heuristic algorithms namely simulated annealing (SA) and genetic algorithm (GA). Additionally, Taguchi method is used to calibrate the parameters of the meta-heuristic algorithms. Computational results on a set of randomly generated instances show that the VDO algorithm compared with the other algorithms can obtain appropriate solutions.

1.   Introduction

Lot-sizing problem (LSP) is a crucial step and well-known optimization problem in production planning which involved time-varying demand for a set of *N* items over *T* periods. It is a class of production planning problems in which the available amounts of the production plans are always considered as decision variables. Two versions of the lot-sizing problems are capacitated and uncapacitated lot-sizing problems. The uncapacitated single-item problem can be solved efficiently using dynamic programming Aksen, et al addressed a profit maximization version of the well-known Wagner–Whitin model for the deterministic single-item uncapacitated lot-sizing problem with lost sales [1]. They proposed an $O(T^2)$ forward dynamic programming algorithm to solve the problem. Montgomery et al presented several single-echelon, single-item, static demand inventory models for situations in which, during

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the stock out period, a fraction *b* of the demand is backordered and the remaining fraction 1 - *b* is lost forever. On the other hand, backlogging, safety stocks and limited outsourcing are three complicated constraints to reach desired solutions in lot-sizing problem. Also, both deterministic and stochastic demands are considered. They considered only part of the fixed costs, which associated with decision making in improving programming in order to develop a model which can cover periods beyond the planning horizon and can be applicable to the wide range of decision making models. He also proved that if the periods of the planning horizon are not fixed, dynamic optimization methods will not function non-optimally [2]. Abad considered the problem of determining the optimal price and a lot-size for a reseller. He assumed that demand can be backlogged and the selling price is constant within the inventory cycle [3]. Karimi, et al considered a single level lot-sizing problems, their types and solution approaches [4]. Berretta and Rodrigues developed methods based on evolutionary metaheuristic to solve a complex problem in production planning, the multi-stage lot-sizing problem with capacity constraints [5].

Ahmadi et al investigates integrated production-inventory models with backorder. A single supplier and a single buyer were considered and shortage as backorder was allowed for the buyer. The proposed models determine optimal order quantity, optimal backorder quantity and optimal number of deliveries on the joint total cost for both buyer and supplier. Two cases were discussed: single-setup-single-delivery (SSSD) case and single-setup-multiple-deliveries (SSMD) case. Two algorithms were applied for optimizing SSMD case: Gradient search and particle swarm optimization (PSO) algorithms [6].

Robinson et al updated at 1988 review of the coordinated lot-sizing problem and complemented reviews on the single-item lot-sizing problem and the capacitated lot-sizing problem [7]. Akbalik and Penz studied a special case of the single-item capacitated lot-sizing problem, where alternative machines are used for the production of a single-item. They proposed an exact pseudo-polynomial dynamic programming algorithm which made it NP-hard in the ordinary sense. They also proposed three mixed integer linear programming (MILP) formulations [8]. Wang, et al addressed the single-item, dynamic lot-sizing problem for systems with remanufacturing and outsourcing. They proposed

a dynamic programming approach to derive the optimal solution in the case of large amounts of the returned product [9]. Kovalyov et al proposed a straightforward $O(nLogn)$ time algorithm for the single-item capacitated lot-sizing problem with linear costs and without backlogging [10]. Tang provides a brief presentation of the simulated annealing techniques and their application in lot-sizing problems [11].

Tarokh et al developed an integrated JIT lot-splitting model for a single supplier and a single buyer for only one product. In the model they analyzed the effect of setup time reduction in the integrated lot splitting strategy. Two cases, Single Delivery (SD) case, and Multiple Delivery (MD) case were investigated before and after setup time reduction. The Gradient Search (GS) and Particle Swarm Optimization (PSO) were used in the proposed model to determine the optimal order quantity of decision variables [12].

Mehdizadeh et al proposed a mixed integer programming model for single-item capacitated lot-sizing problem with setup times, safety stock, demand shortages, outsourcing and inventory capacity in crisp environment. Due to the complexity of the problem, three meta-heuristics algorithms named simulated annealing (SA), vibration damping optimization (VDO) and harmony search (HS) have been used to solve the model [13].

Rodado et al presented a mathematical model for the product mixing and lot-sizing problem by considering stochastic demand. They developed a general model for the product-mix, and lot size decision within a stochastic demand environment, by introducing the Economic Value Added (EVA) as the objective function of a product portfolio selection. The proposed stochastic model has been solved by using a Sample Average Approximation (SAA) scheme [14]. Boonmee and Sethanan presented a computational tool for the multi-level capacitated lot-sizing and scheduling problem in hen egg production planning with the aim of minimizing the total cost. A mixed-integer programming model was developed to solve small-size problems. For large-size problems, particle swarm optimization (PSO) was firstly applied. However, the component of traditional PSO for social learning behavior includes only personal and global best positions. Therefore, a variant of PSO such as the particle swarm optimization with combined gbest, lbest and nbest social structures

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(GLNPSO) which considers multiple social learning terms was proposed [15].

