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RESEARCH PAPER

A Fuzzy Goal Programming for Dynamic Cell Formation and Production Planning Problem Together with Pricing and Advertising Decisions

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ABSTRACT
In this paper, an integrated mathematical model of the dynamic cell formation and production planning, considering the pricing and advertising decision, is proposed. This paper puts emphasis on the effect of demand aspects (e.g., pricing and advertising decisions) along with the supply aspects (e.g., reconfiguration, inventory, backorder, and outsourcing decisions) in the developed model. Due to the imprecise and fuzzy nature of input data such as unit costs, capacities and processing times in practice, a fuzzy multi-objective programming model is proposed to determine the optimal demand and supply variables simultaneously. For this purpose, a fuzzy goal programming method is applied to solve the equivalent defuzzified multi-objective model. The objective functions are to maximize the total profit for firms and maximize the utilization rate of machine capacity. The proposed model and solution method are verified by a numerical example.

KEYWORDS: *Dynamic cell formation, Production planning, Fuzzy goal programming, Pricing and advertising, Multi-objective model.*

1. Introduction

While many companies that are using the cellular manufacturing system (CMS) focus on supply, it is essential to consider the demand side as tools to improve profit and efficiency. The supply sides in a CMS are categorized into cell formation (CF), intra-cell and inter-cell layout, group scheduling, and resource allocation decisions; in addition, the demand sides include pricing and advertising. Thus, for the manufacturing industries with CMS, the coordination of demand aspects can be not only useful but also essential. The integration of pricing, advertising, and supply side decisions is still in its early stages in many firms, but has the potential to basically improve supply efficiency.

With the high variability of demand mix and volume for different periods in a horizon planning, the manufacturer should have an optimal plan to minimize the costs related to dynamic cell formation problem (DCFP). The objective of DCFP is to divide the machines into distinctive cells as machine groups and categorize similar parts in terms of manufacturing, design, etc. as part families so as to assign them to the cells for each period. In DCFP, the reconfigurations of manufacturing cells for each period can be a good solution in order to deal with fluctuating and dynamic demands. The reconfiguration consists of relocating existing machines in a cellular system, adding new machines to cells, and removing existing machines from cells. However, the optimal solution for DCFP in a current period may not be optimal for the next period. An example of the reconfiguration of manufacturing cells in two successive periods is represented in Figure 1.

In each company, both pricing and advertising and reconfiguration decisions need to be made. Pricing and advertising decisions are aimed at controlling the demand side, while reconfigurations for DCFP decisions are used to control the supply side. The central problem is to optimally integrate the demand and supply decisions.

In an integrated pricing/advertising and DCFP, to determine the price of each product and

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advertising cost in each period; to generate machine cell and job family in each period, inventory and backorder level at the end of each period, the outsourced quantity of products in each period, adding and removing and relocating the machines for each period are the most important decisions. These activities are often conducted either individually or sequentially with the poor overall performance for the whole industrial environments, resulting in deficiencies and disadvantages. Therefore, to ensure a better environment, the integration of both demand and supply aspects in an integrated manufacturing environment is of particular interest. In addition, in real life, most of the input data and related parameters are not known with certainty because of incompleteness and/or unavailability of required data over the planning horizon. Moreover, often the decision-maker cannot fit some probability distribution for uncertain parameters with certainty. Therefore, the critical data cannot be represented in a deterministic or stochastic formulation and, as a result, the corresponding optimal results may not serve the real purpose of modeling.

The aim of this paper is to propose an integrated pricing/advertising, reconfiguration, inventory, backorder and outsourcing for a manufacturing factory using a CMS in a fuzzy environment. The two objective functions that include maximizing total profit and minimizing the total utilization rate of machine capacity variation together with a number of constraints are jointly considered in this paper. Accordingly, this paper presents a fuzzy multi-objective programming method to capture the inherent fuzziness in the parameters and objectives. In summary, this paper covers the following contributions.

- To present a novel bi-objective integrated non-linear mixed-integer mathematical model for CMS design considering the demand management by pricing and advertising.
- To obtain the optimal price and advertising cost for products together with cell design and production planning.
- To consider two objectives for the developed model and use the goal programming.
- To present the imprecise parameters by fuzzy concept and develop a fuzzy programming.
- To demonstrate the applicability of the proposed model through generated data.

The structure of this paper is as follows. Section 2 includes a review of the relevant literature. Section 3 describes the problem clearly and, then, formulates it as an MINLP model in Section 4. Next, several numerical experiments are presented, and the analysis of the obtained results is carried out. Section 5 concludes the paper with final remarks.

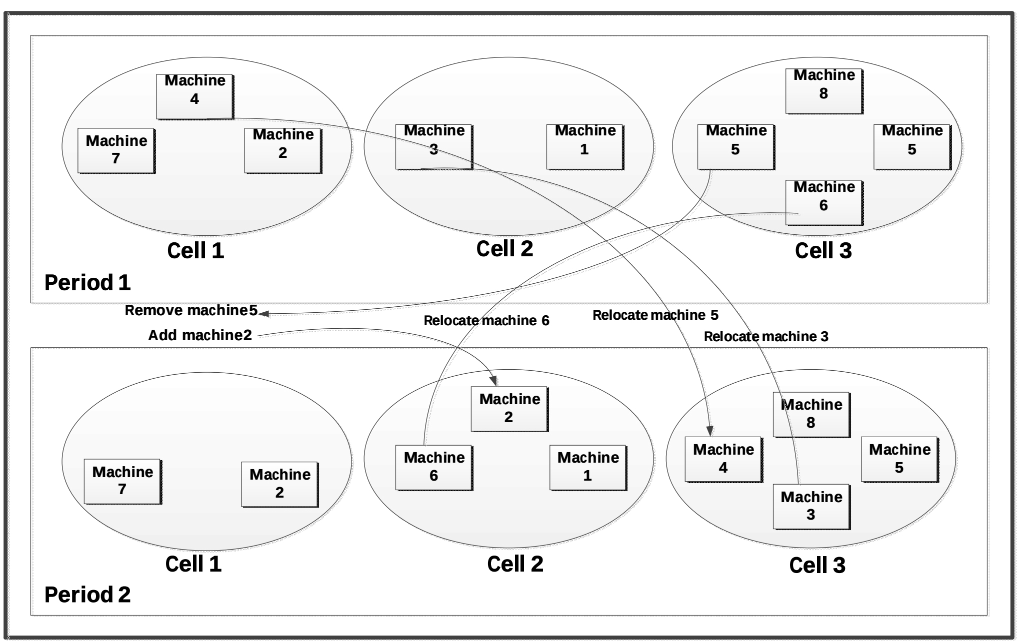

Fig. 1. An example of the reconfiguration of manufacturing cells in two successive periods.

2. Literature Review

Although many papers on CMS are published all over the world every year such as Solimanpur et al. [1], Tavokkoli-Moghaddam et al. [2], Spiliopoulos and Sofianopoulou [3], Wu et al. [4], Nalluri et al. [5], Behnia et al. [6], Fattahi and Ismailnezhad [7], etc., different studies have been done in the field of DCFP in recent years. The dynamic cell formation problem (DCFP) was introduced by Rheault et al. [8] for the first time. After their paper, a growing interest of research has been observed in developing models and solution procedures for DCFP. For example, Chen [9] developed an integer mathematical model for DCFP in which the objective function was aimed at minimizing material handling and machine costs and reducing cell reconfiguration cost. Since the developed model was NP-hard, a decomposition approach was developed to solve the sub-problems with less computational effort and a dynamic programming was then employed to find a solution to the main problem.

In another paper, Taboun et al. [10] developed a mathematical model that includes the subcontracting concept for the first time. They solved the model by a heuristic approach that is based on the criteria of maximum cell similarity and minimum number of machines. They concluded that the proposed heuristic was a powerful procedure in order to achieve near-optimal solutions.

