International Journal of Industrial Engineering & Production Research March 2020 Vol. 31, No. 1: 35- 50
DOI: 10.22068/ijiepr.31.1.35

RESEARCH PAPER

Analysis of the Hardening and Resilience Ability in Location-Allocation Problems

Zahra Esfandiari[1] , Mahdi Bashiri [*2] & Reza Tavakkoli-Moghaddam[3]

Received 05 August 2018; Revised 23 September 2019; Accepted 08 October 2019; Published online 31 March 2020

ABSTRACT

Although disruptions in supply chains occur at a low probability, they may cause huge financial effects whose recovery will not be quickly achieved. Designing a resilient supply network may be an effective way to hedge against these disruption effects. This feature represents the ability of the network to maintain its operation and connection despite the absence of some chain members. Facility hardening is one of the strategies used for designing a resilient network. In this paper, three different non-linear resilient capacitated fixed-charge location-allocation models are developed for hedging the network against failure by assuming that when a random disruption occurs at a facility, the facility fails to deliver any product. In the first model, both facility hardening and equipping the network with backup facilities for disrupted elements are considered together to prevent the supply network from failure due to random disruption. The recovery time of the facility after a disruption event is one of the significant features that has been considered in the second and third proposed models, capable of being implemented in real-world applications. The sensitivity analysis confirms that the proposed models are valid as expected. In addition, in a low failure probability condition, the superiority of the proposed models is confirmed by comparing it to the classical model using a low additional investment. A Lagrangian decomposition algorithm is developed to solve large-scale instances. Computational results confirm the high efficiency of the proposed solution approach, compared to those obtained from the classical solution approaches, in dealing with large-scale problems.

KEYWORDS: *Reliable; Random disruption; Hedging system; Hardening; Resilience; Lagrangian Relaxation.*

1. Introduction

Facilities represent physical components of interrelated systems that are designed, created, and equipped with special equipment or personnel to serve a particular function. Social activities are extensively dependent on network systems. Most of our basic daily activities require interaction with a diversity of impressible facility systems. For example, telecommunication facilities are used in order to fulfill our financial dealing and protect our contact with family and friends. In addition, energy facilities are utilized to heat our homes, power local industries, etc. While these basic mentioned activities are usual, the importance of a utilized facility system is rarely tangible. For instance, according to the U.S department report, over 19 billion tons of shipment valued at 13 trillion dollars was transferred through the U.S. multi-modal transportation system in 2002. Critical facility systems specify an industrial degree of a society so that their importance would not be undervalued [1]. Since system operations can be disrupted in random disruption or intentional harm, there is a need to perceive how network systems and their performance can be affected by the failure of their facilities in disruption events. A common subject, in which network elements are debilitated causing disruption in the flow of goods and services through the network, is interdiction. For example, failure or decline of power supply facilities eventuates dramatic consequences for a society and national economy; therefore, these facilities are considered as vital entities. The interconnectedness of continuous power supply

* Corresponding author: *Mahdi Bashiri*
bashiri@shahed.ac.ir

1. *Science and Research Branch, Islamic Azad University IE dep.*
2. *Shahed University, IE dep.*
3. *University of Tehran -IE dep.*

with our daily life, industrial production, and electronics has made today's society more vulnerable due to power supply stoppages. In all countries, even a short power outage is not acceptable because it has a significant impact on the entire economy. Natural disasters can also have significant impact on these network performances and their functionalities. In 2010, 450 natural disasters were recorded across the world. In 2011, only the USA sustained over 55 billion economic harms that resulted from natural disasters [2].

The above-mentioned instances and their severe consequences present an extensive geographical dimension of supply network disruption that made decisionmakers consider the possibility of disruption in a facility while designing their networks and preventing them from failure. One of the most important features is the resilience ability to avoid failures in systems. The resilience of a supply network against disruptions depends on in its ability to conserve system operations and connection, while there is a lack in some components or parts. There are three strategies to improve the supply network resiliency, namely considering the topological characteristics of the network, providing backup components with flexible capacity, and hardening or protection of critical elements. In this study, the two last strategies are assumed to improve the resiliency of the designed network. We take a facility location problem into consideration that incorporates different facility characteristics such as capacity limitation, facility random disruption probability, facility hardening, and resilience ability. Three integer programming models are proposed for preventing the system from failure. In the first proposed model, hardening decisions are emphasized, while, in the second one, the resilience ability is regarded. Finally, in the last one, hardening and resilience abilities are considered simultaneously. A Lagrangian decomposition algorithm (LDA) was developed for the first and third proposed models, and the computational results demonstrating the efficiency of the developed algorithm were consequently presented.

The structure of this paper is as follows: the next section gives a brief review of the previous studies. Section 3 presents the proposed models and their corresponding Lagrangian decomposition algorithms. Section 4 gives an analysis of numerical examples, including the sensitivity analysis and computational results of the solution algorithm. Finally, the last section concludes the paper with final remarks.

2. Literature Review

A facility location-allocation problem has been expansively considered in the literature, and many different types of facilities in both private and public service systems with different conditions have been modeled. This study seeks to determine the location of facilities and assign customers to these locations with the minimum cost or maximum profit. The classical facility location-allocation models assume that facilities are always operational, never fail during their operation, and do not need care in the case of a possible disruption. In other words, although it is far from real world, it is assumed that they are completely reliable. Because of severe consequences arising from the facility disruption in supply networks as explained earlier, the possibility of disruption is one of the recent assumptions in reliability consideration of facility location-allocation decisions.

Snyder (2003) proposed an initial facility location-allocation model with reliability consideration. He incorporated facility failures directly in *p*-median and fixed charge facility location problems [3]. Recently, reliable facility location models have been developed to design facility locations and customer assignment plans when facility disruptions are possible. In the literature, the disruption risks and the corresponding objective functions that are incorporated in most of reliable facility location-allocation models are divided into two categories based on their sources: (1) random disruption risk by minimizing the expected cost objective function and (2) premeditated or intentional disruption risk by minimizing the maximum possible cost of the objective function. The former may occur at any point in the supply network (e.g., natural hazards like earthquakes). However, the latter includes components, more probable to be targeted, whose failure will bring about the maximum damage in a supply network.

As stated in the previous section, the resilience ability is one way to reduce system vulnerability and increase its reliability in order to prevent the system from failure. Resiliency of a supply network against disruptions depends on its ability to conserve system operations and connections despite the lack of some components or parts. This is why the reliable facility location-allocation models in supply network design decisions have been developed based on two basic approaches: determining the backup facility and protecting the facility. Some researchers have integrated these two approaches together. In the first approach, the disruption probability is generally incorporated in the integer programming model, in which the existing facilities are used as a backup facility and these facilities are reassigned to customers after their primary facility failure. Their objective function often minimizes the total expected cost after disruption. Some studies in these approaches were included in the literature [4-8]. Studies relevant to the second approach are divided into two categories. Models of the first category are based on interdiction and involve only decision-making about allocating the protection resources among the located facilities in order to fortify these facilities in supply networks [9]. In contrast, models of the other category often make a decision about facility locations, customer assignment, and allocation of protection resources among facilities in order to minimize the worst-case cost [10-11].

In the compound approach, two types of facilities (i.e., unreliable and hardened or totally reliable facility with an extra cost) should be located in the supply network, and each customer can be assigned to a primary facility and a hardened backup facility [12-16]. In this group, only three papers make a decision about facility protection by distinct decision variables [17-18]. Table 1 gives a brief review of the studies with reliability consideration in designing a supply network with their basic characteristics.

