Regulating the Bullwhip Effect in Supply Chain with Hybrid Recycling Channels Using Linear Quadratic Gaussian Controller

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ABSTRACT

In this paper, a discrete time state space model for a four-level supply chain system including factory, wholesaler, retailer, and customer is designed with a recovery center as recycling hybrid channels. Due to the lack of coordination between the chain level and the unhealthy exchange of information in the system, almost all supply chain dynamics involve stochastic noise. For the first time, in this paper, we investigated that stochastic noise led to the bullwhip effect. Also, we probed the effect of lead time, various forecasting methods, and aggressive ordering on the bullwhip effect in the proposed supply chain. In order to mitigate the bullwhip effect, the Kalman filter method was proposed. Therefore, by using linear quadratic Gaussian controllers, not only the effect of the bullwhip effect was adjusted but also the system became stable. Eventually, the simulation results of Meshkin match factory indicate the efficiency of the proposed method.

1. Introduction

Highly competitive ambience not only offers new business chances for firms but also challenges companies to optimize their business activities to remain competitive. Competition is not between single organizations but between competing supply chains [28]. The concept of supply chain management is becoming more popular today, given the progressive global competition in the global marketplace [1]. Supply Chain Management (SCM) may be defined as a set of relationships between suppliers, manufacturers, distributors, and retailers, which transformed the row materials to the end products [5]. Modeling and controlling such systems include taking all commercial components into account in this complexity [1]. Supply chain management has recently attracted much attention among engineering engineers in the processing system [27]. The tendency of orders to increase in diversity in proportion to the supply chain

movement is commonly known as bullwhip effect. Dejonckheere et al. [13] conducted analysis of the variance amplification phenomenon. His work has inspired many authors to develop business games to demonstrate the bullwhip effect. In this regard, complex dynamics are one of the most important factors which lead to variance amplification (Aslani et al., 2019 [3]). Other factors such as replenishment policy (Gaffari et al., 2014), demand process (Chaharsooghi, S, K et al. [8]), and forecasting method are vital for affecting the efficiency of the supply chain system. Time is another factor that causes unrealistic bullwhip effect. The results of Aslani et al. showed that time was an unfair factor in the magnitude of the bullwhip effect [4].

Choosing inventory management policies is known as an important factor in controlling supply chain costs and one of the promising approaches to improving the dynamics of the supply chain is the use of formal control theory methods [34]. The linear control theory has been widely used in engineering for many years. To give an introduction to linear control theory, we refer the reader to Franklin et al. [17], Nise [26], and Benett [6]. For a brief overview of its historic background, linear control theory was first applied by Nise in an inventory management

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field. While Nise takes into account the continuous representation of time for studying the steady-state behavior of an existing system, a discrete time display was introduced to apply for order based on customer's current order and current inventory.

Meyer et al. [25] were the first to apply linear control theory to the analysis of multi-echelon problems. In recent years, linear control theory has been widely used to develop existing policies and investigate bullwhip effects. Dejonckheere et al. [11] investigated the order amplification induced by order-up-to policies and the APIOBCS policy. They expanded their scope of research by analyzing the role of sharing information to reduce order changes. In addition to order variability, Disney et al. [16] studied the inventory variability induced by an APIOPBCS-related inventory policy. Agaran et al. [1] investigated modern control theory and LQG controllers to regulate bullwhip effect phenomenon. Their work provides an opportunity to extend the modeling and control work to nonlinear, time-varying, stochastic, adaptive, and large-scale systems. Pin et al. [27] reduced bullwhip effect for supply chain using z-transform analysis. Ignacuk et al. [20] designed an optimal strategy with LQI controller in supply chain to mitigate demand variability. Chandra et al. [9] designed a controllable, observable, and stable supply chain system representation of a generalized order-up-to policy. Dongfei et al. [15] quantified and mitigated the bullwhip effect on the benchmark supply chain by the extended prediction self-adaptive control. In hybrid approach methods, in supply chain with three echelons, Devika et al. [14] regulated the bullwhip effect using evolutionary multi-objective metaheuristics. They showed that in a decentralized chain, the order batching factor and the demand signal processing in wholesaler are the most important factors in bullwhip effect. Conversely, in a centralized chain, factors such as rationing, shortage gaming, and lead time are the most effective in reducing the bullwhip effect. Keshari et al. [22] proposed a multiple order-up-to policy-based inventory replenishment scheme to mitigate the bullwhip effect in a multi-stage supply chain scenario, where various transportation modes are available between Supply Chain (SC) participants. They found that the implementation of a new emerging optimization algorithm named bacterial foraging algorithm (Bullwhip effect) presented superior optimization performances.

Social economy is developing in parallel to the growing world population. To improve living standards, the available natural resources are not enough to meet the public needs and the earth's ecosystems are facing much more threats [2]; as a result, many companies are actively planning their supply chain planning schemes to prevent the flow of current and reverse flow of the product better [18]. In fact, some manufacturers are responsible for recycling the third party. Therefore, the recovery process can be completed by the manufacturer and the third-party recovery provider simultaneously, which is called hybrid recycling channels.

For a hybrid recycling channel, Li et al. [24] analyzed the stable operation of a closed-loop supply chain with mixed recycling channels using game theory. Yi et al. [31] constructed three hybrid recycling methods for the product remanufacturing closed-loop supply chain under premium and penalty mechanisms. For Hang et al. [19], one of the key issues was the selection of a suitable channel template among three hybrid gathering channels. Zhang et al. [33] and Xiu et al. [30] attempted to reduce the fluctuation resulting from the complexity of hybrid recycling by analyzing robust stability of closed-loop forms. Zhang et al. applied fuzzy control method to control bullwhip effect in uncertain closed-loop supply chains with hybrid recycling channels [33]. The bullwhip effect was constrained by considering the uncertainties in the closed-loop supply chain system and fuzzy control method (Takagi-Sugeno). Their method not only constrained the bullwhip effect but also made the supply chain robust. Teimoury, E et al. [29] analyzed the chain performance with the objective function of defining holding and lost sales costs. At the end, the proposed algorithm namely Best Neighborhood (BN) was used to find a good solution for inventory and the results were compared with Simulated Annealing (SA) solutions. The rest of the paper is organized as follows: Section 2 develops a dynamic model of supply chain with hybrid recycling channel. The model is analyzed and in Section 4, LQG optimal control is designed to restrain the bullwhip effect. Numerical simulations are provided based on the proposed method in Section 5. Finally, the conclusions of the paper are discussed in Section 6.

