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A bi-level Mathematical Programming for Cell Formation Problem Considering Workers' Interest

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ABSTRACT

Nowadays, the necessity of manufacturers' response to their customers' demands and their spheres of actions has extended widely. Staff creativity is an important factor in product evolution, and their interest in collaborating with each other in the working environment can contribute to the growth and maturity of this component. In this research, two important aspects of cellular manufacturing are taken into consideration: Cell formation and workforce planning. Cell formation is a strategic decision, and workforce planning is a tactical decision. Practically, these two sectors cannot be planned simultaneously, and decision making in this regard is decentralized. For this reason, a bi-level mathematical model is proposed. The first level aims to minimize the number of voids and exceptional elements. The second level tends to promote the sense of interest between the workforces for working together, which will result in synergy and growth of the organization. To solve the proposed bi-level model, it was first changed to a mathematical model with one level using KKT method and then solved by LINGO software. To verify the proposed model, a real case study was studied, and the results were presented. This bi-level method makes it possible to find the balance point of the two objectives.

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1. Introduction

A system is a purposeful combination of staff and managers, facilities, information, and operational methods in the environment. Transforming raw materials, capital, information, and other resources into products or value-added services is the base of a manufacturing system. Today, achieving customer satisfaction and competitive advantage requires the evolution of products and services, or even introducing new items in the manufacturing portfolio of diverse industries and business. Various manufacturing systems are planned due to diverse demands and nature of actions. Since cellular manufacturing systems have adopted reduced cost price from mass-production systems and high flexibility from job-shop manufacturing systems, they are very popular amongst other manufacturing systems. One of the major concerns in cellular manufacturing is the layout of the machines and parts inside the cells. The Cell formation should be done in a way that all processes of manufacturing a part fit in the same cell.

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Consequently, the most important objective could be reducing voids and exceptional elements. Moreover, the role of the workforce in manufacturing systems cannot be denied. Planning the facilities could be centralized or decentralized. In centralized planning and several sizes of facilities, manufacturing policies is designed in an integrated manner, but in decentralized cases, this activity could change depending on the requirements and policies taken for different facilities even in one unit. Decentralized planning has been recently taken into consideration in supply chain and production planning problems. Cell formation has been the subject of various studies for a long time, but workforce planning in this system has attracted attention in recent years. In the few studies conducted in this regard, Cell formation and workforce planning are centralized while Cell formation is a strategic decision and workforce planning is a key decision. Practically, these two sectors cannot be planned centrally, and decision making in this regard is decentralized. For the same reason, this article points to provide a scientific and meanwhile practical solution to cell manufacturing problems in real conditions.

2. Literature Review

Cellular manufacturing is a combination of mass-production and job-shop manufacturing. This method is based on group technology and is proposed to increase efficiency using similar specifications of the fabricated parts. In cellular manufacturing, a group of parts which are of similar manufacturing requirements are considered a family. The system also tends to fit all the skills and equipment required for manufacturing the products of the same family in one cell. Increasing intracellular movement increases manufactory traffic and wasted time, and consequently, imposes excessive costs on the system. Chandrasekharan and Rajagopalan (1989) defined a criterion named "grouping efficiency" which is a function of the number of voids and exceptional elements and has been used as the criterion for evaluating the quality of answers for a long time [1]. Of course, the bigger the part-machine matrixes are, the less the effect of exceptional elements in the quality of answers will be; this is one of the faults of this criterion. For this reason, after a while, Kumar and Chandrasekharan (1990) used a new criterion named "grouping efficacy" to distinguish the quality of results [2].

The cell formation problem is the core of cellular manufacturing. Machines and parts are classified based on the similarity of their design, form, and performance to simplify the manufacturing system and benefit from concealed advantages of similarity. Chen and Chang (1995) used the neural network algorithm to determine the cell formation. They used "adaptive resonance theory" based on the neural network in cellular manufacturing [3]. Cheng et al. (1998) formulated the cell formation problem as the traveling salesman problem and provided a methodology based on genetic algorithms to solve this problem [4]. Onwubolu and Mutingi (2001) provided a genetic algorithm method for determining the cell formation in cellular manufacturing systems [5]. Goncalves and Rezende (2004) created an approach to determining product family and machine cell. This method was a combination of an innovative local algorithm and genetic algorithm [6]. Al-Badwi et al. (2005) proposed a mathematical model for manufacturing cell formation. Their approach consists of two phases. In the first phase, machine cells are determined by the coefficient of proportional similarity matrix factors. In the second phase, a mathematical model is used to allocate part of the cells [7]. Mahdavi et al. (2009) suggested a mathematical model for the cell formation problem based on the concept of using cells in cellular manufacturing systems. This model was proposed to minimize the number of voids and exceptional elements in the cells. They also planned an effective method for solving the mathematical model based on the genetic algorithm [8]. Anvari et al. (2010) developed a particle swarm optimization algorithm to determine part families using the coefficient of proportional similarity, and determine the machine group aiming to minimize voids and exceptional elements [9].

