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RESEARCH PAPER

A Study of Probabilistic Multi-Objective Linear Fractional Programming Problems Under Fuzziness

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ABSTRACT
This paper investigates a multi-objective linear fractional programming problem that involves probabilistic parameters on the right-hand side of constraints. Probabilistic parameters are randomly distributed with known means and variances through Uniform and Exponential Distributions. After converting the probabilistic problem into an equivalent deterministic problem, a fuzzy programming approach is applied by defining a membership function. A linear membership function is used for obtaining an optimal compromise solution. The stability set of the first kind without differentiability corresponding to the obtained optimal compromise solution is determined. A solution procedure for obtaining an optimal compromise solution and the stability set of the first kind is presented. Finally, a numerical example is given to clarify the practicality and efficiency of the study.

KEYWORDS: *Multi-objective linear fractional programming; Uniform distribution; Exponential distribution; Linear membership function; Fuzzy programming; optimal compromise solution; parametric study.*

1. Introduction

Fractional problem (FP) is a decision problem that aims to optimize a ratio subject to constraints. In real-world decision cases, a decision-maker (DM) may sometimes need to evaluate the ratio among inventory and sales, actual cost and standard cost, output, etc. while both denominator and numerator are linear. If only one ratio is considered as an objective function, then a problem is said to be a linear fractional programming (LFP) problem under linear constraints. The fractional programming problem, i.e., the maximization of a fraction of two functions subject to given conditions, arises in various decision-making situations; for instance, fractional programming is applied to the fields of traffic planning (Dantzig et al. [11]), network flows (Arisawa and Elmaghraby [5]), and game theory (Isbell and Marlow [17]). In this respect, a review of different applications was given by Schaible [36- 37]. Ammar and Khalifa [4] studied the LFP problem with fuzzy parameters. Ammar and Khalifa [3] introduced a parametric approach to solve the multi-criteria linear fractional programming problem. Tantawy [40- 41] introduced two approaches to solve the LFP problem: a feasible direction approach and a duality approach. Odior [28] introduced an algebraic approach based on the duality concept and the partial fractions to solve the LFP problem. Pandey and Punnen [31] introduced a procedure based on the Simplex method, developed by Dantzig [11], to solve the LFP problem. Gupta and Chakraborty [14] solved the LFP problem based on the sign in the numerator under the assumption that the denominator is non-vanishing in a feasible region using the fuzzy programming approach. Chakraborty [8] studied a nonlinear fractional programming problem with multiple constraints under a fuzzy environment. Stanojevic and Stancu- Minasian [39] proposed a method for solving a fully fuzzified LFP problem. Buckley and Feuring [7] studied the fully fuzzified linear

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programming involving coefficients and decision variables as fuzzy quantities. Li and Chen [23] introduced a fuzzy LFP problem with fuzzy coefficients and presented the concept of fuzzy optimal solution. Sakawa and Kato [33] introduced an interactive satisficing method for solving multi-objective fuzzy LFP problems with fuzzy parameters both in the objective functions and constraints. Pop and Stancu [32] studied the LFP problem with all parameters and decision variables being triangular fuzzy numbers. Gupta and Chakraborty [15] applied the fuzzy programming approach for solving a restricted class of multi-objective linear fractional programming (MOLFP) problems, such that certain values of decision variables exist for which the numerator and denominator are positive for all values of decision variables. Nykowski and Zolkiewski [27] solved the MOLFP problem by converting it into a multi-objective linear programming (MOLP) problem. Dutta et al. [12] applied the fuzzy programming approach for solving the bi-objective linear programming problem. Charnes and Cooper [9] optimized the LFP problem by solving two linear programs. Luhandjula [24] solved the MOLFP problem by the fuzzy compromise approach. Three main approaches to stochastic programming (Goicoechea et al. [13]) are recognized, of which one is the risk programming in linear programming models that include chance-constrained programming. The chance-constrained programming solves problems that involve chance constrains. Leclercq et al. [22] and Teghem et al. [42] introduced interactive methods in stochastic programming. Sinha et al. [38] studied multi-objective probabilistic linear programming with only the right-hand side of the constraints distributed with known means and variances and, then, applied the fuzzy programming approach to obtain an optimal compromise solution.