Balakrishnan and Cheng [11] designed a flexible framework for modeling cellular manufacturing in a DCFP. In order to deal with dynamic demands, they used two stages where the optimal cell configuration was obtained in a static environment in the first stage and the dynamic programming was applied in the second stage using the optimal material handling cost of the first stage to obtain an optimal solution in dynamic conditions. They indicated that with an increase in the reconfiguration cost, the job shop may be preferred to the dynamic cellular manufacturing system.

Tavakkoli-Moghaddam et al. [12] presented a mathematical model for DCFP with demands being dynamic, but they are deterministic too. They added alternative process plans (routing flexibility) and a variable number of cell concepts within the DCFP model for the first time. They proposed a memetic algorithm to find near-optimal solutions in a reasonable amount of CPU time.

In another paper written by Tavakkoli-Moghaddam et al. [13], they used the same previous model used in [14] by appending the operating cost to each machine. They used the genetic algorithm (GA), simulated annealing (SA), and tabu search (TS) for solving the non-linear model, and found that SA outperformed the two other meta-heuristic approaches in both quality of solutions and computational times in most of the test problems.

In a relatively more comprehensive paper, Defersha and Chen [14] considered a model that incorporates dynamic cell configuration, alternative routing, sequence of operations, multiple units of identical machines, machine capacity, workload balancing among cells, operation cost, subcontracting cost, tool consumption cost, setup cost, and other practical constraints. The authors developed a genetic-based heuristic algorithm and compared its results with lower bound determined by LINGO. The results of the three examples showed that heuristic was a proper approach as a solution procedure for solving this problem.

Defersha and Chen [15] developed their previous DCFP model, in which lot splitting and machine adjacency constraints were inserted. In other words, two concepts are used in this paper: 1) large orders can be divided into smaller batches to provide an opportunity for simultaneous processing of orders to more than one work center (lot splitting) and 2) a set of machine pairs should be placed in the same cell (machine adjacency).

Saidi-Mehrabad and Safaei [16] applied a neural network approach (NNA) for DCFP, in which the number of formed cells is a decision variable in the proposed model. With the comparison of results in NAA and optimum solutions of LINGO, they claimed that NNA could be a reliable and powerful method.

The human issues such as hiring, firing, and training can be integrated into DCFP. In this way, a new model for the simultaneous work assignment and DCFP was found by Aryanezhad et al. [17]. They considered both machine level and skill level in their new model. Moreover, the demands were both dynamic and deterministic. The single-objective nonlinear integer model was transformed into a linear one. Eventually, the developed integrated model was compared with a separate model.

The multi-objective models also have been presented for DCFP in literature. In this section, a paper done by Wang et al. [18] is the first paper that has considered a three-objective model with the objective functions of minimizing the relocation costs, maximizing the utilization rate of machine capacity, and minimizing the total number of inter-cell moves. They applied Scatter Search (SS) method, since the DCFP was a hard problem and that the SS showed satisfactory results regarding the percentage gaps and execution time even for large-scale test problems. Bajestani et al. [19] developed a two-objective model, where the first objective is to minimize costs like machine depreciation, inter-cell material handling, and machine relocation cost; the second objective is to minimize the total cell load variation. A multi-objective scatter search (MOSS) is designed and compared with two salient multi-objective genetic algorithms, i.e., SPEA-II and NSGA-II, based on some comparison metrics and statistical approach. The experimental results imply that MOSS is superior to two other approaches.

Furthermore, Ghotboddini et al. [20] developed a two-objective model that consists of minimizing the sum of miscellaneous costs and maximizing the sum of minimum labor ratio for the entire periods. They used GAMS to validate the model and the benders decomposition approach to solve large-sized test problems with an exact method.

Some authors have taken into consideration the integrated model of DCFP and production planning in their researches. In other words, they formulated an integrated model that considers the inventory, production, outsourcing levels, and costs together with parameters and variables of DCFP. In this group, Bulgak and Bektas [21] considered an integrated approach to CMS design, where production planning and system reconfiguration decision were incorporated. The proposed mixed-integer non-linear model was converted into a linearized mixed-integer programming formulation. The computational results of the selected dataset existing in literature solved by CPLEX for small- and medium-sized are reported. Safaei and Tavakkoli-Moghaddam [22] also proposed production planning and DCFP simultaneously. They added inter- and intra-cell material handling by assuming the operation sequence and, also, partial subcontracting by assuming a lead time for ordered items. The performance of their model was verified by two numerical examples. The results showed that inventory, subcontracting, and backorder could significantly affect the cell configuration throughout the horizon planning. Khaksar-Haghani et al. [23] presented a model for designing DCFP and production planning, considering the inflation rate and budget constraints.

A comprehensive model including DCFP, production planning, and worker assignment problem was presented by Mahdavi et al. [124]. To illustrate the validity of the proposed model, two examples were solved by the branch-and-bound method using Lingo 8.0 Software. CPU time required to reach the optimal solution for relatively large-sized problems was computationally intractable. For this reason, they suggested a heuristic or meta-heuristic approach to solve the developed model for large-sized examples. Saxena and Jain [25] incorporated important manufacturing attributes such as machine breakdown effect, production planning, transfer batch size for both intra-cell and inter-cell travels, lot splitting, alternative process plan, and so on.

In the multi-objective integrated model of DCFP and production planning, Javadian et al. [26] considered two objectives in their model. The first objective is to minimize a specific set of costs and the second is to minimize total cell load variation. They solved this complex problem in small- and large-sized examples by Lingo 9 and NSGAII approach, respectively.

In some papers, it is observed that the supply chain system design has been integrated into DCFP. For example, Saxena and Jain [27] proposed an integrated model of dynamic cellular manufacturing and supply chain design with respect to different issues such as multi-plant locations, multiple markets, multi-time periods, reconfiguration, etc. For solving the model, they proposed two solution procedures, artificial immune system (AIS), and hybrid artificial immune system (HAIS) algorithm. The results showed that HAIS generally outperformed the LINGO and AIS approaches.

Some authors have worked on layout issues and DCFP, simultaneously. Kia et al. [28] presented a group layout design model of the dynamic cellular manufacturing system with alternative process routing, lot splitting, and flexible reconfiguration. They assumed that there were a number of locations in each cell, where the machines should be assigned to them. This model can be very complex, which motivated Kia et al. to develop a simulated annealing (SA) algorithm with a straightforward, but effective solution structure and neighborhood generation mechanism. Finally, they compared the performance of SA with that of Lingo software for several test problems taken from the literature. Bagheri and Bashiri [29] also considered a DCFP with the inter-cell layout problem and worker assignment in a dynamic environment. They assumed that there were a number of candidate locations to be a manufacturing cell. This non-linear model is converted into a linear model, and an LP-metric approach is applied for solving it. Golmohammadi et al. [30] presented a nonlinear mathematical model with cell layout and machine reliability.

Moreover, there are papers with uncertainty in parameters. For example, Safaei et al. [31] used a fuzzy programming approach to a DCFP with uncertain demands and available machine capacity. The uncertain parameters were given a piecewise fuzzy number. The main deficiency of the developed fuzzy approach is its intensively increase in computational efforts when the problem size increases. Thus, they devised a heuristic or meta-heuristic method to solve the maximizing decision problem of large-sized problems. Arzi et al. [32] also studied the effect of lumpy demand on DCFP. They modeled the weekly lumpy demand as a random variable from a normal distribution. Notably, the estimates of the distribution parameters and the correlation parts were derived from historical weekly demand. They inserted the expected value and variance of demands into the proposed model and made endeavor to reduce the capacity requirement variability within each cell as a part of the cell design. Moreover, a genetic algorithm was proposed and examined for designing large-scale systems. An integrated DCFP and production planning with unreliable machines was given by Sakhaii et al. [33]. They solved this integrated mixed-integer linear programming using the robust optimization approach.

Renna and Ambrico [34] proposed three mathematical models to support the design, reconfiguration, and scheduling of the manufacturing system. They used a simulation to investigate the DCFP with reconfigurable machines.

