RESEARCH PAPER

Multi-Period Sustainability Performance in the Presence of Discrete and Bounded Measures

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ABSTRACT

Sustainability performance assessment is a significant aspect of making sustainable decisions for organizations. Measuring sustainability performance of firms in a time span, covered in several periods, leads to more rational decision-making and planning by managers. Furthermore, in many application fields, there are discrete and bounded measures. However, there has been no systematic effort to analyze sustainability performance of Decision-Making Units (DMUs) in multiple periods of time when discrete and bounded factors are presented. Therefore, approaches based on Data Envelopment Analysis (DEA) are proposed in this paper to tackle this problem. To illustrate this issue in more detail, the performance of systems is measured for all dimensions, including economic, social, and environmental ones and for each period. Moreover, the overall multi-period sustainability performance and sustainability performance of each period are estimated using the suggested one-stage methods. Then, the sustainability performance is investigated for conditions in which internal relationships among economic, social, and environmental indicators are presented. Moreover, sustainability performance changes and performance changes of dimensions are addressed. An example and a case study are provided to explain our proposed approach. Results show that the introduced ideas are practical and effective.

KEYWORDS: *Sustainability Performance, Data Envelopment Analysis, Multi-Period, Discrete, Bounded.*

1. Introduction

Sustainable development is an important aspect in today's societies. Forwarding sustainable growth needs to follow economic, social, and environmental goals, simultaneously. Measuring the sustainability of systems can facilitate determining the sustainability level and the efficiency in each sustainability dimension. Obviously, more beneficial information can be obtained from sustainability performance analysis in a time span containing multiple periods of time.

In the literature, there are approaches to measuring sustainability performance. Readers can refer to [1] for more information. One of the prevailing approaches for this purpose is the non-parametric Data Envelopment Analysis (DEA) approach, originally proposed by Charnes et al. [2]. Zhou et al. [3] provided an organized investigation on DEA utilizations in sustainability. Galán-Martín et al. [4] proposed an enhanced DEA method that applied the concept of 'order of efficiency' to assess sustainability. In their approach, sustainability has been analyzed in a specific period of time. Zhao et al. [5] developed a method based upon DEA to measure the performance of systems that each of them designed as an economic and environmental subunit and a social subunit. Hassanzadeh et al. [6] introduced input-oriented and output-oriented inverse semi-oriented models and measured the sustainability of countries in the presence of negative data. Tajbakhsh and Shamsi [7] designed a non-parametric index to assess the sustainability performance of countries in one period when undesirable and integer materials are presented. Amirteimoori et al. [8] assessed the sustainability

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performance in the existence of undesirable outputs over time.

In the DEA context, there are methods for measuring the efficiency of Decision-Making Units (DMUs) in several periods of time. Park and Park [9] evaluated the aggregative efficiency of multi-period systems. Kao and Liu [10] utilized and adopted a relational network model to calculate the overall and period efficiencies of multi-period processes. Afterwards, Kao and Hwang [11] computed the multi-period efficiency of two-stage systems. Esmaeilzadeh and Kazemi Matin [12] extended multi-period DEA models considering more complex internal relations for the sub-processes of each entity. Kordrostami and Jahani Sayyad Noveiri [13] proposed a DEA-based method to analyze the efficiency of the multi-period production systems when imprecise data are presented. Jablonsky [14] presented efficiency and super-efficiency multi-period DEA models. Kordrostami et al. [15] investigated multi-period efficiency scores and efficiency changes where there were undesirable outputs. Furthermore, the multi-period efficiency of firms with negative factors is measured in the study by Kordrostami and Jahani Sayyad Noveiri [16]. According to the DEA literature, there is no investigation to analyze the multi-period sustainability performance of systems with discrete and bounded data. Nevertheless, there are occasions in the real world that the sustainability of systems should be measured in multiple periods while discrete and bounded measures are appeared. For instance, the number of employees and the number of products are integer variables. In addition, the satisfaction degree and the rate of on-time delivery can be taken into account as bounded factors.

Through a survey of the literature, one can found that there are studies within DEA that have addressed the efficiency of DMUs with integer and/or bounded factors.

Lozano and Villa [17] suggested DEA patterns to appraise the performance of DMUs where integer inputs and outputs appeared. Subsequently, Kuosmanen and Kazemi Matin

[18] described principles to evaluate the efficiency and finding targets under constant returns to scale (CRS) case when integer factors were detected. Then, Kazemi Matin and Kuosmanen [19] developed integer-valued DEA models under alternative returns to scale assumptions including variable, non-increasing and non-decreasing. Chen et al. [20] measured efficiency and super-efficiency in the presence of integer undesirable factors. Further to that, Chen et al. [21] designed models based on DEA to analyze the relative efficiency of DMUs in the presence of bounded, discrete data and Likert scales. Then, after that, Chen et al. [22] evaluated efficiency and super-efficiency of NBA players utilizing bounded integer DEA models. A mixed-integer linear programming problem has been developed by Kazemi Matin and Emrouznejad [23] to estimate the performance of DMUs with integer input-output variables and bounded outputs.