Tab. 1. A brief review of Studies with reliability consideration in the supply network design

Reference	Hardening	Backup	Disruption	Disruption probability	Resilient Facility	Objective Function	Decision Variable
[3]	-----	Multiple	Random	ID	-----	MTEC	FL , PBA
[19]	CF	One	Random	-----	-----	MTEC	FL , PBA
[6]	-----	Multiple	Random	ND ,ID		MTEC	FL , PBA
[7]		Multiple	Random , Intentional	SD , SC	-----	MTEC	FL , PBA
[13]	-----	One	Random	-----	-----	MTEC	FL , PBA
[20]	-----	One	-----	ID	-----	MTEC	FL , PBA
[9]	WDV	-----	Intentional	-----	SD	MTEC	PA , PRA
[21]	-----	One	-----	-----	RE	MENR	FL , PBA , NTV
[22]	CF	One	Random	ND , SD	-----	MTEC	FL , PBA
[23]		Multiple	Random	ND , SD	------	MTEC	FL , PBA
[17]	WDV	One	Random	ND , SD	-----	MTEC	FL , FH , PBA
[14]	CF	One	Random	SD	-----	MTEC	FL , PBA
[24]	-----	One	Random	ID	-----	MIMEC	FL , PBA
[10]	WDV	-----	Intentional	-----	-----	MIMTC	FL , PA , PRA
[18]	WDV	One	Random	ND , ID	-----	MTEC	FL , FH , PBA
Current research	WDV	One	Random	SD , SC	SD	MTEC	FL , FH , PBA

FL: Facility location, PBA: Primary and backup assignment, FH: Facility hardening, PRA: Protection resource allocation, NTV: Number of transportation vehicles, PA: Primary allocation, MTEC: Minimization of total expected cost, MENR: Maximization of expected net revenue, MIMEC: Minimization of maximum expected cost, MIMTC: Minimization of maximum total

Most of the existing studies on the facility location emphasizing on the the network resilience do not consider the physical resilience of each component and its recovery time in problem modeling. Only two researchers included these features in their proposed models, especially in intentional disruption conditions and discrete time periods. They do not make decisions about the facility location and only consider the allocation of limited protection resources among located facilities in order to reduce their recovery time. To the best knowledge of us, no study has simultaneously considered the hardening decisions and resilience of facilities with continuous facility recovery time incorporated in the facility location model in random disruption conditions. The current study covers the mentioned research gaps to address the non-intentional disruptions. The main contributions of this study that differentiate it from the previous studies in the related literature can be summarized as follows:

- Non-linear integer programming models are developed for the facility location-allocation problem in the supply chain network design, considering capacity limitation, facility hardening, and facility resiliency to optimize the total expected cost in disruption events
- A comparison of the priorities of designed networks is made with those of classical network in facility location problems in different facility failure probabilities;
- The nonlinear proposed models are linearized by suitable techniques.
- A Lagrangian decomposition algorithm (LDA) is developed to obtain a solution to large-scale problems in a reasonable amount of time.

3. Problem Development

3.1. Problem description and assumptions

This section first introduces some common assumptions and notations and, then, proposes three formulations for the reliable capacitated fixed charge facility location problem under their special assumptions and random disruption risk. In all of them, there is a set of potential locations for establishing capacitated facilities and a set of demand centers that must be served by these capacitated facilities.

The main assumptions of this study can be as follows: 1) Customer demands are deterministic; 2) the transportation cost is related to the distance between a customer and its assigned facility; 3) each facility can be hardened with an extra cost during its establishment; 4) the disruption probability in facilities is independent of each other. 5) Hardened facilities are completely reliable and are immune against failure. 6) Once a facility fails, it becomes unavailable and its assigned customers should be reassigned to the nearest hardened facility.

3. 2. Indices and sets

i	*Potential facility location*
j	*Customer location*

3. 3. Model parameters

a_i	*Opening cost of facility i*
p_i	*Failure probability of facility i*
b_i	*Hardening cost of facility i*
h_{ij}	*Distance of customer j from facility i*
tr_i	*Recovery time of facility i*
c_i	*Capacity of facility i*
rc_i	*Recovery cost of facility i*
s_i	*Penalty cost of facility i*
d_j	*Demand of customer j*
m_{ij}	*Partial demand of customer j assigned to facility i that Supply from another one*
K	*Total hardening budget*
B	*Total penalty budget*
U	*Total recovery cost budget*

3. 4. Decision variables

x_i	*Opening facility in location i*
z_i	*Hardening facility in location i*
x_{0ij}	*Primary assignment of facility i to customer j*
x_{1ij}	*Backup assignment of facility i to customer j*

3.5. Problem formulations

This section proposes three formulations for the reliable capacitated fixed charge facility location allocation problem considering facility hardening and resiliency as the two main features. In the first one, only the hardening ability in the network design is considered, and its corresponding model is called "RFLA&H" (i.e., reliable facility location-allocation with the hardening ability). In the second one, another model that enjoys the resilience ability is proposed, which is called "RFLA&RE". The last one includes an integrated model, which contains both hardening and resilience abilities called "RFLA&H,RE".

3.5.1. Reliable capacitated facility location allocation problem using hardening ability

This problem is called RFLA&H as a nonlinear integer programming model that minimizes the total expected cost by opening optimal facilities, deciding about hardening of opened facilities and allocating customers to normal and hardened facilities as primary and backup assignments. The RFLA&H model is presented below:

$$\min W = \sum_i a_i \times x_i + \sum_i \sum_j x_{0ij} \times h_{ij} \times d_j \times (1 - p_i \times (1 - z_i)) + \sum_i \sum_j x_{1ij} \times h_{ij} \times d_j \times \sum_{r \neq i} x_{0rj} \times p_r \times (1 - z_r) + \sum_i z_i \times b_i \quad (1)$$

Subject to

$$x_{0ij} \leq x_i \qquad \forall i,j \qquad (2)$$

$$x_{1ij} \leq x_i \qquad \forall i,j \qquad (3)$$

$$\sum_j x_{0ij} + x_{1ij} \geq x_i \qquad \forall i \qquad (4)$$

$$\sum_i x_{0ij} = 1 \qquad \forall j \qquad (5)$$

$$\sum_i x_{1ij} \leq 1 \qquad \forall j \qquad (6)$$

$$x_{1ij} \leq Z_i \qquad \forall i,j \qquad (7)$$

$$\sum_{i \neq r} x_{1ij} \geq x_{0rj} \times (1 - z_r) \qquad \forall r,j \qquad (8)$$

$$\sum_{i \neq r} x_{1ij} \leq 1 - (x_{0rj} \times z_r) \qquad \forall r,j \qquad (9)$$

$$z_i \leq x_i \qquad \forall i \qquad (10)$$

$$\sum_j (x_{0ij} \times d_j + (d_j \times x_{1ij} \times \sum_{r \neq i} x_{0ij} \times (1 - z_r))) \leq c_i \qquad \forall i \qquad (11)$$

$$\sum_j x_{0ij} \geq x_i \qquad \forall i \qquad (12)$$

$$\sum_i z_i \times b_i \leq K \qquad (13)$$

$$x_i, x_{0ij}, x_{1ij}, z_i \in \{0,1\} \qquad \forall i,j \qquad (14)$$

Equation (1) is the objective function that minimizes the total expected cost. This cost includes the fixed cost of opening and hardening of facilities and variable transportation costs. Constraints (2) and (3) guarantee that only an open facility can serve as a supplier. Constraint (4) assures that the opening facility can serve customers as either primary or backup supplier. Constraints (5) and (6) assure that each customer should be supplied from only one facility as either primary or backup. Constraint (7) assures that only hardened facility can be assigned to customers as the backup supplier. Constraints (8) and (9) assure that only customers of failed facilities can be reassigned to a backup facility. Constraint (10) guarantees that only an open facility can be hardened. Constraint (11) considers the capacity limitation of each facility to serve customers. Constraint (12) guarantees that a backup assignment can occur if it has at least one primary assigned customer, and Constraint (13) limits the hardening budget. Constraint (14) defines decision variables type.