Bootaki et al. [35] studied a bi-objective robust design for DCFP with fuzzy random demands. They developed a new goal programming method, namely percentage multi-choice goal programming (PMCGP), to solve their bi-objective model. In their paper, the first objective function is to minimize the inter-cell movements, and the second one is to maximize the machine and worker utilization.

Niakan et al. [36] proposed a multi-objective mathematical model with demand and cost uncertainty. The contribution of their work is to consider social criteria such as potential machine hazards and job opportunities in their developed model.

Moreover, Zohrevand et al. [37] developed a bi-objective stochastic model. The first objective function of their developed model was to minimize total costs, while the second objective function maximizes the labor utilization of cellular manufacturing system. They considered the demand for various periods to be uncertain.

Shirzadi et al. [38] investigated the reliability of processing routes in their paper. This concept was inserted in the second objective function. They applied ε-constraint to demonstrate the conflict between two objectives of maximum value of system reliability and the total costs of the system. For large-sized problems, they presented a multi-objective imperialist competitive algorithm (MOICA) and NSGAII.

Azadeh et al. [39] presented a new three-objective model with considering human factors. They used NSGA-II and multi-objective particle swarm optimization (MOPSO) to solve their model.

To the best of our knowledge, there is no research considering the DCFP together with production planning and pricing and advertising in a fuzzy environment. Hence, this study develops a comprehensive fuzzy multi-objective model for a manufacturing company that faces dynamic demand in various periods to determine the optimal cell configuration, inventory and backorder level, and price and advertising cost in each period.

The main contributions of this paper can be summarized as follows. First, it introduces a fuzzy multi-objective model for making the demand decisions such as pricing and advertising along with the supply decisions. Second, it optimized two important objective functions that include total profit and utilization rate of machine capacity. To consider these two objective functions can improve the efficiency of companies. Third, many practical situations involve uncertain input data. This paper presents most of data as a fuzzy number. Table 1 shows the features of some corresponding papers. As shown in this table, our model has the following contributions in which these contributions have rarely appeared in the literature.

- To consider DCFP, inventory, backorder, and outsourcing simultaneously.
- To investigate a two-objective uncertain environment.
- To insert the demand side decisions together with the supply side.
- To apply a multi-objective goal programming in order to solve the proposed model.

3. Problem Definition and Formulation

3.1. Problem definition

Consider a cellular manufacturing system consisting of a number of cells, products, and machines to process different products. A product may require several operations in a given sequence. The manufacturing system is considered for a number of time periods. One time period could be a month, a season, or a year. Each machine has s capacity to operate in hours during each time period. Machines can be duplicated to meet capacity requirements and reduce or remove the inter-cell movement. Moreover, inventory, backorder, and outsourcing policy can be activated in each period. It is assumed that the demands for the products vary with time in a fuzzy manner. Machines are to be grouped into relatively independent cells for each period with minimum costs, especially inter-cell movement costs. On the other hand, determining the optimal price and advertising cost of products in each period can mitigate the demand fluctuations and lower the imposed costs. In addition to maximizing the total profit for manufacturers, they tend to minimize the maximum deviation between the available capacity and the workload assigned to each machine for increasing the utilization rate of machine capacity. This important criterion should be minimized because the low utilization rate increases the investment cost of machines and the labor cost and reduces the return on asset. Thus, the following two important objectives are considered as follows:

- Maximization of total profit for the manufacturing company.
- Minimization of maximum deviation between the available capacity and the workload assigned to each machine.

When designing an integrated model, various conflicting objective functions and different constraints should be considered simultaneously. Moreover, in practice, the parameters are often imprecise and, to cope with this imprecision, the traditional stochastic programming approaches are usually computationally inefficient. In these circumstances, assigning a set of crisp values to such ambiguous input data is not appropriate and rational. Thus, a fuzzy modeling approach is used to handle these imprecise parameters. In this paper, the fuzzy parameters have linear membership functions in which, after the defuzzification process, the equivalent crisp model is solved by a fuzzy priority goal programming method.

3.2. Problem assumptions

The main assumptions and properties used in the problem are as follows:

- Each product includes a number of operations that must be processed as numbered, respectively.
- The processing time for all operations of a product on different machine types is known and fuzzy.
- Each machine has fuzzy capabilities and time capacities throughout the planning horizon.

Tab. 1. Characteristics of the reviewed articles in the DCFP.

Resource	Kinds of problem				Environment		Objective function			Decision			Solution procedure
	DCFP	Production Planning Problem									Demand side		
		Inventory	Backorder	Outsourcing	Certain	uncertain	One	Two	Three	Supply side	Pricing	Advertising	
[3]	✔			✔	✔		✔			✔			Heuristic
[4]	✔				✔		✔			✔			Two-stage procedure

[5]	✔				✔		✔			✔	memetic
[6]	✔				✔		✔			✔	Gams + SA + GA +TS
[7]	✔			✔	✔		✔			✔	GA-based heuristic
[8]	✔			✔	✔		✔			✔	Lingo
[9]	✔				✔		✔			✔	Gams + NNA
[10]	✔				✔		✔			✔	Lingo
[11]	✔				✔				✔	✔	SS + GA
[12]	✔				✔			✔		✔	MOSS + SPEA-II + NSGA-II
[13]	✔				✔			✔		✔	Gams + benders
[14]	✔	✔		✔	✔		✔			✔	Lingo
[15]	✔	✔	✔	✔	✔		✔			✔	Lingo
[16]	✔	✔	✔	✔	✔		✔			✔	Lingo
[17]	✔	✔		✔	✔		✔			✔	Lingo
[18]	✔	✔	✔	✔	✔			✔		✔	NSGA-II + Gams
[19]	✔	✔		✔	✔		✔			✔	Gams + AIS + HAIS
[20]	✔				✔		✔			✔	Lingo + SA
[21]	✔				✔		✔			✔	Lingo
[22]	✔					✔	✔			✔	Lingo+ fuzzy programming
[24]	✔	✔	✔			✔	✔			✔	Robust optimization + Cplex
[25]	✔					✔	✔			✔	Simulation
[26]	✔					✔		✔		✔	PMCGP+ Lindo
[27]	✔				✔			✔		✔	NSGA-II + MOSA+ ε-constraint
[28]	✔					✔		✔		✔	Stochastic programming + SA-GA
[29]	✔				✔			✔		✔	NSGA-II + MOICA+ ε-constraint +

[30]	✔				✔			✔	✔			Goal programming NSGA-II + MOPSO
This paper	✔	✔	✔	✔		✔	✔		✔	✔	✔	Fuzzy goal programming + GA toolbox

- The constant cost of each machine is fuzzy. This cost includes maintenance, other over-head, rent, and overall service cost of each machine. Herein, the buying or selling cost of machines is not considered.
- The variable cost of each machine type is fuzzy. This cost is dependent on the workload allocated to the machine.
- The relocation cost for each machine type from one cell to another is fuzzy. All machines are able to be moved toward any cell. This cost is the sum of uninstalling, shifting, and installing costs. The time needed for relocation is assumed to be zero.
- Products are moved in a batch between and within cells. Moreover, inter- and intra-cell batches related to the product are characterized by different sizes and costs. It is assumed that the distance between each pair of cells and each pair of machines at each cell is the same.
- The maximum number of cells that can be formed in each period is known in advance.
- The maximal and minimal cell size is fuzzy.
- Outsourcing is finite. It is at most 10 percent of demands.
- Each product has a minimum amount of demand in each period. This means that each product has a demand greater than zero in each period.
- Holding and backorder inventories are allowed between periods with known costs.
- Partial subcontracting is allowed. In other terms, the total or a portion of the demand for the product can be subcontracted in each period. Moreover, the time-gap between releasing and receiving orders (lead time) is considered fuzzy.
- Customers are myopic, i.e., they do not exhibit strategic behavior.
- Due to unavailability and/or incompleteness of required parameters, critical parameters (such as unit costs and capacities) are imprecise (fuzzy) in nature. Moreover, a triangular or trapezoidal fuzzy number pattern is adopted to represent each fuzzy parameter. A trapezoidal fuzzy number $\tilde{a} = (a_1, a_2, a_3, a_4)$ is represented in Figure 2. The parameters $\tilde{a}$ are estimated by the decision-maker. Noteworthy, a triangular fuzzy number could be obtained for $a_2 = a_3$.