As literature reviews and Table 1 show, no study has estimated multi-period sustainability performance in the presence of discrete and bounded measures. In this paper, approaches on the basis of DEA are proposed to analyze the sustainability performance of systems in a time span where discrete and bounded measures appeared. To illustrate, the relative efficiency is evaluated for each sustainability dimension and each period while bounded and discrete measures are available. The overall multi-period sustainability is also calculated. Indeed, the approaches proposed by Jablonsky [14] and Chen et al. [21] are extended to assess multi-period sustainability performance in the presence of discrete and bounded factors. Next, the presented method is generalized for situations where internal relationships among dimensions are observed. After introducing two models, input-oriented and output-oriented, the efficiency changes between years are addressed. These variations between the whole sustainability performance and each dimension of the sustainability in economic, environmental, and social terms are explored.

Tab 1. Comparative review of some studies explored in Section 1

Source	Three dimensions of sustainability	Bounded	Integer-valued	Multi-period
Tajbakhsh and Shamsi [7]	*		*	
Galán-Martín et al. [4]	*			
Lozano and Villa [17]			*	
Kuosmanen and Kazemi Matin [18]			*	
Kazemi Matin and Kuosmanen [19]			*	
Chen et al. [20]			*	
Chen et al. [21]		*	*	
Chen et al. [22]		*	*	
Kazemi Matin and Emrouznejad [23]		*	*	
Jablonsky [14]				*
Park and Park [9]				*
Kao and Liu [10]				*
Amirteimoori et al. [8]	*			*
Our approach	*	*	*	*

The paper is organized as follows: the approach of Chen et al. [21] to address bounded and discrete data and, also, Jablonsky's technique [14] to study the relative efficiency of systems in several years are reviewed in Section 2. Then, approaches are proposed to assess the multi-period sustainability performance with bounded and discrete measures in Section 3. Afterwards, an example and a case study of gas companies are provided in Section 4 to exemplify the suggested method herein. Finally, conclusions are revealed in Section 5.

2. Preliminaries
This section begins with the explanation of the approach proposed by Chen et al. [10] to deal with discrete and bounded items. Then, Jablonsky's method [8] to analyze the relative efficiency of DMUs in several periods of time is investigated.

2-1. Approach of Chen et al. [21] for handling discrete and bounded factors
By considering x_{io} and y_{ro} as the ith input and the rth output of the unit under consideration, DMU_o, Chen et

al. [21] provided the following model to measure the relative efficiency of DMU_o and determine the projection points when discrete and bounded data are presented:

$$ef^* = Min \quad ef = \frac{1}{m}\sum_{i=1}^{m}\alpha_i$$

$$
\begin{aligned}
s.t. \quad & \sum_{j=1}^{n}\lambda_j x_{ij} \leq \tilde{x}_{io}, i = 1,2,...,m, \\
& \tilde{x}_{io} \leq \alpha_i x_{io}, i = 1,2,...,m, \\
& L_{i_{Bnd}} \leq \tilde{x}_{i_{Bnd}o} \leq U_{i_{Bnd}}, i_{Bnd} \in I_{Bnd}, \quad (1)\\
& \tilde{x}_{i_{Int}o} \text{ integer}, i_{Int} \in I_{Int}, \\
& \sum_{j=1}^{n}\lambda_j y_{rj} \geq y_{ro}, r = 1,2,...,s, \\
& \lambda_j \geq 0, j = 1,...,n.
\end{aligned}
$$

in which $\lambda_j (j = 1,...,n)$ and $\alpha_i (i = 1,...,m)$ are intensity and efficiency variables, respectively. For $DMU_j (j = 1,...,n)$, inputs $x_{ij} (i = 1,...,m)$ may be integer, non-integer, bounded and unbounded variables. Therefore, inputs are split into integer and continuous factors as indicated by $x_{i_{int}j}$ and $x_{i_{cont}j}$ such that $I_{int} \cup I_{cont} = \{1,...,m\}$; bounded and unbounded items are symbolized by $x_{i_{bnd}j}$ and $x_{i_{Unb}j}$ while $I_{Bnd} \cup I_{Unb} = \{1,...,m\}$. I_{Int} and I_{Bnd} may have common components. Moreover, $y_{rj}(r = 1,...,s)$ show outputs. $\tilde{x}_{i_{Int}o}, i_{Int} \in I_{Int}$ are integer variables. $\tilde{x}_{i_{Bnd}o}, i_{Bnd} \in I_{Bnd}$ also indicate bounded variables. It is clear that $L_{i Bnd}$ and

U_{iBnd} show lower and upper bounds of input i, respectively. The unit under evaluation is called efficient if and only if $ef^* = 1$.

Notice that Model (1) is a non-radial and input-oriented model. For investigating some other forms, readers can refer to [21, 22].

