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An Approach to Optimizing the Water Resources Management Problem in a Fuzzy Environment

El- Saed Ebrahim Ammar & Hamiden Abd Elwahed Khalifa*

El- Saed Ebrahim Ammar, *Department of Mathematics, Faculty of Science, Tanta University, Tanta, Egypt*
Hamiden Abd Elwahed Khalifa*, Department of Operations Research, Institute of Statistical Studies and Research, Cairo University, Giza, Egypt*

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ABSTRACT

Fully fuzzy linear programming is applied to water resources management due to its close connection with human life, which is considered to be of great importance. This paper investigates the decision-making concerning water resources management under uncertainty based on two-stage stochastic fuzzy linear programming. A solution method for solving the problem with fuzziness in relations is suggested to prove its applicability. The purpose of the method is to generate a set of solutions for water resources planning that helps the decision-maker make a tradeoff between economic efficiency and risk violation of the constraints. Finally, a numerical example is given and is approached by the proposed method.

1. Introduction

Water Resources Management is an integrating concept for a number of water sub-sectors such as hydropower, water supply and sanitation, irrigation and drainage, and environment (Gasinov and Yenilmez[7]).

The water resources management includes (Jairaj and Vedula[10]):

- The quantitative and qualitative exploration of water resources;
- Water requiring inventory records;
- Measurement and matching of the water resources and water needs (demands) in a special system;
- Decision support depending on the results.

Up to now, fuzzy set theory has been applied to broad fields. Fuzzy set theory introduced by Zadeh [28] creates a model that is set up using approximately known data. Fuzzy numerical data can be represented by means of fuzzy subsets of the real line, known as fuzzy numbers. For the fuzzy set theory development, we may refer to the papers of Kaufmann [12] and Dubois and Prade [3]. They extended the application of algebraic operations of real numbers to fuzzy numbers by using a fuzzy principle. Fuzzy linear constraints with fuzzy numbers were studied by Dubois and Prade [3]. Lu et al. [16] introduced the definition of an inexact rough interval fuzzy linear programming method and investigated the allocation of generated water to agricultural irrigation system. In the real-world problems, uncertainties may be estimated as intervals. Shaocheng [20] studied two kinds of linear programming with fuzzy numbers called interval numbers and fuzzy number linear programming. Tanaka et al. [22] formulated and proposed a method for solving linear programming with fuzzy coefficients. Wang and Huang [25] developed interactive two-stage stochastic fuzzy programming for managing water resources. They proposed an interactive resolution method

* Corresponding author: *Hamiden Abd Elwahed Khalifa*
Email: hamiden_2008@yahoo.com
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within inexact two-stage stochastic programming. A two-stage optimization framework for planning reservoir operations was proposed by Wang and Adams [23], where hydrologic uncertainty and seasonal reservoir inflows have been modeled as in a periodic Markov process. Through a two-stage dynamic programming approach, a long-term hydrothermal scheduling of multi-reservoir systems was examined by Ferrero et al. [5]. Bellman and Zadeh [1] introduced the concept of a maximizing decision-making problem. Zhao et al. [29] introduced a complete solution set for fuzzy linear programming problems using linear and nonlinear membership functions. For water resources management, Huang and Loucks [9] proposed inexact two-stage stochastic programming. An interactive fuzzy resolution method for solving linear programming problems with fuzzy parameters was proposed by Jimenez et al. [11]. For developing water resources management, a number of optimization techniques were developed (Slowinski [21], Wu et al. [26], Jairaj and Vedula [10], and Maqsood et al. [17]). A model for obtaining an optimal multi-period operation within a multi-reservoir system was developed by Eiger and Shamir [4]. Xu et al. [27] investigated and applied an inexact two-stage fuzzy gradient chance-constrained programming method to the water resources management in Heshui River Basin, Jiangxi Province. To quantify the economic trade-offs when reducing groundwater abstraction to a sustainable level, Martinsen et al. [18] applied a multi-objective multi-temporal deterministic hydro economic optimization approach for this purpose. Fu et al. [6] proposed a two-level symmetric Nash-Harany leader-follower game model to resolve the conflict that arises when different water users compete for a limited water supply. Khalifa [14] studied the water allocation problem using the two-stage fuzzy random programming. An interval-valued fuzzy linear programming method for modeling parameters with high vagueness was represented by Wang et al. [24], Goralczany [8], and Cai et al. [2]. Khalifa and Al-Shabi [15] developed an approach for optimizing the water resources management problem based on the weighting method.