3.5.2 Reliable capacitated facility location allocation problem with resilience ability

In this section, a capacitated fixed charge facility location-allocation problem is formulated with the resilience ability in a random disruption condition called RFLA&RE as a linear integer programming model with the following special additional assumptions. 1) The events of facility failures occur before servicing. 2) Each facility has an associated penalty cost, recovery time, and recovery cost that it takes to become fully operational after a failure. 3) Recovery cost of a facility is related to its capacity.4) penalty cost of a facility is related to its customers demand. 5) Backup allocation is not allowed in this model; therefore, customers of a failed facility will receive a penalty cost during the facility recovery time to be served instantaneously. The RFLA&RE model is presented below.

$$\min W = \sum_i a_i \times x_i + \sum_i \sum_j x_{0ij} \times h_{ij} \times c \qquad (15)$$
$$+ \sum_i \sum_j x_{0ij} \times p_i \times tr_i \times s_i \times d_j$$
$$+ \sum_i x_i \times rc_i \times p_i \times c_i$$

Subject to Constraint (2),(5),

$$\sum_i x_i \times rc_i \times c_i \le U \qquad (16)$$
$$\sum_i \sum_j x_{0ij} \times tr_i \times s_i \times d_j \le B \qquad (17)$$
$$\sum_j x_{0ij} \times d_j \le c_i \qquad \forall i \qquad (18)$$
$$x_i, x_{0ij} \in \{0,1\} \qquad \forall i,j \qquad (19)$$

The objective function minimizes the total expected cost, including the fixed cost of opening facilities, variable costs of transportation, penalty, and recovery costs. Constraints (16) and (17) are the recovery and penalty cost budget constraints, respectively. Constraint (18) limits the capacity of serving facility. Constraint (19) defines the decision variables type.

3.5.3 Reliable capacitated facility location allocation problem of resilient facilities with hardening ability

In this section, an integrated integer nonlinear programming model called RFLA&H,RE is presented. In this proposed model, it is assumed that customers are served gradually with the fixed velocity in unit of time; therefore, the penalty cost should be paid for their non-served part of demands during the facility recovery time. This model is presented below.

$$\min W = \sum_i a_i \times x_i + \sum_i \sum_j x_{0ij} \times h_{ij} \times d_j \times (1 - p_i \times (1 - z_i))$$
$$+ \sum_i \sum_j x_{1ij} \times h_{ij} \times \sum_{r \ne i} x_{0rj} \times m_{rj} \times p_r \times (1 - z_r) \qquad (20)$$
$$+ \sum_i \sum_j x_{0ij} \times p_i \times (1 - z_i) \times h_{ij} \times (d_j - m_{ij})$$
$$+ \sum_i \sum_j x_{0ij} \times p_i \times (1 - z_i) \times m_{ij} \times s_i$$
$$+ \sum_i x_i \times rc_i \times p_i \times c_i + \sum_i z_i \times b_i$$

Subject to Constraints(2)-(13), (16),

$$\sum_i \sum_j x_{0ij} \times m_{ij} \times s_i \times (1 - z_i) \le B \qquad (21)$$
$$\sum_j ((x_{0ij} \times d_j) + (x_{1ij} \times \sum_{r \ne i} x_{0ij} \times (1 - z_r) \times m_{rj})) \le c_i \qquad \forall i \qquad (22)$$
$$x_i, x_{0ij}, x_{1ij}, z_i \in \{0,1\} \qquad \forall i, j \qquad (23)$$

Constraint (21) limits the penalty cost to its budget, and Constraint (22) is the facility capacity constraint. Constraint (23) defines the decision variables type. Table 2 shows a comparison of three proposed models with basic characteristics.

Tab. 2. A comparison of three proposed models considering a basic characteristic

Model	Decision Variables	Objective Function	Customer Information About Facility Status	Penalty Payment	Facility Features	Facility Disruption
RFLA&H	OF,HF,PBA	TEC	Incomplete	-----	HA	RIS
RFLA&RE	OF,PA	TEC	Incomplete	Facility incurs penalty during recovery time	REA	RIS
RFLA&H,RE	OF,HF,PBA	TEC	Incomplete	Facility incurs penalty during recovery time	HA REA	RIS

HA: Hardening ability, REA: Resilience ability, RIS: Random, Independent, Site dependent, OF: Opening facility, HF: Hardening facility, PBA: Primary and backup assignment, PA: Primary assignment, TEC: Total expected cost

4. Solution Algorithm

Since the classic solution algorithms for the capacitated location-allocation problem need a huge amount of computational time, especially in large cases, it seems to be essential to propose an efficient solution approach for solving the models in large instances. Here, a Lagrangian decomposition-based algorithm is proposed to solve the problem efficiently.

Lagrangian decomposition algorithm

Lagrangian relaxation is one of the important and popular techniques for solving integer programming problems. In Lagrangian relaxation algorithm, the main issue is to identify a set of complicated constraints of a general integer programming that increase the computational complexity of the solution approach and add them to the objective function by multiplying by Lagrangian coefficients. On the other hand, the decomposition approach is a general procedure for solving a problem by decomposing it into smaller ones and solving each of them separately. Because of the ability of the Lagrangian Decomposition Algorithm (LDA) to provide equal or better lower bounds to/than Lagrangian relaxation, in this paper, by the relaxation of constraints that relate allocation and hardening variables to location ones, the problem is decomposed into two sub problems. Both sub problems can be solved optimally to generate lower and upper bounds concurrently. The penalties are adjusted based on the violation of relaxed constrains, and the process is repeated until achieving a deterministic stopping criterion.

4.1. Lagrangian formulation for RFLA&H model

In the RFLA&H model, by the relaxation of constraints (2),(3),(4),(10),(12), the relaxed problem can be expressed as follows:

$$\begin{aligned}\min w_{LR} = &\sum_i a_i \times x_i + \sum_i \sum_j x_{0ij} \times h_{ij} \times d_j \times (1 - p_i \times (1 - z_i)) \\ &+ \sum_i \sum_j x_{1ij} \times h_{ij} \times d_j \times \sum_{r \neq i} x_{0rj} \times p_r \times (1 - z_r) \\ &+ \sum_i z_i \times b_i + \sum_i \sum_j u_{1ij} \times (x_{0ij} - x_i) \\ &+ \sum_i \sum_j u_{2ij} \times (x_{1ij} - x_i) + \sum_i u_{3i} \times (x_i - \sum_j (x_{0ij} + x_{1ij})) \\ &+ \sum_i u_{4i} \times (x_i - \sum_j x_{0ij}) + \sum_i u_{5i} \times (z_i - x_i)\end{aligned} \tag{24}$$

Subject to: Constraints (5)-(9), (11), (13), (14).