Tab. 2. Basic data for the illustrative example of all products as a trapezoidal fuzzy number.

i	$\tilde{B}_i^{inter}$	$\tilde{B}_i^{intra}$	$\tilde{\lambda}_i$	$\tilde{\eta}_i$	$\tilde{\rho}_i$
1	(25,30,35,40)	(7,8,8,9)	(70,80,90,100)	(8,10,10,13)	(30,34,38,42)
2	(30,40,45,55)	(10,12,12,14)	(56,82,112,122)	(12,14,14,16)	(30,34,34,38)
3	(15,20,20,25)	(4,6,6,8)	(65,80,95,110)	(10,15,17,20)	(40,42,42,44)

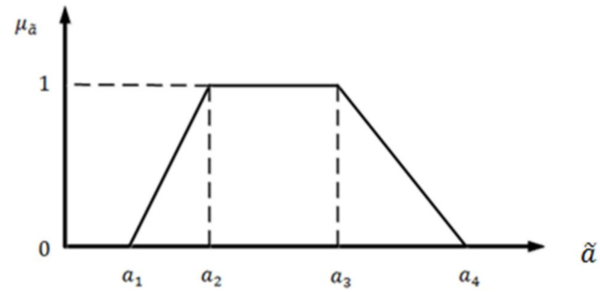

Fig. 2. The membership function of trapezoidal fuzzy number $\tilde{a}$

3.3. Problem notations

The indices, parameters, and decision variables used to formulate the problem mathematically are as follows:

Indices

i Index for products ($i = 1, \dots, I$)

j Index for operations of product i ($j = 1, \dots, O_i$)

m Index for machine types ($m = 1, \dots, M$)

c Index for manufacturing cells ($c = 1, \dots, C$)

t Index for time periods ($t = 1, \dots, H$)

Parameters

I Number of products

O_i Number of operations for product i

M Number of machine types

C Number of cells

D_{it} Demand for product i in period t

$\tilde{B}_i^{inter}$ Batch size for inter-cell movement of product i

$\tilde{B}_i^{intra}$ Batch size for intra-cell movement of product i

$\tilde{\gamma}^{inter}$ Inter-cell movement cost per batch

$\tilde{\gamma}^{intra}$ Intra-cell movement cost per batch. To justify the CMS, it is assumed that $(\tilde{\gamma}^{intra} / \tilde{B}_i^{intra}) < (\tilde{\gamma}^{inter} / \tilde{B}_i^{inter})$.

$\tilde{\alpha}_m$ Constant cost of machine type m in each period

$\tilde{\beta}_m$ Variable cost of machine type m for each unit time

$\tilde{\delta}_m$ Relocation cost of each machine type m

$\tilde{T}_m$ Time-capacity of machine type m in each period

$\widetilde{UB}$ Maximal cell size

$\tilde{D}^{min}$ The lower bound for demand of products

$\widetilde{pt}_{jim}$ Processing time required to perform operation j of product i on machine m

a_{jim} If operation j of product i can be done on machine m equal to 1; otherwise 0

$\tilde{\lambda}_i$ Unit cost of subcontracting product i

$\tilde{\eta}_i$ Inventory carrying cost per unit product i during each period

$\tilde{\rho}_i$ Backorder cost per unit part i during each period

$\tilde{l}$ Lead time where $l \le H - 1$

$\tilde{A}_t^{max}$ The maximum advertising cost in period t

Z Large positive number

Decision variables

N_{mct} Number of machine type m assigned to cell c in period t

${K_{mct}}^+$ Number of machines of type m added in cell c in period t

${K_{mct}}^-$ Number of machines of type m removed from cell c in period t

x_{jimct} If operation j of product i done on machine type m in cell c in period t equal to 1; otherwise 0

Q_{it} Number of products i produced in period t

y_{it} If $Q_{it} > 0$ equal to 1; otherwise 0

S_{it} Number of products i subcontracted in period t

P_{it} The sale price of product i at the end of period t

${In_{it}}^+$ Inventory level of product i at the end of period t

${In_{it}}^-$ Backorder level of product i at the end of period t

A_{it} The advertising cost per unit of product i in period t

3.4. Model formulation

3.4.1. Demand function

There are many forms of demand functions. In these functions, D must increase as P decreases and A increases. Hence, the demand function is proposed in Eq. (1):

$$D_i(P, A) = \omega_{it} A_{it} + (a_{it} - b_{it} P_{it}) \tag{1}$$

where $D_i(P, A)$ denotes the demand function for the i^{th} product. In other terms, the demand is defined as a function of unit price (P) and unit advertising cost (A), where $a_{it}, b_{it} > 0$, and the parameter of advertising $\omega_{it} > 0$.

Proposition: The demand function (1) satisfies the following assumptions:

(i) $D_i(P, A)$ is finite.

(ii) $D_i(P, A)$ is decreasing P in and increasing A.

(iii) $D_i(P, A)$ is nonnegative, changing continuously with price and advertising.

Proof:

(i) $lim_{P\to 0}\, D(P, A) = lim_{P\to 0}\, \omega A + (a - bP) = \omega A + a < \infty$.

(ii) $\frac{\partial D(P,A)}{\partial P} = -b < 0$; $\frac{\partial D(P,A)}{\partial A} = \omega > 0$.

(iii) If $(a - bP) > -\omega A$ or in other words $P \leq \frac{a+\omega A}{b}$ it is clear.

3.4.2. Objective functions

The total profit of manufacturing is equal to the revenue of the manufacturer minus total cost. The first objective function is to maximize the total profit as in Eq. (2):

$$\begin{aligned}
\max \quad f_1 = & \sum_{i=1}^{I}\sum_{t=1}^{H} P_{it}D_{it} - [\sum_{t=1}^{H}\sum_{m=1}^{M}\sum_{c=1}^{C} N_{mch}\tilde{\alpha}_m + \sum_{t=1}^{H}\sum_{m=1}^{M}\sum_{c=1}^{C}\sum_{i=1}^{I}\sum_{j=1}^{O_i} \tilde{\beta}_m Q_{it}\, \widetilde{pt}_{jim} x_{jimct} \\
& + 1/2 \sum_{t=1}^{H}\sum_{c=1}^{C}\sum_{i=1}^{I}\sum_{j=1}^{O_i-1} \left\lceil \frac{Q_{it}}{\tilde{B}_i^{inter}} \right\rceil \tilde{\gamma}^{inter} \left| \sum_{m=1}^{M} x_{(j+1)imct} - \sum_{m=1}^{M} x_{jimct} \right| \\
& + 1/2 \sum_{t=1}^{H}\sum_{c=1}^{C}\sum_{i=1}^{I}\sum_{j=1}^{O_i-1} \left\lceil \frac{Q_{it}}{\tilde{B}_i^{intra}} \right\rceil \tilde{\gamma}^{intra} \Big(\sum_{m=1}^{M} \left| x_{(j+1)imct} - x_{jimct} \right| \\
& - \left| \sum_{m=1}^{M} x_{(j+1)imct} - \sum_{m=1}^{M} x_{jimct} \right| \Big) \\
& + 1/2 \sum_{t=1}^{H}\sum_{m=1}^{M}\sum_{c=1}^{C} \tilde{\delta}_m (K_{mct}^{+} + K_{mct}^{-}) + \sum_{t=1}^{H}\sum_{i=1}^{I} (\tilde{\eta}_i In_{it}^{+} + \tilde{\rho}_i In_{it}^{-} + \tilde{\lambda}_i S_{it})] \\
& - \sum_{i=1}^{I}\sum_{t=1}^{H} A_{it}D_{it} \qquad (2)
\end{aligned}$$