This paper aims to introduce and solve the problem of water resources management as two-stage stochastic fuzzy linear programming. The problem is considered by incorporating fuzzy numbers. A solution method for solving the problem with fuzziness in relations is suggested to demonstrate its applicability.

The remainder of the paper is organized as follows. Some preliminaries are given in Section 2. In Section 3, a water resources management problem introduced by Huang and Loucks[9] and Wang and Huang [25] is introduced; hence, the problem is investigated in a fuzzy environment. Characterization of $\alpha-$fuzzy optimal solution of the problem is presented in Section 4. A solution method for solving the problem is proposed in Section 5. In Section 6, a numerical example is given for illustration. Finally, some concluding remarks are reported in Section 7.

2. Preliminaries

Some basic concepts and results related to fuzzy numbers as well as some of their arithmetic operations, triangular fuzzy numbers, and some of algebraic operations are recalled in this section (Kaufmann and Gupta [13], Sakawa [19] and Zimmermann[30]) .

Definition 1.

A fuzzy number $\tilde{p}$ is a mapping:

$\mu_{\tilde{p}}: R \to [0, 1]$, with the following properties:

(i) $\mu_{\tilde{p}}(x)$ is an upper semi-continuous membership function;

(ii) $\tilde{p}$ is a convex set, i. e., $\mu_{\tilde{p}}(\lambda x + (1-\lambda)y) \geq \min\{\mu_{\tilde{p}}(x), \mu_{\tilde{p}}(y)\}$ for all $x, y \in R, 0 \leq \lambda \leq 1$;

(iii) $\tilde{p}$ is normal, i.e., $\exists x_0 \in R$ for which $\mu_{\tilde{p}}(x) = 1$;

Suppose that $(\tilde{p}) = \{x : \mu_{\tilde{p}}(x) > 0\}$ is the support of a fuzzy set $\tilde{p}$.

Let $F_0(R)$ denote the set of all compact fuzzy numbers on R , that is, for any $g \in F(R), g$ satisfies the following:

1. $\exists x \in R : g(x) = 1$;
2. For any $0 < \alpha \leq 1, g_\alpha = [g_\alpha^L, g_\alpha^U]$ is a close interval number on R.

Definition 2.

The $\alpha-$ level set of $\tilde{p} \in F(R), 0 \leq \alpha \leq 1$ is denoted by $(\tilde{p})_\alpha$ and is defined as follows:

$$(\tilde{p})_\alpha = \begin{cases} \{ x \in R : \mu_{\tilde{a}}(x) \geq \alpha, \ 0 < \alpha \leq 1 \\ \textit{closure}\,(\text{support}\,(\tilde{p})), \qquad \alpha = 0 \end{cases}$$

Definition 3.

A fuzzy number $\widetilde{A}$ on R is called triangular fuzzy number, if there exist real a, b, c, and $b, c \geq 0$ such that:

$$\widetilde{A}(x) = \begin{cases} \dfrac{x}{b} + \dfrac{b-a}{b}, & b-a \leq x \leq a, \\ \dfrac{-x}{c} + \dfrac{a+c}{c}, & a \leq x \leq a+c, \\ 0, & eleswhere \end{cases}$$

Let the triangular fuzzy number denoted by $\widetilde{A} = (a, b, c)$ and $F(R)$ be the set of all $L-R$ fuzzy numbers on R.

Definition 4.

$\widetilde{A} = (a\ , b, c)$ is called non-negative triangular fuzzy number if $a \geq 0$.

Definition 5.