2-2. Multi-period efficiency

Let us consider n DMUs; their performance should be evaluated in T periods of time, i.e., $t = 1, ..., T$. Each observed DMU_j $(j = 1, ..., n)$ is represented by a set of inputs $x_{ij}^{(t)}$ $(i = 1, ..., m)$ and outputs $y_{rj}^{(t)}$ $(r = 1, ..., s)$ for period t. Jablonsky [14] introduced the following model to analyze the efficiency of the unit under evaluation p in several periods of time:

$$e_p^* = Max \quad e_p = \sum_{t=1}^{T} \theta_p^t / T$$

$$s.t. \quad \sum_{j=1}^{n} \lambda_j^t x_{ij}^t \leq x_{ip}^t, i = 1, 2, ..., m; t = 1, ..., T, \quad (2$$

$$\sum_{j=1}^{n} \lambda_j^t y_{rj}^t \geq \theta_p^t y_{rp}^t, r = 1, 2, ..., s; t = 1, ...,$$

$$\lambda_j^t \geq 0, j = 1, ..., n; t = 1, ..., T.$$

in which λ_j^t shows intensity variables for the unit j and the period t. The optimal objective function value of Model (2) is more than or equal to 1, that is, $e_p^* \geq 1$. $e_p^* = 1$ implies that DMU_p is the whole efficient. In another way, it is inefficient at least in one period when $e_p^* > 1$. To illustrate, the multi-period efficiency is defined as the average of the efficiency of periods. It is noted that Model (2) is an output-oriented and radial form. Readers can refer to [14] for further details in this regard.

In the next section, the extension of the approach of Chen et al. [21] and, also,

Jablonsky's method [14] is provided to measure multi-period sustainability performance in the presence of discrete and bounded factors.

3. The Proposed Approach

In what follows, approaches based on DEA are provided to measure the sustainability performance of entities comprising three dimensions of economic, social, and environmental where integer and bounded values are presented. In doing so, we take n DMUs, DMU_j $(j = 1, ..., n)$, that use m^d inputs $x_{ij}^{(t)d}$ $(i = 1, ..., m^d)$, produce s^d desirable outputs $y_{rj}^{(t)d}$ $(r = 1, ..., s^d)$, and emit B^d undesirable outputs $z_{bj}^{(t)d} (b = 1, ..., B^d)$ while they should be evaluated in T periods of time $(t = 1, ..., T)$. It is assumed that undesirable outputs are weakly disposable by the following [24]. Weak disposability of outputs means that $\theta y (0 \leq \theta \leq 1)$ can be generated proportionally if the output vector y can be generated. Firstly, to analyze the multi-period sustainability performance with discrete and bounded factors, Model (2) is introduced in which $d \in D = \{economic, environmental, social\}$.

It is assumed that inputs may be integer, non-integer, bounded or unbounded in this stage. Therefore, inputs for dimension d and period t have been split into integer and continuous factors that are shown by $x_{i_{Int}j}^{(t)d}$ and $x_{i_{cont}j}^{(t)d}$ that $I_{Int}^{(t)d} \cup I_{cont}^{(t)d} = \{1, ..., m^d\}$. Moreover, bounded and unbounded inputs have been denoted by $x_{i_{bnd}j}^{(t)d}$ and $x_{i_{Unb}j}^{(t)d}$ while $I_{Bnd}^{(t)d} \cup I_{Und}^{(t)d} = \{1, ..., m^d\}$. $\tilde{x}_{i_{Int}O}^{(t)d}$, $i_{Int}^{(t)d} \in I_{Int}^{(t)d}$, and $\tilde{x}_{i_{Bnd}O}^{(t)d}, i_{Bnd}^{(t)d} \in I_{Bnd}^{(t)d}$ indicate integer and bounded variables for period t and dimension d.

In summary, all notations and terms used in this section are described as follows:

$d \in D$: sustainability dimensions $\qquad D = \{economic, environmental, social\}$,

$t = 1, ..., T$: periods,

λ_j^t : intensity variables for economic dimension in period t,

β_j^t : intensity variables for environmental dimension in period t,

γ_j^t : intensity variables for social dimension in period t,

Ω^* : the overall multi-period sustainability variable in the input orientation,

$\theta_o^{t\,d}$: the performance of DMU_o for dimension d and period t,

α_o^t : sustainability performance of DMU_o for period t,

$L_{i\,Bnd}^{(t)\,(d)}$: lower bound of input i for dimension d and period t,

$U_{i\,Bnd}^{(t)\,(d)}$: upper bound of input i for dimension d and period t,

$L_{r\,Bnd}^{(t)\,(d)}$: lower bound of desirable output r for dimension d and period t,

$U_{r\,Bnd}^{(t)\,(d)}$: upper bound of desirable output r for dimension d and period t,

$L_{b\,Bnd}^{(t)\,(d)}$: lower bound of undesirable output b for dimension d and period t,

$U_{b\,Bnd}^{(t)\,(d)}$: upper bound of undesirable output b for dimension d and period t,

$x_{ij}^{(t)\,(d)}$: i th input of DMU_j for dimension d and period t,

$x_{io}^{(t)\,(d)}$: i th input of the unit under evaluation DMU_o for dimension d and period t,