The utilization rate of machine capacity is the deviation between the available capacity and the workload assigned to each type of machine. The second objective function is to minimize the maximum of these deviations as Eq. (3):

$$\min \quad f_2 = max_m \left\{ \sum_{c=1}^{C}\sum_{t=1}^{H} \tilde{T}_m N_{mct} - \sum_{i=1}^{I}\sum_{j=1}^{O_i}\sum_{t=1}^{H} D_{it}\widetilde{pt}_{jim} \right\} \qquad (3)$$

3.4.3. Model constraints

$$\sum_{c=1}^{C}\sum_{m=1}^{M} a_{jim} x_{jimct} = y_{it} \qquad \forall j, i, t \qquad (4)$$

$$\sum_{i=1}^{I}\sum_{j=1}^{O_i} Q_{it}\widetilde{pt}_{jim} x_{jimct} \leq \tilde{T}_m N_{mct} \qquad \forall m, c, t \qquad (5)$$

$$\sum_{m=1}^{M} N_{mct} \leq \widetilde{UB} \qquad \forall c, t \qquad (6)$$

$$N_{mc(t-1)} + K_{mct}^{+} - K_{mct}^{-} = N_{mct} \qquad \forall m, c, t \qquad (7)$$

$$In_{it}^{+} - In_{it}^{-} = In_{i(t-1)}^{+} - In_{i(t-1)}^{-} + Q_{it} + S_{i(t-\tilde{l})} - D_{it} \quad \forall i,t \tag{8}$$

$$In_{iH}^{+} = 0 \quad \forall i, \tag{9}$$

$$In_{iH}^{-} = 0 \quad \forall i \tag{10}$$

$$Q_{it} \leq Z y_{it} \quad \forall i,t \tag{11}$$

$$Q_{it} \geq y_{it} \quad \forall i,t \tag{12}$$

$$S_{it} \leq 0.1 \times D_{it} \quad \forall i,t \tag{13}$$

$$D_{it} \geq \widetilde{D}^{min} \quad \forall i,t \tag{14}$$

$$D_{it} = \omega_{it} A_{it} + (a_{it} - b_{it} P_{it}) \quad \forall i,t \tag{15}$$

$$\sum_{i=1}^{I} A_{it} D_{it} \leq \tilde{A}_t^{max} \quad \forall c,t \tag{16}$$

$$P_{it} \leq \frac{a_{it}}{b_{it}} \quad \forall i,t \tag{17}$$

$$y_{it}, x_{jimct} \in \{0,1\}, N_{mct}, K_{mct}^{+}, K_{mct}^{-}, Q_{it}, S_{it}, In_{it}^{+}, In_{it}^{-} \geq 0 \; and \; integer, A_{it}, P_{it} \geq 0 \quad \forall\, i,j,m,c,t \tag{18}$$

Eq. (4) ensures that the operation of each product is assigned to one machine and one cell in one time period if the number of the same products produced in the same time period is greater than zero. Eq. (5) guarantees that machine capacity in each cell in each time period is not exceeded and must satisfy the demand. Eq. (6) ensures that the maximum cell size is not violated. It means that the total number of used machines in each cell in each time period should be less than or equal to maximum bound predefined for all cells. Eq. (7) is called a balancing constraint on machines, ensuring that the number of machines in the current period is equal to the number of machines in the previous period plus the number of machines being moved in and minus the number of machines being moved out. Eq. (8) shows the balancing inventory constraint between periods for each product in each period. In other words, the inventory/backorder level of the product at the end of the each period is equal to the inventory/backorder level of the same product at the end of the previous period plus the number of production produced in current period plus number of productions subcontracted considering lead time, received in the current period minus the product demand rate in the current period. Notably, since the inventory and backorder level variables in each period appear with opposite signs and the model will be transformed into a linear one, one of these variables (inventory or backorder level variable) can be greater than zero in each period and another will be zero. Eqs. (9) and (10) assure that the inventory and backorder levels in the last period ($t = H$) are equal to zero, respectively. These two constraints are inserted into the model because the total demand of all products must be met during the planning horizon. Eq. (11) says that the number of products produced in each period can be greater than zero if $y_{it} = 1$ while this value will be zero if $y_{it} = 0$. Eq. (12) states that if $y_{it} = 1$ for each product in each period, the number of the same products produced in the same period must be greater than zero. Eq. (13) ensures that the amount of outsourcing is less than or equal to 10 percent of the demand. Eq. (14) states that the demand for products has a lower bound. Eq. (15) shows the demand function. Eq. (16) shows the limitation of advertising costs in each period. Eq. (17) assures that the demand function is non-negative. Finally, the values of the decision variables are restricted by constraints in Eq. (18).

4. The proposed Possibilistic Programming Model

The above-mentioned model is a multi-objective possibilistic nonlinear programming model. At first, this model must be converted to an equivalent crisp one. Here, the developed methods by Jimenez et al. [41] and Parra et al. [42] should be used because of computational efficiency and the ease of implementation. In this line, the total expected value of $\tilde{f}$ with an index of optimism $\alpha \in [0,1]$ is defined as Eq. (19):

$$EV_\alpha(\tilde{f}) = \alpha\, EI(\tilde{f})^R + (1-\alpha)\, EI(\tilde{f})^L \qquad (19)$$

where $EI(\tilde{f})^R$ and $EI(\tilde{f})^L$ are the right and left expected values of $\tilde{f}$, respectively. The parameter reflects the decision-maker's degree of optimism. The total expected values of trapezoidal possibility distribution of $\tilde{f}_1$and $\tilde{f}_2$ are calculated through Eqs. (20)-(22):

$$EV_\alpha(\tilde{f}_1) = \alpha\, \frac{f_1^3 + f_1^4}{2} + (1-\alpha)\, \frac{f_1^1 + f_1^2}{2} \qquad (20)$$

$$EV_\alpha(\tilde{f}_2) = \alpha\, \frac{f_2^3 + f_2^4}{2} + (1-\alpha)\, \frac{f_2^1 + f_2^2}{2} \qquad (21)$$

Where, as an example, $\tilde{f}_2 = (f_2^1, f_2^2, f_2^3, f_2^4)$:

$$f_2^1 = max_m \left\{ \sum_{c=1}^{C} \sum_{t=1}^{H} T_m^1 N_{mct} - \sum_{i=1}^{I} \sum_{j=1}^{O_i} \sum_{t=1}^{H} D_{it} pt_{jim}^1 \right\}$$

$$f_2^2 = max_m \left\{ \sum_{c=1}^{C} \sum_{t=1}^{H} T_m^2 N_{mct} - \sum_{i=1}^{I} \sum_{j=1}^{O_i} \sum_{t=1}^{H} D_{it} pt_{jim}^2 \right\}$$

$$f_2^3 = max_m \left\{ \sum_{c=1}^{C} \sum_{t=1}^{H} T_m^3 N_{mct} - \sum_{i=1}^{I} \sum_{j=1}^{O_i} \sum_{t=1}^{H} D_{it} pt_{jim}^3 \right\}$$

$$f_2^4 = max_m \left\{ \sum_{c=1}^{C} \sum_{t=1}^{H} T_m^4 N_{mct} - \sum_{i=1}^{I} \sum_{j=1}^{O_i} \sum_{t=1}^{H} D_{it} pt_{jim}^4 \right\}$$

Similarly, $\tilde{f}_1$ can be defined as above.