Kargari et al combined Genetic algorithm and Lagrange multiplier to solve lot size determination problems in a complex multi-stage production planning problem with production capacity constraint. The problems have multiple products with sequential production processes which are manufactured in different periods to meet customer's demand. By determining the decision variables, machinery production capacity and the customer's demand, an integer linear program with the objective function of minimization of total costs of setup, inventory and production is achieved [16].

1-1. Fuzzy lot-sizing problem

Fuzzy set theory was first proposed by Zadeh [17]. It is a mathematical tool to describe the imprecision in the fuzzy environment. Imprecision refers to the sense of vagueness rather than the lack of knowledge about the value of parameters. The vagueness is due to the unique experiences and judgments of decision makers. Fuzzy mathematical programming or fuzzy optimization, which was proposed by Zimmermann, is one application of fuzzy set theory. There has been a lot of research which deals with vagueness in the lot-sizing models as one of the fuzzy mathematical programming models [18]. For example, Yao and Lee investigated a group of computing schemas for the economic order quantity as fuzzy values of the inventory with/without backorder [19]. Pai applied the fuzzy set theory to solve the capacitated lot-size problem and by using numerical examples [20]. Guillaume et al investigated lot-sizing problem with fuzzy demands [21]. Mandala et al investigated multi-item multi-objective inventory model with shortages and demand dependent unit cost has been formulated along with storage space, number of orders and production cost restrictions. They imposed the cost parameters, the objective functions and constraints are in fuzzy environment. They used geometric programming method to solve the model [22]. Chang et al presented a fuzzy extension of the economic lot-size scheduling problem (ELSP) for fuzzy demands. They used a genetic algorithm to solve the problem [23]. Chen and Chang studied Fuzzy Economic Production Quantity (FEPQ) model with defective productions which cannot be repaired, fuzzy opportunity cost. They used function principle as arithmetical operations of

fuzzy total inventory cost, and used the graded mean integration representation method to defuzzify the fuzzy total production and inventory cost [24]. Halim et al considered a single-unit unreliable production system which produces a single-item. They developed two production planning models on the basis of fuzzy and stochastic demand patterns and defuzzified by using the graded mean integration representation method [25]. Ketsarapong et al proposed a single-item lot-sizing problem with fuzzy parameters, which is called the fuzzy single-item lot-sizing problem. They used the possibility approach to convert the fuzzy model to the equivalent crisp single-item lot-sizing problem (EC-SILSP) [26]. Sahebjamnia et al developed a fuzzy stochastic multi-objective linear programming model for a multi-level, capacitated lot-sizing problem (ML-CLSP) in a mixed assembly shop. The proposed model aims to minimize the total cost consisting of total variable production cost, inventory cost, backorder cost, and setup cost while maximizing the resource utilization rate simultaneously. To cope with inherent mixed fuzzy stochastic uncertainty associated with the input data, e.g., the demand and process-related parameters, they were treated as fuzzy stochastic parameters. To validate the expediency of the proposed ML-CLSP and solution method, a real case study was executed in a furniture company [27].

In this paper, a vibration damping optimization algorithm to solve a fuzzy mathematical model for the single-item capacitated lot-sizing problem is proposed. A fuzzy mathematical model is presented for the single-item capacitated lot-sizing problem. The possibility approach is chosen to convert the fuzzy mathematical model to crisp mathematical model. To verify the performance of the proposed algorithm, we computationally compared the results obtained by the VDO algorithm with the results of the branch-and-bound method and two other well-known meta-heuristic algorithms, namely simulated annealing (SA) and genetic algorithm (GA) and Taguchi method is used to calibrate the parameters of the meta-heuristic algorithms.

The remaining of this paper is organized as follows: Section 2 describes the single-item capacitated lot-sizing problem with fuzzy parameters and equivalent crisp of the single-item capacitated lot-sizing problem. The solution approach to solve the proposed model introduced in Section 3. The Taguchi method for tuning the parameters and computational experiments is

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presented in Section 4. The conclusions and suggestions for future studies are included in Section 5.

2. Problem Formulation

The single-item capacitated lot-sizing problem with backlogging, safety stocks, limited outsourcing and fuzzy parameters is a production planning problem in which there is a time-varying demand for an item over *T* periods. In this section, we present a formulation of the problem. First, the problem assumptions, parameters, and decision variables have thoroughly been introduced and then the proposed model has been defined.