In his earlier work, Osman [29] analyzed the notions of solvability set, the stability set of the first kind, and the stability set of the second kind for parametric convex nonlinear programming problems. Kassem [18] determined the stability set of the first kind for the interactive multi-objective nonlinear programming problems involving fuzzy parameters in the constraints. Kassem and Ammar [19] studied the stability of multi-objective nonlinear programming problems with fuzzy parameters in the constraints. Osman and El-Banna [30] presented the stability of multi-objective nonlinear programming problems involving fuzzy parameters.

Despite considerable decision-making experience, a decision-maker cannot always live up to predefined goals precisely. Decision-making in a fuzzy environment, as developed by Bellman and Zadeh [6], has improved considerably that, in turn, helps deal with management decision problems. The fuzzy nature of a goal-programming problem was first discussed by Zimmermann [44], followed by Narasimhan [25] and Hanan [16]. Using the main operator and linear and special membership functions, Leberling [21] showed that compromise solutions could always be derived from the original multi-criteria problem. Khalifa [20] studied a linear fractional programming problem with inexact rough intervals in the parameters. Nasseri and Bavandi [26] studied the fuzzy stochastic linear fractional programming in which the coefficients and scalars in the objective function were triangular fuzzy numbers and technological coefficients and the quantities on the right-hand side of the constraints were fuzzy random variables with specific distributions. Ren et al. [33] developed a multi-objective stochastic fractional goal programming for the optimal allocation of water resources based on analysis of water resources quantity, quality, and uncertainty. Acharya et al. [1] proposed a solution methodology for the multi-objective probabilistic fractional programming, where parameters on the right-hand side of constraints follow Cauchy distribution.

The remainder of the paper is organized as follows: In section 2, a probabilistic multi-objective linear fractional programming problem is introduced with specific definitions and properties. In Section 3, a fuzzy programming approach to solving the problem is given. The stability set of the first kind without differentiability is determined in Section 4. In Section 5, a solution procedure for obtaining an optimal compromise solution and the stability set of the first kind corresponding to the resulted solution is presented. In Section 6, an illustrative numerical example is given to clarify the obtained results. Finally, some concluding remarks are reported in Section 7.

2. Problem Statement and Solution Concepts

In chance-constrained programming, a stochastic multi-objective linear fractional programming problem can be stated as follows:

$$\max F^k(x) = \frac{N^k(x)}{Q^k(x)} = \frac{\sum_{j=1}^{n} p_j^k x_j + p_0^k}{\sum_{j=1}^{n} q_j^k x_j + q_0^k}, k = 1, 2, ..., K \quad (1)$$

subject to:

$$prob\left[\sum_{j=1}^{n} a_{ij} x_j \le b_i\right] \ge 1 - \alpha_i,\ i = 1, 2, \ldots, m, \quad (2)$$

$$x_j \ge 0,\ j = 1, 2, ..., n \quad (3)$$

where $F^k(x) = \{F^1(x), F^2(x), ..., F^K(x)\}$ is a vector of K objectives, and the subscript on $F^k(x)$ represents the number of objectives $(k = 1, 2, ..., K)$, p_j^k, q_j^k $(j = 1, 2, \ldots, n; k = 1, 2, \ldots, K)$, p_0^k, q_0^k $(k = 1, 2, ..., K)$, a_{ij} $(i = 1, 2, ..., m; j = 1, 2, ..., n)$, and $b_i (i = 1, 2, \ldots, m)$ are random variables, and $\alpha_i \in (0,1)$ represents specified probabilities. It is assumed that the decision variables $(x_j, j = 1, 2, ..., n)$ are deterministic.

It is clear that the notion of Pareto optimal solution to the probabilistic MOLFP problems (1)-(3) cannot be applied. For this, the following distributions are introduced as follows:

(a) Uniform Distribution,
(b) Exponential Distribution.