The defuzzification process of fuzzy linear and nonlinear constraints is given through Eqs. (22)-(26):

$$\sum_{i=1}^{I} \sum_{j=1}^{O_i} \left[(1-\beta) \left(\frac{pt_{jim}^1 + pt_{jim}^2}{2} \right) + \beta \left(\frac{pt_{jim}^3 + pt_{jim}^4}{2} \right) \right] Q_{it} x_{jimct} \le \left[(\beta) \left(\frac{T_m^1 + T_m^2}{2} \right) + (1-\beta) \left(\frac{T_m^3 + T_m^4}{2} \right) \right] N_{mct} \qquad \forall m,c,t \qquad (22)$$

$$\sum_{m=1}^{M} N_{mct} \le \left[(\beta) \left(\frac{UB^1 + UB^2}{2} \right) + (1-\beta) \left(\frac{UB^3 + UB^4}{2} \right) \right] \qquad \forall c,t \qquad (23)$$

$$In_{it}{}^{+} - In_{it}{}^{-} = In_{i(t-1)}{}^{+} - In_{i(t-1)}{}^{-} + Q_{it} + S_{i\left(t-(\beta)\left(\frac{l^1+l^2}{2}\right)+(1-\beta)\left(\frac{l^3+l^4}{2}\right)\right)} - D_{it} \qquad \forall i,t \qquad (24)$$

$$D_{it} \ge \left[(\beta) \left(\frac{D^{1,min} + D^{2,min}}{2} \right) + (1-\beta) \left(\frac{D^{3,min} + D^{4,min}}{2} \right) \right] \qquad \forall i,t \qquad (25)$$

$$\sum_{i=1}^{I} A_{it} D_{it} \leq \left[(\beta)\left(\frac{A_t^{max,1}+A_t^{max,2}}{2}\right)+(1-\beta)\left(\frac{A_t^{max,3}+A_t^{max,4}}{2}\right)\right] \quad \forall t \qquad (26)$$

where β denotes the minimum acceptable degree of the decision vector.

5. A Fuzzy Goal Programming Model

The proposed fuzzy model was transformed into an equivalent crisp multiple objective mathematical programming in the previous section. To reduce the equivalent multiple objective mathematical programming into a single-objective formulation, the fuzzy goal programming (FGP) is given. In FGP, the decision-maker usually expresses types of vague goals by linguistic statements so that they can be quantified by membership functions carrying the preference concept. These membership functions are constructed by the decision-maker's judgments. An example of this membership function is as follows:

For each fuzzy goal $EV_\alpha(\tilde{f}_1)(\upsilon) \gtrsim g_k$, the respective membership function is:

$$\mu_k(\upsilon) = \begin{cases} 1, & EV_\alpha(\tilde{f}_1)(\upsilon) \geq g_k \\ \dfrac{EV_\alpha(\tilde{f}_1)(\upsilon)-l_k}{g_k-l_k} & L_k \leq EV_\alpha(\tilde{f}_1)(\upsilon) \leq g_k; \quad \forall k \\ 0 & EV_\alpha(\tilde{f}_1)(\upsilon) \leq L_k \end{cases}$$

where L_k is the lower tolerance limit for the fuzzy goal $EV_\alpha(\tilde{f}_1) \gtrsim g_k$, and $\mu_k(\upsilon)$ represents the achievement degree of the k^{th} fuzzy goal for the given solution vector υ. The membership function is depicted in Figure 3.

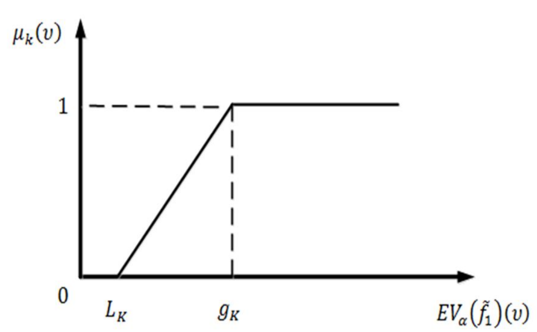

Fig. 3. Linear membership function for the fuzzy goal.

In order to solve the FGP, there are many procedures in the literature such as a simple and weighted additive model, preemptive structure, etc. In many decision problems, some goals are more important than others. In our model, the profit maximization is more important than the minimization deviation between the available capacity and the workload assigned to each type of machine. Among the different papers, Chen and Tsai [43] developed an efficient formulation because it not only satisfies the desired structure, but also optimizes the sum of each fuzzy goal's achievement degree only by a single problem. Therefore, the model will be converted to a crisp one with one objective, i.e., maximizing the summation of achievement degrees of all fuzzy goals. The crisp non-linear model is formulated as follows:

$$\max \quad \sum_{i=1}^{2} \mu_i$$

Subject to:
(4), (7),(9)-(13), (15),(17),(18), (23)-(27)
the membership functions ($\mu_i, i = 1,2$)
$\mu_1 \geq \mu_2$
$y_{it}, x_{jimct} \in \{0,1\}, N_{mct}, K_{mct}^{+}, K_{mct}^{-}, Q_{it}, S_{it}, In_{it}^{+}, In_{it}^{-} \geq 0$ *and integer*, $A_{it}, P_{it} \geq 0 \quad \forall i,j,m,c,t$

6. An Illustrative Example

The following numerical example demonstrates the feasibility and efficiency of the proposed model and the solution method.

Tables 2 and 3 include the basic parameters of the numerical example.

6.1. Data for the numerical example

1. There is a four-period planning horizon.
2. A three-product is considered.
3. It is assumed that each product has three operations.
4. There are five machine types.
5. The manufacturing firm has three cells.
6. The initial inventory for this company is 50, 0, and 100 for Products 1, 2, and 3, respectively.
6. The Inter-cell movement cost per batch is $\tilde{\gamma}^{inter}$ = (120, 130, 140, 150).
7. The Intra-cell movement cost per batch is $\tilde{\gamma}^{intra}$ = (1, 3, 3, 5).
8. The Maximal cell size for each cell is $\widetilde{UB}$ = (3, 5, 6, 7).
9. The lead time is $\tilde{l}$ = (0, 1, 2, 4).
10. The maximum marketing and advertising cost in different periods are $\tilde{A}_1^{max}$ = (16000, 18000, 18000, 20000), $\tilde{A}_2^{max}$ = (20000, 24000, 26000,

28000), $\tilde{A}_3^{max}$ = (20000, 22000, 24000, 26000), $\tilde{A}_4^{max}$= (12000, 16000, 17000, 19000).
11. The minimum demand for all products for all periods is $\widetilde{D}^{min}$ = (180,200,260,300).

12. The parameters of the demand function are summarized in Table 4.
13. The fuzzy processing times are summarized in Table 5.

Tab. 3. Basic data for the illustrative example of all machines as a trapezoidal fuzzy number

m	$\tilde{\alpha}_m$	$\tilde{\beta}_m$	$\tilde{\delta}_m$	$\tilde{T}_m$(hour)
1	(1500,1800,1800,2000)	(5,8,8,10)	(810,850,860,900)	(400,450,450,600)
2	(1000,1200,1400,1700)	(10,12,12,15)	(600,750,900,1050)	(400,450,450,600)
3	(1550,1850,1850,2100)	(3,4,5,6)	(800,840,900,980)	(400,450,450,600)
4	(800,1000,1200,1400)	(5,6,6,7)	(700,800,900,1000)	(400,450,450,600)
5	(2500,2700,2700,2900)	(1,4,8,10)	(750,850,950,1050)	(400,450,450,600)

Tab. 4. Demand function parameters

Periods	Product 1			Product2			Product3		
	a_{it}	b_{it}	ω_{it}	a_{it}	b_{it}	ω_{it}	a_{it}	b_{it}	ω_{it}
1	1000	5	9	1150	5.5	11	900	6.5	10
2	1100	5.5	11	1000	5	8	900	6	10
3	1100	5	10	950	7	10	1000	5	11
4	1000	6	10	1000	5	10	1000	5	9

Tab. 5. The processing time for operations of products on machines (hour).

i	j	m_1	m_2	m_3	m_4	m_5
	1	---	---	(0.65,0.69,0.69,0.74)	---	---
1	2	(0.5,0.55,0.55,0.67)	---	---	---	---
	3	---	(0.75,0.8,0.8,0.9)	---	---	---
	1	---	---	---	---	(0.78,0.8,0.8,0.83)
2	2	---	---	---	(0.6,0.65,0.65,0.7	---
	3	(0.51,0.55.0.56,0.6)	---	---	---	---
	1	---	---	(0.4,0.44,0.46,0.48)	---	---
3	2	---	---	---	---	(0.9,0.9,0.93,0.94)
	3	---	---	---	(0.7,0.8,0.85,0.9)	---