2-1. Assumptions

Before presenting the formulation, the following assumptions are made on the problem:

- The shortage is backlogged.
- The shortage, inventory, safety stock deficit costs, variable cost, setup cost, out-sourcing cost and demand are non-deterministic.
- Shortage and inventory costs must be taken into consideration at the end.
- Raw material resource with given capacities are considered.
- The quantity of inventory and shortage at the beginning of the planning horizon are zero.
- The quantity of inventory and shortage at the end of the planning horizon are zero.

2-2. Parameters

T: Number of periods in the planning horizon, t=1, …, T

J: Number of production manner, j=1, …, J

$\tilde{C}_{jt}$: Production cost of each unit in period t through manner j

$\tilde{A}_{jt}$: Setup cost of the production in period t through manner j

$\tilde{h}_t^+$: Unit holding cost in period t

$\tilde{h}_t^-$: Unitary safety stock deficit cost in period t

$\tilde{d}_t$: Demand in period t

$\tilde{L}_t$: Quantity of the safety stock of product in the period t

$\tilde{\partial}_t$: Unitary shortage cost in period t

B_{kt}: Capacity of the K source at hand in period t

α_k : Quantity of the K source used by each unit of product

f_{jk}: Quantity of wasted K source produced through manner j

$\tilde{\gamma}_t$: Unit out-sourcing cost at period t

v : Space need for per unit

φ_t : Total available space in period t

2-3. Decision Variables

X_{it}: Production quantity in the period t through the manner j

y_{it}: Binary variable; 1 if the produced in the period t through the manner j, otherwise $y_{jt} = 0$.

U_t: Out-sourcing level at period t

I_t^- : Quantity of shortage in the period t

S_t^+ : Quantity of overstock deficit in the period t

S_t^- : Quantity of safety stock deficit in the period t

2.4. The proposed Model

$$MinZ = \sum_{t=1}^{T}\left(\sum_{j=1}^{J}(\tilde{C}_{jt} X_{jt} + \tilde{A}_{jt} y_{jt}) + \tilde{\partial}_t I_t^- + \tilde{h}_t^+ S_t^+ + \tilde{h}_t^- S_t^- + \tilde{\gamma}_t U_t \right) \tag{1}$$

Subject to:

$$S_{t-1}^+ - S_{t-1}^- - I_{t-1}^- + I_t^- + \sum_{j=1}^{J} X_{jt} + U_t = S_t^+ - S_t^- + \tilde{d}_t + \tilde{L}_t - \tilde{L}_{t-1} \quad \forall \ t = 1,2,....,T \tag{2}$$

$$S_T^+ = 0 \tag{3}$$

$$I_T^- = 0 \tag{4}$$

$$\sum_{j=1}^{J}\left(\alpha_k X_{jt} + f_{jk} y_{jt}\right) \leq B_{kt} \qquad \forall \qquad k = 1,2,.....,K \ , \ t = 1,2,....,T \tag{5}$$

$$X_{jt} \leq M_t y_{jt} \qquad \forall \quad j = 1,2,....,J \ , \quad t = 1,2,....,T \tag{6}$$

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$$I_t^- \leq \tilde{d}_t \qquad \forall \, t = 1, 2, ..., T-1 \qquad (7)$$

$$S_t^- \leq \tilde{L}_t \qquad \forall \, t = 1, 2, ..., T \qquad (8)$$

$$0 \leq U_t \leq I_{t-1}^- + S_{t-1}^- + \tilde{d}_t + \tilde{L}_t \qquad \forall \, t = 1, 2, ..., T \qquad (9)$$

$$v \, (\sum_{j=1}^{J} X_{jt} + U_t) \leq \varphi_t \qquad \forall \, t = 1, 2, ..., T \qquad (10)$$

$$y_{jt} \in \{0,1\} \qquad \forall \, j = 1, 2, ..., J \;, \; t = 1, 2, ..., T \qquad (11)$$

$$X_{jt}, I_t^-, S_t^-, S_t^+ \geq 0 \qquad \forall \, j = 1, 2, ..., J \;, \; t = 1, 2, ... T \qquad (12)$$

The objective function (1) minimizes the total cost considered by the production plans which include unit production costs with different production manner, inventory costs, shortage costs, setup costs and outsourcing. Constraints (2) are the inventory flow conservation equations through the planning horizon. Constraints 3 and 4 define respectively the demand shortage and the safety stock deficit for items at the end period are zero. Constraint 5 is the capacity constraints; the overall consumption must remain lower than or equal to the available capacity. If we produce an item at period t, then constraint (6) impose that the produced quantity must not exceed a maximum production level M_t. Constraints (7) and (8) define respectively upper bounds on the demand shortage and the safety stock deficit for an item in period t. Constraint (9) ensure that outsourcing level U_t at period t is non-negative and cannot exceed the sum of the demand, safety stock of the period t and the quantity backlogged, safety stock deficit from previous periods. Constraint (10) is the maximum space available for storage of the items in excess. Constraints (11) and (12) characterize y_{jt} is a binary variable and the variable's domains: $X_{jt}, I_t^-, S_t^-, S_t^+$ are non-negative for j ∈ J and t ∈ T.