The parameter u stands for the array of Lagrangian multipliers. In order to apply Lagrangian decomposition, the proposed model is modified by adding a new set of binary variables $y_{1ij}, y_{2ijr}, y_{3ijr}$ that are equal to variables $(x_{0ij} \times z_i)$, $(x_{1ij} \times x_{0rj})$, $(x_{1ij} \times y_{1ij})$ and new constraints (29)-(37) in the following formulation:

$$\begin{aligned}\min w_{LR} = &\sum_i a_i \times x_i + \sum_i \sum_j x_{0ij} \times h_{ij} \times d_j - \sum_i \sum_j x_{0ij} \times h_{ij} \times d_j \times p \\ &+ \sum_i \sum_j y_{1ij} \times h_{ij} \times d_j \times p_i + \sum_i \sum_j \sum_{r \neq i} y_{2ijr} \times p_r \times h_{ij} \times d_j \\ &- \sum_i \sum_j \sum_{r \neq i} y_{3jr} \times p_r \times h_{ij} \times d_j + \sum_i z_i \times b_i + \sum_i \sum_j u_{1ij} \times (x_{0ij} - x_i) \\ &+ \sum_i \sum_j u_{2ij} \times (x_{1ij} - x_i) + \sum_i u_{3i} \times (x_i - \sum_j (x_{0ij} + x_{1ij})) \\ &+ \sum_i u_{4i} \times (x_i - \sum_j x_{0ij}) + \sum_i u_{5i} \times (z_i - x_i)\end{aligned} \tag{25}$$

Subject to: Constraint (5)-(7),(13),

$$\sum_{i \neq r} x_{1ij} \geq x_{1rj} - y_{1rj} \qquad \forall r, j \tag{26}$$

$$\sum_{i \neq r} x_{1ij} \geq 1 - y_{1rj} \qquad \forall r, j \tag{27}$$

$$\sum_j ((x_{0ij} \times d_j) + (d_j \times \sum_{r \neq i} (y_{2ijr} - y_{3ijr}))) \leq c_i \qquad \forall i \tag{28}$$

$$y_{1ij} \leq z_i \qquad \forall i, j \tag{29}$$

$$y_{1ij} \leq x_{0ij} \qquad \forall i, j \tag{30}$$

$$y_{1ij} \geq z_i + x_{0ij} - 1 \qquad \forall i, j \tag{31}$$

$$y_{2ijr} \leq x_{0rj} \qquad \forall i, j, \tag{32}$$

$$y_{2ijr} \leq x_{1ij} \qquad \forall i, j, \tag{33}$$

$$y_{2ijr} \geq x_{0rj} \times x_{1ij} - 1 \qquad \forall i, j, \tag{34}$$

$$y_{2ijr} \leq x_{0rj} \qquad \forall i, j, \tag{35}$$

$$y_{3ijr} \leq y_{1rj} \qquad \forall i, j, \tag{36}$$

$$y_{3ijr} \geq x_{1ij} \times y_{1rj} - 1 \qquad \forall i, j, \tag{37}$$

$$x_0, x_{0ij}, x_{1ij}, z_i, y_{1ij}, y_{2ijr}, y_{3ijr} \in \{0,1\} \qquad \forall i, j, \tag{38}$$

The above formulation can be decomposed into two sub-sub-problems to be solved in a separable and easier manner, referred to here as Sub1 and Sub2, respectively. The first sub-problem, Sub1, which is expressed in terms of the x binary variables only, is given below:

$$\begin{aligned}\min W_{sub1} = &\sum_i a_i \times x_i - \sum_i \sum_j u_{1ij} \times x_i \\ &- \sum_i \sum_j u_{2ij} \times x_i + \sum_i u_{3i} \times x_i + \sum_i u_{4i} \times x_i \\ &- \sum_i u_{5i} \times x_i\end{aligned} \tag{39}$$

Subject to:

$$x_i \in \{0,1\} \qquad \forall i \tag{40}$$

The second sub-problem (Sub2), which is defined in terms of the x_{0ij}, x_{1ij}, z_i

$y_{1ij}, y_{2ijr}, y_{3ijr}$ binary variables, is given as follows:

$$\begin{aligned} \min W_{sub2} &= \sum_i \sum_j x_{0ij} \times h_{ij} \times d_j - \sum_i \sum_j x_{0ij} \times h_{ij} \times d_j \times p_i \\ &+ \sum_i \sum_j y_{1ij} \times h_{ij} \times d_j \times p_i + \sum_i \sum_j \sum_{r \neq i} y_{2ijr} \times p_r \times h_{ij} \times d_j \\ &- \sum_i \sum_j \sum_{r \neq i} y_{3ijr} \times p_r \times h_{ij} \times d_j + \sum_i z_i \times b_i + \sum_i \sum_j u_{1ij} \times x_{0ij} \\ &+ \sum_i \sum_j u_{2ij} \times x_{1ij} - \sum_i u_{3i} \times \sum_j (x_{0ij} + x_{1ij}) - \sum_i u_{4i} \times \sum_j x_{0ij} \\ &+ \sum_i u_{5i} \times z_i \end{aligned} \quad (41)$$

Subject to: Constraints (5)-(7), (13), (26)-(37),

$$x_{0ij}, x_{1ij}, z_i, y_{1ij}, y_{2ijr}, y_{3ijr} \in \{0,1\} \qquad \forall i, j, r \quad (42)$$

4.2 Lagrangian formulation for RFLA&H,RE model

In order to apply the Lagrangian decomposition for RFLA&H,RE model, our proposed model is modified by adding a new additional binary variable y_{4i} (equivalent to $(x_i \times z_i)$), previous additional variables $y_{1ij}, y_{2ijr}, y_{3ijr}$, and additional Constraints (44)-(50); then, Constraints(2)-(4),(7),(12),(29), and (31) are relaxed in the following formulation:

$$\begin{aligned} \min w_{LR} &= \sum_i a_i \times x_i + \sum_i \sum_j x_{0ij} \times h_{ij} \times d_j - \sum_i \sum_j x_{0ij} \times h_{ij} \times d_j \times \\ &+ \sum_i \sum_j y_{1ij} \times h_{ij} \times d_j \times p_i \times \sum_i \sum_j \sum_{r \neq i} y_{2ijr} \times h_{ij} \times p_r \times m_{rj} \\ &- \sum_i \sum_j \sum_{r \neq i} y_{3ijr} \times h_{ij} \times p_r \times m_{rj} + \sum_i \sum_j x_{0ij} \times h_{ij} \times p_i \times (d_j - m_{ij}) \\ &- \sum_i \sum_j y_{1ij} \times h_{ij} \times p_i \times (d_j - m_{ij}) + \sum_i \sum_j x_{0ij} \times p_i \times m_{ij} \times s_i \quad (43) \\ &- \sum_i \sum_j y_{1ij} \times p_i \times m_{ij} \times s_i + \sum_i x_i \times p_i \times rc_i \times c_i - \sum_i y_{4i} \times p_i \times rc_i \times c_i \\ &+ \sum_i z_i \times b_i + \sum_i \sum_j u_{1ij} \times (x_{0ij} - x_i) + \sum_i \sum_j u_{2ij} \times (x_{1ij} - x_i) \\ &+ \sum_i \sum_j u_{3ij} \times (x_{1ij} - z_i) + \sum_i u_{4i} \times (x_i - \sum_j x_{0ij}) + \sum_i u_{5i} \times (x_i - \sum_j (x_{0ij} + x_1 \\ &+ \sum_i \sum_j u_{6ij} \times (y_{1ij} - z_i) + \sum_i \sum_j u_{7ij} \times (z_i + x_{0ij} - y_{1ij} - 1) \end{aligned}$$