Mahdavi et al. (2010) proposed a mathematical model for the common problem of machine layout and cell formation problem. They minimized the enterprise cost of machines and the total cost of forward and backward movements [10]. Paydar et al. (2011) proposed a model for determining part families and machine groups at the same time in the cell formation problem. This model proposed to minimize exceptional elements and intracellular empty spaces at the same time [11]. An advantage of this model is the number of manufacturing cells as a decision variable. Therefore, the optimum number of cells is determined by the model.

Arcat et al. (2011) proposed a dual-purpose model for minimizing the number of voids and exceptional elements in the cells [12]. They developed a multi-objective genetic algorithm for solving large-scale problems. Elbanani et al. (2012) proposed a local search method for working out the cell formation problems in which each cell contains at least one machine and one part. The proposed method successively uses resonance strategy to optimize the answer locally and also uses destruction strategy for gaining a new resolution from the previous result [13]. They proved the efficiency of this method by solving 35 problems of the subject literature and comparing the result to the best answers achieved by that time using group efficiency method. In his article, Brown (2015) tried to work out a long-term sustainable cell formation in cellular manufacturing systems with exceptional elements by developing mathematical models and cutting the total cost down through minimizing intracellular movements and the number of similar machinery [14]. Sakhaie et al. (2016) used mixed integer programming model to consider the reduction of shortage costs, costs of moving the machinery, and labor costs in a cellular manufacturing system with unreliable machinery to program the production [15]. Deep and Singh (2015) proposed a mathematical model for minimizing manufacturing costs in dynamic cellular manufacturing systems and solved it using a genetic algorithm, considering factors such as the capacity of the machine, multiple routing, production capacity, and accessibility of the raw material [16]. Egilmez et al. (2014) studied the issue of allocating workforce to different cells in their article [17]. They proposed a mathematical model with a random approach for allocating the workforce to different cells based on the capabilities of the workforces. Fattahi and Ismailnezhad (2016) proposed a mathematical method for a stochastic cell formation problem considering the queuing theory and reliability concept. They developed two algorithms for resolving this problem. The algorithms are based on genetic and modified particle swarm optimization algorithms [18]. Bootaki et al. (2016) studied about configuring manufacturing cells when product mix variation occurs. The nature of CMS in manufacturing products in mid-variety and mid-volume, the product mix variation, is not too far-fetched. Product mix variation causes the part-machine incidence matrix to change. They formulated the problem with two different criteria in which one of the two relates to worker experts and another to worker relations [19] and [20].

Multi-level optimization problems (MLOP) have been built up for distributed planning problems in a hierarchical system with many decision-makers. The decisions are made in a sequential manner and without any cooperation. These MLOPs are characterized by a hierarchy of plans; each plan has independently controlled a subset of decision variables, disjoint from the others. A bi-level optimization problem (BOP) is a multi-level problem with two levels, upper-level or leader, and lower-level or follower problems [21].

In this problem, the objectives are reducing the number of voids and exceptional elements and maximizing the workers' interest for working in each cell. Obviously, these two objectives are not in cooperation, and hence, must be addressed in two dissimilar sections. For this reason, it is necessary to use a bi-level solution to solve the problem. Although the first level contains cooperated objectives and can be modeled and optimized through methods such as multi-objective planning, considering what was mentioned above, the final solution to the subjected problem should be done utilizing a bi-level planning model.

3. Decentralized Decision Making

After 1990 and in modern management, decision making is the main function and necessary process for the manager represented in every field (Androniceanu and Ristea, 2014). Decision-making is known as the thought process of making a logical choice out of available choices. The negatives and positives of each choice must be noted and all the alternatives should be observed, when trying to make a good decision [22]. The outcome of each alternative must be forecasted for efficient decision making and ascertain the best choice is to that circumstance based on all these items. Stackelberg game theory improved multi-level decision-making techniques [23]. Decentralized decision making has been modernized to address compromises between the interactive decision entities that are spread throughout a hierarchical organization and presented by multi-level programming models [24]. Multilevel decision problems have newly appeared in decentralized management situations more and more in the real world. Bracken and McGill presented multi-level decision-making in a paper authored in 1973, and since 1980s, a wide scope of relevant research under the following designations has been undertaken: multi-level programming, multi-level optimization, and multi-level decision-making [25].