2. Proposed Supply Chain and State-Space Model

This study investigated a supply chain model with four levels of one factory, wholesaler,

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retailer, and customer including the recovery center as a hybrid recycling channel of an inventory system. The supply chain designed is intended to be controllable. The information between the nodes is visible. For this purpose, it is assumed that the nodes share their information throughout the chain.

Our target is to provide a sustainable model of fluctuation while modifying the bullwhip effect and pursuing a specific index. To achieve the desired performance, in this model, the retailer can return some of its inventory to the wholesaler for any reason. The recovery center focuses on incoming products to return damaged products to the factory or deliver them to retailers after recovery. Indeed, the customer is not directly connected to the recovery center. Note that the designed supply chain system is discrete and time invariant. As mentioned earlier, it is assumed that all levels share their information and, thus, all state variables are measurable.
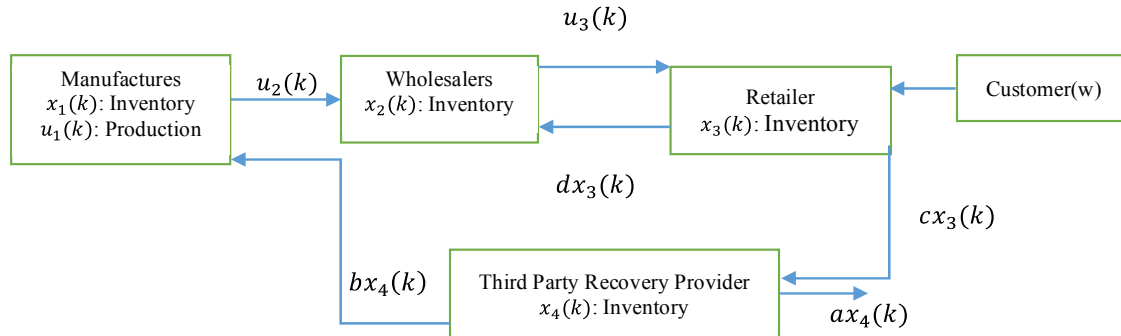

Fig. 1. Supply chain system with hybrid recycling channel

Third recovery provider as a recycling channel receives detective goods and applies rebuilding measures. It returns the repaired goods to the customer and other parts of the goods that cannot be repaired return to the factory. In the current period, retailers will adjust their stocks by sending orders to wholesalers after the forecast of demand. In this way, retailers send some of their inventory to the recovery center and wholesaler. After preparing the retailer's order by the wholesaler, the wholesaler will send an order to set up his inventory. As shown in Figure 1, there are many uncertain factors in the internal and external systems, and this parameter may cause the changes of manufacturer's inventory and the retailer's inventory. The linear supply chain model with hybrid recycling channels is as follows:

In Fig. 1, $x_1(k), x_2(k), x_3(k), x_4(k)$ are manufacturer inventory, the wholesaler inventory, the retailer inventory, and the third-party recovery provider inventory in period k, respectively, which are the state variables. $u_1(k), u_2(k)$, and $u_3(k)$ are the manufacturer production, the wholesaler order, and the retailer ordering quantities which are the control variables. In this model, we can observe all state variables. a is disposal rate, b is the remanufacturing rate, c is recycling rate, and d is return rate of the new product without any reason. In addition, it is assumed that the processes of ordering and production can be complete in one period; therefore, the lead time will not be considered. As shown in Equation (1), there are many uncertain factors in the internal and external systems and this parameter may cause changes in manufacturer inventory and the retailer inventory. Real supply chain systems include many customers, products, retailers, and wholesalers. Although the state-space model of the supply chain in this paper seems simple, this model can be the basis for real models. In the decentralized system, the inventory dynamics do not depend on how many customers the node has, since all customer demands can be lumped into an aggregate demand. If all nodes have enough inventories, then the order cannot disrupt the system dynamics. In fact, if the processing of

$$x_1(k+1) = x_1(k) + u_1(k) + bx_4(k) - u_2(k)$$
$$x_2(k+1) = x_2(k) + u_2(k) + dx_3(k) - u_3(k)$$
$$x_3(k+1) = x_3(k) + u_3(k) - dx_3(k) - cx_3(k)$$
$$x_4(k+1) = x_4(k) + cx_3(k) - ax_4(k) - bx_4(k)$$
$$y_1(k+1) = x_1(k+1)$$
$$y_2(k+1) = x_2(k+1)$$
$$y_3(k+1) = x_3(k+1)$$
$$y_4(k+1) = x_4(k+1)$$
$$a + b = 1$$
$$c + d \leq 1 \qquad (1)$$

orders and the amount of goods exchanged do not interfere, then for each product, a similar supply chain can be modeled.

3. Main Result

Before designing the LQG optimal feedback controller, controllability, observability, and stability of the system need to be analyzed. For dynamical systems, controllability and observability are the most important indicators in system analysis. One of the most important methods for proving the stability of a linear system is to check the special values of its characteristic equation [4], as obtained using the following equation:

$$\det(\lambda I - A) = 0$$
$$\det(\lambda I - A) = \lambda(\lambda - 1)(\lambda - 1)(\lambda - 1 + c + d) = 0$$
$$\lambda = 0; \lambda = 1, 1; \lambda = 1 - (c + d).$$

Clearly, the system cannot be stable for any of all parameters. Therefore, the state space model can be useful if it is controllable and observable. One of the most important methods for checking the controllability and observability is to use the following theorem:

Theorem1: Time-invariant linear system is fully controllable if and only if the controllable matrix column vectors [AB ... $A^{n-1}B$] be full rank [23].

Theorem2: Time-invariant linear system is fully observable if and only if the observable matrix vectors $[C\ CA\ ...\ CA^{n-1}]^T$ be full rank [23].

3.1. Bullwhip effect

The bullwhip effect can be shown as fortification of demand oscillation from downstream to upstream. Note that propagation of demand oscillation is only possible when every node has adequate stock. The important reasons for the occurrence of bullwhip effect are as follows [27]:
1.   Aggressive ordering.
2.   Demand process and forecasting methods.
3.   Stochastic lead time.
4.   Inventory control policy.
5.   Etc.