Subject to: Constraints (5),(6),(10),(13),(26),(27),(30), (32)-(37),

$$y_{4i} \le z_i \qquad \forall i \quad (44)$$

$$y_{4i} \le x_i \qquad \forall i \quad (45)$$

$$y_{4i} \ge x_i + z_i - 1 \qquad \forall i \quad (46)$$

$$\sum_j ((x_{0ij} \times d_j) + (\sum_{r \neq i} (y_{2ijr} - y_{3ijr}) \times m_{rj})) \le c_i \qquad \forall i \quad (47)$$

$$\sum_i x_i \times rc_i \times c_i - \sum_i y_{4i} \times rc_i \times c_i \le U \quad (48)$$

$$\sum_i \sum_j x_{0ij} \times m_{ij} \times s_i - \sum_i \sum_j y_{1ij} \times m_{ij} \times s_i \le B \quad (49)$$

$$x_i, x_{0ij}, x_{1ij}, z_i, y_{1ij}, y_{2ijr}, y_{3ijr}, y_{4i} \in \{0,1\} \qquad \forall i, j \quad (50)$$

The above formulation can be decomposed into two separable sub problems, referred to as sub1and sub2, respectively. Sub 1, which is expressed in terms of *x* and *z*, binary variables, is given below:

$$\begin{aligned} \min W_{sub1} &= \sum_i a_i \times x_i + \sum_i x_i \times p_i \times rc_i \times c_i - \sum_i y_{4i} \times p_i \times rc_i \times c_i \\ &+ \sum_i z_i \times b_i - \sum_i \sum_j u_{1ij} x_i - \sum_i \sum_j u_{2ij} \times x_i - \sum_i \sum_j u_{3ij} \times z_i \\ &+ \sum_i u_{4i} \times x_i + \sum_i u_{5i} \times x_i - \sum_i \sum_j u_{6ij} \times z_i - \sum_i \sum_j u_{7ij} \times z_i \end{aligned} \quad (51)$$

Subject to: Constraints (10),(13),(44)-(46),(48),

$$z_i, x_i, y_{4i} \in \{0,1\} \qquad \forall i \quad (52)$$

The second sub-problem (Sub2), which is defined in terms of the binary variables $x_{0ij}, x_{1ij}, y_{1ij}, y_{2ijr}, y_{3ijr}$ only, is given as follows:

$$\begin{aligned} \min W_{sub2} &= \sum_i \sum_j x_{0ij} \times h_{ij} \times d_j - \sum_i \sum_j x_{0ij} \times h_{ij} \times d_j \times p_i \\ &+ \sum_i \sum_j y_{1ij} \times h_{ij} \times d_j \times p_i + \sum_i \sum_j \sum_{r \neq i} y_{2ijr} \times h_{ij} \times p_r \times m_{rj} \\ &- \sum_i \sum_j \sum_{r \neq i} y_{3ijr} \times h_{ij} \times p_r \times m_{rj} + \sum_i \sum_j x_{0ij} \times h_{ij} \times p_i \times (d_j - m_{ij}) \\ &- \sum_i \sum_j y_{1ij} \times h_{ij} \times p_i \times (d_j - m_{ij}) + \sum_i \sum_j x_{0ij} \times p_i \times m_{ij} \times s_i \\ &- \sum_i \sum_j y_{1ij} \times p_i \times m_{ij} \times s_i + \sum_i \sum_j u_{1ij} \times x_{0ij} \\ &+ \sum_i \sum_j u_{2ij} \times x_{1ij} + \sum_i \sum_j u_{3ij} \times x_{1ij} - \sum_i u_{4i} \times \sum_j x_{0ij} \\ &- \sum_i u_{5i} \times \sum_j (x_{0ij} + x_{1ij}) + \sum_i \sum_j u_{6ij} \times y_{1ij} + \sum_i \sum_j u_{7ij} \times (x_{0ij} - y_{1ij} - 1) \end{aligned} \quad (53)$$

Subject to: Constraints (5),(6), (26),(27),(30),(32)-(37),(47),(49),

$$x_{0ij}, x_{1ij}, y_{1ij}, y_{2ijr}, y_{3ijr} \in \{0,1\} \quad (54)$$
$$\forall i, j, r$$

4.3. Sub-gradient optimization

In order to solve the mentioned decomposed problems, a sub-gradient algorithm is used to calculate the Lagrangian multipliers. The sub-gradient optimization is applied by using the following notations and steps.

W_{LB}:	*Best lower bound*
W_{UB}:	*Best upper bound*
$£_M$:	*Max of relative gap*
L:	*Stepsize*
μ:	*Stepsize coefficient*
t	*Number of iterations*
$t_{\max}$	*Max iteration*
ε_q^t	*Violation of relaxed constraint q from sub1, sub2 solutions in iteration t*
IM_0	*Max iteration without improvement*

Step 0: Initialize the parameters (u_q , μ , t_{Max} , IM_0 , $£_M$)

1. Set $W_{LB} \longrightarrow -\infty$ and $W_{UB} \longrightarrow +\infty$

Step 1:

1. Solve *sub1*
2. Solve *sub2*
3. Set $W^*_{LR} = W^*_{sub1} + W^*_{sub2}$
4. If $W^*_{LR} \geq W_{LB}$ Set $W_{LB} = W^*_{LR}$
   Otherwise $IM = IM+1$
5. Fix the *sub1* solution in the main problem and get (W^*)
6. If $(W^* < W_{UB}) \rightarrow$ set $W_{UB} = W^*$
7. Set $£ = \frac{(W_{UB} - W_{LB})}{W_{LB}}$

Step 2:

1. If ($t \geq t_{\max}$ Or $£ \leq £_M$) stop
   Otherwise, go to the next step

Step 3:

1. If $IM \geq IM_0$, then μ=μ/2
2. *set* $L = \mu \times \frac{(W_{UB} - W_{LB})}{\sum_q (\varepsilon^t{}_q)^2}$
3. Set $u_q{}^{t+1} = Max\ (0, u_q^t + (L \times \varepsilon_q^t))$
4. Set $t \longleftarrow t+1$ and return to $Step_1$

5. Result

Some numerical analyses from different aspects are described in two parts. In the first one, the validity of the proposed mathematical models is evaluated. In the second one, the performance of the proposed model is compared with that of the classic model in different failure probabilities. Then, in the next part, the efficiency of the proposed solution algorithm (LDA) is considered in terms of both solution quality and computational time.

5.1. Sensitivity analysis

In order to analyze the validity of the proposed models, some instances with different sizes are considered. The small-sized one comprises 7 facilities and 5 customers. For the mentioned instance, different parameters (e.g., capacity, facility construction cost, facilities failure probability, facility hardening cost, recovery time, recovery cost, and penalty cost) are analyzed individually on the model behavior. Then, the effect of simultaneous changes of these parameters is analyzed, too. The results are reported in Tables 3 to 11.