6.2. Corresponding mathematical model

Assuming an optimistic DM, we set $\alpha = 0.7$. As a result, the membership functions for two objectives are constructed as follows:

$$\mu_{f_1} = \begin{cases} 1, & EV_{0.7}(\tilde{f}_1) \geq 450000 \\ \dfrac{EV_\alpha(\tilde{f}_1) - 80000}{450000 - 80000} & 80000 \leq EV_{0.7}(\tilde{f}_1) \leq 450000; \quad \forall k \\ 0 & EV_{0.7}(\tilde{f}_1) \leq 80000 \end{cases}$$

$$\mu_{f_2} = \begin{cases} 1, & EV_{0.7}(\tilde{f}_2) \leq 300 \\ \dfrac{800 - EV_\alpha(\tilde{f}_2)}{800 - 300} & 300 \leq EV_{0.7}(\tilde{f}_2) \leq 800; \quad \forall k \\ 0 & EV_{0.7}(\tilde{f}_2) \geq 800 \end{cases}$$

As seen, the membership functions include the maximization and minimization of $f_1$and f_2, respectively. Figures 4 and 5 depict these two membership functions.

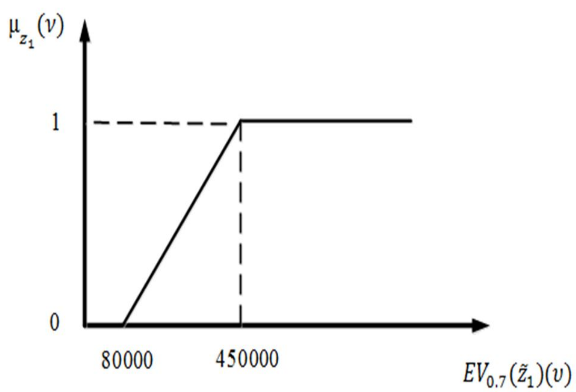

Fig. 4. The linear membership function for the first objective function in the numerical example

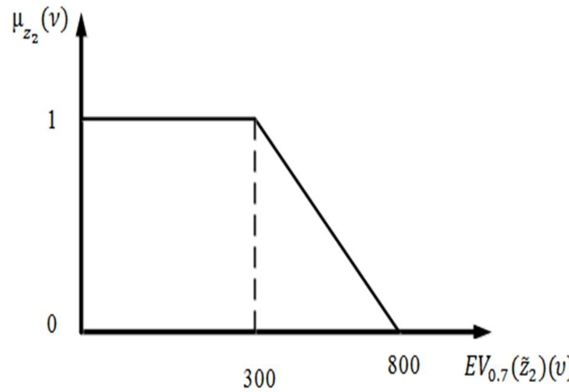

Fig. 5. The Linear membership function for the second objective function in the numerical example.

Finally, the equivalent crisp non-linear model for the numerical example is given as follows. It is assumed that the feasibility degree of the decision vector is equal to 0.8 ($\beta = 0.8$).

$$\max \quad \sum_{i=1}^{2} \mu_i$$

Subject to:

$$\sum_{c=1}^{C} \sum_{m=1}^{M} a_{jim} x_{jimct} = y_{it} \qquad \forall j, i, t$$

$$\sum_{i=1}^{I} \sum_{j=1}^{O_i} \left[0.2 \left(\frac{pt_{jim}^1 + pt_{jim}^2}{2} \right) + 0.8 \left(\frac{pt_{jim}^3 + pt_{jim}^4}{2} \right) \right] Q_{it} x_{jimct} \leq \left[0.8 \left(\frac{T_m^1 + T_m^2}{2} \right) + 0.2 \left(\frac{T_m^3 + T_m^4}{2} \right) \right] N_{mct} \qquad \forall m, c, t$$

$$\sum_{m=1}^{M} N_{mct} \leq 5 \quad \forall c,t$$

$$N_{mc(t-1)} + K_{mct}{}^{+} - K_{mct}{}^{-} = N_{mct} \quad \forall m,c,t$$

$$In_{it}{}^{+} - In_{it}{}^{-} = In_{i(t-1)}{}^{+} - In_{i(t-1)}{}^{-} + Q_{it} + S_{i(t-1)} - D_{it} \quad \forall i,t$$

$$In_{iH}{}^{+} = 0 \quad \forall i$$

$$In_{iH}{}^{-} = 0 \quad \forall i$$

$$Q_{it} \leq Z y_{it} \quad \forall i,t$$

$$Q_{it} \geq y_{it} \quad \forall i,t$$

$$S_{it} \leq 0.1 \times D_{it} \quad \forall i,t$$

$$D_{it} = A_{it}^{\gamma}(a_{it} - b_{it}P_{it}) \quad \forall i,t$$

$$ob2 \geq \sum_{c=1}^{C}\sum_{t=1}^{H}\left[0.3\left(\frac{T_m^1 + T_m^2}{2}\right) + 0.7\left(\frac{T_m^3 + T_m^4}{2}\right)\right]N_{mct} - \sum_{i=1}^{I}\sum_{j=1}^{O_i}\sum_{t=1}^{H} D_{it}\left[0.3\left(\frac{pt_{jim}^1 + pt_{jim}^2}{2}\right) + 0.7\left(\frac{pt_{jim}^3 + pt_{jim}^4}{2}\right)\right] \quad \forall m$$

$$D_{it} \geq \left[(\beta)\left(\frac{D^{1,min} + D^{2,min}}{2}\right) + (1-\beta)\left(\frac{D^{3,min} + D^{4,min}}{2}\right)\right] \quad \forall i,t$$

$$\sum_{i=1}^{I} A_{it}D_{it} \leq \left[0.8\left(\frac{A_t^{max,1} + A_t^{max,2}}{2}\right) + 0.2\left(\frac{A_t^{max,3} + A_t^{max,4}}{2}\right)\right] \quad \forall t$$