(i)When b_i's are uniformly distributed continuous random variables Let b_i's be uniform random variables. Then,

$$g(b_i) = \begin{cases} \frac{1}{\beta_i - \gamma_i}, & \gamma_i < b_i < \beta_i, i = 1, 2, ..., m \\ 0, & otherwise \end{cases} \quad (4)$$

where $\mu = \frac{\beta_i + \gamma_i}{2}$ and $\sigma^2 = \frac{\beta_i^2 - \gamma_i^2}{12}$. It follows that the Constraints (2) become

$$\left[\int_{\sum_{j=1}^{n} a_{ij} x_j}^{\beta_i} \frac{1}{\beta_i - \gamma_i} db_i\right] \ge 1 - \alpha_i = \left[\frac{b_i}{\beta_i - \gamma_i}\right]_{\sum_{j=1}^{n} aij\, x_j}^{\beta_i} \ge 1 - \alpha_i,$$

Or $$\frac{\beta_i - \sum_{j=1}^{n} a_{ij} x_j}{\beta_i - \gamma_i} \ge 1 - \alpha_i \quad \text{or} \quad \sum_{j=1}^{n} a_{ij} x_j \le c_i, i = 1, 2, ..., n \quad (5)$$

where $c_i = \gamma_i + \alpha_i(\beta_i - \gamma_i), i = 1, 2, \ldots, m.$

Therefore, the probabilistic MOLFP problems (1)-(3) become

$$(\mathrm{P}_1)\ \max F^k(x) = \frac{N^k(x)}{Q^k(x)} = \frac{\sum_{j=1}^{n} p_j^k\, x_j + p_0^k}{\sum_{j=1}^{n} q_j^k\, x_j + q_0^k}, k = 1, 2, \ldots, K$$

Subject to

$$\sum_{j=1}^{n} a_{ij}\, x_j \le c_i, i = 1, 2, \ldots, m,$$

$$x_j \ge 0,\ j = 1, 2, \ldots, n\ .$$

Definition1. (Nondominated solution). A feasible solution $x^* \in G$ (G is a feasible domain) is said to be the nondominated solution of (P_1) if and only if there is no other feasible solution $x \in G$ such that $\left(\frac{\sum_{j=1}^{n} p_j^k\, x_j^* + p_0^k}{\sum_{j=1}^{n} q_j^k\, x_j^* + q_0^k}\right) \le \left(\frac{\sum_{j=1}^{n} p_j^k\, x_j + p_0^k}{\sum_{j=1}^{n} q_j^k\, x_j + q_0^k}\right)$, for all k's and $\left(\frac{\sum_{j=1}^{n} p_j^k\, x_j^* + p_0^k}{\sum_{j=1}^{n} q_j^k\, x_j^* + q_0^k}\right) \ne \left(\frac{\sum_{j=1}^{n} p_j^k\, x_j + p_0^k}{\sum_{j=1}^{n} q_j^k\, x_j + q_0^k}\right)$, for some $k, k = 1, 2, \ldots, K.$

Definition2. (Compromise solution). A feasible solution $x^\circ \in G$ is said to be a compromise solution of (P_1) if and only if $x^\circ \in H$ and $F(x^\circ) \ge \vee_{x \in G} F(x),$ where $\vee$ and H represent maximum and a set of efficient solutions, respectively.

(ii) When b_i's are exponential random variables, let b_i's be exponential random variables. Then, we get

$$f(b_i) = \begin{cases} \lambda_i\, e^{(-\lambda_i b_i)}, i = 1,2,\ldots,m \\ 0, \qquad otherwise \end{cases} \tag{6}$$

where $\mu = \frac{1}{\lambda_i}$ and $\sigma^2 = \frac{1}{\lambda_i^2}$. It follows that Constraints (2) become

$$\int_{\sum_{j=1}^{n} a_{ij} x_j}^{\infty} \lambda_i e^{(-\lambda_i b_i)} db_i \ge 1 - \alpha_i = e^{(-\lambda_i \sum_{j=1}^{n} a_{ij} x_j)} \ge 1 - \alpha_i, i = 1,2,\ldots m. \tag{7}$$