Bi-level programming problems are hierarchical optimization problems with two levels; each level has its own constraints and objective functions [26]. In this model, decision entities at the first and second levels are respectively named the leader and follower. Leader and follower make their individual decisions with the goal of optimizing their relative objectives in sequence. The leader takes action first, and the follower responds to the leader's decision, implying that the leader has the priority in reaching its own decision and the follower reacts after and in full cognition of the leader's decision [27]; nevertheless, the leader's decision is implicitly affected by the follower's reaction.

The bi-level programming is used frequently by problems with decentralized planning structure. It is specified as follows [28]:

$$
\begin{aligned}
&\min_x\ F(x,y)=c_1x+d_1y\\
&\quad s.t.\\
&\qquad A_1x+B_1y\le b_1\\
&\min_y\ F(x,y)=c_2x+d_2y\\
&\quad s.t.\\
&\qquad A_2x+B_2y\le b_2,\\
&\qquad x,y\ge 0
\end{aligned}
\tag{1}
$$

With $\mathbb{R}$ as the feasible region of the bi-level programming problem, every point, such as (x*, y*), is an optimal solution problem if:

$$F(x^*,y^*)\le F(x,y)\qquad \forall(x,y)\in\mathbb{R} \tag{2}$$

The primary aim of solving the bi-level programming problem is to discover a point of the accessible area where the value of the first level objective function is optimized on the accessible field.

For solving the bi-level programming problems, many algorithms have been presented, categorized into the following groups: global techniques, primal–dual interior methods, enumeration methods, transformation methods, fuzzy methods, and metaheuristic approaches [29]. In this paper, we used Karush-Kahn-Tucker (KKT) method for solving bi-level programming problem.

4. Mathematical Model

A mathematical model is designed to achieve an optimal solution. The indices used in the mathematical model are:

i: index for part type ($i=1, 2, \ldots, P$);
w: index for worker ($w=1, 2, \ldots, W$);
m: index for machine type ($m=1, 2, \ldots, M$);
k: *index for cell* ($k=1, 2, \ldots, C$).

The parameters used in the mathematical model are:

A_{im} = 1 if part type i needs machine type m; 0 otherwise
B_{imw} = 1 if part type i can be processed on machine type m with worker w; =0 otherwise
LM_k = minimum size of cell k in terms of the number of machine types;
LP_k = minimum size of cell k in terms of the number of part types;
LW_k = minimum size of cell k in terms of the number of workers;
UW_k = maximum size of cell k in terms of the number of workers;
$R_{ww'}$ = 1 if worker w is interested to work with worker w'; =0 otherwise.

Decision variables in the model are:

x_{mk} =1 if machine type m is assigned to cell k; =0 otherwise;
y_{ik} =1 if part i is assigned to cell k; =0 otherwise;
z_{wk} = 1 if worker w is assigned to cell k; =0 otherwise;
d_{imwk}=1 if part i is processed by machine type m with worker w in cell k; =0 otherwise;

According to the desired goals of the model optimization, two objective functions in two levels were considered. At the top level and leader's objective, we are looking for minimizing voids and inter cell movements.

$Min =$

$$\sum_{k=1}^{C}\left[\sum_{i=1}^{P}\sum_{m=1}^{M}\sum_{w=1}^{W}y_{ik}x_{mk}z_{wk}-\sum_{i=1}^{P}\sum_{m=1}^{M}\sum_{w=1}^{W}y_{ik}x_{mk}z_{wk}d_{imwk}\right] \tag{3}$$

$$+\sum_{i=1}^{P}\sum_{k=1}^{C}\sum_{m=1}^{M}\sum_{w=1}^{W}\left[y_{ik}x_{mk}(1-z_{wk})d_{imwk}\right] \tag{4}$$

$$+\sum_{i=1}^{P}\sum_{k=1}^{C}\sum_{m=1}^{M}\sum_{w=1}^{W}\left[2\times x_{mk}(1-y_{ik})(1-z_{wk})d_{imwk}\right] \tag{5}$$

$$+\sum_{i=1}^{P}\sum_{k=1}^{C}\sum_{m=1}^{M}\sum_{w=1}^{W}\left[x_{mk}(1-y_{ik})z_{wk}d_{imwk}\right] \tag{6}$$