$\tilde{x}_{io}^{(t)\,(d)}$: i th input variable of the unit under evaluation DMU_o for dimension d and period t,

$\tilde{x}_{i_{\mathrm{Int}}o}^{(t)\,(d)}$: i th integer input variable of the unit under evaluation DMU_o for dimension d and period t,

$\tilde{x}_{i_{Bnd}o}^{(t)\,(d)}$: i th bounded input variable of the unit under evaluation DMU_o for dimension d and period t,

$y_{rj}^{(t)\,(d)}$: r th desirable output of DMU_j for dimension d and period t,

$y_{ro}^{(t)\,(d)}$: r th desirable output of the unit under evaluation DMU_o for dimension d and period t,

$\tilde{y}_{ro}^{(t)\,(d)}$: r th desirable output variable of the unit under evaluation DMU_o for dimension d and period t,

$\tilde{y}_{r_{\mathrm{Int}}o}^{(t)\,(d)}$: r th integer desirable output variable of the unit under evaluation DMU_o for dimension d and period t,

$\tilde{y}_{r_{Bnd}o}^{(t)\,(d)}$: r th bounded desirable output variable of the unit under evaluation DMU_o for dimension d and period t,

$z_{bj}^{(t)\,(d)}$: b th undesirable output of DMU_j for dimension d and period t,

$z_{bo}^{(t)\,(d)}$: b th undesirable output of the unit under evaluation DMU_o for dimension d and period t,

$\tilde{z}_{bo}^{(t)\,(d)}$: b th integer undesirable output variable of the unit under evaluation DMU_o for dimension d and period t,

$\tilde{z}_{b_{\mathrm{Int}}o}^{(t)\,(d)}$: b th integer undesirable output variable of the unit under evaluation DMU_o for dimension d and period t,

$\tilde{z}_{b_{Bnd}o}^{(t)\,(d)}$: b th bounded undesirable output variable of the unit under evaluation DMU_o for dimension d and period t,

w_d^t : the predefined preference by the decision-maker for dimension d and period t,

Φ^* : the variable for obtaining the overall multi-period sustainability index in the output orientation.

Therefore, the following input-oriented model, which is under the constant returns to scale assumption, is proposed:

$$\Omega^* = \underset{\theta^{td}, \lambda_j^t, \beta_j^t, \gamma_j^t}{Min} \quad \Omega = \sum_{t=1}^{T} \alpha^t / T$$

(3)

s.t. (*economic*)

$$\sum_{j=1}^{n} \lambda_j^t x_{ij}^{(t)ec} \leq \tilde{x}_{io}^{(t)ec}, \quad i = 1,...,m^{ec}, t = 1,...,T,$$

$$\tilde{x}_{io}^{(t)ec} \leq \theta_o^{t\ economic} x_{io}^{(t)ec}, i = 1,...,m^{ec}, t = 1,...,T,$$

$$L_{i_{Bnd}}^{(t)ec} \leq \tilde{x}_{i_{Bnd}o}^{(t)ec} \leq U_{i_{Bnd}}^{(t)ec}, i_{Bnd}^{(t)ec} \in I_{i_{Bnd}}^{(t)ec},$$

$$\tilde{x}_{i_{Int}o}^{(t)ec} \text{ integer}, i_{Int}^{(t)ec} \in I_{Int}^{(t)ec},$$

$$\sum_{j=1}^{n} \lambda_j^t y_{rj}^{(t)ec} \geq y_{ro}^{(t)ec}, \quad r = 1,...,s^{ec}, t = 1,...,T,$$

$$\sum_{j=1}^{n} \lambda_j^t z_{bj}^{(t)ec} = \theta_o^{t\ economic} z_{bo}^{(t)ec}, \quad b = 1,...,B^{ec}, t = 1,...,T,$$

(*environmental*)

$$\sum_{j=1}^{n} \beta_j^t x_{ij}^{(t)en} \leq \tilde{x}_{io}^{(t)en}, \quad i = 1,...,m^{en}, t = 1,...,T,$$

$$\tilde{x}_{io}^{(t)en} \leq \theta_o^{t\ environmental} x_{io}^{(t)en}, i = 1,...,m^{en}, t = 1,...,T,$$

$$L_{i_{Bnd}}^{(t)en} \leq \tilde{x}_{i_{Bnd}o}^{(t)en} \leq U_{i_{Bnd}}^{(t)en}, i_{Bnd}^{(t)en} \in I_{i_{Bnd}}^{(t)en},$$

$$\tilde{x}_{i_{Int}o}^{(t)en} \text{ integer}, i_{Int}^{(t)en} \in I_{Int}^{(t)en},$$

$$\sum_{j=1}^{n} \beta_j^t y_{rj}^{(t)en} \geq y_{ro}^{(t)en}, \quad r = 1,...,s^{en}, t = 1,...,T,$$

$$\sum_{j=1}^{n} \beta_j^t z_{bj}^{(t)en} = \theta_o^{t\ environmental} z_{bo}^{(t)en}, \quad b = 1,...,B^{en}, t = 1,...,T,$$