Tab. 3. Analysis of capacity effect on the model's performance

Model	Capacity		
	300	600	800
RFLA&H	(0,0,0,1,1,0,1)	(0,0,0,0,0,1,1)	(0,0,0,1,0,0,0)
RFLA&RE	(1,0,1,0,1,0,0)	(1,0,1,0,0,0,0)	(0,1,0,0,0,0,0,)
RFLA&H,RE	(1,1,0,0,1,0,0,)	(1,0,0,0,1,0,0)	(1,0,0,0,0,0,0)
Number of opened facilities	3	2	1

Tab. 4. Effect of construction cost on the model's behavior

Model	a_i	x_i
RFLA&H	(100,200,200,200,100,200,200)	(1,0,0,1,1,0,0)
RFLA&RE	(100,200,100,200,100,200,200)	(1,0,1,0,1,0,0)
RFLA&H,RE	(200,200,200,100,100,100,200)	(0,0,0,1,1,1,0)

Tab. 5. Analysis of facility failure probability on the model's performance

Model	p_i	x_i
RFLA&H	(0.001,0.3,0.6,0.001,0.5,0.1,0.4	(1,0,0,1,0,1,0)
RFLA&RE	(0.001,0.3,0.6,0.001,0.5,0.1,0.4)	(1,0,0,1,0,1,0)
RFLA&H,RE	(0.001,0.3,0.6,0.001,0.5,0.1,0.4)	(1,0,0,1,0,0,1)

Tab. 6 Analysis of hardening cost on the behavior of three models

Model	b_i	x_i
RFLA&H	(270,100,150,180,120,300,300)	(0,1,1,0,1,0,0)
RFLA&RE	--------	--------
RFLA&H,RE	(110,100,150,180,120,300,300)	(0,1,1,0,1,0,0)

Tab. 7. Analysis of recovery time on the behavior of three models

Model	tr_i	x_i
RFLA&H	-----	-----
RFLA&RE	(2,6,8,5,1,3,6)	(1,0,0,0,1,0,0)
RFLA&H,RE	(4,6,10,2,1,8,6)	(0,0,0,1,1,0,0)

Tab. 8. Analysis of recovery cost on the behavior of three models

Model	rc_i	x_i
RFLA&H	-----	-----
RFLA&RE	(10,5,2,9,4,3,7)	(0,0,1,0,0,1,0)
RFLA&H,RE	(10,8,2,1,4,5,7)	(0,0,0,1,1,0,0)

Tab. 9. Analysis of penalty cost on the behavior of three models

Model	s_i	x_i
RFLA&H	-----	-----
RFLA&RE	(10,3,5,9,1,2,6)	(0,0,0,0,1,1,0)
RFLA&H,RE	(4,10,7,9,11,5,2)	(1,0,0,0,0,0,1)

Tab. 10. Pair wise analysis of hardening cost, recovery time and recovery cost on the model's performance

Model	(b_i, tr_i, x_i)	(c_i, rc_i, x_i)
	-----	(600,300,500,300,600,300,400)
RFLA&R	--------------	(10,10,20,40,10,30,10)
		(0,1,0,0,1,0,0)
	(10,60,15,50,60,20,20)	(600,300,500,300,280,300,400)
RFLA&H,	(6,1,7,2,1,5,5)	(10,13,20,40,15,30,10)
	(1,1,1,0,0,0,0)	(0,1,0,0,0,0,1)

Tab. 11. Pair wise analysis of failure probability, hardening cost, and recovery time on the models' performance

Model	(p_i, b_i, x_i)	(p_i, tr_i, x_i)
	(0.9.0.1,0.001,0.0001,.01,0.9,0.9)	
RFLA&H	(25,25,15,15,15,30,25)	---------------
	(0,0,1,1,1,0,0)	
		(0.95,0.001,0.99,0.1,0.9,0.1,0.008)
RFLA&RE	----------------	(2,9,1,4,3,6,10)
		(0,1,1,0,0,0,1)
	(0.0001,0.9,0.0001,0.00001,0.99,0.01	(0.0001,0.1,0.0001,0.99,0.8,0.001,0.02)
RFLA&H,	(30,70,30,90,60,30,30)	(10,6,5,1,3,2,9)
	(0,0,1,1,0,0,0,)	(0,0,0,1,0,1,0)

5.2. Comparison of networks designed of the proposed model and the classic network

To compare the behavior of the proposed models with that of the classic model, which is called CFLA model, this paper compares their objective function values in different failure probability conditions. Moreover, they are compared in a condition where the hardening cost is less than the recovery and penalty costs (i.e., case 1), in a condition with the similar hardening, recovery, and penalty costs (i.e., case 2), and finally in a condition that we impose with a higher hardening cost rather than others (i.e., case 3). The results are reported in Figures (1)-(3), respectively.

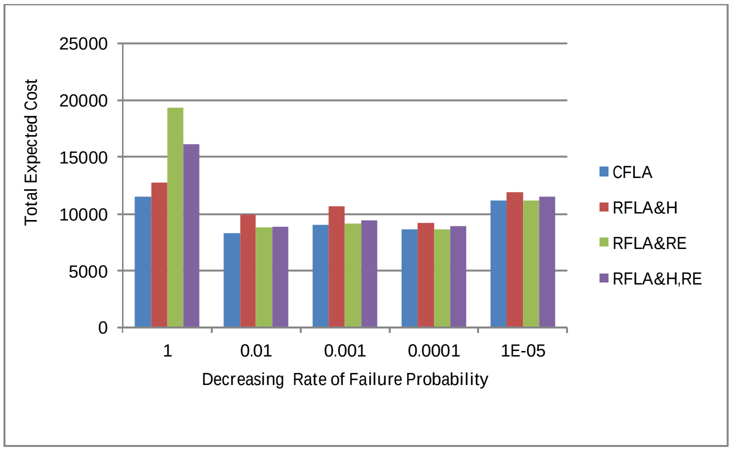

Fig. 1. Comparison of objective function values of CFLA, RFLA&H, RFLA&RE, RFLA&H,RE models in Case 1

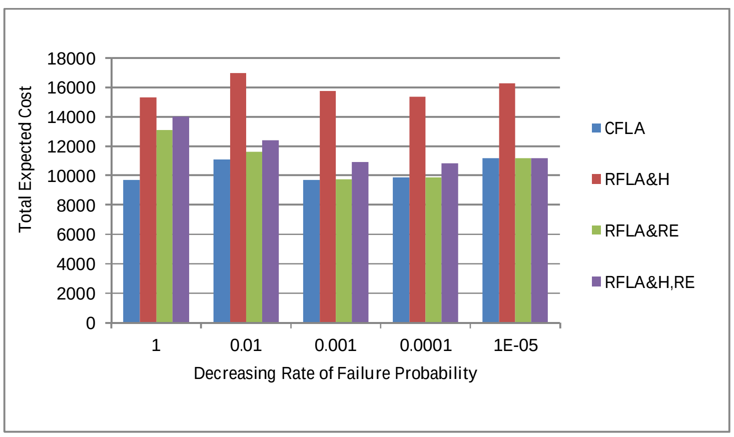

Fig. 2. Comparison of objective function values of CFLA, RFLA&H, RFLA&RE, RFLA&H,RE models in Case 2

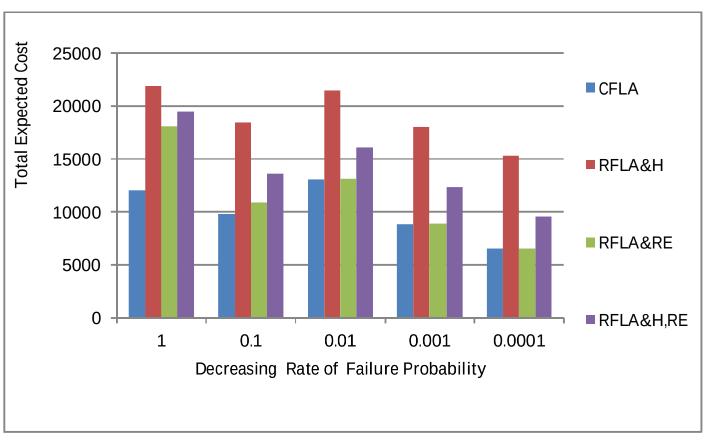

Fig. 3. Comparison of objective function values of CFLA, RFLA&H, RFLA&RE, RFLA&H,RE models in Case 3

The results shown in Tables (3) to (9) confirm that the proposed models are valid. These results show that all the models tend to open facilities with a smaller number of the mentioned parameters in variation of the facility failure probability, construction cost, recovery time, hardening cost, penalty cost, and recovery cost. Moreover, it can be concluded that a smaller number of facilities will be opened by increasing the facilities capacity. Tables (10) and (11) show a pairwise analysis of the failure probability, hardening cost, recovery time, and recovery cost on the models' performance.