2-5. Equivalent crisp single-item capacitated lot-sizing problem

The possibility approach in the context of fuzzy set theory was introduced by Zadeh to deal with non-stochastic imprecision and vagueness [35]. According to the Dubois and Prade and Dubois, the possibility approach appropriately was used to model various kinds of information, such as linguistic information and uncertain formulate, in logical settings. In this section, the possibility approach will be used to convert the fuzzy model to the equivalent crisp model [28, 29].

In this paper, Chance-Constrained Programming (CCP) by Charnes and Cooper, which is normally used to confront stochastic linear programming)SLP(, is adopted as a way to convert the fuzzy single-item capacitated lot-sizing problem to the Equivalent crisp single-item capacitated lot-sizing problem. The concept of CCP guarantees that the probability of stochastic constraints is greater than or equal to a pre-specified minimum probability [30]. Lertworasirikul et al proved and proposed a Lemma which the single-item capacitated lot-sizing problem with fuzzy parameters is transformed into the equivalent crisp single-item capacitated lot-sizing problem by the mentioned Lemma as follows [31]

$$Min Z = \sum_{t=1}^{T} \left(\sum_{j=1}^{J} \left((\alpha(\tilde{C}_{jt})_1^L + (1-\alpha)(\tilde{C}_{jt})_0^L) X_{jt} + (\alpha(\tilde{A}_{jt})_1^L + (1-\alpha)(\tilde{A}_{jt})_0^L) Y_{jt} \right) + \right.$$

$$(\alpha(\tilde{\delta}_t)_1^L + (1-\alpha)(\tilde{\delta}_t)_0^L) I_t^- + (\alpha(\tilde{h}_t^+)_1^L + (1-\alpha)(\tilde{h}_t^+)_0^L) S_t^+ + (\alpha(\tilde{h}_t^-)_1^L + (1-\alpha)(\tilde{h}_t^-)_0^L) S_t^-$$

$$+ (\alpha(\tilde{\gamma}_t)_1^L + (1-\alpha)(\tilde{\gamma}_t)_0^L) S_t^+ \;) \qquad (13)$$

Subject to:

$$S_{t-1}^{+} - S_{t-1}^{-} - I_{t-1}^{-} + I_{t}^{-} + \sum_{j=1}^{J} X_{jt} + U_{t} \geq S_{t}^{+} - S_{t}^{-} + \left(\alpha(\tilde{d_t})_1^{L} + (1-\alpha)(\tilde{d_t})_0^{L} \right)$$
$$+ \left(\alpha(\tilde{L_t})_1^{L} + (1-\alpha)(\tilde{L_t})_0^{L} \right) - \left(\alpha(\tilde{L}_{t-1})_1^{L} + (1-\alpha)(\tilde{L}_{t-1})_0^{L} \right) \qquad \forall \ t = 1,2,....,T \quad (14)$$

$$S_{t-1}^{+} - S_{t-1}^{-} - I_{t-1}^{-} + I_{t}^{-} + \sum_{j=1}^{J} X_{jt} + U_{t} \leq S_{t}^{+} - S_{t}^{-} + \left(\alpha(\tilde{d_t})_1^{U} + (1-\alpha)(\tilde{d_t})_0^{U} \right)$$
$$+ \left(\alpha(\tilde{L_t})_1^{U} + (1-\alpha)(\tilde{L_t})_0^{U} \right) - \left(\alpha(\tilde{L}_{t-1})_1^{U} + (1-\alpha)(\tilde{L}_{t-1})_0^{U} \right) \qquad \forall \ t = 1,2,....,T \quad (15)$$

$$S_{T}^{+} = 0 \qquad (16)$$

$$I_{T}^{-} = 0 \qquad (17)$$

$$\sum_{j=1}^{J} \left(\alpha_k X_{jt} + f_{jk} y_{jt} \right) \leq B_{kt} \qquad \forall \ k = 1,2,.....,K \ , \ t = 1,2,....,T \qquad (18)$$

$$X_{jt} \leq M_t y_{jt} \qquad \forall \ j = 1,2,....,J \ , \ t = 1,2,....,T \qquad (19)$$