Let $\widetilde{A} = (a, b, c) \geq \widetilde{0}$, $\widetilde{B} = (d, e, f) \geq \widetilde{0}$, and $x \in R$; the formulas for the addition, subtraction, scalar multiplication, and multiplication can be defined as follows:

1. Addition:

$$\widetilde{A} \oplus \widetilde{B} = (a, b, c) \oplus (d, e, f) = (a+d, b+e, c+f).$$

2. Subtraction:

$$\widetilde{A}(-)\widetilde{B} = (a, b, c) \oplus (d, e, f) = (a-f, b-e, c-d).$$

3. Multiplication:

$$\widetilde{A} \otimes \widetilde{B} = \begin{cases} (a\,d, b\,e, c\,f), & a \geq 0 \\ (a\,f, b\,e, c\,d), & a < 0, c \geq 0 \\ (a\,f, b\,e, c\,d), & c < 0 \end{cases}$$

4. Scalar multiplication

$$x\,\widetilde{A} = \begin{cases} (x\,a, x\,b, x\,c), & x \geq 0, \\ (x\,c, x\,b, x\,a), & x < 0. \end{cases}$$

Remark1. Let $\widetilde{0} = (0,0,0)$ represent a zero triangular fuzzy number.

Remark2. $\widetilde{A} \geq \widetilde{0}$ if and only if $a \geq 0, a-b \geq 0, a+c \geq 0$.

3. Water Resources Management Problem

In this section, some of the notations needed in the problem formulation are introduced.

3-1. Notations

The following notations are needed in the formulation

f: A benefit of system (\$);

B_j: Net benefit to user j per m^3 of water allocated (\$/m^3)

(First-stage revenue parameters)

T_j: Allocation target for water that is promised to the user j (m^3)

(First-Stage decision variables)

$E[.]$: Expected value of a random variable;

C_j: Loss to user j per m^3 of water not delivered, $C_j > NB_j$ (\$/m^3)

(Second-Stage cost parameters)

S_{jQ}: Shortage of water to user j when the seasonal flow is Q (m^3)

(Second-Stage decision variables))

Q: Total amount of seasonal flow (m^3) (random variables);

δ: Rate of water loss during transportation;

$T_{j\max}$: Maximum allowable allocation amount for user j (m^3);

m: Total number of water users;

i: Water user, $i = 1, 2, 3$, where $i = 1$ for municipality, $i = 2$ for the industrial user, and $i = 3$ for the agricultural sector.

The typical two-stage stochastic programming for the water resources management problem, introduced by Huang and Loucks [9], Wang and Huang [25], is considered.

$$\max f = \sum_{j=1}^{n} \widetilde{B}_j T_j - E\left[\sum_{j=1}^{n} C_j\, S_{jQ}\right] \tag{1}$$

subject to

$$\sum_{j=1}^{n} \left(T_j - S_{jQ}\right)(1+\delta) \leq Q, \tag{2}$$

(Water availability constraints)

$$S_{jQ} \leq T_j \leq T_{j\max}; \forall j, \tag{3}$$

(Water-allocation target constraints)

$$S_{jQ} \geq 0; \forall j \tag{4}$$

(Non-negative and technical constraints)
Problems (1)- (4) can be reformulated as in the following form (Huang and Loucks [9]):

$$\max f = \sum_{j=1}^{n} B_j T_j - \sum_{i=1}^{m}\sum_{j=1}^{n} p_i C_j S_{ij} \tag{5}$$

Subject to

$$\sum_{j=1}^{n}\left(T_j - S_{ij}\right)\left(1+\delta\right) \le q_i ;\forall i, \tag{6}$$

(Water availability constraints)

$$S_{ij} \le T_j \le T_{j\max} ;\forall, j \tag{7}$$

(Water-allocation target constraints)

$$S_{ij} \ge 0; \forall i, j \tag{8}$$

(Non-negative and technical constraints)
where S_{ij} is the amount by which water-allocation target T_j is not met when the seasonal flow is q_i with probability p_i.