It is obvious that (7) can be rewritten as follows:

$$\sum_{j=1}^{n} a_{ij}\, x_j \le d_i, i = 1,2,\ldots,m.$$

where $d_i = -\frac{\ln(1-\alpha_i)}{\lambda_i}, i = 1, 2, \ldots, m.$

Thus, the probabilistic problems (1)- (3) are converted into the following deterministic problem:

(P_2)

$$\max F(x) = \frac{N(x)}{Q(x)} = \frac{\sum_{j=1}^{n} p_j^{(k)}\, x_j + p_0^{(k)}}{\sum_{j=1}^{n} q_j^{(k)}\, x_j + q_0^{(k)}}, k = 1, 2, \ldots, K$$

Subject to

$$\sum_{j=1}^{n} a_{ij}\, x_j \le d_i, i = 1,2,\ldots,m$$

$$x_j \ge 0,\ j = 1, 2, \ldots, n\ .$$

Let the MOLFP problem of the type be

$$\max F(x) = \{F_1(x), F_2(x), \ldots, F_k(x)\} \tag{8}$$

Subject to

$$x \in \Omega = \{x \in R^n : Ax \le b, x \ge 0\} \tag{9}$$

It is clear that Problems (8)-(9) are equivalent to the following multi-objective linear programming:

(MOLP) problem (Schaible [34])

$$\max\{t\,N_1(y/t), t\,N_2(y/t), ..., t\,N_K(y/t)\}$$

Subject to (10)

$$tQ_k(y/t) \le 1, k = 1,2,...,K; A(y/t) - b \le 0, t > 0, y \ge 0.$$

3. Fuzzy Programming Approach for Solving MOLFP Problem

Bellman and Zadeh [6] introduced three basic concepts: fuzzy goal (G), fuzzy constraints (C), and fuzzy decision (D) and explored the application of these concepts to decision-making processes under fuzziness.

The fuzzy decision is a fuzzy set and is defined as follows:

$$D = G \cap C. \tag{11}$$

The fuzzy decision is characterized by its membership function:

$$\mu_D(x) = \min(\mu_G(x), \mu_C(x)). \tag{12}$$

The Membership function of each objective function can be constructed as follows:

$$\mu_k(tN^k(y/t)) = \begin{cases} 0, & tN^k(y/t) \le \underline{F}^k \\ \dfrac{tN^k(y/t) - \underline{F}^k}{\bar{F}^k - \underline{F}^k}, & \underline{F}^k < tN^k(y/t) < \bar{F}^k \\ 1, & tN^k(y/t) \ge \bar{F}^k \end{cases} \tag{13}$$

Where

$$\bar{F}^k = \max\left\{ \frac{p_j^k}{q_j^k}, \frac{p_0^k}{q_0^k}, j = 1,2,...,n; k = 1,2,...,K \right\}$$

, and (14)

$$\underline{F}^k = \min \left\{ \frac{p_j^k}{q_j^k}, \frac{p_0^k}{q_0^k}, j = 1,2,...,n; k = 1,2,...,K \right\}. \tag{15}$$

Based on Zadeh's min operator [43], the fuzzy problem (10) is reduced to the following ordinary model as follows:

$$\max v$$

Subject to (16)

$$\mu_k(t\,N^k(y/t)) \ge v,\ t\,Q^k(y/t) \le 1, k = 1,2,...,K,$$
$$A(y/t) - b \le 0, t > 0, y, v \ge 0.$$

4. Determination of The Stability Set of The First Kind Without Differentiability

In this section, the stability set of the first kind corresponding to the obtained optimal compromise solution x^* of the deterministic MOLFP problem is obtained under the effect of the probability distributions on the probabilistic MOILFP problem. Let us consider the deterministic MOLFP problem below

(p_3)

$$\max F^k(x) = \frac{N^k(x)}{Q^k(x)} = \frac{\sum_{j=1}^{n} p_j^k x_j + p_0^k}{\sum_{j=1}^{n} q_j^k x_j + q_0^k}, k = 1,2,...,K$$