In this objective function, the first term, i.e., (3), which minimizes the entire number of voids, plus the terms (4), (5), and (6) are to work out the number of exceptional elements. The exceptional

elements for parts are estimated based on the status, availability of corresponding worker and machine.
The constraints are:

$$\sum_{k=1}^{C} y_{ik} = 1 \qquad \forall i\,; \tag{7}$$

$$\sum_{m=1}^{M} x_{mk} \geq LM_k \qquad \forall k\,; \tag{8}$$

$$d_{imwk} \leq B_{imw} x_{mk} \qquad \forall i, m, w, k\,; \tag{9}$$

$$\sum_{k=1}^{C} \sum_{w=1}^{W} d_{imwk} = A_{im} \qquad \forall i, m\,; \tag{10}$$

$$\sum_{i=1}^{P} y_{ik} \geq LP_k \qquad \forall k\,; \tag{11}$$

$$x_{mk}, y_{ik}, z_{wk}, d_{imwk} \in \{0,1\} \qquad \forall i, m, w, k. \tag{12}$$

In this mathematical model, Equation (7) ensures that a specific part is assigned to one cell only. Inequality (8) is defined to control the assignment of minimum machines to a cell. Constraint (9) ensures that when machine type *m* is not in cell *k*, then $d_{imwk} = 0$. Equation (10) ensures that if the part *i* required to process by machine *m*, there is a cell and just one like *k* which contain this machine and worker *w* whom work on it to processing part *i* on this cell. Inequality (11) is determined to hold the minimum number of parts which processed in each cell. Finally, Equation (12) suggests that x_{mk}, y_{ik}, z_{wk} and d_{imwk} are binary decision variables.
In the follower objective as the second level, we are looking for maximizing interesting to work together between workers who work in a particular cell.

$$Max = \sum_{w=1}^{W} \sum_{w'=1}^{W} \sum_{k=1}^{C} R_{ww'} z_{wk} z_{w'k} \tag{13}$$

S.t.:

$$\sum_{k=1}^{C} z_{wk} = 1 \qquad \forall\, w\,; \tag{14}$$

$$\sum_{w=1}^{W} z_{wk} \leq UW_k \qquad \forall\, k; \tag{15}$$

$$\sum_{w=1}^{W} z_{wk} \geq LW_k \qquad \forall k\,; \tag{16}$$

$$\sum_{i=1}^{P} \sum_{m=1}^{M} d_{imwk} \geq z_{wk} \qquad \forall\, w,k; \tag{17}$$

In the second level of the mathematical model, Equation (13) ensures that a specific worker is assigned to one cell only. Constraints (14) and (15) are defined to control the assignment of minimum and maximum workers to a cell. Finally, Equation (16) is defined to ensure if worker *w* assigned to cell *k* there is at least one part, such as *I*, in this cell which processed by the machine type *m* by working the worker *w* on that machine. Inequality (17) defines the relation of two decision variables.
The nonlinear terms in the first level objective function can be linearized with $s1_{imwk}$, $s2_{imwk}$, $q1_{imwk}$, $q2_{imwk}$, and $q3_{imwk}$.

$$s1_{imwk} = x_{mk} . d_{imwk}$$

$$s2_{imwk} = x_{mk} . y_{ik} . z_{wk}$$

$$q1_{imwk} = s2_{imwk} . d_{imwk}$$

$$q2_{imwk} = y_{ik} . s1_{imwk}$$

$$q3_{imwk} = z_{wk} . s1_{imwk}$$

The following sets of constraints for linearization are added to the model too:

$$s1_{imwk} - x_{mk} - d_{imwk} + 1.5 \geq 0 \qquad m, w, k \tag{18}$$

$$s2_{imwk} - x_{mk} - y_{ik} - z_{wk} + 2.5 \geq 0 \qquad m, w, k \tag{19}$$

$$q1_{imwk} - s2_{imwk} - d_{imwk} + 1.5 \geq 0 \qquad m, w, k \tag{20}$$

$$1.5 \times q1_{imwk} - s2_{imwk} - d_{imwk} \leq 0 \qquad m, w, k \tag{21}$$

$$q2_{imwk} - y_{ik} - s1_{imwk} + 1.5 \geq 0 \qquad m, w, k \tag{22}$$

$$1.5 \times q2_{imwk} - y_{ik} - s1_{imwk} \leq 0 \qquad m, w, k \tag{23}$$

$$q3_{imwk} - z_{wk} - s1_{imwk} + 1.5 \geq 0 \qquad m, w, k \tag{24}$$

$$1.5 \times q3_{imwk} - z_{wk} - s1_{imwk} \leq 0 \qquad m, w, k \tag{25}$$

After the linearization of the objective function, the first level is presented as follows:

$$Min = \sum_{k=1}^{C} \sum_{i=1}^{P} \sum_{m=1}^{M} \sum_{w=1}^{W} (s2_{imwk} - q1_{imwk}) + \sum_{k=1}^{C} \sum_{i=1}^{P} \sum_{m=1}^{M} \sum_{w=1}^{W} (2 \times s1_{imwk} - q2_{imwk} - q3_{imwk})$$

subject to constraints (7) - (12) and (18) - (25).

5. KKT Method and Model Reformulation

Using the KKT conditions is one of the popular methods to solve the bi-level optimization problems. In this approach, the original problem changes to its first level of subsidiary problem [30]. In this way, the problem is reduced to a regular mathematical programming problem; the lower level problem is replaced by its KKT conditions. However, the transformed problem or the subsidiary problem is hard to resolve due to nonlinearity, which was brought out through the supplementary slackness conditions.

We should use KKT conditions on the follower objective function. Let u and v be the dual variables associated with two sets of constraints.

$$\begin{aligned} &\underset{y}{Min}\ \ F(x,y) = c_2 x + d_2 y \\ &\quad s.t. \\ &\qquad b_2 - A_2 x - B_2 y \ge 0 \quad (u) \\ &\qquad y \ge 0 \quad (v) \end{aligned} \tag{26}$$

Then:

$$\begin{aligned} &d_2 + uB_2 - v = 0 \\ &u(b_2 - A_2 x - B_2 y) + vy = 0 \\ &u, v \ge 0 \end{aligned} \tag{27}$$

For solving the bi-level programming problem, the second level should be replaced with these steps [31]:

- Constraints of the original second level;
- Constraints of dual problem of the second level;
- Equality constraint of strong duality theory;

Problem (1) transforms to the following problem after using KKT conditions on the lower level:

$$\begin{aligned} &\underset{x}{Min}\ \ c_1 x + d_1 y \\ &s.t. \\ &\qquad A_1 x + B_1 y \le b_1 \\ &\qquad uB_2 - v = -d_2 \\ &\qquad u(b_2 - A_2 x - B_2 y) + vy = 0 \\ &\qquad A_2 x + B_2 y \le b_2 \\ &\qquad x, y, u, v \ge 0 \end{aligned} \tag{28}$$

Before applying the KKT conditions on the second level, the non-linear term in the objective function of this level could be linearized with $p_{ww'k} = z_{wk} . z_{w'k}$ under the following constraints:

$$z_{wk} + z_{w'k} - p_{ww'k} \le 1 \tag{29}$$

$$2p_{ww'k} - z_{wk} - z_{w'k} \le 0 \tag{30}$$

We now present the linear mathematical model of the second level as follows:

$$Max = \sum_{w=1}^{W} \sum_{w'=1}^{W} \sum_{k=1}^{C} R_{ww'} . p_{ww'k} \qquad w \ne w'$$

subject to constraints (14) - (17) and (29) – (30)

$$p_{ww'k} \in \{0,1\} \quad \forall\ i, m, w, k; \tag{31}$$

For the first step and before using the KKT conditions, duality of the second level should be calculated. The dual model is presented as follows:

$$Min = \sum_{w=1}^{W} U_w + \sum_{k=1}^{C} (UW_k) \times \mathrm{V}_k - \sum_{k=1}^{C} (\mathrm{L}W_k) \times L_k \tag{32}$$

$$+ \sum_{w=1}^{W} \sum_{k=1}^{C} \sum_{i=1}^{P} \sum_{m=1}^{M} h_{wk} d_{imwk} + \sum_{w=1}^{W} \sum_{\substack{w'=1, \\ w \ne w'}}^{W} \sum_{k=1}^{C} n_{ww'k} \tag{33}$$

s.t.