The concurrent consideration of the failure probability and hardening cost in RFLA&H, RFLA&H,RE models shows that the models tend to open facilities with the lowest hardening cost, and if there is a need to open extra facilities because of capacity limitations, models will open facilities with a lower failure probability to decrease the total hardening expenses. Simultaneous consideration of the failure probability and recovery time in the RFLA&RE model shows that facilities with lower failure probability have a greater chance to be opened because of the existing penalty cost. In contrast, the RFLA&H,RE model tends to open facilities with lower recovery time. By considering the hardening cost and recovery time simultaneously, the RFLA&H,RE model tends to open facilities with lower hardening cost; then, if opening more facilities is needed, the model establishes facilities with a shorter recovery time among others. Finally, simultaneous consideration of the capacity and recovery cost confirms that the RFLA&RE model tends to open facilities with the lowest total recovery cost; however, the REFLA&H,RE model tends to open facilities with the lower recovery cost per unit of the lost capacity. The mentioned studies confirm that the proposed models are valid corresponding to their expectation. For more analysis, the problem in medium- and large-sized instances and the results of our analysis confirm the validity of the proposed models.

The results of the comparison of the designed networks of the proposed models and the classic network showed the following:

- Where the hardening costs of facility are high, the RFLA&RE and RFLA&H,RE models are more appropriate, respectively, because they propose a reliable network with lower additional cost than the classical network.
- While the hardening costs of facility are low, in the high facility failure probabilities, the RFLA&H and the RFLA&H,RE models are more appropriate; however, in the low facility failure probabilities, the RFLA&RE and the RFLA&H,RE models propose a secure network with very low additional cost over the classic network.

As for managerial implications, it can be noted that these proposed models can be used in the supply chain network design to decrease the negative effects of disruptions.

5.3. Lagrangian decomposition algorithm computational results

In this section, in order to compare the exact solution approach with Lagrangian decomposition algorithm, some instances with different sizes from small to large sizes are considered. The results are reported in Tables (12), (13), and Figures (4)-(7), respectively. The exact solution approach, which is the binary integer programming model, was coded in GAMS and solved by CPLEX solver.

Tab. 12. The computational results of LDA for the RFLA&H model

Instance	(F-C)	CPLEX	LB	UB	RG	CPU(s) LDA	CPU(s) CPLEX
1	(7-12)	14325.100	14299.600	14415.200	0.008	1.3	2.5
2	(9-15)	18101	18099.700	18101.000	0.000	0.03	0.07
3	(10-17)	17501.000	17431.895	17501.000	0.004	31.1	143
4	(12-20)	21101.000	21019.895	21101.000	0.004	47	1001
5	(15-25)	28625.100	28574.600	28625.100	0.002	39.4	189
6	(17-30)	34901.000	34779.895	34901.000	0.003	877	1001
7	(20-35)	39625.100	39553.600	39625.100	0.002	126	1002
8	(23-40)	46901.000	46739.895	46901.000	0.003	126	1002

F: Number of facilities, C: Number of customers, LB: Lower bound of LDA, UB: Upper bound of LDA, RG: Relative gap, CPU(s): Computational time

Tab. 13. The computational results for the RFLA&H,RE model

Instance	(F-C)	CPLEX	LB	UB	RG	CPU(s) LDA	CPU(s) CPLEX
1	(7-12)	14450.200	14295.800	14549.800	0.018	0.85	0.58
2	(9-15)	17750.200	17568.100	19225.100	0.094	1.29	1.56
3	(10-17)	19950.200	19747.100	19950.200	0.010	1.67	2.84
4	(12-20)	23250.200	23017.800	23474.800	0.020	3.33	7.34
5	(15-25)	28750.200	28495.800	28974.800	0.017	7.405	1000.385
6	(17-30)	34250.200	33969.300	34474.800	0.015	19.796	1000.951
7	(20-35)	39725.700	39430.800	39926.300	0.013	127.572	865.313
8	(23-40)	45225.700	44902.700	45426.300	0.012	44.057	808.916
9	(30-50)	56150.200	55849.100	56426.300	0.010	115.753	1004.388

F: Number of facilities, C: Number of customers, LB: Lower bound of LDA, UB: Upper bound of LDA, RG: Relative gap, CPU(s): Computational time

The results presented in Tables (12), (13) show that, in both RFLA&H and RFLA&H,RE models, the Lagrangian Decomposition solutions are closer to their optimal solutions. In addition, in RFLA&H,RE and RFLA&H models, the Lagrangian Decomposition generates a very good near-optimal solutions in a shorter computational time than the exact solution.

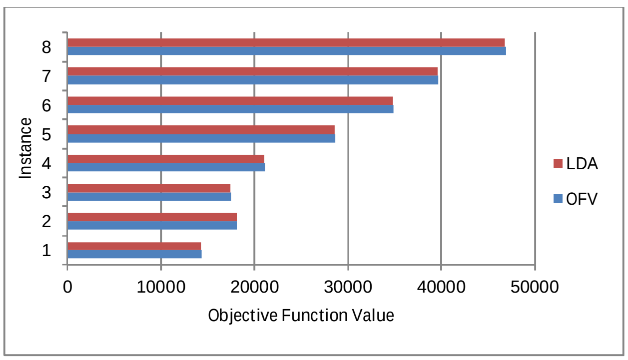

Fig. 4. Comparison of objective function values of exact and LDA solutions (LB) in RFLA&H model

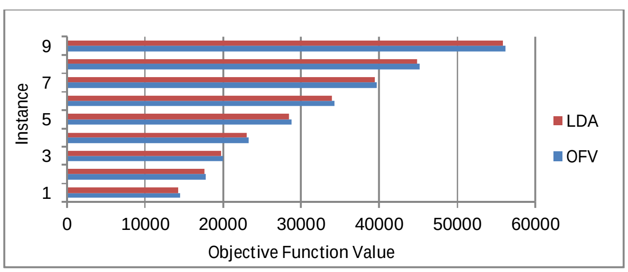

Fig. 5. Comparison of objective function values of exact and LDA solutions (LB) in RFLA&H,RE model

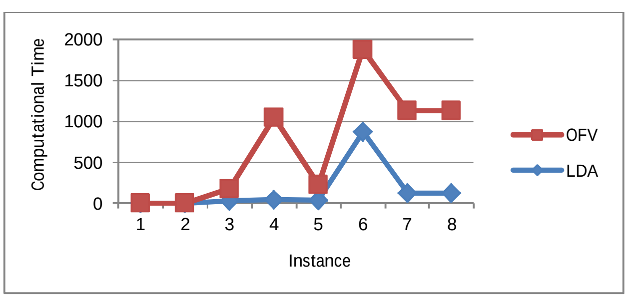

Fig. 6. Comparison of the computational time of exact and LDA solutions (LB) in RFLA&H model