Subject to (17)

$$x \in X(v) = \left\{ x \in R^n : g_r(x) \le v_r, r = 1,2,3,...,m \right\}$$

The above problem (p_3) can be rewritten according to Problem (16) as follows:

$$\max v$$

Subject to

$$G(y/t) = \begin{cases} \mu_k(t\,N^k(y/t)) \ge v,\ t\,Q^k(y/t) \le 1, k = 1,2,...,K; \\ g_r(y/t) - v_r \le 0, r = 1,2,3,...,m;\ t > 0, y \ge 0,\ v \ge 0. \end{cases} \tag{18}$$

Let $F(v^*)$ be a subset of efficient solutions to the problem (p_3) corresponding to $v^* \in R^m$.

Definition3. (Osman [29]). The stability set of the first kind of problem (p_3) corresponding to

$F(v^*)$, as denoted by $S(F(v^*))$, is defined as follows:
$S(F(v^*)) = \{v \in R^m : F(v^*) \subseteq F(v)\}$, where $F(v)$ is the set of all efficient solutions to the problem (p_3) corresponding to $v \in R^m$.
It is known that x^* is an efficient solution to the problem (p_3) if v^* exists such that (y^*, t^*) is the unique optimal solution to Problem (18) (Chankong and Haimes [10]). Let x^* be an efficient solution to the problem (p_3) corresponding to v^*; based on the stability of problem (p_3), we have $u_r^* \geq 0, r = 1, 2, \ldots, m$; $w_k^*, \beta_k^*, k = 1,2,\ldots, K$, such that (y^*, t^*, u^*) solves the following Kuhn-Tucker Saddle point problem (Chankong and Haimes, 1983)

$\psi(y^*, t^*, u, w, \beta, v^*) \leq \psi(y^*, t^*, u^*, w^*, \beta^*, v^*) \leq \psi(y, t, u^*, w^*, \beta^*, v^*); \forall\, y \in R^n, t \in R, u \in R^m$; $w, \beta \in R^K$ with $u \geq 0$, $w \geq 0, \beta \geq 0$, where

$\psi(y,t,u,,w,\beta,v) = v + \sum_{r=1}^{m} u_r\,(g_r(x) - v_r) + \sum_{k=1}^{K} w_k \left(-\mu_k(t\,N^k(y/t)) + v\right) + \sum_{k=1}^{K} \beta_k \left(Q^k(y/t) - 1\right)$, is the Lagrangian function of Problem (18) and $v \in R^m$.

Let (y^*, t^*) be a unique optimal solution to Problem (18) corresponding to $v^* \in R^m$. The Kuhn-Tucker Saddle point conditions of Problem (18) can be formulated as follows:

$$
\begin{aligned}
& v + \sum_{r=1}^{m} u_r\,(g_r(y^*/t^*) - v_r^*) + \sum_{k=1}^{K} w_k \left(-\mu_k(t^* N^k(y^*/t^*)) + v\right) + \sum_{k=1}^{K} \beta_k \left(Q^k(y^*/t^*) - 1\right) \\
& \leq v + \sum_{r=1}^{m} u_r^*\,(g_r(y^*/t^*) - v_r^*) + \sum_{k=1}^{K} w_k^* \left(-\mu_k(t^* N^k(y^*/t^*)) + v\right) + \sum_{k=1}^{K} \beta_k^* \left(Q^k(y^*/t^*) - 1\right) \\
& \leq v + \sum_{r=1}^{m} u_r^*\,(g_r(y/t) - v_r^*) + \sum_{k=1}^{K} w_k^* \left(-\mu_k(t\,N^k(y/t)) + v\right) + \sum_{k=1}^{K} \beta_k^* \left(Q^k(y/t) - 1\right), \forall\, y \in R^n, t \in R, \\
& u \in R^m, w, \beta \in R^K, u, w, \beta \geq 0,
\end{aligned}
$$