$$\begin{aligned} &U_w + V_k - L_k + h_{wk} + \\ &\sum_{\substack{w'=1, \\ w \ne w'}}^{W} n_{ww'k} + \sum_{\substack{w'=1, \\ w \ne w'}}^{W} n_{w'wk} - \\ &\sum_{\substack{w'=1, \\ w \ne w'}} n'_{ww'k} - \sum_{\substack{w'=1, \\ w \ne w'}} n'_{ww'k} \ge 0 \end{aligned} \qquad \forall\ w, k; \tag{34}$$

$$2n'_{ww'k} - n_{ww'k} \ge r_{ww'} \qquad \forall w, w', k\ \&\ w \ne w'; \tag{35}$$

$$n_{ww'k}, n_{w'wk}, n'_{ww'k} \ge 0 \qquad \forall w, w', k\ \&\ w \ne w'; \tag{36}$$

$$n'_{w'wk}, \mathrm{V}_k, \mathrm{L}_k, h_{wk} \ge 0 \qquad \forall w, w', k\ \&\ w \ne w'; \tag{37}$$

$$d_{imwk} \in \{0,1\} \qquad \forall\ i, m, w, k; \tag{38}$$

$$U_w \ \ unsigned \qquad \forall\ w; \tag{39}$$

The nonlinear term in Equation (33) could be linearized with $g_{imwk} = d_{imwk} \times h_{wk}$ under the following constraints:

$$g_{imwk} \le h_{wk} \qquad \forall i, m, w, k \tag{40}$$

$$g_{imwk} \le M \times d_{imwk} \qquad \forall i, m, w, k \tag{41}$$

$$g_{imwk} \ge h_{wk} - M(1 - d_{imwk}) \qquad \forall i, m, w, k \tag{42}$$

$$g_{imwk} \ge 0 \qquad \forall i, m, w, k \tag{43}$$

M in equations (40) and (43) is a positive large number.

After using the KKT conditions for the second level and linearization of the first and second levels, the mathematical model is presented as follows:

$$Min = \sum_{k=1}^{C}\sum_{i=1}^{P}\sum_{m=1}^{M}\sum_{w=1}^{W}(s2_{imwk} - q1_{imwk}) + \sum_{k=1}^{C}\sum_{i=1}^{P}\sum_{m=1}^{M}\sum_{w=1}^{W}(2\times s1_{imwk} - q2_{imwk} - q3_{imwk})$$

Subject to constraints (7) - (12), (14) - (25), (29) - (31), and (34) - (43).

$$\sum_{w=1}^{W}\sum_{\substack{w'=1,\\ w\neq w'}}^{W}\sum_{k=1}^{C} R_{ww'}.p_{ww'k} - \left(\sum_{w=1}^{W}U_w + \sum_{k=1}^{C}(UW_k)\times \mathrm{V}_k - \sum_{k=1}^{C}(\mathrm{L}W_k)\times L_k + \sum_{w=1}^{W}\sum_{k=1}^{C}\sum_{i=1}^{P}\sum_{m=1}^{M} g_{imwk} + \sum_{w=1}^{W}\sum_{\substack{w'=1,\\ w\neq w'}}^{W}\sum_{k=1}^{C} n_{ww'k}\right) = 0$$

6. Case Study

Alyagkaran Manufacturing Company is a producer of brake and clutch pads. Products of this company are brake shoes for agricultural machines, brake pads for heavy vehicles, disk brake pads, and industrial brake and clutch pads for industrial machinery. Five different types of machines and equipment are used to manufacture these products: hydraulic pressing machine, mixer, molding machine, rubbing machine, and furnace. The machines are set up in two halls and totally 9 workers are operating them. Workers in each hall must be capable of performing tasks of the other workers of the same hall in emergency cases, in addition to their own tasks. Products are coded by P, machines are coded by M and indexed by numbers 1-5, and workers are coded by W and indexed by numbers 1-9.

To solve the model in the case study environment, Part-Machine matrixes showing the machines' need of manufacturing each part as shown in Table 1, Machine-Worker matrixes showing workers' capability of working with several machines as shown in Table 2, and Worker-Worker matrixes showing each worker's interest in working with other workers were completed. The worker - worker matrix is worked out based on each worker's "yes/no" response to this question – asked in private – as shown in Table 3.