$$I_{t}^{-} \leq \left(\alpha(\tilde{d_t})_1^{U} + (1-\alpha)(\tilde{d_t})_0^{U} \right) \qquad \forall \ t = 1,2,....,T-1 \qquad (20)$$

$$S_{t}^{-} \leq \left(\alpha(\tilde{L_t})_1^{U} + (1-\alpha)(\tilde{L_t})_0^{U} \right) \qquad \forall \ t = 1,2,....,T \qquad (21)$$

$$0 \leq U_t \leq I_{t-1}^{-} + S_{t-1}^{-} + \left(\alpha(\tilde{d_t})_1^{U} + (1-\alpha)(\tilde{d_t})_0^{U} \right) + \left(\alpha(\tilde{L_t})_1^{U} + (1-\alpha)(\tilde{L_t})_0^{U} \right) \ \forall \ t = 1,2,....,T \qquad (22)$$

$$v \left(\sum_{j=1}^{J} X_{jt} + U_t \right) \leq \varphi_t \qquad \forall \ t = 1,2,....,T \qquad (23)$$

$$y_{jt} \in \{0,1\} \qquad \forall \ j = 1,2,....,J \ , \ t = 1,2,....,T \qquad (24)$$

$$X_{jt}, I_{t}^{-}, S_{t}^{-}, S_{t}^{+} \geq 0 \qquad \forall \ j = 1,2,....,J \ , \ t = 1,2,....T \qquad (25)$$

The objective function (13) minimizes the total cost considered by the production plans which include unit production costs with different production manner, inventory costs, shortage costs, setup costs and outsourcing. Constraints (14) and (15) are the inventory flow conservation equations through the planning horizon. Constraints (16) and (17) define respectively, that the demand shortage and the safety stock deficit for item at end period is zero. Constraint (18) is the capacity constraints; the overall consumption must remain lower than or equal to the available capacity. If we produce an item at period t, then constraint (19) impose that the produced quantity must not exceed a maximum production level M_t. M_t could be set to the minimum between the total demand requirement for item on section [1, T] on the horizon and the highest quantity of items that could be produced regarding the capacity constraints, then M_t can be shown as Equation (26).

$$M_t = Min \left(\frac{B_{kt} - f_{jk}}{\alpha_k} , \alpha \sum_{t=1}^{T} (\tilde{d_t})_1^{U} + (1-\alpha) \sum_{t=1}^{T} (\tilde{d_t})_0^{U} \right)$$
$$(26)$$

Constraints (20) and (21) define respectively upper bounds on the demand shortage and the safety stock deficit for an item in period t. Constraints (22) ensure that outsourcing level U_t at period t is nonnegative and cannot exceed the sum of the demand, safety stock of period t and the quantity backlogged, safety stock deficit from previous periods. Constraint 23 is the maximum space available for storage of items in excess. Constraints (24) and (25) characterize y_{it} as a binary variable and the variable's domains: $X_{jt}, I_{t}^{-}, S_{t}^{-}, S_{t}^{+}$ are non-negative for j ∈ J and t ∈ T.

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3. Vibration Damping Optimization Algorithm

Recently, a new heuristic optimization technique based on the concept of the vibration damping on mechanical vibration was introduced by Mehdizadeh and Tavakkoli-Moghaddam named vibration damping optimization (VDO) algorithm. At first, they utilized this algorithm to solve parallel machine scheduling problem [40] and after that, this algorithm was applied in various fields [32-35].

The VDO algorithm is illustrated in the following steps:

Step 1: Generating feasible initial solution.

Step 2: Initializing the algorithm parameters which consist of: initial amplitude (A_0), maximum of iteration at each amplitude (l_{max}), damping coefficient (γ), and standard deviation (σ). Finally, parameter t is set in one (t=1)

Step 3: Calculating the objective value U_0 for initial solution.

Step 4: Initializing the internal loop

In this step, the internal loop is carried out for l =1 and repeat while $l < l_{max}$.

Step 5: Neighborhood generation.

In this paper, we use mutation.

Step 6: Accepting the new solution

Set $\Delta = U - U_0$ Now, if, $\Delta < 0$, accept the new solution, else if $\Delta > 0$ generate a random number r between (0, 1);

If $r < 1 - \exp\left(\dfrac{-A_t^2}{2\sigma^2}\right)$, then accept a new solution;

otherwise, reject the new solution and accept the previous solution.

If $l > l_{max}$, then $t + 1 \rightarrow t$ and go to step 7; otherwise $l + 1 \rightarrow l$ and go back to step 5.