$$
\begin{aligned}
& \mu_k(t^* N^k(y^*/t^*)) \geq v, k = 1, 2, \ldots, K, \\
& t\,Q^k(y^*/t^*) \leq 1, k = 1, 2, \ldots, K, \\
& g_r(y^*/t^*) - v_r^* \leq 0, r = 1, 2, 3, \ldots, m, \\
& u_r^*(g_r(y^*/t^*) - v_r^*) = 0, r = 1, 2, \ldots, m; \\
& w_k^*\left(-\mu_k(t^* N^k(y^*/t^*)) + v\right) = 0, k = 1, 2, \ldots, K; \\
& \beta_k^*\left(Q^k(y^*/t^*) - 1\right) = 0, k = 1, 2, \ldots, K, \\
& u_r^* \geq 0, r = 1, 2, \ldots, m; w_k^*, \beta_k^* \geq 0, k = 1, 2, \ldots, K.
\end{aligned}
$$

To determine $S((y/t)^{*i})$, let us apply the following condition:

$$
\begin{aligned}
& u_r^*(g_r(y^*/t^*) - v_r^*) = 0, r = 1, 2, \ldots, m; \\
& w_k^*\left(-\mu_k(t^* N^k(y^*/t^*)) + v\right) = 0, k = 1, 2, \ldots, K; \\
& \beta_k^*\left(Q^k(y^*/t^*) - 1\right) = 0, k = 1, 2, \ldots, K;
\end{aligned}
$$

Considering the following three cases:

(i) $u_r^* > 0, r \in I = \{1,2,...,m\}; w_k^*, \beta_k^* > 0, kJ = \{1,2,...,K\}, u_r^* = 0, r \notin I, w_k^* = \beta_k^* = 0, k \notin J.$

Let M be all proper subsets of $\{1,2,...,m\}$, and $\{1,2,..,K\}$. Then, the stability set of the first kind corresponding to the subsets I and J is given by

$$S_{I,J}\left((y^*/t^*)^i\right) = \begin{Bmatrix} v \in R^m : g_r((y^*/t^*)^i) = v_r\ r \in I,\ g_r((y^*/t^*)^i) \le v, r \notin I, \\ \mu_k(t^* N^k(y^*/t^*)) \ge v, k \in J, t^* Q^k(y^*/t^*) \le 1, k \in K \end{Bmatrix}$$

Then,

$$S_1\left((y^*/t^*)^i\right) = \bigcup_{I,J \in M} S_{I,J}\left((y^*/t^*)^i\right)$$

(ii) $u_r^* = 0, r \in I = \{1,2,...,m\}; w_k^*, \beta_k^* > 0, k \in J = \{1,2,...,K\},\ w_k^* = \beta_k^* = 0, k \notin J.$

Then,

$$S_2\left((y^*/t^*)^i\right) = \begin{Bmatrix} v \in R^m : g_r((y^*/t^*)^i) \le v_r\ r \in I,\ \mu_k(t^* N^k(y^*/t^*)) \ge v, \\ k \in J, t^* Q^k(y^*/t^*) \le 1, k \in K \end{Bmatrix}$$

(iii) $u_r^* > 0, r \in I = \{1,2,...,m\}; w_k^*, \beta_k^* > 0, k \in J = \{1,2,...,K\},\ w_k^* = \beta_k^* = 0, k \notin J.$

Then,

$$S_3\left((y^*/t^*)^i\right) = \begin{Bmatrix} v \in R^m : g_r((y^*/t^*)^i) = v_r\ r \in I,\ \mu_k(t^* N^k(y^*/t^*)) \ge v, \\ k \in J, t^* Q^k(y^*/t^*) \le 1, k \in K \end{Bmatrix}$$

Hence,

$$S\left((y^*/t^*)^i\right) = \bigcup_{l=1}^{3} S_l\left((y^*/t^*)^i\right).$$

The stability set of the first kind $S(F(v^*))$ is determined as follows:

$$S(F(v^*)) = \bigcap_{i \in L} S\left((y^*/t^*)^i\right).$$

5. Solution Method

In this section, a methodology for the probabilistic MOLFP problem through the fuzzy programming approach is presented as in the following steps:

Step1: Convert a given probabilistic MOLFP problem into the corresponding deterministic MOLFP problem based on the chance-constrained programming technique, illustrated above.