By taking z transform, we have:

$$X_1(z) = \frac{1}{z-1}(U_1(z) + bX_4(z) - U_2(z))$$

$$X_2(z) = \frac{1}{z-1}(U_2(z) + dX_3(z) - U_3(z))$$

$$X_3(z) = \frac{1}{z-1+c+d}(U_3(z))$$

$$X_1(z) = \frac{1}{z}(cX_3(z)) \tag{2}$$

Then,

$$U_1(z) = (z-1)X_1(z) - (z-1)X_2(z) - (z-1+c)X_3(z) - bX_4(z)$$

$$U_2(z) = (z-1)X_2(z) + (z-1+c)X_3(z)$$

$$U_3(z) = (z-1+c+d)X_3(z) \tag{3}$$

Therefore, we can write:

$$U_2(z) = (z-1)X_2(z) + (z-1+c)\frac{U_3(z)}{(z-1+c+d)} \tag{4}$$

Transfer function for the wholesaler echelon is as follows:

$$\frac{U_2(z)}{U_3(z)} = (z-1)\frac{X_2(z)}{U_3(z)} + \frac{(z-1+c)}{(z-1+c+d)} \tag{5}$$

3.1.1. Aggressive ordering.

One factor to which bullwhip effect is usually attributed is aggressive ordering [27]. We have shown that the system could become unstable for every parameter (**a,b,c,d**); then, we show that if we do not change retailer inventory and retailer takes an aggressive order, then the bullwhip effect will be found in wholesaler nodes and spread through the chain. A necessary and sufficient condition for creating bullwhip effect is as follows:

$$\frac{U_2(z)}{U_3(z)} \geq 1 \tag{6}$$

By using Equation (5) and applying aggressive order, we obtain:

$$\left|\frac{U_2(z)}{U_3(z)}\right| = \left|\frac{(z-1+c)}{(z-1+c+d)}\right| \tag{7}$$

We can have:

$$\left|\frac{U_2(z)}{U_3(z)}\right|^2 = \left|\frac{(z-1+c)}{(z-1+c+d)}\right|^2 = \left|\frac{(e^{iw}-1+c)(e^{-iw}-1+c)}{(e^{iw}-1+c+d)(e^{-iw}-1+c+d)}\right| \tag{8}$$

Thus, we obtain the following through mathematic manipulation:

$$\left|\frac{U_2(z)}{U_3(z)}\right|^2 = \left|\frac{c^2 + 2(\cos w - 1)(c-1)}{c^2 + 2(\cos w - 1)(c-1) + d^2 + 2cd + 2d(\cos w - 1)}\right| \quad (9)$$

As a result,

$$\left|\frac{U_2(z)}{U_3(z)}\right|^2 \geq 1 \square \quad d^2 + 2cd + 2d(\cos w - 1) < 0 \quad (10)$$

Through manipulation and from the mathematical viewpoint, at frequency zero, the system does not obviously lead to bullwhip effect. At frequencies $\frac{\pi}{2}$ and $\frac{3\pi}{2}$, the system leads to the bullwhip effect if $2c + d < 2$; however, if c=1 and d=0, then the system cannot head toward the bullwhip effect (As seen in Figure 2). At frequency π, the system does not lead to bullwhip effect.
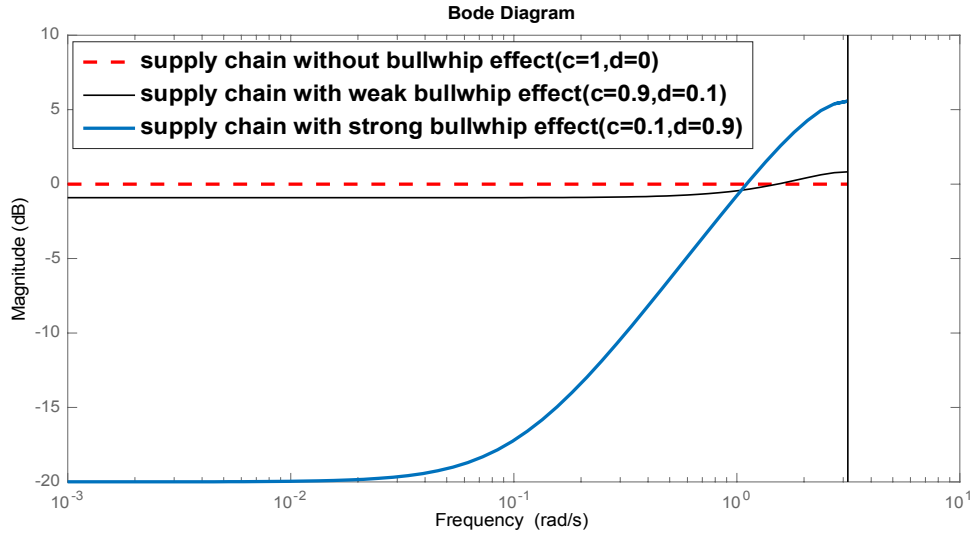

Bode Diagram

- - - supply chain without bullwhip effect(c=1,d=0)
—— supply chain with weak bullwhip effect(c=0.9,d=0.1)
—— supply chain with strong bullwhip effect(c=0.1,d=0.9)

Fig. 2. The magnitude ratio of $\left|\frac{U_2(z)}{U_3(z)}\right|$ for various values of parameters

In fact, after making changes in the parameters of the system and taking aggressive orders, the phenomenon of bullwhip effect occurs. Such changes in parameters can cause complex and non-linear dynamics of the system as well as chaos.

3.1.2. Demand forecasting methods and effect of lead time

When every echelon attempts to forecast and anticipate customer demand and regulate the inventory position target accordingly, the order of the supplier becomes as follows:

$$U_j(z) = k_j(S_j(z) - X_j(z)), \qquad j = 1,2,3 \quad (11)$$

where $S_j(z)$ is the set point of inventory position at nodes j and k_j is the gain of proportional controller. For example, for the wholesaler echelon, Equation (11) becomes:

$$U_2(z) = k_2(S_2(z) - X_2(z))$$

$$U_2(z) = k\left[LF(z)U_2(z) - \frac{1}{z-1}U_2(z) - \frac{d}{z-1}X_3(z) + \frac{1}{z-1}U_3(z)\right] \quad (12)$$

In Equation (12), *L* is the lead time. Equation (1) and simple manipulation Equation (12) become:

$$U_2(z) = \frac{k}{z-1+k}\left(\frac{LF(z)(z-1)(z-1+c+d)+(z-1+c+d)-d}{z-1+c+d}\right)U_3(z) \quad (13)$$

Transfer function for wholesaler echelon becomes:

$$\frac{U_2(z)}{U_3(z)} = \frac{k}{z-1+k}\left(\frac{LF(z)(z-1)(z-1+c+d)+(z-1+c+d)-d}{z-1+c+d}\right) \quad (14)$$