Tab. 1. Part-machine matrix

	M_1	M_2	M_3	M_4	M_5
P_1	1	1	0	0	1
P_2	0	1	1	0	1
P_3	1	1	1	0	1
P_4	1	0	0	1	1
P_5	0	0	1	1	1

Tab. 2. Machine-worker matrix

	W_1	W_2	W_3	W_4	W_5	W_6	W_7	W_8	W_9
M_1	1	0	0	1	1	0	0	1	0
M_2	1	1	0	1	1	0	1	1	0
M_3	0	0	0	0	1	0	0	1	1
M_4	1	0	1	1	0	0	1	0	1
M_5	1	1	1	0	0	1	0	0	1

Tab. 3. Workers interest matrix

	W_1	W_2	W_3	W_4	W_5	W_6	W_7	W_8	W_9
W_1	1	1	0	1	0	1	0	1	1
W_2	1	1	0	0	0	1	0	0	1
W_3	0	0	1	1	0	0	0	1	1
W_4	1	0	0	1	0	0	1	0	1
W_5	1	0	1	0	1	1	0	1	0
W_6	0	0	1	0	0	1	0	1	0
W_7	0	1	0	1	1	0	1	0	1
W_8	0	0	1	0	1	1	0	1	1
W_9	1	1	1	0	1	0	1	0	1

Several methods can be used for layout machines and workers, which we will discuss later. In the first case, only the workers' interest in working with each other has been taken into consideration. Regarding the fact that the workers must be capable of performing tasks of the other workers in the same hall, it is essentially important to encourage a sense of cooperation and coordination amongst them. To achieve this goal, the workers' interest in working with each other is of great importance, because it is not always easy to exchange knowledge between the workers, and so friendly relationship between the workers of each section can facilitate solving this problem. The consequences of the proposed mathematical model, considering only the second level and its constraints, are provided in Fig.1 and Table 4, based on which the value of interest is 32, and the number of voids and exceptional elements equals 177.

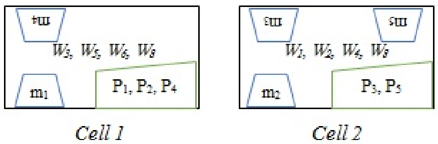

Fig. 1. Cells formation in the first case

Tab. 4. Workers interest matrix after forming the cells basis of the first case

		Cell 1				Cell 2				
		W_3	W_5	W_6	W_8	W_1	W_2	W_4	W_7	W_9
Cell 1	W_3	1	0	0	1	0	0	1	0	1
	W_5	1	1	1	1	1	0	0	0	0
	W_6	1	0	1	1	0	0	0	0	0
	W_8	1	1	1	1	0	0	0	0	1
Cell 2	W_1	0	0	1	1	1	1	1	0	1
	W_2	0	0	1	0	1	1	0	0	1
	W_4	0	0	0	0	1	0	1	1	1
	W_7	0	1	0	0	0	1	1	1	1
	W_9	1	1	0	0	1	1	0	1	1

It must be also mentioned that although the workers' interest in playing with each other is at the highest possible degree, the number of voids and exceptional elements is also big and unacceptable. For the same reason, and because in solving cellular manufacturing problems, cutting the number of voids and exceptional elements is of a high priority, only the proposed mathematical model is taken into consideration in the second case. Of course, for full allocation of the required workforce, limitations of the second level have been added to those of the first level. The answers are presented in Fig.2.

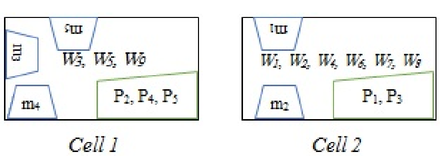

Fig. 2. Cells formation in the second case

In this case, the best answer and minimal number of voids and exceptional elements will be 45. Regarding the constraints of the model in the first level and limitations of the second level which has been added to this level, the value of the workers of two halls interest in working with each other is calculated as 25, as shown in Table 5.

Tab. 5. Workers interest matrix after forming the cells basis of the second case

		Cell 1				Cell 2				
		W_3	W_5	W_6	W_8	W_1	W_2	W_4	W_7	W_9
Cell 1	W_3	1	0	1	0	0	1	0	0	1
	W_5	1	1	0	1	0	0	1	0	1
	W_6	1	1	1	1	1	0	0	1	0
	W_8	0	0	1	1	1	1	1	0	1
Cell 2	W_1	0	0	1	1	1	0	1	0	0
	W_2	0	0	1	1	0	1	0	1	0
	W_4	1	0	0	0	0	0	1	0	1
	W_7	0	1	1	0	1	1	0	1	0
	W_9	0	1	1	0	0	0	1	0	1

However, since types of tasks and activities of staff in an organization have changed a lot and management and exchange of knowledge are essentially important in the modern world, the workers' interest in collaborating and coordinating with each other cannot be disregarded. Considering various potential problems and circumstances, employers and managers must pay excessive attention to the workers' capability of handling each other's task, in addition to their knowledge concerns.