3-2. Fuzzy model for water resources management problem

Consider the fuzzy model for problems (5)- (8) as follows:

$$\max f(S_{ij}, \widetilde{B}, \widetilde{C}, \widetilde{T}) = \sum_{j=1}^{n} \widetilde{B}_j \otimes \widetilde{T}_j - \sum_{i=1}^{m}\sum_{j=1}^{n} p_i\left(\widetilde{C}_j \otimes S_{ij}\right)$$

Subject to

$$S_{ij} \in \widetilde{X} = \begin{cases} \sum_{j=1}^{n}\left(\widetilde{T}_j - S_{ij}\right)\left(1+\widetilde{\delta}\right) \le \widetilde{q}_i ;\forall \\ S_{ij}(\le)\widetilde{T}_j(\le)\widetilde{T}_{j\max} ;\forall, j \\ S_{ij} \ge \widetilde{0}; \forall i, j \end{cases} \tag{9}$$

where $\widetilde{B}_j$, $\widetilde{C}_j$, $\widetilde{\delta}$, $\widetilde{q}_i$, $\widetilde{T}_{j\max}$, and $\widetilde{T}_j$ are triangular fuzzy numbers.

Definition 6.
(Optimal fuzzy solution). S_{ij}^{*} that satisfies the conditions in (9) is called a fuzzy optimization solution.
By using the representation of the fuzzy number as mentioned before, Problem (9) becomes

$$\max f(S_{ij}, \widetilde{B}, \widetilde{C}, \widetilde{T}) = \sum_{j=1}^{n}\left(c_j, b_j, t_j\right)\otimes\left(u_j, v_j, w_j\right) - \sum_{i=1}^{m}\sum_{j=1}^{n} p_i\left(\left(d_j, e_j, g_j\right)S_{ij}\right)$$

Subject to

$$S_{ij} \in \widetilde{X} = \begin{cases} \sum_{j=1}^{n}\left(\left(u_j, v_j, w_j\right) - S_{ij}\right)\left(1+\left(\delta_1, \delta_2, \delta_3\right)\right) \le \left(q_i^1, q_i^2, q_i^3\right);\forall \\ S_{ij} \le \left(u_j, v_j, w_j\right)(\le)\left(T_{j\max}^1, T_{j\max}^2, T_{j\max}^3\right);\forall, j \\ S_{ij} \ge 0; \forall i, j \end{cases} \tag{10}$$

Based on the arithmetic operations of fuzzy numbers, Problem (10) can be rewritten as follows:

$$\max f(S_{ij}, \widetilde{B}, \widetilde{C}, \widetilde{T}) = \sum_{j=1}^{n}\left(c_j, b_j, t_j\right)\otimes\left(u_j, v_j, w_j\right) - \sum_{i=1}^{m}\sum_{j=1}^{n} p_i\left(\left(d_j, e_j, g_j\right)\otimes S_{ij}\right)$$

Subject to

$$S_{ij} \in X^{'} = \begin{cases} \sum_{j=1}^{n}\left(\left(T_j\right)_\alpha - S_{ij}\right)\left(1+\delta_1\right) \le q_i^1, i = 1,2,...,m \\ \sum_{j=1}^{n}\left(\left(T_j\right)_\alpha - S_{ij}\right)\left(1+\delta_1-\delta_2\right) \le \left(q_i^1 - q_i^2\right), i = 1,2,...,m \\ \sum_{j=1}^{n}\left(\left(T_j\right)_\alpha - S_{ij}\right)\left(1+\delta_1+\delta_3\right) \le \left(q_i^1 + q_i^3\right), i = 1,2,...,m \\ S_{ij} \le \left(T_j\right)_\alpha \le T_{j\max}^1, j = 1,2,...,\mathrm{n} \\ S_{ij} \le \left(T_j\right)_\alpha \le T_{j\max}^1 - T_{j\max}^2, j = 1,2,...,n \\ S_{ij} \le \left(T_j\right)_\alpha \le T_{j\max}^1 + T_{j\max}^3;, j = 1,2,...,n \\ S_{ij} \ge 0; \forall i, j \end{cases} \tag{11}$$

In problem (11), $\widetilde{B}_j$, $\widetilde{C}_j$, and $\widetilde{T}_j$ are the fuzzy variables of R characteristics based on membership functions $\mu_{\widetilde{B}_j}$, $\mu_{\widetilde{C}_j}$, and $\mu_{\widetilde{T}_j}$, respectively.