$F(z)$ is the forecaster filter used to predict current demand. Pin et al. [27] demonstrated that any forecaster would lead inevitably to the bullwhip effect.

$$F(z) = \frac{1}{z},$$

$$\frac{U_2(z)}{U_3(z)} = \frac{k}{z-1+k}\left[\frac{L(z-1)(z-1+c+d)+z(z-1+c+d)-zd}{(z-1+c+d)z}\right]$$

(15)

In other research, Chen et al. [10] suggested the use of exponential filters to predict demand:

$$F(z) = \frac{\gamma}{z-1+\gamma}$$

(16)

Figure 3 shows the Bode plot of magnitude ratio $\frac{U_2(z)}{U_3(z)}$ using two prediction filters for $k = 1.9$ and 0.1 and $\gamma = 0.1$. Results show that for $k = 1.9$ in both filters, bullwhip effect occurs. However, if gain decreases, the bullwhip effect does not happen. In general, we can conclude that as the gain increases, the effect of bullwhip also increases.
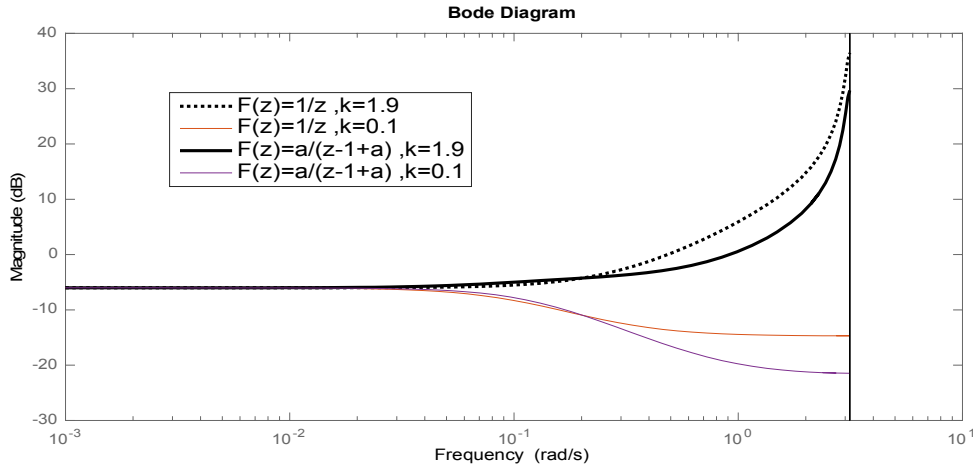
Fig. 3. Frequency responses of $\frac{U_2(z)}{U_3(z)}$ in the case of demand forecasting for various filters and gains

Figure 4 demonstrates that a longer lead time will lead to a stronger bullwhip effect. However, results show that without demand forecasting, lead time will not be a factor in the bullwhip effect.
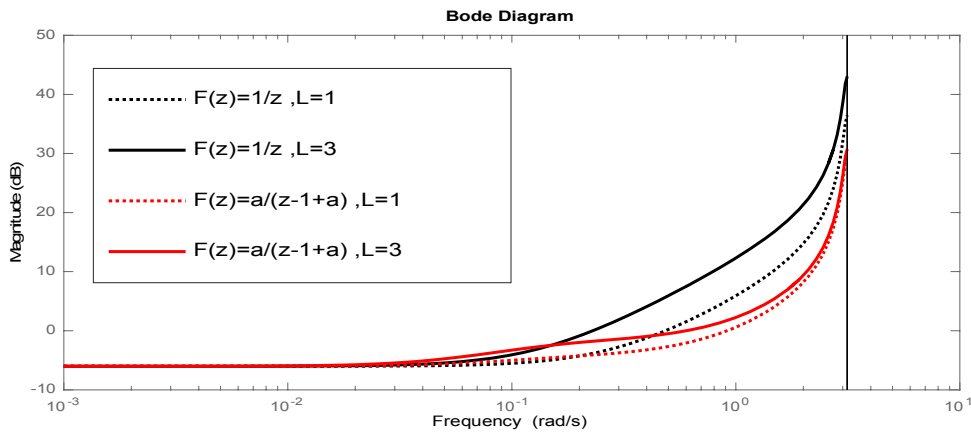
Fig. 4. Frequency responses of $\frac{U_2(z)}{U_3(z)}$ in the case of lead time for various times and filters

Moreover, with a comparison of the bode diagram in Figures 3 and 4, the results show that in the case of forecasting that does not lead to bullwhip ($k = 0.1$), by increasing the lead time, the forecasting filters will lead to the bullwhip effect. Also, we can conclude that as the lead time rises, the effect of bullwhip also strongly increases. In general, Figure 4 shows that

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$F(z) = \frac{1}{z}$ more than $F(z) = \frac{\gamma}{z-1+\gamma}$ causes the bullwhip effect.

3.1.3. Effect of stochastic noises on the bullwhip effect

We intend to indicate that the stochastic noisy information in the chain leads to the bullwhip effect. In this case, the supply chain with stochastic noise becomes:

$$x_1(k+1) = x_1(k) + u_1(k) + bx_4(k) - u_2(k) + w(k)$$
$$x_2(k+1) = x_2(k) + u_2(k) + dx_3(k) - u_3(k) + w(k)$$
$$x_3(k+1) = x_3(k) + u_3(k) - dx_3(k) - cx_3(k) + w(k)$$
$$x_4(k+1) = x_4(k) + cx_3(k) - ax_4(k) - bx_4(k) + w(k)$$
$$y_1(k+1) = x_1(k+1) + v_1(k)$$
$$y_2(k+1) = x_2(k+1) + v_2(k)$$
$$y_3(k+1) = x_3(k+1) + v_3(k)$$
$$y_4(k+1) = x_4(k+1) + v_4(k)$$
$$a + b = 1$$
$$c + d \leq 1$$