Step 7: Adjusting the amplitude

In this step, $A_t = A_0 \exp(\dfrac{-\gamma t}{2})$ is used for reducing the amplitude at each iteration of the outer cycle of the algorithm. If $A_t = 0$ return to step 8; otherwise, go back to step 4.

Step 8: Stopping criteria

In this step, the proposed algorithm will be stopped after a predetermined number of iterations. At the end, the best solution is obtained.

3-1. Representation schema

To design the vibration damping optimization algorithm to solve the mentioned problem, a suitable representation scheme that shows the solution characteristics is needed. In this paper, the general structure of the solution representation performed for running the vibration damping for four periods with two production methods is shown in Fig 1.

Y_{11}	Y_{21}	Y_{12}	Y_{22}	Y_{13}	Y_{23}	Y_{14}	Y_{24}
0	1	1	0	1	0	0	1

Fig.1. Solution representation

4. Results

In this paper, all test problems are conducted on a not book with Intel Core i5 Processor 2.53 GHz and 4 GB of RAM and the proposed algorithms namely SA, VDO and GA are coded in Visual Basic 2000.

4-1. Parameter calibration

Appropriate design of parameters has significant impact on the efficiency of meta-heuristics. In this paper the Taguchi method applied to calibrate the parameters of the proposed methods, namely SA, VDO and GA algorithms. The Taguchi method was developed by Taguchi [36]. This method is based on maximizing performance measures called signal-to-noise ratios in order to find the optimized levels of the effective factors in the experiments. The S/N ratio refers to the mean-square deviation of the objective function that minimizes the mean and variance of quality characteristics to make them closer to the expected values. For the factors that have significant impact on S/N ratio, the highest S/N ratio provides the optimum level of that factor. As mentioned before, the purpose of Taguchi method is to maximize the S/N ratio. In this subsection, the parameters for experimental analysis are determined.

Table 1 lists different levels of the factors for SA, VDO and GA. In this paper according to the levels and the number of the factors, respectively the Taguchi method L_9 is used for the adjustment of the parameters for the SA and L_{27} are used for the VDO and GA.

Figures 2, 3 and 4 show S/N ratios. According to these figures 1500, 60, 0.99, 10, 60, 0.1, 1200, 0.5, 450, 0.3, 0.1, 0.95, 200 are the optimal level of the factors T_0, L, α, A_0, L_{max}, γ, t, σ, Npop, P_m, P_C, Sm and Iteration.

Tab. 1. Factors and their levels

Factor	Algorithm	Notation	Level	Value
Initial temperature		T_0	3	1000, 1500, 2000
Rate cooling	SA	α	3	0.9, 0.95, 0.99
Number of iteration at each temperature		L	3	20, 40, 60
Initial amplitude		A_0	3	6, 8, 10
Max of iteration at each amplitude	VDO	L_{max}	3	20, 40, 60
Damping coefficient		γ	3	0.01, 0. 1, 0.5
external loop		t	3	600, 800, 1200
standard deviation		σ	3	٠.٥, 1.5, 2
Number of population		Npop	3	250, 350, 450
Probability of mutation	GA	P_m	3	0.3, 0.6, 0.9
Probability of crossover		P_C	3	0.05, 0.075, 0.1
Strongly mutation		Sm	3	0.35, 0.65, 0.95
Stop criteria		Iteration	3	100, 200, 300
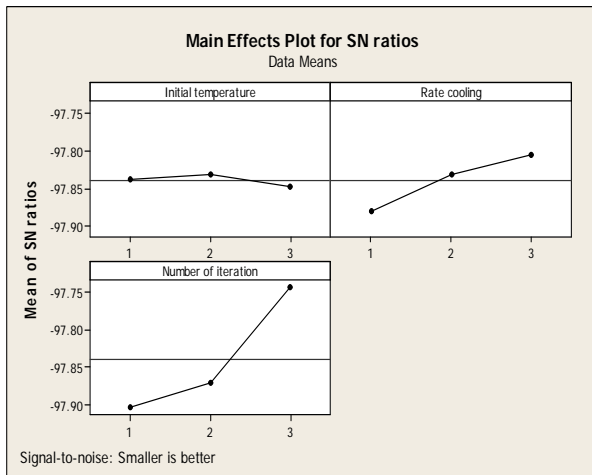
Fig. 2. The SN ratios for Simulated Annealing Algorithm
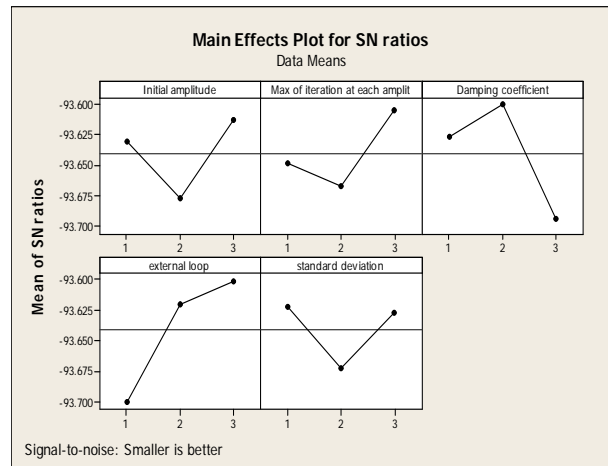
Fig .4. The SN ratios for Genetic Algorithm
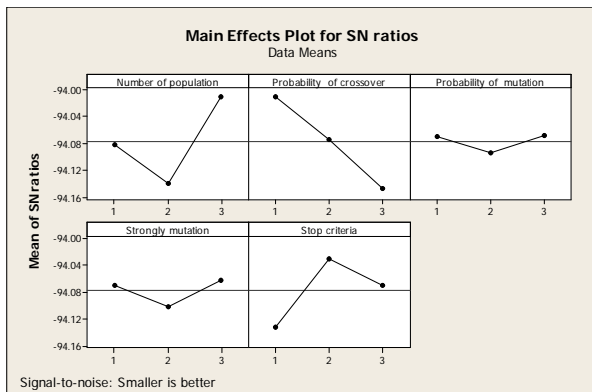
Fig. 3. The SN ratios for Vibration Damping Algorithm