(*social*)

$$\sum_{j=1}^{n} \gamma_j^t x_{ij}^{(t)so} \leq \tilde{x}_{io}^{(t)so}, \quad i = 1,...,m^{so}, t = 1,...,T,$$

$$\tilde{x}_{io}^{(t)so} \leq \theta_o^{t\ social} x_{io}^{(t)so}, i = 1,...,m^{so}, t = 1,...,T,$$

$$L_{i_{Bnd}}^{(t)so} \leq \tilde{x}_{i_{Bnd}o}^{(t)so} \leq U_{i_{Bnd}}^{(t)so}, i_{Bnd}^{(t)so} \in I_{i_{Bnd}}^{(t)so},$$

$$\tilde{x}_{i_{Int}o}^{(t)so} \text{ integer}, i_{Int}^{(t)so} \in I_{Int}^{(t)so},$$

$$\sum_{j=1}^{n} \gamma_j^t y_{rj}^{(t)so} \geq y_{ro}^{(t)so}, \quad r = 1,...,s^{so}, t = 1,...,T,$$

$$\sum_{j=1}^{n} \gamma_j^t z_{bj}^{(t)so} = \theta_o^{t\ social} z_{bo}^{(t)so}, \quad b = 1,...,B^{so}, t = 1,...,T,$$

$$\theta_o^{t\ d} \leq 1,$$

$$\alpha_o^t = \frac{\sum_{d \in D} w_d^t \theta_o^{td}}{\sum_{d \in D} w_d^t}$$

$$\lambda_j^t, \beta_j^t, \gamma_j^t \geq 0.$$

Definition 1. A production system DMU_o is called overall multi-period sustainable if and only if $\Omega^* = 1$; in other words, it is efficient in each period and each dimension. Thus, the system is said to be overall multi-period

unsustainable if $\Omega^* < 1$, i.e., it is inefficient at least in one period or one dimension.

Values α_o^t that denote the sustainability performance for each period are less than or equal to one. In other words, $\alpha_o^t \leq 1$. For each

period t, the index α_o^t is obtained as the weighted average of the efficiency scores of dimensions. As defined, we have also the optimal $\theta_o^{*t\,d} \leq 1$ which indicates that the efficiency for each dimension d and each period t is less than or equal to 1.

According to [8], some definitions are described related to sustainability as follows:

Definition 2. If $\theta_o^{t\ economic} = \theta_o^{t\ environmental} = 1$ for $t = 1,...,T$, the unit under examination is called economic-environmental sustainable, and vice versa.

Definition 3. If $\theta_o^{t\ economic} = \theta_o^{t\ social} = 1$ for $t = 1,...,T$, the unit under assessment is said to be economic-social sustainable, and vice versa.

Definition 4. DMU_o is called social-environmental sustainable if and only if $\theta_o^{t\ social} = \theta_o^{t\ environmental} = 1$ for $t = 1,...,T$.

Theorem 1. Model (3) is always feasible.

Proof. See

$$\lambda_o^t = 1, \lambda_j^t = 0, j \neq o, \theta_o^{t\,economic} = 1, \beta_o^t = 1, \beta_j^t = 0, j \neq o, \theta_o^{t\,environmental} = 1,$$

$$\gamma_o^t = 1, \gamma_j^t = 0, j \neq o, \theta_o^{t\,social} = 1,\ \alpha_o^t = 1,$$

$$\Omega^* = 1, x_{ij}^{(t)\,d} = \tilde{x}_{ijo}^{(t)\,d},\ i = 1,...,m^t, d \in D, t = 1,...,T.$$

It is clear that it is a feasible solution for Model (3). Therefore, this model is always feasible. $\square$

As mentioned earlier, Model (3) is an input-oriented multi-period form.

In this stage, an extended output-oriented form is provided to analyze the sustainability performance of systems with input and output measures that may be bounded and/or integer. Thus, we have:

$$\Phi^* = \underset{\theta^{t\,d}, \lambda_j^t, \beta_j^t, \gamma_j^t}{Max}\ \Phi = \sum_{t=1}^{T} \alpha^t / T$$

$$s.t.\quad (economic) \tag{4}$$

$$\sum_{j=1}^{n} \lambda_j^t x_{ij}^{(t)ec} \leq \tilde{x}_{io}^{(t)ec},\ i = 1,...,m^{ec}, t = 1,...,T,$$

$$\tilde{x}_{io}^{(t)ec} \leq x_{io}^{(t)ec}, i = 1,...,m^{ec}, t = 1,...,T,$$

$$L_{i_{Bnd}}^{(t)ec} \leq \tilde{x}_{i_{Bnd}o}^{(t)ec} \leq U_{i_{Bnd}}^{(t)ec}, i_{i_{Bnd}}^{(t)ec} \in I_{i_{Bnd}}^{(t)ec},$$

$$\tilde{x}_{i_{Int}o}^{(t)ec}\ integer, i_{Int}^{(t)ec} \in I_{Int}^{(t)ec},$$

$$\sum_{j=1}^{n} \lambda_j^t y_{rj}^{(t)ec} \geq \tilde{y}_{ro}^{(t)ec},\quad r = 1,...,s^{ec}, t = 1,...,T,$$