With views of what mentioned above, it can be said that going through any of the above factors alone cannot fully solve the problem in the real world. Therefore, we have tried to use the bi-level planning method to develop the mathematical model in a way that the most optimum result, suitable for practical applications, is achieved. Accordingly, we considered the more important objective to be minimizing the number of voids and exceptional elements in the first level, and promoting the workers' interest in working with each other on the second level. Based on this and after solving the proposed mathematical model, the new layout is worked out, as shown in Fig.3.

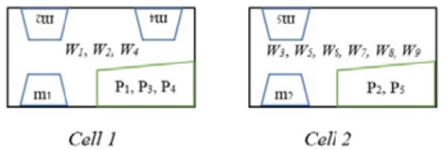

Fig. 3. Cells formation after solving the bi-level mathematical model

In the new layout, the number of voids and exceptional elements equals 49 as shown in the above figure, and value of the workers' interest in working with each other equals 29, as indicated in Table 6.

Tab. 6. Workers interest matrix after forming the cells basis of the fully solved mathematical model

		Cell 1				Cell 2				
		W_3	W_5	W_6	W_8	W_1	W_2	W_4	W_7	W_9
Cell 1	W_3	1	1	1	0	0	1	0	1	1
	W_5	1	1	0	0	0	0	0	1	1
	W_6	1	0	1	0	0	0	1	0	1
	W_8	0	0	1	1	0	0	0	1	1
Cell 2	W_1	1	0	0	1	1	1	0	1	0
	W_2	0	0	0	1	0	1	0	1	0
	W_4	0	0	1	0	1	0	1	0	1
	W_7	0	1	0	1	1	1	0	1	1
	W_9	1	0	0	1	1	0	1	0	1

Therefore, although every single objective is far from its optimal value, considering the decentralized decision making and necessity of moving both objectives mutually in the field of manufacturing and priorities set forth in this regard, the values are calculated in a way that both the employer's requirements are met mutually and at a proper level.

For the mathematical model solving, LINGO 16 software is used on a computer with the hardware and software specifications, shown as follows:

- HP ProLiant DL380 Gen9 Server
- CPU: Intel XEON E5-2600 2.60 GHz, 4 processors with ؟6 cores total.
- Installed memory (RAM): 96 GB DDR4.
- Operating system: Microsoft Windows Server 2012 R2 (64 Bits).
- Run times for three noted situations of the case study are indicated in Table 7.

Tab. 7. Run times for each situation of the mathematical model.

Case#	Description	Run Time (H:M:S)
1	Second Level (Follower Function)	00:00:01
2	First Level (Leader Function)	88:58:20
3	The Fully Bi-Level Model	97:29:03

7. Conclusions

As mentioned before, cellular manufacturing problems aim to achieve the best layout for the facilities. Facilities planning can be centralized or decentralized. In centralized planning, manufacturing policies are designed in an integrated manner. In decentralized cases, facility planning could change depending on the requirements and policies adopted for different facilities, even in one unit. Considering the current dynamic and constantly changing business environment, the systems have to adapt themselves to the changes of the environment and seek innovation and competitive advantages for survival. To have an innovative and dynamic organization, it is necessary to promote the sense of cooperation and coordination between the workforces because the cooperation of the workforces results in a more effective and realistic exchange of knowledge and consecutively optimizes the group performance. Therefore, workforce planning in cellular manufacturing must focus on promoting learning interactions between the staff.

Cell formation is a strategic decision and workforce planning is a tactical decision. Practically, these two aspects cannot be planned centrally, and decision-making in this regard is decentralized. For this reason and aiming to make a decentralized, yet integrated, decision, a bi-level approach has been provided in this article. The first level addresses the more important problem, which is minimizing the number of voids and exceptional elements. The results of this level in the case of this article are shown in Fig.2 and Table 5. In the second level, promoting a sense of collaboration between the workers has been taken into consideration to maintain an innovative and dynamic organization in the long term. The results of this level, in Fig.1 and Table 4, are shown. Based on the proposed bi-level mathematical model, machinery and workforces are laid out in a way that the sense of cooperation between the workforces is increased in proportion to the decrease in the number of voids and exceptional elements. The obtained results based on the proposed bi-level mathematical model in Fig.3 and Table 6 are shown. This method, instead of applying one-aspect approaches to solving two different and non-aligned problems, is a series of optimum answers that are worked out, directing the organization towards realizing both of its objectives.

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