Definition 7.
On the account of the extension principle,

$$\mu\left(f(S^*, \widetilde{B}, \widetilde{C}, \widetilde{T}) \le f(S^*, \widetilde{B}, \widetilde{C}, \widetilde{T})\right) = \sup_{B,C,T} \min\left(\mu_{\widetilde{B}}(B), \mu_{\widetilde{C}}(C), \mu_{\widetilde{T}}(T)\right), \tag{13}$$

S^* is an $\alpha-$fuzzy optimal solution for problem (11) if

$\mu\left(f(S^*, BC,T) \ge f(S,B,C,T)\right) \ge \alpha$, for all S_{ij}. (12)

4. Characterization of $\alpha-$ Fuzzy Pptimal Solution Problem (11)

For deducing the $\alpha-$ fuzzy optimal solution for problem (11), let us consider the following α-parametric problem:

Model 1:

$$\max f(S_{ij}, B, C, T) = \sum_{j=1}^{n} B_j T_j - \sum_{i=1}^{m}\sum_{j=1}^{n} p_i\left(C_j S_{ij}\right)$$

Subject to

$S_{ij} \in X^{'}, B_j \in \left(\widetilde{B}_j\right)_\alpha, C_j \in \left(\widetilde{C}_j\right)_\alpha, T_j \in \left(\widetilde{T}_j\right)_\alpha$

where $\left(\widetilde{B}_j\right)_\alpha$, $\left(\widetilde{C}_j\right)_\alpha$, and $\left(\widetilde{T}_j\right)_\alpha$ denote the $\alpha-$cut sets of $\widetilde{B}_j, \widetilde{C}_j$, and $\widetilde{T}_j$, respectively. Based on the convexity of the problem, $\mu_{\widetilde{B}_j}(B_j), \left(\widetilde{B}_j\right)_\alpha; \mu_{\widetilde{C}_j}(C_j), \left(\widetilde{C}_j\right)_\alpha\ \mu_{\widetilde{T}_j}(T_j), \left(\widetilde{T}_j\right)_\alpha$ are real intervals that are denoted by $\left[\left(\widetilde{B}_j(\alpha)\right)^-, \left(\widetilde{B}_j(\alpha)\right)^+\right]$, $\left[\left(\widetilde{C}_j(\alpha)\right)^-, \left(\widetilde{C}_j(\alpha)\right)^+\right]$, and $\left[\left(\widetilde{T}_j(\alpha)\right)^-, \left(\widetilde{T}_j(\alpha)\right)^+\right]$, respectively. Let Ω_α, Δ_α, and Ψ_α be the set of $1\times n$ matrices $B = \left(B_j\right)$, $C = \left(C_j\right)$, and $T = \left(T_j\right)$ with $B_j \in \left[\left(\widetilde{B}_j(\alpha)\right)^-, \left(\widetilde{B}_j(\alpha)\right)^+\right]$, $C_j \in \left[\left(\widetilde{C}_j(\alpha)\right)^-, \left(\widetilde{C}_j(\alpha)\right)^+\right]$, and $T_j \in \left[\left(\widetilde{T}_j(\alpha)\right)^-, \left(\widetilde{T}_j(\alpha)\right)^+\right]$. It is obvious that problem (14) may be rewritten as follows:

Model. 2.

$$\max f(S_{ij}, B, C, T) = \sum_{j=1}^{n} B_j T_j - \sum_{i=1}^{m}\sum_{j=1}^{n} p_i\left(C_j S_{ij}\right)$$

Subject to

$S_{ij} \in X^{'}, B_j \in \Omega_\alpha, C_j \in \Delta_\alpha, T_j \in \Psi_\alpha$.

Definition 8. $S_{ij}^* \in X^{'}$ is called an $\alpha-$parametric optimal solution for Model 2, if

$$f(S^*_{ij},B,C,T) \ge f(S_{ij},B,C,T);\forall S_{ij} \in X',B_j \in \Omega_\alpha ,C_j \in \Delta_\alpha ,T_j \in \Psi_\alpha$$

Theorem1. $S_{ij}^* \in \widetilde{M}$ is an $\alpha -$fuzzy optimal solution to problem (9) if and only if $S_{ij}^* \in X'$ is an $\alpha -$parametric optimal solution to problem Model 2.