$$\tilde{y}_{ro}^{(t)ec} \geq \theta_o^{t\ economic} y_{ro}^{(t)ec},$$

$$L_{r_{Bnd}}^{(t)ec} \leq \tilde{y}_{r_{Bnd}jo}^{(t)ec} \leq U_{r_{Bnd}}^{(t)ec}, r_{r_{Bnd}}^{(t)ec} \in O_{r_{Bnd}}^{(t)ec},$$

$$\tilde{y}_{r_{Int}o}^{(t)ec}\ integer, r_{Int}^{(t)ec} \in O_{Int}^{(t)ec},$$

$$\sum_{j=1}^{n} \lambda_j^t z_{bj}^{(t)ec} = \tilde{z}_{bo}^{(t)ec},\quad b = 1,...,B^{ec}, t = 1,...,T,$$

$$z_{bjo}^{(t)ec} = \tilde{z}_{bo}^{(t)ec}, b = 1,...,B^{ec}, t = 1,...,T,$$

$$L_{b_{Bnd}}^{(t)ec} \leq \tilde{z}_{b_{Bnd}o}^{(t)ec} \leq U_{b_{Bnd}}^{(t)ec}, b_{b_{Bnd}}^{(t)ec} \in B_{b_{Bnd}}^{(t)ec},$$

$$\tilde{z}_{b_{Int}o}^{(t)ec}\ integer, b_{Int}^{(t)ec} \in B_{Int}^{(t)ec},$$

(environmental)

$$\sum_{j=1}^{n} \beta_j^t x_{ij}^{(t)en} \le \tilde{x}_{io}^{(t)en}, \ i=1,...,m^{en}, t=1,...,T,$$

$$\tilde{x}_{io}^{(t)en} \le \theta_o^{t \ environmental} x_{io}^{(t)en}, i=1,...,m^{en}, t=1,...,T,$$

$$L_{i_{Bnd}}^{(t)en} \le \tilde{x}_{i_{Bnd}o}^{(t)en} \le U_{i_{Bnd}}^{(t)en}, i_{i_{Bnd}}^{(t)en} \in I_{i_{Bnd}}^{(t)en},$$

$$\tilde{x}_{i_{Int}o}^{(t)en} \ \text{integer}, i_{Int}^{(t)en} \in I_{Int}^{(t)en},$$

$$\sum_{j=1}^{n} \beta_j^t y_{rj}^{(t)en} \ge \tilde{y}_{ro}^{(t)en}, \quad r=1,...,s^{en}, t=1,...,T,$$

$$\tilde{y}_{ro}^{(t)ec} \ge \theta_o^{t \ environmental} y_{ro}^{(t)ec}, r=1,...,s^{en}, t=1,...,T,$$

$$L_{r_{Bnd}}^{(t)en} \le \tilde{y}_{r_{Bnd}o}^{(t)en} \le U_{r_{Bnd}}^{(t)en}, r_{r_{Bnd}}^{(t)en} \in O_{r_{Bnd}}^{(t)en},$$

$$\tilde{y}_{r_{Int}o}^{(t)en} \ \text{integer}, r_{Int}^{(t)en} \in O_{Int}^{(t)en},$$

$$\sum_{j=1}^{n} \beta_j^t z_{bj}^{(t)en} = \tilde{z}_{bo}^{(t)en}, \quad b=1,...,B^{en}, t=1,...,T,$$

$$z_{bjo}^{(t)en} = \tilde{z}_{bo}^{(t)en}, b=1,...,B^{en}, t=1,...,T,$$

$$L_{b_{Bnd}}^{(t)en} \le \tilde{z}_{b_{Bnd}o}^{(t)en} \le U_{b_{Bnd}}^{(t)en}, b_{b_{Bnd}}^{(t)en} \in B_{b_{Bnd}}^{(t)en},$$

$$\tilde{z}_{b_{Int}o}^{(t)en} \ \text{integer}, b_{Int}^{(t)en} \in B_{Int}^{(t)en},$$

(social)

$$\sum_{j=1}^{n} \gamma_j^t x_{ij}^{(t)so} \le \tilde{x}_{io}^{(t)so}, \ i=1,...,m^{so}, t=1,...,T,$$

$$\tilde{x}_{ijo}^{(t)so} \le x_{io}^{(t)so}, i=1,...,m^{so}, t=1,...,T,$$

$$L_{i_{Bnd}}^{(t)so} \le \tilde{x}_{i_{Bnd}o}^{(t)so} \le U_{i_{Bnd}}^{(t)so}, i_{i_{Bnd}}^{(t)so} \in I_{i_{Bnd}}^{(t)so},$$

$$\tilde{x}_{i_{Int}o}^{(t)so} \ \text{integer}, i_{Int}^{(t)so} \in I_{Int}^{(t)so},$$

$$\sum_{j=1}^{n} \gamma_j^t y_{rj}^{(t)so} \ge \tilde{y}_{ro}^{(t)so}, \quad r=1,...,s^{so}, t=1,...,T,$$