4-2. Computational results

Computational experiments are conducted to validate and verify the behavior and the performance of the meta-heuristic algorithms employed to solve the considered single-item capacitated lot-sizing problem. We try to test the performance of the VDO, SA and GA in finding good quality solutions in reasonable time for the problems. For this purpose, 30 problems with different sizes are generated. For these problems, we used the trapezoidal fuzzy setup cost and unit price which would be classified at three levels cheap, normal and expensive. It is respectively denoted by:

$$\tilde{A_C} = (17000, 18000, 19000, 20000)$$,

$$\tilde{A_N} = (19000, 20000, 21000, 22000)$$,

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$$A_E = (21000, 22000, 23000, 24000) \quad ,$$

$$C_C = (60, 65, 70, 75) \quad , \quad C_N = (70, 75, 80, 85)$$

and $C_E = (70, 75, 80, 85)$.

The trapezoidal fuzzy demand and safety stock are classified at three levels, low, medium, and high. It is respectively denoted by:

$$d_L = (8, 9, 10, 11) \quad , \quad d_M = (10, 11, 12, 13) \quad ,$$

$$d_H = (13, 14, 15, 16) \quad , \quad L_L = (1, 2, 3, 4) \quad ,$$

$$L_M = (3, 4, 5, 6) \text{ and } L_H = (5, 6, 7, 8).$$

The Triangular fuzzy inventory holding cost, shortage cost, safety stock deficit cost and outsourcing cost are classified at three levels cheap, normal and expensive.

$$h_C^+ = (6, 7, 8) \quad , \quad h_N^+ = (7, 8, 9) \quad , h_E^+ = (8, 9, 10),$$

$$\delta_C = (16, 17, 18) \quad , \delta_N = (17, 18, 19) \quad ,$$

$$\delta_E = (18, 19, 20) \quad , \quad h_C^- = (11, 12, 13) \quad ,$$

$$h_N^- = (12, 13, 14) \quad , h_E^- = (13, 14, 15)$$

$$u_C = (30000, 35000, 40000)$$

$$u_N = (35000, 40000, 45000) \quad ,$$

$$u_E = (40000, 45000, 50000) \quad ,$$

These test problems are classified into three classes: small size, medium size and large size. The number of manners and periods has the most impact on the problem hardness. The proposed model coded with Lingo (Ver.8) software using for solving the instances. The best results are recorded as a measure of the related problem. Table 2 shows details of computational results obtained by solution methods for all test problems.

The results of running VDO, SA, and GA are compared with the optimal solution of the instances, obtained from Lingo software, in row 1 to 10 of Table 2. As Table 2 shows, the total cost of VDO algorithm is smaller than those SA and GA algorithms. To prove this matter, the presented statistical analysis (the variance analysis outcome) was reported for problems with small, medium, and large dimensions. In Tables 3, 4, and 5, according to the values of the survey (or *P-Value*) we can get the conclusion that the VDO has a better performance for these sizes of problem sizes. The SA, VDO and GA can find good quality solutions for small size problems. The VDO can find good quality solutions for all size problems. The objective values obtained by SA and VDO are close to each other for small and medium size problems. The VDO has a better performance for problems with all sizes compared to the SA and GA.

Tab. 2. Details of computational results for all test problems.