(17)

where $w(k)$ is the process noise and $V(k)$ is the measurement noise vector, while usually assuming normal white noise. Figure 5 shows that the stochastic noise is one of the factors that creates bullwhip effect. Figure 5 simulates the impulse response of open-loop supply chain in four cases of stochastic noise: without noise, large variance noise ($Q_w = 10, R_w = 10eye(3)$), medium variance noise ($Q_w = 1, R_w = 1eye(3)$), and low variance noise $\left(Q_w = 0.1, R_w = 0.1eye(3)\right)$, where Q_w and R_w are covariances of process and measurement noises.
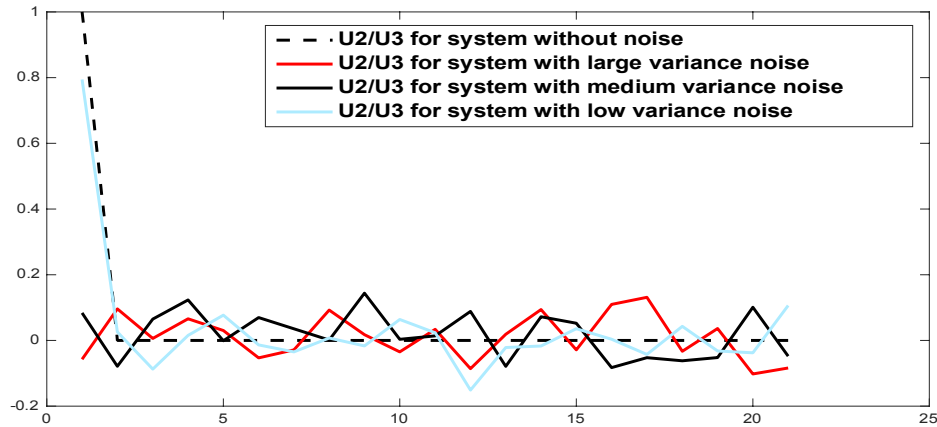

Fig. 5. Frequency responses of $\frac{U_2(z)}{U_3(z)}$ for various variance noises (process and measurement noises)

Results show that the supply chain without noise becomes out of bullwhip effect form after a short time. However, with the addition of noises to the system, the bullwhip effect appears little by little. This phenomenon is a direct relationship. As the noise increases, the bullwhip effect also increases. In general, due to the noise of information in engineering and statistics, we conclude that noise is an important factor in bullwhip effect.

In the next section, we will show how optimal ordering policy can be synthesized using modern control theory.

3.2. Optimal control synthesis

In the previous section, it was proved that noisy information in the supply chain was very dangerous, which has the potential to lead to a catastrophe in supply chains. Therefore, filtering information is the first attempt to regulate the bullwhip effect. One of the optimal methods for reducing the noise effect is Kalman filter. One of the objectives of the optimal controller design is speed as it is important for the simultaneous design of the controller. High speed can be achieved with great gain, which causes greater sensitivity to the system in the presence of noise. The Kalman filter is indeed the state observer,

with a difference in the optimal compromise between minimizing the error and reducing the effects of noise (speed of controller and minimized sensitivity to the stochastic noise). The proposed supply chain is complex and nonlinearly dynamic. So, we use an extended Kalman filter. By filtering the information from the beginning of the chain (demand) to the end (the information received by the factory) of the chain, EKF reduces one of the most destructive factors in the supply chain.

3.2.1. Optimal filtering

The Kalman filter is an optimal recursive filter that estimates the internal state of a linear dynamic system from a series of noisy measurements. Kalman filter has two steps: the prediction step, in which the next state of the system is predicted given the previous measurements; the update step, in which the current state of the system is estimated given the measurement at that time step [21]:

$$X(k+1) = AX(k) + BU(k) + Gw(k)$$
$$Y(k+1) = CX(k) + V(k) \tag{18}$$

Prediction dynamic for the supply chain model in Equation (18) becomes:

$$X^-(k+1) = AX(k) + BU(k)$$
$$P^-(k+1) = AP(k)A^T + GQ_w G^T \tag{19}$$

Also, the update step for the supply chain system becomes:

$$X^+(k+1) = X^-(k+1) + K(k)(Y(k) - CX^-(k+1))$$
$$P^+(k+1) = P^-(k+1) + K(k+1)CP^-(k+1)$$
$$K(k+1) = P^-(k+1)C^T(R_w + CP^-(k+1)C^T)^{-1} \tag{20}$$

where $X^-(k+1)$ and $P^-(k+1)$ are the predicted mean and covariance of the state, respectively, in the time step $k+1$ before seeing the measurement. $X^+(k+1)$ and $P^+(k+1)$ are the estimated mean and covariance of the state, respectively, in the time step $k+1$ after seeing the measurement. $K(k)$ is the filter gain, which tells how much the predictions should be corrected in time step $k+1$. $w(k)$ is the process noise which is applied to be drawn from a zero mean multivariate normal distribution with covariance Q_w. $V(k)$ is the observation noise vector which is assumed to be zero mean Gaussian white noise with covariance R_w.

3.2.2. Optimal controller

In the following, control-theoretic approach is employed to design a discrete – time controller for a considered inventory system with white noise. In order to regulate the bullwhip effect, we propose that the dynamic optimization with a quadratic quality scale and filtering noisy information be used.

LQG concerns uncertain linear systems disturbed by additive white Gaussian noise with incomplete state information (i.e., not all the state variables are measured and available for feedback) and enduring control subject to quadratic costs.

Equation (21) is a general quadratic cost function which not only states excursions but controls excursions and state-control products as well. So, for discrete time linear system (Equations (20)), the quadratic cost function to be minimized is as follows [23]:

$$J = E\left[\sum_{k=1}^{N} (X(k)QX^T(k) + U^T(k)RU(k)) \right] \tag{21}$$

where E is the expected value and Q and R are positive semidefinite matrices for regulating the evaluation outputs and weighting the control force, respectively. Thus, this weighting matrix needs to be carefully selected in order to achieve a desired performance of the regulator.

In the following instead of J, two Riccati equations are used, without loss of generality one may assume that the active control force $U(t)$ is proportional to the estimated state variable $X(t)$. So, to design discrete-time LQG controller by using Kalman filter (Equation (20)) and feedback control low, we have:

$$X(k+1) = AX(k) + BU(k) + K(k)(Y(k+1) - C(AX(k) + BU(k)))$$
$$U(k) = -L(k)X(k)$$

The feedback Gain matrix equals to:

$$L(k) = R^{-1}B^T P(k) \tag{22}$$

Where $P(k)$ is determined by the following Riccati matrix difference equation that runs backward in time:

$$P(k+1) + P(k)A - P(k)BR^{-1}B^T P(k) + Q + A^T(k) = 0 \tag{23}$$

This equation is solved through the introduction of known parameters in the MATLAB Control toolbox; by different Q matrices, a set of different