$$\tilde{y}_{ro}^{(t)so} \ge \theta_o^{t \ social} y_{ro}^{(t)so}, r=1,...,s^{so}, t=1,...,T,$$

$$L_{r_{Bnd}}^{(t)so} \le \tilde{y}_{r_{Bnd}o}^{(t)so} \le U_{r_{Bnd}}^{(t)so}, r_{r_{Bnd}}^{(t)so} \in O_{r_{Bnd}}^{(t)so},$$

$$\tilde{y}_{r_{Int}o}^{(t)so} \ \text{integer}, r_{Int}^{(t)so} \in O_{Int}^{(t)so},$$

$$\sum_{j=1}^{n} \gamma_j^t z_{bj}^{(t)so} = \tilde{z}_{bo}^{(t)so}, \quad b=1,...,B^{so}, t=1,...,T,$$

$$z_{bo}^{(t)so} = \tilde{z}_{bo}^{(t)so}, b=1,...,B^{so}, t=1,...,T,$$

$$L_{b_{Bnd}}^{(t)en} \le \tilde{z}_{b_{Bnd}o}^{(t)so} \le U_{b_{Bnd}}^{(t)so}, b_{b_{Bnd}}^{(t)so} \in B_{b_{Bnd}}^{(t)so},$$

$$\tilde{z}_{b_{Int}o}^{(t)so} \ \text{integer}, b_{Int}^{(t)so} \in B_{Int}^{(t)so},$$

$$\theta_o^{t \ d} \ge 1,$$

$$\alpha_o^t = \frac{\sum_{d \in D} w_d^t \theta_o^{td}}{\sum_{d \in D} w_d^t}$$

$$\lambda_j^t, \beta_j^t, \gamma_j^t \ge 0.$$

Definition 5. The overall multi-period sustainability in Model (4) is defined as $1/\Phi^*$. The unit under consideration is said to be overall multi-period sustainable if and only if $1/\Phi^* = 1$; otherwise, it is unsustainable for at least one period and one dimension.

Furthermore, the sustainability performance for each period t can be determined by $1/\alpha_o^{t*}$. The entity is called sustainable in period t if and only if $1/\alpha_o^{t*} = 1$. In another respect, it is

unsustainable in period t if and only if $1/\alpha_o^{t*} < 1$. Similarly, the efficiency of the unit under evaluation for each period and each dimension is defined as $1/\theta_o^{*td}$, and it is efficient if and only if $1/\theta_o^{*td} = 1$.

Moreover, $(\tilde{x}_{io}^{*(t)d}, \tilde{y}_{ro}^{*(t)d}, \tilde{z}_{bo}^{*(t)d}), d \in D$ shows the target point of DMU_o.

Likewise, economic-environmental, economic-social, and social-environmental sustainability can be defined in this case. It can also be shown that Model (4) is always feasible.

Due to the possibility of internal relationships among economic, social, and environmental items, we investigate the following cases for economic and social issues:

- $d_{lj}^{(t)d}$ is treated as a desirable integer item l for DMU_j $(j = 1, ..., n)$ in both economic and social respects.

o If it is the input for both perspectives, the next constraints are included in Model (4):

Economic member

$$\sum_{j=1}^{n} \lambda_j^t d_{lj}^{(t)ec} \leq \tilde{d}_{lo}^{(t)ec}, t = 1, ..., T,$$

$$\tilde{d}_{ljo}^{(t)ec} \leq d_{lo}^{(t)ec}, t = 1, ..., T,$$

$$\tilde{d}_{l_{Int}o}^{(t)ec} \text{ integer}, l_{Int}^{(t)ec} \in I_{Int}^{(t)ec},$$

Social member

$$\sum_{j=1}^{n} \gamma_j^t d_{lj}^{(t)so} \leq \tilde{d}_{lo}^{(t)so}, t = 1, ..., T,$$

$$\tilde{d}_{lo}^{(t)so} \leq d_{lo}^{(t)so}, t = 1, ..., T,$$

$$\tilde{d}_{l_{Int}o}^{(t)so} \text{ integer}, l_{Int}^{(t)so} \in I_{Int}^{(t)so},$$

o If it is the desirable output for both perspectives, the following expressions are included in Model (4):

Economic member

$$\sum_{j=1}^{n} \lambda_j^t d_{lj}^{(t)ec} \geq \tilde{d}_{lo}^{(t)ec}, t = 1, ..., T,$$

$$\tilde{d}_{lo}^{(t)ec} \geq \theta_o^{t \ economic} d_{lo}^{(t)ec},$$

$$\tilde{d}_{l_{Int}o}^{(t)ec} \text{ integer}, l_{Int}^{(t)ec} \in O_{Int}^{(t)ec},$$

Social member

$$\sum_{j=1}^{n} \gamma_j^t d_{lj}^{(t)so} \geq \tilde{d}_{lo}^{(t)so}, t = 1, ..., T,$$

$$\tilde{d}_{lo}^{(t)so} \geq \theta_o^{t \ social} d_{ro}^{(t)so}, t = 1, ..., T,$$

$$\tilde{d}_{l_{Int}o}^{(t)so} \text{ integer}, l_{Int}^{(t)so} \in O_{Int}^{(t)so},$$

- It is considered as an undesirable integer output item l in both respects.