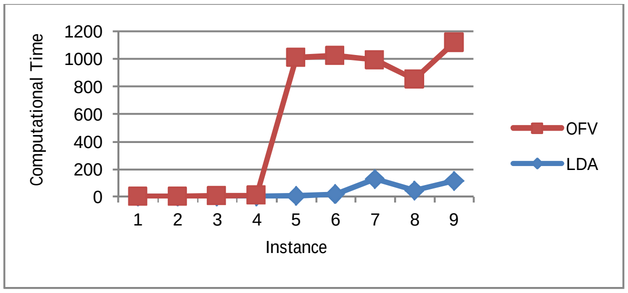

Fig. 7. Comparison of computational time of exact and LDA solutions (LB) in RFLA&H,RE model

6. Conclusions

Today, many supply chain networks face disruptions and other unexpected events throughout their chain. Although some of these disruptions are short-term, they can lead to serious negative financial and operational consequences. It should be noted that, due to the inability of the classical design networks against disruption, this problem should be considered when designing these networks. Designing a resilient network is one of the disruption management strategies that has been considered for this purpose during design of supply networks, demonstrating the ability of the network to conserve operations and network connections despite the lack of some parts of the chain. This paper considered the capacitated fixed charge facility location-allocation problem in the supply network design in a random disruption condition. For this purpose, three integer programming models with different facility abilities were proposed that minimized the total cost and the expected transportation cost after facility failures. In the first model, the hardening ability was considered, while, in the second one, the resilience ability was investigated; moreover, in the last one, the hardening and resilience abilities were considered together. In order to analyze the validity of the presented models, this study considered some instances of different sizes. The results of the analysis confirmed the validity of the proposed models. The comparison of these models and CFLA showed that the network designed by the CFLA model was not reliable; however, in the lower facility failure probabilities, the system reliability could have increased by very low additional investment by adding the hardening or resilience abilities to the facilities, while, in the higher failure probabilities, to ensure a reliable network, the difference between the cost of the proposed model and that of the CFLA model would increase. Computational results of the presented solution approach confirm that the Lagrangian decomposition algorithm in RFLA&H and RFLA&H,RE models has a good ability to generate very near-optimal solutions to the exact ones in a shorter computational time. Consideration of uncertainty of customer demands can be a new direction for the related future study. Additionally, considering the possibility of allocating more than one facility to a customer as a primary or backup assignment can be as another future work.

References

[1] Powell ,R.," Defending against terrorist attacks with limited resources," American Political Sciece Review, Vol. 101, (2007), pp. 527-541.

[2] Amoaning, y., "A resiliency framework for planning in state transportation agencies,". Georgia Institute of Technology, (2013).

[3] Snyder, L., “Supply chain robustness and reliability: Models and algorithms,” Cite seer, (2003).

[4] Snyder, L., Daskin, M., "Reliability models for facility location: the expected failure cost case,"Transcience, Vol. 39, (2005), pp. 400-416.

[5] Aboolian, R., Cui, T., Max, S., Zuo, J.,” An efficient approach for solving reliable facility location models,” Informs Journal on Computing, Vol. 25, (2012), pp.720-729.

[6] Berman, O., Dmitry, K., Mozart, M., “Locating Facilities in the Presence of Disruptions and Incomplete Information,” Decision Scienc, Vol. 40, (2009), pp. 845-868.

[7] Li, X., "A continuum approximation approach to reliable facility location design under correlated probabilistic disruptions," Transportation Research Part B Methodological, Vol. 44, No. 4, (2010), pp. 535-548.

[8] Chen, Q., Xiaopeng, L., Ouyang, Y.," Joint inventory-location problem under the risk of probabilistic facility disruptions," Transportation Research Part B Methodological, Vol. 45, No. 7, (2011), pp. 991-1003.

[9] Losada, C., Scaparra, M.P., Jesse, R.O.H.," Optimizing system resilience: a facility protection model with recovery time," European Journal of Operational Research, Vol. 217, No. 3, (2012), pp. 519-530.

[10] Medal, H.R., Edward, A.P., Rossetti, M.D., “A multi-objective integrated facility location-hardening model: Analyzing the pre-and post-disruption tradeoff,” European Journal of Operational Research, Vol. 237, No. 1, (2014), pp. 257-270.

[11] Aksen, D., Piyade, N., Necati, A., "The budget constrained r-interdiction median problem with capacity expansion," Central European Journal of Operational Research, Vol.18, No. 3, (2010), pp. 269-291.

[12] Lim, M., Daskin, M.S., Bassamboo, A., Chopra, S**.,”** A Facility Reliability Problem: Formulation, Properties, and Algorithm,” Naval Research Logistic, Vol. 57, (2010), pp. 58-70.

[13] Shishebori, D., Snyder, L.V., Jabalameli, M.S.," A reliable budget-constrained facility location/network design problem with unreliable facilities," Networks and Spatial Economics, Vol. 14, (2012), pp. 549-580.

[14] Azad, N., Davoudpour, H., Saharidis, G.K.D., Shiripour, M.," A new model to mitigating random disruption risks of facility and transportation in supply chain network design,"International Journal of Advance Manuf acturing Technology, Vol. 70, (2014_, pp. 1757-1774.

[15] Jabbarzadeh, A., Fahimnia, B., Sheu, J.B., Shahmoradi Moghadam, H., ” Designing a supply chain resilient to major disruptions and supply/demand interruptions," Transportation Research Part B, Vol. 94, (2016), PP. 121-149.

[16] Fattahi, M., Govindan, K., Keyvanshokooh, E., ”Responsive and resilient supply chain network design under operational and disruption risks with delivery lead-time sensitive customers,"Transportation Research Part E, Vol. 101, (2017), pp. 176-200.

[17] Li, Q., Zeng, B., Savachkin, A.,” Reliable facility location design under disruptions,” Computers and Operations Research, Vol. 40, No. 4, (2013), pp. 901-909.

[18] Li, Q., Savachkin, A.,” Reliable distribution networks design with nonlinear fortification function,”International Journal of Systems Science (ahead-of-print), (2014), pp. 805-813.

[19] Ratick, S., Brian, M., Yuko, A.,” Locating backup facilities to enhance supply chain disaster resilience,” Growth and Change, Vol. 39, No. 4, (2008), pp. 642-666.

[20] Seifbarghy, M., Jalali, S., Rahmati, S.H.," Distribution Centers Reliability Cost in Capacitated Facility Location Problem,"World Academy of Science, Engineering and Technology, Vol. 6, (2012), pp. 1-26.

[21] Klibi, W., Martel, A.,” Modeling approaches for the design of resilient supply networks under disruptions,” International Journal of Production Economics, Vol. 135, No. 2, (2012), pp. 882-898.

[22] Ghomi-Avili, M., Makui, A., Naghibi, S.B., Yadegari, E.,” Facility Location with the Disruption Risk in the Supply Chain Network Design,” Journal of Basic and Applied Scientific Research, Vol. 3, No. 7, (2013), pp. 898-910.

[23] Wang, X., Yanfeng, O.,” A continuum approximation approach to competitive facility location design under facility disruption risks,”Transportation Research Part B Methodological, Vol. 50, (2013), pp. 90-103.

[24] Puerto, J., Federica, R., Scozzari, A., "Unreliable point facility location problems on networks,"Discrete Applied Mathematics, Vol. 166, (2014), pp. 188-203.

Follow This Article at The Following Site:

Esfandiari Z, Bashiri M, Tavakkoli-Moghaddam R. Analysis of the Hardening and Resilience Ability in Location-Allocation Problems. IJIEPR. 2020; 31 (1) :35-50
URL: http://ijiepr.iust.ac.ir/article-1-841-en.html