No	Class	M.P	Lingo		SA		VDO		GA	
			O.V.	T	O.V.	T	O.V.	T	O.V.	T
1		2.2	18981.4	0	18981.4	2	18981.4	6	18981.4	28
2		3.3	19801.6	3	19801.6	2	19801.6	6	19801.6	28
3		4.4	20679	3	20679	2	20679	9	20679	30
4	Small size	4.5	21756.6	2	21756.6	2	21756.6	9	21756.6	30
5		5.6	22986	3	22986	2	22986	11	22986	31
6		6.4	20845.2	4	20870.6	2	20845.2	8	20845.2	28
7		9.5	22095.4	3	22095.4	3	22095.4	13	22095.4	33
8		8.6	23544	3	23773.2	3	23773.2	14	23773.2	33
9		6.7	23265.2	784	24685.2	4	24685.2	14	24685.2	34
10		3.11	41420.6	3600	44104.6	4	43428.4	19	44919.4	34
11		6.11	-----	----	44965	5	44987.2	20	45876.2	38
12	Medium	5.12	-----	----	45953.6	5	45885	18	47045.4	40
13		2.13	-----	----	48489.2	5	46880	29	48094.8	38
14		7.13	-----	----	47485.6	6	46961.6	30	48124.8	38

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15		8.13	-----	----	46390.8	6	46203.8	30	48105.4	38
16		7.14	-----	----	47326.8	6	47012.2	30	48874.8	40
17		8.14	-----	----	46541.8	6	46683.8	30	47758	40
18		7.15	-----	----	47866.6	7	47878.2	33	48559	36
19		8.15	-----	----	52290.4	7	49078.2	33	52832.2	43
20		9.15	-----	----	47485.8	7	47352	34	49086.4	44
21		7.16	-----	----	52127.6	7	49702.4	30	68736.4	37
22		8.16	-----	----	53205.4	7	49615.4	31	55669.8	37
23		9.16	-----	----	52965.2	7	50160.2	32	57410	36
24		7.17	-----	----	55981.2	7	51060.2	32	67986	36
25	Large	8.17	-----	----	68672.6	8	52970.6	33	68721.2	40
26		9.17	-----	----	55041.2	8	51445.2	33	54436.8	36
27		7.18	-----	----	70133.8	8	59688.8	34	71547.4	36
28		9.18	-----	----	68404	8	51168.2	34	71294	38
29		8.20	-----	----	79349.6	9	70978.4	41	75744.8	42
30		9.20	-----	----	78939.4	9	71131.8	41	74985.4	38

Tab. 3. Analysis of variance for small size problem between SA, VDO and GA

Source	DF	SS	MS	F	P
Small size	2	111283	55641	0.00	0.999
Error	27	1441003765	53370510		
Total	29	1441115047			

Tab. 4. Analysis of variance for medium size problem between SA, VDO and GA

Source	DF	SS	MS	F	P
Small size	2	12138626	6069313	2.17	0.133
Error	27	75421354	2793383		
Total	29	87559980			

Tab. 5. Analysis of variance for large size problem between SA, VDO and GA

Source	DF	SS	MS	F	P
Small size	2	623844204	311922102	3.69	0.038
Error	27	2280744285	84472011		
Total	29	2904588489			

5. Conclusion

In this paper, we proposed a mathematical model for the single-item capacitated lot-sizing problem with backlogging, safety stocks, limited outsourcing with different production methods, limited warehouse space and fuzzy parameters. We used possibility approach to convert the fuzzy single-item capacitated lot-sizing model, to an equivalent crisp single-item capacitated lot-sizing model. Due to the complexity of the problem, a vibration damping optimization (VDO) algorithm was used to solve the problem. In table 2 the proposed VDO algorithm was compared with two well known algorithms, namely simulated annealing (SA) and genetic algorithm (GA). Additionally, in table 1 and figures 2-4 an extensive parameter setting by performing the Taguchi method was conducted for selecting the optimal levels of the factors that effect on the algorithm's performance. In table 3, 4 and 5 to verify the performance of the

algorithms, the results obtained by the algorithms computationally compared with the results of branch-and-bound method. The experiment results showed that the vibration damping optimization (VDO) algorithm has a better performance than other two presented algorithms for solving this model, especially for large size problems.

The first opportunity for future research is developing multi-item capacitated lot-sizing problem with backlogging, safety stocks, and outsourcing with different production methods, limited warehouse space and fuzzy parameters. Secondly opportunity is using other more sophisticated lot sizing models, such as the single-item capacitated lot-sizing problem in closed loop supply chain where returned products are collected from customers, etc. Also, developing new heuristic or meta-heuristic algorithms to make better solutions can be suggested.

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