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controllers is gained. In order to gain a suitable controller for the system, it is needed to give proper weights to Q matrix. In this regard, we can weigh important evaluation parameters for the system and place them at one diagonal matrix, this matrix is a weighted matrix.
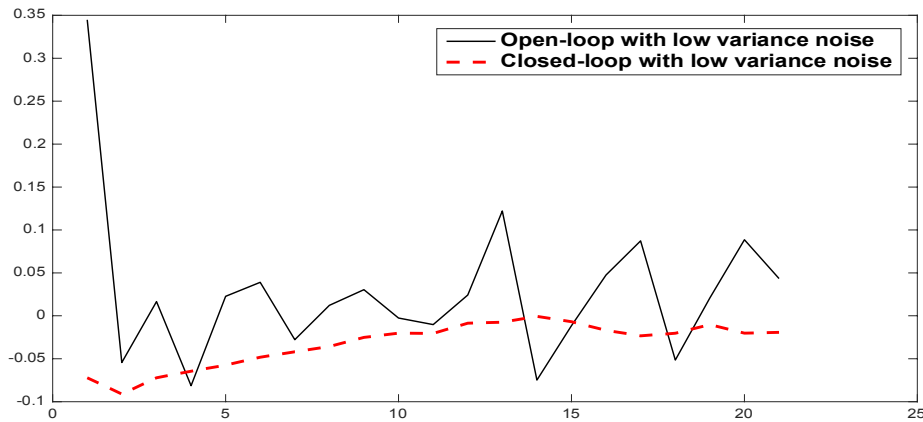

Fig. 6. Frequancy responses of $\frac{U_2(z)}{U_3(z)}$ for noisy open-loop and noisy closed-loop supply chain

Figure 6 shows that a designed controller can regulate the bullwhip effect in the supply chain with low variance noise. Open-loop supply chain has a high level of bullwhip at early times. However, after a while, the bullwhip effect is reduced. In contrast, the closed-loop supply chain prevents bullwhip effect.
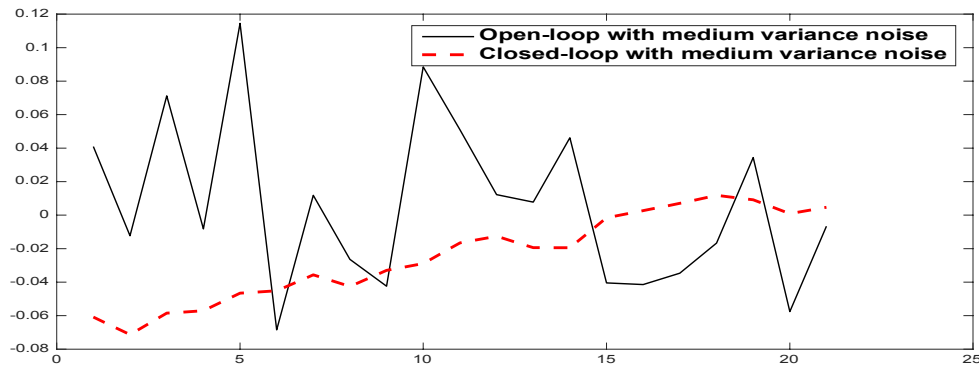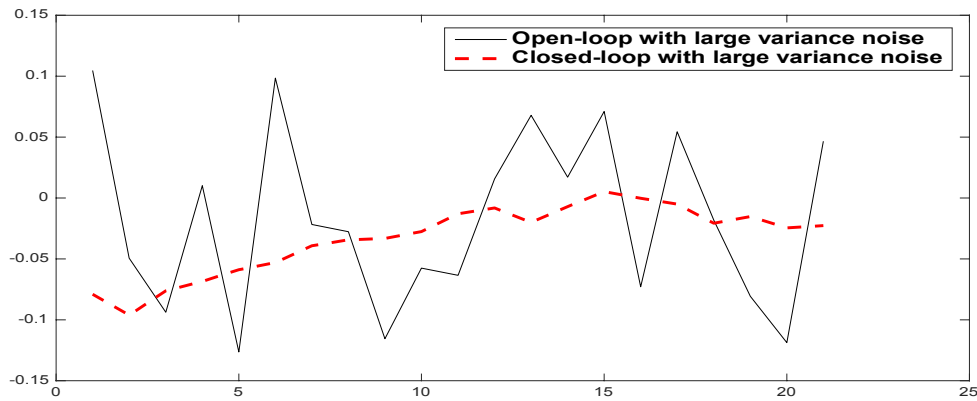

Fig. 7. Frequency responses of $\frac{U_2(z)}{U_3(z)}$ for noisy open-loop and noisy closed-loop supply chains

Figure 7 shows that a designed controller can regulate the bullwhip effect in the supply chain with medium variance noise. As seen in Figure 7, open-loop systems have strong bullwhip effects at all frequencies. In contrast, LQG controllers can regulate this phenomenon.

Fig. 8. Frequency responses of $\frac{U_2(z)}{U_3(z)}$ for noisy open-loop and noisy closed-loop supply chains

Figure 8 shows that the designed controller can regulate the bullwhip effect in the supply chain with large variance noise.

4. Case Study

In this section, we are simulating the system by using the information of Meshkin matches as a real supply chain. This information involves noise in inventory data and measurement.

We assume that the inventory information obtained from the retailer, the recovery center, the wholesaler, and the factory contains normal white noise with variance Q_w. Also, the measurement information contains white noise with covariance R_w. The results for each level indicated an optimal and accurate estimate of inventories. In this supply chain, the retailer returns one tenth of the inventory to the wholesaler for any reason and sends the same amount of inventory to the recovery center for repair. Therefore, the recovery center is returning half of the stock to the factory. We can consider the parameters as follows:

$$a = 0.5;\ b = 0.5;\ c = 0.1;\ d = 0.1, Q_w = 0.01,\quad R_w = \begin{pmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0.1 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 & 0 & 0.5 \\ 0 & 1 & 0.1 & 0 \\ 0 & 0 & 0.8 & 0 \\ 0 & 0 & 0.1 & 0 \end{pmatrix}, B = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix},$$

$$G = (1\ \ 1\ \ 1\ \ 1)^T, C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},\quad Kalman\ gain{:}\ K = \begin{pmatrix} 0.2432 & 0.2050 & 0.1468 & 0.0896 \\ 0.1874 & 0.1574 & 0.1081 & 0.0644 \\ 0.0907 & 0.0788 & 0.0763 & 0.0533 \\ 0.0113 & 0.0098 & 0.0095 & 0.0066 \end{pmatrix}$$