$$P_{it} \leq \frac{a_{it}}{b_{it}} \quad \forall i,t$$

$$\mu_1 \geq \mu_2$$

$$\mu_1 \leq 4.76 \times 10^{-6} \Bigg(\sum_{i=1}^{I}\sum_{t=1}^{H} P_{it}D_{it} - \sum_{t=1}^{H}\sum_{m=1}^{M}\sum_{c=1}^{C} N_{mch}\left[0.3\left(\frac{\alpha_m^1+\alpha_m^2}{2}\right)+0.7\left(\frac{\alpha_m^3+\alpha_m^4}{2}\right)\right]$$
$$+\sum_{t=1}^{H}\sum_{m=1}^{M}\sum_{c=1}^{C}\sum_{i=1}^{I}\sum_{j=1}^{O_i}\left[0.3\left(\frac{\beta_m^1+\beta_m^2}{2}\right)+0.7\left(\frac{\beta_m^3+\beta_m^4}{2}\right)\right]Q_{it}\left[0.3\left(\frac{pt_{jim}^1+pt_{jim}^2}{2}\right)\right.$$
$$\left.+0.7\left(\frac{pt_{jim}^3+pt_{jim}^4}{2}\right)\right]x_{jimct}$$
$$+{}^1\!/_2\sum_{t=1}^{H}\sum_{c=1}^{C}\sum_{i=1}^{I}\sum_{j=1}^{O_i-1}\left\lceil\frac{Q_{it}}{\left[0.3\left(\frac{B_i^{inter.1}+B_i^{inter.2}}{2}\right)+0.7\left(\frac{B_i^{inter.3}+B_i^{inter.4}}{2}\right)\right]}\right\rceil\left[0.3\left(\frac{\gamma^{inter.1}+\gamma^{inter.2}}{2}\right)\right.$$
$$\left.+0.7\left(\frac{\gamma^{inter.3}+\gamma^{inter.4}}{2}\right)\right]\left|\sum_{m=1}^{M}x_{(j+1)imct}-\sum_{m=1}^{M}x_{jimct}\right|$$
$$+{}^1\!/_2\sum_{t=1}^{H}\sum_{c=1}^{C}\sum_{i=1}^{I}\sum_{j=1}^{O_i-1}\left\lceil\frac{Q_{it}}{\left[0.3\left(\frac{B_i^{intra.1}+B_i^{intra.2}}{2}\right)+0.7\left(\frac{B_i^{intra.3}+B_i^{intra.4}}{2}\right)\right]}\right\rceil\left[0.3\left(\frac{\gamma^{intra.1}+\gamma^{intra.2}}{2}\right)\right.$$
$$\left.+0.7\left(\frac{\gamma^{intra.3}+\gamma^{intra.4}}{2}\right)\right]\Bigg(\sum_{m=1}^{M}\left|x_{(j+1)imct}-x_{jimct}\right|$$
$$-\left|\sum_{m=1}^{M}x_{(j+1)imct}-\sum_{m=1}^{M}x_{jimct}\right|\Bigg)$$
$$+{}^1\!/_2\sum_{t=1}^{H}\sum_{m=1}^{M}\sum_{c=1}^{C}\left[0.3\left(\frac{\delta_m^1+\delta_m^2}{2}\right)+0.7\left(\frac{\delta_m^3+\delta_m^4}{2}\right)\right]\left(K_{mct}{}^{+}+K_{mct}{}^{-}\right)$$
$$+\sum_{t=1}^{H}\sum_{i=1}^{I}\Bigg(\left[0.3\left(\frac{\eta_i^1+\eta_i^2}{2}\right)+0.7\left(\frac{\eta_i^3+\eta_i^4}{2}\right)\right]In_{it}{}^{+}+\left[0.3\left(\frac{\rho_i^1+\rho_i^2}{2}\right)+0.7\left(\frac{\rho_i^1+\rho_i^1}{2}\right)\right]In_{it}{}^{-}$$
$$+\left[0.3\left(\frac{\lambda_i^1+\lambda_i^2}{2}\right)+0.7\left(\frac{\lambda_i^3+\lambda_i^4}{2}\right)\right]S_{it}\Bigg)+\sum_{i=1}^{I}\sum_{t=1}^{H}A_{it}D_{it}-80000\Bigg)$$

$$\mu_2 \leq 3.57 \times 10^{-3}(680 - ob2)$$

$$y_{it}, x_{jimct} \in \{0,1\}, N_{mct}, K_{mct}{}^{+}, K_{mct}{}^{-}, Q_{it}, S_{it}, In_{it}{}^{+}, In_{it}{}^{-} \geq 0 \text{ and integer and } \mu_1, \mu_2 \geq 0 \qquad \forall\, i, j, m, c, t$$

This non-linear model is solved by Genetic Algorithm toolbox in MATLAB. The obtained DCFP reconfiguration during four periods is depicted in Figure 6. Moreover, Tables 6 and 7 show the DCFP assignment variables and production planning and marketing variables, respectively. With respect to the results, Cell 1 should be removed from planning for this industrial manufacturer. In Figure 6 (a), in the first period, the machines M1, M2, M3, and M4 are assigned to Cell 2 (C2) and the machine M5 to C3. Notably, the number of machines is equal to 1 for each machine. In addition, Product I1 is allocated to C2 and I2 to C3. In the second period (Figure 6 (b)), machines M1, M2, and M4 are allocated to C2; M3 and M5 are allocated to C3; I1 and I3 are allocated to C3. The number of machines M1, M2, and M3 is 2 and for M4 and M5 is 1. For the third period (Figure 6 (c)), machines M3, M4, M5 and product I3 are located in C2. The number of M4 and M5 is 2 and for M3 is 1. In the fourth period (Figure 6 (d)), machines M1 and M4 are in C2; machine M5 and product I2 are in C3. There is 1 machine from M1, M4, and M5 in this period. In the final solution, the values of achievement degrees for the first and second objectives are μ_1 =0.881 and

$\mu_2 =$ 0.628 and with the total profit equal to 406114 and the utilization rate of machine capacity equal to 486.

Tab. 6. The obtained DCFP assignment variables for the numerical example.

$x(1,1,3,2,1) = 1$	$x(2,1,1,2,1) = 1$	$x(3,1,2,2,1) = 1$
$x(1,1,3,3,2) = 1$	$x(2,1,1,2,2) = 1$	$x(3,1,2,2,2) = 1$
$x(1,2,5,3,1) = 1$	$x(2,2,4,2,1) = 1$	$x(3,2,1,2,1) = 1$
$x(1,2,5,3,2) = 1$	$x(2,2,4,2,2) = 1$	$x(3,2,1,2,2) = 1$
$x(1,2,5,3,4) = 1$	$x(2,2,4,2,4) = 1$	$x(3,2,1,2,4) = 1$
$x(1,3,3,2,3) = 1$	$x(2,3,5,2,3) = 1$	$x(3,3,4,2,1) = 1$
$x(j,i,m,c,t) = 0$ $\quad \forall\, other\ j,i,m,c,t$		

Tab. 7. The obtained production planning and marketing variables for the numerical example.

periods	products	Q_{it}	S_{it}	P_{it}	${In_{it}}^{+}$	${In_{it}}^{-}$	A_{it}
1	1	314	36	127.2	---	---	---
	2	436	43	202.54	6	---	35.81
	3	---	25	100	---	150	---
2	1	950	38	263.51	602	---	66.67
	2	537	43	114	156	---	---
	3	---	25	108.33	---	375	---
3	1	---	43	134	210	---	---
	2	---	32	90	---	121	---
	3	937	35	277.1	237	---	66.85
4	1	---	---	124.5	---	---	---
	2	537	---	188.97	---	---	39.29
	3	---	---	145.6	---	---	---

(a)

			C2 I1	C3 I2
C2	1	M1	3	3
	1	M2	2	---
	1	M3	1	---
	1	M4	---	2
C3	1	M5	---	1

Period 1

(b)

			C3 I1	C3 I2
C2	2	M1	2	3
	2	M2	3	---
	1	M4	---	2
C3	2	M3	1	---
	1	M5	---	1

Period 2

(c)

			C2 I3
C2	1	M3	1
	2	M4	3
	2	M5	2

Period 3

(d)

			C3 I2
C2	1	M1	3
	1	M4	2
C3	1	M5	1

Period 4

Fig. 6. Solution representation for DCFP reconfiguration in periods 1, 2, 3, and 4 in (a), (b), (c), and (d), respectively, for the numerical example.

7. Conclusion and Future Research

In recent years, different papers associated with the dynamic cell formation problem (DCFP) as a design problem of cellular manufacturing systems have increased in number. All authors have formulated this problem and made endeavor to minimize the total cost including the fixed and variable machine costs, inter-cell and intra-cell movement costs, reconfiguration cost, inventory, backorder, and outsourcing costs by supply side decisions (i.e., adding, removing, and relocating the machines, and holding the additive products in the warehouse at the end of period, backordering, and subcontracting with other companies). In principle, the supply side decisions help companies to manage the effect of demand fluctuations in different periods. On the other hand, the demand side decisions such as pricing and advertising can also cope with the growing demand changes. Moreover, most of parameters are uncertain in practice; therefore, a fuzzy goal programming was formulated in order to consider two objectives including maximizing the total profit of the firm and maximizing the utilization rate of machine capacity in this paper. In this model, the fuzzy parameters and goals were represented by appropriate linear membership functions. After the defuzzification process, the equivalent crisp one was solved by a fuzzy goal programming. Some future researches can be considered for this study. For example, the demand function can be constructed by fitting the best regression equations on the real data in practice. To integrate DCFP into other important problems such as inter-cell and intra-cell layout, human aspects and environmental issues can be other researches. Moreover, to develop an efficient solving method such as benders decomposition, column generation and even metaheuristics can be other interesting issues.

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