Step2: From the obtained deterministic MOLFP problem, determine $\bar{F}_k$ and $F_{\underline{k}}$ as defined in (14) and (15), respectively.

Step3: Using a membership function defined as in (13), find a corresponding fuzzy linear programming, which is discussed as in (16).

Step4: Solve Problem (16) using any computer package to obtain an optimal compromise solution that is an efficient solution to the deterministic MOLFP problem.

Step 5: Determine the stability set of the first kind corresponding to the optimal compromise solution obtained in Step 4.

6. Numerical Example

(i)When b_i's are uniformly distributed continuous random variables

$$\max F(x) = \left[F^1(x) = \frac{5x_1 + 3x_2}{5x_1 + 2x_2 + 1}, F^2(x) = \frac{7x_1 + x_2}{x_1 + 9x_2 + 1} \right]$$

Subject to

$$prob.\left[3x_1 + 5x_2 \le 7.6\right] \ge 1 - \alpha_1,$$
$$prob.\left[5x_1 + 2x_2 \le 7.2\right] \ge 1 - \alpha_2,$$
$$x_j \ge 0, j = 1,2.$$

where $E(b_i) = 6$, $V(b_i) = 4$, $\alpha_1 = 0.95$, and $\alpha_2 = 0.8$.

From Step1, the following deterministic MOLFP problem is obtained:

$$\max F(x)=\left[F^1(x)=\frac{5x_1+3x_2}{5x_1+2x_2+1}, F^2(x)=\frac{7x_1+x_2}{x_1+9x_2+1}\right]$$

Subject to

$$3x_1+5x_2\le 0.76,$$
$$5x_1+2x_2\le 0.72,$$
$$x_1,x_2\ge 0.$$

It is clear that $\bar{F}^1=\frac{3}{2}, \bar{F}^2$ and $\underline{F}^1=\underline{F}^2=0.$

The membership functions of both $F^1(x)$ and $F^2(x)$ are as follows:

$\mu_1(F^1(y))=(5y_1-3y_2)/1.5, \quad \mu_2(F^2(y))=(7y_1-y_2)/1.5$. Now, by means of the membership functions, the following crisp model is obtained:

$$\max v$$

Subject to

$$10y_1+6y_2-3v\ge 0,\ 7y_1+y_2-7v\ge 0, 5y_1+2y_2+t\le 1,$$
$$y_1+9y_2+t\le 1,\ 3y_1+5y_2-0.76\le 0,\ 5y_1+2y_2-0.72\le 0,$$
$$y_1,y_2,t,v\ge 0.$$

The solution of the above model is given below:

$v=0.144$, $y_1=0.144$, $y_2=0$, $t=01$. For the original problem, the solution is:

$x_1=1.44$, $x_2=0$ $F^1=0.878$, $F^2=0.9097$.

To obtain the stability set of the first kind corresponding to $F(0.76,0.72)$, the following system of equations should be solved:

$$u_1(4.32-v_1)=0,$$
$$u_2(7.2-v_2)=0,$$
$$u_3(-1.44-v_3)=0,$$
$$u_4(0-v_4)=0,$$

We have $S_{I_w}(1.44,0)=\{u\in R^4 : v_r=g_r(1.44,0), r\in I_w, v_r\ge g_r(1.44,0), r\in I\}$, where $I_w\subseteq\{1,2,3,4\}$.