Proof. **Necessity**: Suppose that $(X^*,Y^*) \in \widetilde{M}$ is the $\alpha -$fuzzy optimal solution for problem (2) , and $(X^*,Y^*) \in M'$ is not an $\alpha -$parametric optimal solution for problem Model 2. Then, there are $S_{ij}^\circ \in \widetilde{X}$ and $g \in \Omega_\alpha ,l \in \Delta_\alpha ,h \in \Psi_\alpha$ such that

$f(S_{ij}^\circ ,g,l,h) \ge f(S_{ij}^*,g,l,h)$.

Since $g \in \Omega_\alpha ,l \in \Delta_\alpha$ $h \in \Psi_\alpha$, we have

$\mu\left(f(S_{ij}^\circ,\widetilde{B},\widetilde{C},\widetilde{T}) \ge f(S_{ij}^*,\widetilde{B},\widetilde{C},\widetilde{T})\right) \ge \alpha.$

This contradicts the $\alpha -$fuzzy optimal solution of $S_{ij}^* \in \widetilde{X}$ for problem(9).

Sufficiency: $S_{ij}^* \in X'$ is the $\alpha -$parametric optimal solution for problem Model 2, and $S_{ij}^* \in \widetilde{X}$ is not an $\alpha -$fuzzy optimal solution for problem(9). Then, there are $S_{ij}^* \in X'$ such that: $\mu\left(f(S_{ij}^*,\widetilde{C},\widetilde{D}) \ge f(S_{ij},\widetilde{C},\widetilde{D})\right) \ge \alpha,$ i.e.,

$$\sup_{B',C,T'} \min\left(\mu_{\widetilde{B}}(B),\mu_{\widetilde{C}}(C),\mu_{\widetilde{T}}(T)\right) \ge \alpha, \qquad (14)$$

$$B' = \left\{ B \in R^{(1\times n)} : f(S_{ij}^*,B,C,T) \ge f(S_{ij},B,C,T)\right\},$$

$$C' = \left\{ C \in R^{(1\times n)} : f(S_{ij}^*,B,C,T) \ge f(S_{ij},B,C,T)\right\},$$

$$T' = \left\{ T \in R^{(1\times n)} : f(S_{ij}^*,B,C,T) \ge f(S_{ij},B,C,T)\right\}$$

For the supremum to exist, there are $\phi \in B',\psi \in C'$, and $\varphi \in T'$ with $\min\left(\mu_{\widetilde{B}}(\phi);\mu_{\widetilde{C}}(\psi),\mu_{\widetilde{T}}(\varphi)\right) < \alpha$, then

$$\sup_{B',C,T'} \min\left(\mu_{\widetilde{B}}(\phi),\mu_{\widetilde{C}}(\psi)\,\mu_{\widetilde{T}}(\varphi)\right) < \alpha.$$

This contradicts (14). Then, there are $\phi \in B'$, $\psi \in C'$, and $\varphi \in T'$ that satisfy

$$\min\left(\mu_{\widetilde{B}}(\phi);\mu_{\widetilde{C}}(\psi),\ \mu_{\widetilde{T}}(\varphi)\right) \ge \alpha, \text{ i.e.,} \qquad (15)$$

$$\phi \in \Omega_\alpha ,\psi \in \Delta_\alpha \text{ and } \varphi \notin \Psi_\alpha \qquad (16)$$

Based on (14) and (16), the contradiction of the optimality of $S_{ij}^* \in M'$ for problem Model 2 is clear.