Factory initial inventory (the first day of simulation) is 200 units. Initial wholesaler inventory at the simulation is 170 units. Initial retailer inventory at the simulation is 100 units and the initial inventory of the recovery center at the simulation is 10 units. All of this has been normalized. Fig. 9 shows the actual factory stock and an optimal estimate of this amount using the Kalman filter. Also, the results indicate an estimated error between -0.4 and 0.2, appearing to be reasonable and acceptable, due to the high level of noise. Regarding the distribution of information from the downstream to the highest node of the chain, simulation of the state variables started from the recovery center and retailer echelon.
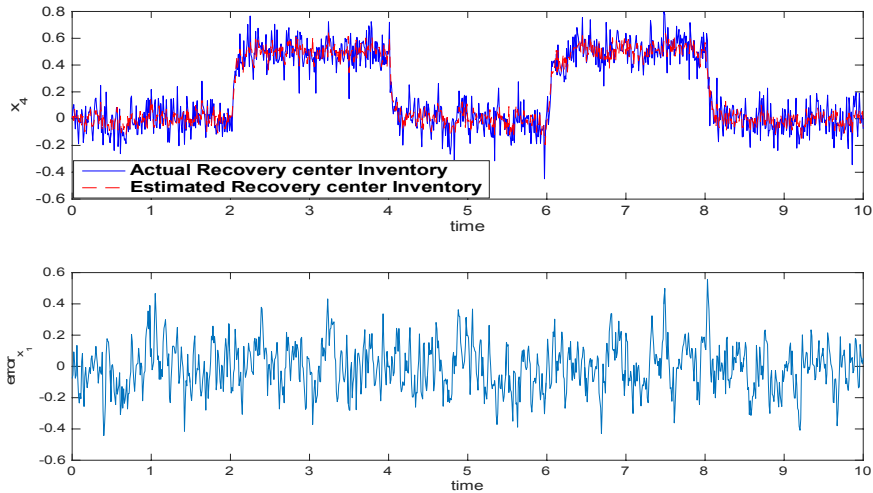
Fig. 9. Estimated recovery center inventory

Figures 9 and 10 show the actual recovery center inventory and retailer inventory in the initial 10-day time. These values correspond to the system with square waves. As seen in the simulation, Kalman filter estimates the amount of retailer and recovery center inventory with acceptable error. However, in real systems, the data are impregnated with noise, causing them to be incorrectly transmitted to the top of the chain. This factor causes the bullwhip effect.
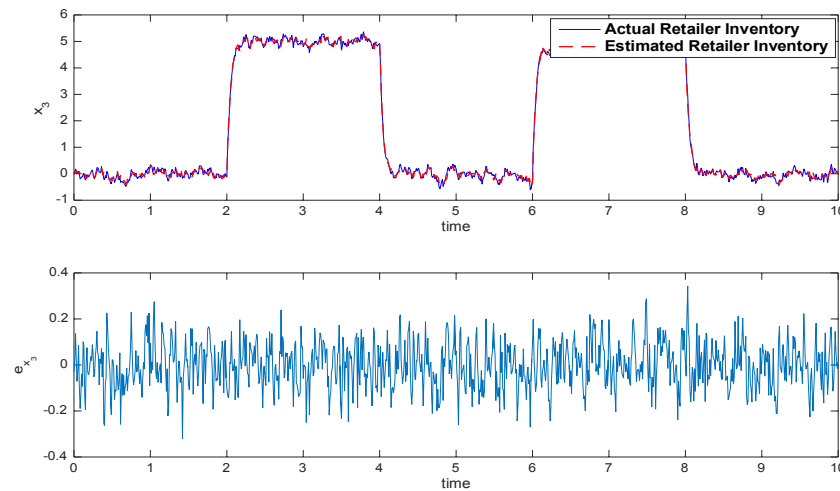

Fig. 10. Estimated retailer inventory

The retailer and recovery center inventory in the open-loop supply chain are finite and pseudo-stepping. As shown in Figures 11 and 12, the amount of inventory for the wholesaler and the factory (upstream nodes) tends to increase abnormally; and if more time is simulated, these states become unstable. In fact, the bullwhip phenomenon in the wholesaler node can happen immediately. So, the first step to adjust and reduce this phenomenon is to prevent the dissemination of false information in the chain.

Kalman filter is capable of performing an optimal estimation of modeling error or random variations in a dynamic model such as uncertainty, error due to request reports, and ordering in the form of possible models. In this case, we get the closest response to the real system (without noise). This effect, in turn, reduces the bullwhip effect significantly and prevents the transmission of noise-impaired information to upper-level through the chain. In this section, the LQG feedback control method, mentioned in the previous section, is utilized to

restraint the bullwhip effect of the open-loop supply chain with hybrid recycling channels. As mentioned in the simulation experiment, the match production enterprise is named "Meshkin match".

$x_1(k)$ is the match manufacturer inventory, I_{1s} is the manufacturer safety inventory, I_{1e} is the match manufacturer expected inventory, I_{1m} is the match manufacturer maximum inventory, $x_2(k)$ is the match wholesaler inventory, I_{2s} is the match wholesaler safety inventory, I_{2e} is the match wholesaler expected inventory, I_{2m} is the match wholesaler maximum inventory, $x_3(k)$ is the match retailer inventory, I_{3s} is match retailer safety inventory, I_{3e} is the match retailer expected inventory, and I_{3m} is the match retailer maximum inventory as given in Table 1.

Tab. 1. Simulation data

Inventory	manufacturer	wholesaler	retailer
Initial inventory	200	170	100
Safety inventory	150	160	90
Expected inventory	400	180	120
Maximum inventory	600	200	130

Q is the cost matrix of inventory and R is the cost matrix of order variables. Figures 14, 15, 16, and 17 show the first output of LQG optimal controller, the second output of LQG optimal controller, the third output of LQG optimal controller, and the latest output of LQG optimal controller, respectively.