Hence,

$S(1.44,0)=\bigcup_{w=1}^{14} S_{I_w}(1.44,0)$ and, thus,

$$S(F(v^*))=\{v\in R^4 : v_1\ge 4.32, v_2\ge 7.2, v_3\ge 1.44, v_4\ge 0\}.$$

(ii)When b_i's are exponential random variables, then

$$\max F(x)=\left[F^1(x)=\frac{x_1-4}{-2x_2+3}, F^2(x)=\frac{-x_1+5}{x_2+1}\right]$$

Subject to

$prob\left[x_1 + x_2 \le b_1\right] \ge 0.94,$
$prob\left[4x_1 + 3x_2 \le b_2\right] \ge 0.93,$
$prob\left[2x_1 + 5x_2 \le b_3\right] \ge 0.91,$
$x_1, x_2, x_3 \ge 0.$

where $E(b_1) = 7,\ E(b_2) = 9,\ E(b_3) = 8,\ \alpha_1 = 0..06, \alpha_2 = 0.07$, and $\alpha_2 = 0.09$

From Step 1, the following deterministic MOLFP problem is obtained:

$$\max\ F(x) = \left[F^1(x) = \frac{x_1 - 4}{-2x_2 + 3}, F^2(x) = \frac{-x_1 + 5}{x_2 + 1} \right]$$

Subject to

$x_1 + x_2 \le 0.433,$
$4x_1 + 3x_2 \le 0.653,$
$2x_1 + 5x_2 \le 0.7545,$
$x_1, x_2 \ge 0.$

It is obvious that $\bar{F}^1 = 1,\ \bar{F}^2 = 5$, and $\underline{F}^1 = \frac{-4}{3}\ \underline{F}^2 = -1.$

The membership functions of both $F^1(x)$ and $F^2(x)$ are given below:

$\mu_1(F^1(y)) = (y_1 - 4t + 1)/(1 + 4/3),\ \mu_2(F^2(y)) = (-y_1 + 5t + 1)/(5 + 1)$

Now, by means of the membership functions, the following crisp model is obtained below:

$\max v$

Subject to

$y_1 - 4t - v \ge 0, -y_1 + 5t - 5v \ge 0, -y_2 + 3t \le 1,$
$y_2 + t \le 1,\ y_1 + y_2 - 0.433t \le 0,$
$4y_1 + 3y_2 - 0.653t \le 0, 2y_1 + 5y_2 - 0.7545t \le 0,$
$y_1, y_2, t, v \ge 0.$

The solution of the above model is given below:

$v = 0.83$, $y_1 = 2.0833$, $y_2 = 0.5$, $t = 0.5.$ The solution to the original problem is given below:

$x_1 = 4.1666$, $x_2 = 1$ $F_1 = 0.1666,$ $F_2 = 0.4167$

To get the stability set of the first kind corresponding to $F(0.94, 0.93, 0.91)$, we get the following system of equations:

$u_1(5.1666 - v_1) = 0,$
$u_2(19.6664 - v_2) = 0,$
$u_3(1.3332 - v_3) = 0,$
$u_4(-4.1666 - v_4) = 0,$
$u_5(1 - v_5) = 0,$

We have
$S_{I_w}(4.1666,1) = \{u \in R^5 : v_r = g_r(4.1666,1), r \in I_w, v_r \geq g_r(4.1666,1), r \in I\},$ where $I_w \subseteq \{1,2,3,4,5\}.$
Hence,
$S(4.1666,1) = \bigcup_{w=1}^{32} S_{I_w}(4.1666,1)$ and, thus,
$S(F(v^*)) = \{v \in R^4 : v_1 \geq 5.1666, v_2 \geq 19.6664, v_3 \geq 1.332, v_4 \geq 4.1666, v_5 \geq -1\}.$

7. Concluding Remarks

In this paper, a multi-objective linear fractional programming problem involving probabilistic parameters on the right-hand side of the constraints was introduced. These probabilistic parameters were randomly distributed with known means and variances through the use of Uniform and Exponential Distributions. Although the probabilistic problem was converted into an equivalent deterministic problem, a fuzzy programming approach was applied by defining a membership function. A linear membership function was applied to obtain an optimal compromise solution. The stability set of the first kind corresponding to the obtained optimal compromise solution was determined. A solution procedure for obtaining an optimal compromise solution and the stability set of the first kind was also presented. An illustrative numerical example was given to clarify the obtained results.

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Conflicts and Interest
The author declares no conflict of interest.

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