Corresponding to Model 2, the following two-level mathematical programming models are structured as follows:

Model. 1-2-1.

$$\max f^+(S_{ij},B,C,T) = \sum_{j=1}^{n}\left(B_j\,T_j\right)^+ - \sum_{i=1}^{m}\sum_{j=1}^{n} p_i\left((C_j)^- S_{ij}\right)$$

Subject to

$$S_{ij} \in X_1 = \begin{cases} \sum_{j=1}^{n}\left((T_j)_\alpha^- - S_{ij}\right)(1+\delta_1) \le q_i^1;\forall \\ \sum_{j=1}^{n}\left((T_j)_\alpha^- - S_{ij}\right)(1+\delta_1-\delta_2) \le \left(q_i^1 - q_i^2\right);\forall \\ \sum_{j=1}^{n}\left((T_j)_\alpha^- - S_{ij}\right)(1+\delta_1+\delta_3) \le \left(q_i^1 + q_i^3\right);\forall \\ S_{ij} \le (T_j)_\alpha^- \le (T_{j\max})_\alpha^-;\forall,j \\ S_{ij} \le (T_j)_\alpha^- \le (T_{j\max})_\alpha^-;\forall,j \\ S_{ij} \le (T_j)_\alpha^- \le (T_{j\max})_\alpha^-;\forall,j \\ B_j \in \Omega_\alpha ,C_j \in \Delta_\alpha,T_j \in \Psi_\alpha; \\ S_{ij} \ge 0;\forall i,j \end{cases}$$

Model. 1-2-2.

$$\max f^-(S_{ij},B,C,T) = \sum_{j=1}^{n}\left(B_j\,T_j\right)^- - \sum_{i=1}^{m}\sum_{j=1}^{n} p_i\left((C_j)^+ S_{ij}\right)$$

Subject to

$$S_{ij} \in X_2 = \begin{cases} \sum_{j=1}^{n}\left(\left(T_j\right)_\alpha^+ - S_{ij}\right)\left(1+\delta_1\right) \le q_i^1;\forall \\ \sum_{j=1}^{n}\left(\left(T_j\right)_\alpha^+ - S_{ij}\right)\left(1+\delta_1-\delta_2\right) \le \left(q_i^1 - q_i^2\right);\forall \\ \sum_{j=1}^{n}\left(\left(T_j\right)_\alpha^+ - S_{ij}\right)\left(1+\delta_1+\delta_3\right) \le \left(q_i^1 + q_i^3\right);\forall \\ S_{ij} \le \left(T_j\right)_\alpha^+ \le \left(T_{j\max}\right)_\alpha^+;\forall, j \\ S_{ij} \le \left(T_j\right)_\alpha^+ \le \left(T_{j\max}\right)_\alpha^+;\forall, j \\ S_{ij} \le \left(T_j\right)_\alpha^+ \le \left(T_{j\max}\right)_\alpha^+;\forall, j \\ B_j \in \Omega_\alpha, C_j \in \Delta_\alpha, T_j \in \Psi_\alpha; \\ S_{ij} \ge 0;\forall i, j \end{cases}$$

Suppose that the $\alpha-$optimal solutions and the corresponding $\alpha-$optimum values of Models 2.1 and Model 2.2 are:

$S_{ij}^{+^*}, (f)_\alpha^{+*};\ S_{ij}^{-^*}, (f)_\alpha^{-*}$.

5. Solution Method

In this section, a solution procedure for solving the problem (9) is introduced as in the following steps:

Step1: Formulate the problem (9),

Step2: Transform the problem (9) into the problem (10) and the corresponding problem (11),

Step3: Ask the decision-maker to specify $\alpha (0 < \alpha < 1)$,

Step4: Convert the problem (11) into Model 2,

Step5: Use the arithmetic operations of fuzzy numbers to obtain the following two auxiliary models: Model2.1, and Model2.2,

Step6: Use any software to obtain the solutions of Model 2.1 and Mode 2.2, hence the optimal fuzzy solution of the problem (9).

6. Numerical Example

Consider the problem introduced by Wang and Huang [25] with triangular fuzzy numbers as:

Table1. Economic data (\$/ m^3) and seasonal flows (in 10^6 m3) at different probability levels

Activity User	Municipal($i=1$)	Industrial($i=2$)	ricultural ($i=3$)
Maximum allowable allocation $\left(\tilde{T}_{\max}\right)$	$(7,8,9)$	$(7,8,9)$	$(7,8,9)$
Target of water allocation $\left(\tilde{T}_j\right)$	$(2,3,5)$	$(2,4,5)$	$(1,2,3)$
Net benefit when water demand is satisfied $\left(\tilde{B}_j\right)$	$(95,100,110)$	$(45,50,70)$	$(28,30,33)$
Reduction of the net benefit when demand is not delivered $\left(\tilde{C}_j\right)$	$(220,250,285)$	$(55,60,90)$	$(45,50,75)$