In all simulation results, in the system without controller, outputs tend to infinity and are open to fluctuation. For instance, retailer inventory or other level inventories become large; therefore, this phenomenon is dangerous for the chain and far from our target. However, in the supply chain with controllers, outputs track the index and we can minimize factory inventory cost and their production cost together (in a quadratic cost function).
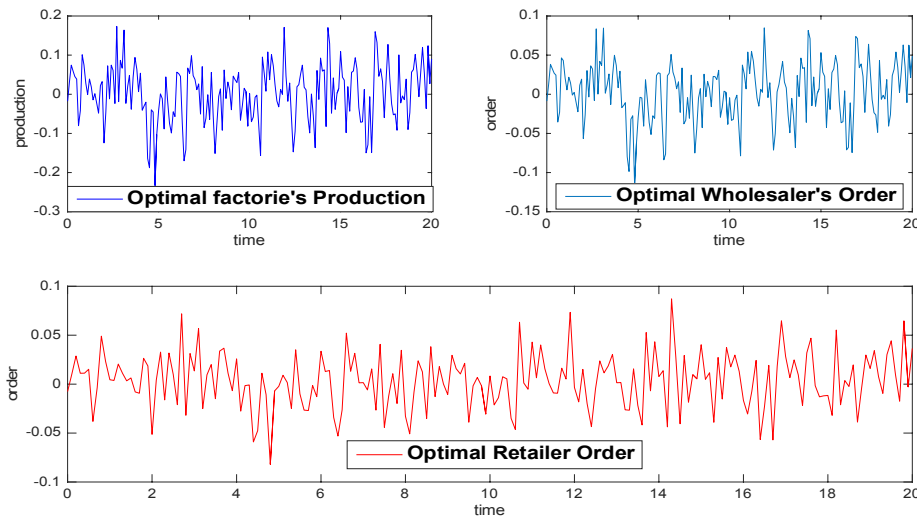
Fig. 11. Optimal policy signals

Given the cost of orders (control variables) and inventories costs (state variables) to achieve the optimal performance, a control effort is required. For this purpose, the retailer adjusts its order in twenty steps according to Figure 11 (last part). After receiving the order of the downstream node, the wholesaler will send the optimal order to the factory in accordance with Figure 11 (Part 2). Ultimately, the factory will work on optimal production in accordance with Figure 11 (Part 1) within this period.

Sending customer periodic demands to the retailer complicates the system, which causes the bullwhip effect. The designed system should track the reference input in an optimal manner. The reference input (demand) provided by the customer in 20 units of the time period is shown in Figure 12. Due to the absence of reference input poles in the open-loop transfer function, all

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inventories as state variables cannot reach the level of demand. Market demand is estimated at

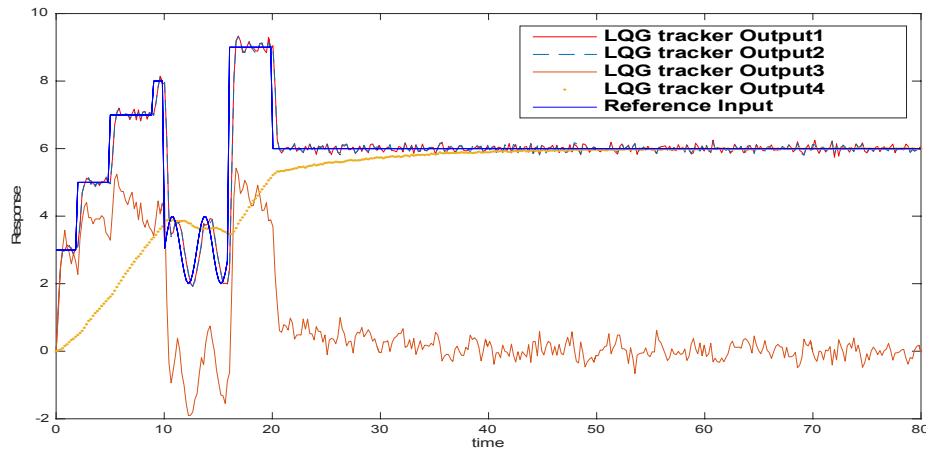80 times, as given in Figure 12. The supply chain must minimize the cost function.

Fig. 12. LQG tracker

Clearly, all outputs of a closed loop system with fixed errors (approximately zero) can track market demand (target input), except for one of the outputs, which track almost 20 steps with a fixed error. Due to the cost of orders and production in index function, control signals (retailer, wholesale, and factory orders) experience significant fluctuations in the first 20 steps. However, the closed-loop system after 20 days is able to keep optimum orders and production at an ideal level. Figure 13 shows the optimal control signals for retailer inventory, wholesaler inventory, and factory production.
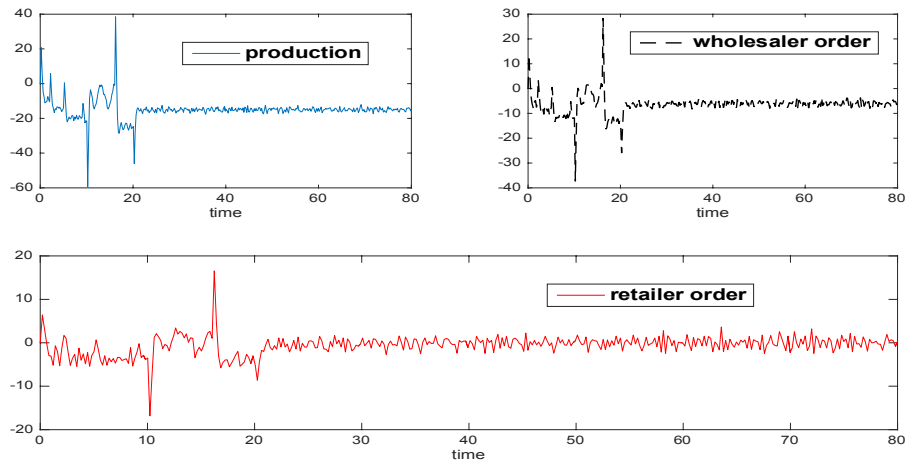

Fig. 13. Optimal control signals

5. Conclusion

In this paper, the model for supply chain with hybrid recycling channel was constructed. Then, the modern control theory method was utilized to design a controllable and observable subsystem of the supply chain. Although the state space model is unstable, the LQG optimal controller guarantees the closed-loop supply chain stabilities. The unstable modes and noisy information caused the bullwhip effect. The important reasons for the bullwhip effect are that

retailers (as downstream level) place aggressive ordering with every level using demand forecasting methods and stochastic lead time. Firstly, the bullwhip effects induced by the use of different forecasting methods, aggressive ordering, and lead time were analyzed. We showed that the bullwhip effect was inevitable in this case. Also, in the next step, we proved that system noise was an important reason for creating the bullwhip effect. Thus, we can add stochastic noise to the list of the bullwhip effect

factors. Using the Kalman filter, the noisy data were filtered. Then, by applying control theory and linear quadratic Gaussian optimal controller, the bullwhip effect variance was regulated. The results showed that the method used for filtering and mitigating bullwhip was efficient. It is also noted that the LQG optimal controller can be easily extended to other multi-echelon supply chains such as power system, economic system, chemical engineering systems, and so on.

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