Flow level	Probability	Seasonal flow(%)
Low($i=1$)	0.3	$(2,3,4)$
Medium($i=2$)	0.5	$(7,9,13)$
High ($i=3$)	0.2	$(14,16,20)$
Water loss($\tilde{\delta}$)		$(0.15,0.20,0.40)$

Model. 2-1.

$$\max f^+(S_{ij}, B, C, T) = 644.25 - \begin{pmatrix} 70.5S_{11} + 17.25S_{12} + 14.25S_{13} \\ +117.5S_{21} + 28.75S_{22} + 23.75S_{23} \\ +47S_{31} + 11.5S_{32} + 9.5S_{33} \end{pmatrix}$$

Subject to

$S_{11}+S_{12}+S_{13}\geq 5.2609,$
$S_{21}+S_{22}+S_{23}\geq 0.9130,$
$-S_{31}-S_{32}-S_{33}\leq 5.1739,$
$S_{11}+S_{12}+S_{13}\geq 8.0526,$
$S_{21}+S_{22}+S_{23}\geq 9.10526,$
$S_{31}+S_{32}+S_{33}\geq 9.10526,$
$S_{11}+S_{12}+S_{13}\geq 3.1290,$
$-S_{21}-S_{22}-S_{23}\leq 5.9032,$
$-S_{31}-S_{32}-S_{33}\leq 14.9355,$
$S_{ij}\geq 0;\forall i,j$

Model. 2-2.

$$\max f^{-}(S_{ij},B,C,T)=352-\begin{pmatrix}80.25S_{11}+22.5S_{12}+18.75S_{13}\\+133.75.S_{21}+37.5S_{22}+31.25S_{23}\\+53.5S_{31}+15S_{32}+12.5S_{33}\end{pmatrix}$$

Subject to

$S_{11}+S_{12}+S_{13}\geq 5.2609,$
$S_{21}+S_{22}+S_{23}\geq 0.9130,$
$-S_{31}-S_{32}-S_{33}\leq 5.1739,$
$S_{11}+S_{12}+S_{13}\geq 8.0526,$
$S_{21}+S_{22}+S_{23}\geq 9.10526,$
$S_{31}+S_{32}+S_{33}\geq 9.10526,$
$S_{11}+S_{12}+S_{13}\geq 3.1290,$
$-S_{21}-S_{22}-S_{23}\leq 5.9032,$
$-S_{31}-S_{32}-S_{33}\leq 14.9355,$
$S_{ij}\geq 0;\forall i,j$

Tab. 3. The solution of Model1.2

Optimal policy	Optimum value
$S_{11}=S_{12}=S_{21}=S_{22}=S_{31}=S$	
$S_{13}=8.0526$	
$S_{23}=9.1053$	$f^{-}=-197.34$
$S_{33}=9.1053$	

Tab. 2. The solution of Model 1.1

Optimal policy	Optimum value
$S_{11}=S_{12}=S_{21}=S_{23}=S_{31}=S_{3}$	$f^{+}=181.224$
$S_{13}=8.0526$	
$S_{22}=9.1053$	
$S_{33}=9.1053$	

Tab. 4. The fuzzy solution of the problem

Optimal policy	Optimum value
$S_{11}=S_{12}=S_{21}=S_{23}=S_{31}=S$	$\widetilde{f}=$
$S_{13}=8.0526$	$(0,0,308.2354$
$S_{22}=9.1053$	
$S_{23}=9.1053$	
$S_{33}=9.1053$	

7. Concluding Remarks

In this paper, the water resources management problem was studied under fuzzy environment. Two auxiliary models were obtained from the proposed approach. Each model was solved using Lingo package computer. The advantage of the approach was significant for its use in interactive methods for making any comment by related managers and achieving the solutions logically. Finally, fully fuzzy linear programming for water resources management is recommended while considering minimal S_{ij} by which the water-allocation target, T_j, is not met when the seasonal flow with the probability of p_i is q_i.

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