Economic member

$$\sum_{j=1}^{n} \lambda_j^t d_{lj}^{(t)ec} = \tilde{d}_{lo}^{(t)ec}, t = 1, ..., T,$$

$$d_{lo}^{(t)ec} = \tilde{d}_{lo}^{(t)ec}, t = 1, ..., T,$$

$$\tilde{d}_{l_{Int}o}^{(t)ec} \text{ integer}, l_{Int}^{(t)ec} \in B_{Int}^{(t)ec},$$

Social member

$$\sum_{j=1}^{n} \gamma_j^t d_{lj}^{(t)so} = \tilde{d}_{lo}^{(t)so}, t = 1, ..., T,$$

$$d_{lo}^{(t)so} = \tilde{d}_{lo}^{(t)so}, t = 1, ..., T,$$

$$\tilde{d}_{l_{Int}o}^{(t)so} \text{ integer}, l_{Int}^{(t)so} \in B_{Int}^{(t)so},$$

- The item l is an integer desirable output item from one perspective and an integer undesirable output from another.

o We suppose that the item l is the integer desirable output of the economic aspect and the integer undesirable output of the social aspect. Thus, we have:

Economic member

$$\sum_{j=1}^{n} \lambda_j^t d_{lj}^{(t)ec} \geq \tilde{d}_{ljo}^{(t)ec}, t = 1, ..., T,$$

$$\tilde{d}_{ljo}^{(t)ec} \geq \theta_o^{t \ economic} d_{ljo}^{(t)ec},$$

$$\tilde{d}_{l_{Int}jo}^{(t)ec} \text{ integer}, l_{Int}^{(t)ec} \in O_{Int}^{(t)ec},$$

Social member

$$\sum_{j=1}^{n} \gamma_j^t d_{lj}^{(t)so} = \tilde{d}_{ljo}^{(t)so}, t = 1, ..., T,$$

$$d_{ljo}^{(t)so} = \tilde{d}_{ljo}^{(t)so}, t = 1, ..., T,$$

$$\tilde{d}_{l_{Int}jo}^{(t)so} \text{ integer}, l_{Int}^{(t)so} \in B_{Int}^{(t)so},$$

o In the same way, the following constraints are added to Model (4), where the item l shows the integer undesirable output of the economic aspect and the integer desirable output of the social aspect:

Economic member

$$\sum_{j=1}^{n} \lambda_j^t d_{lj}^{(t)ec} = \tilde{d}_{ljo}^{(t)ec}, t = 1, ..., T,$$

$$d_{ljo}^{(t)ec} = \tilde{d}_{ljo}^{(t)ec}, t = 1, ..., T,$$

$$\tilde{d}_{l_{Int}jo}^{(t)ec} \text{ integer}, l_{Int}^{(t)ec} \in B_{Int}^{(t)ec},$$

Social member

$$\sum_{j=1}^{n} \gamma_j^t d_{lj}^{(t)so} \geq \tilde{d}_{ljo}^{(t)so}, \quad t = 1, ..., T,$$

$$\tilde{d}_{ljo}^{(t)so} \geq \theta_o^{t\ social} d_{rjo}^{(t)so}, t = 1, ..., T,$$

$$\tilde{d}_{l_{Int}jo}^{(t)so} \ integer, l_{Int}^{(t)so} \in O_{Int}^{(t)so},$$

Note that, analogous to the above, environmental and economic issues and also social and environmental issues can be addressed. Moreover, these cases can conveniently be extended for situations that these items are bounded.

3-1 Performance changes

Apart from the sustainability performance analysis, computing performance changes between the two periods is a considerable issue. Thus, the following formulations are applied for calculating the performance changes of the overall sustainability and dimensions:

$$OS^{t,t+h} = \frac{\alpha_o^{*t+h}}{\alpha_o^{*t}}, \tag{5}$$

$$DS^{t,t+h\ d} = \frac{\theta_o^{*t+h\ d}}{\theta_o^{*td}} \ \forall d \in D, \tag{6}$$

in which $OS^{t,t+h}$ shows the change of the sustainability performance of DMU_o from period t to $t+h$. $DS^{t,t+h\ d}$ indicates the performance change of DMU_o for each

dimension d from period t to $t+h$. Optimal values α_o^{*t+h} and α_o^{*t} are sustainability performance scores of DMU_o for periods $t+h$ and t, respectively. Furthermore, optimal values $\theta_o^{*t+h\ d}$ and θ_o^{*td} are accordingly the dimension performance d of DMU_o for periods $t+h$ and t.

If $OS^{t,t+h}$ is obtained greater than 1, the performance is shown to have improved. If $OS^{t,t+h} < 1$, it has worsened. Moreover, the performance is without change when $OS^{t,t+h} = 1$. There is a similar interpretation about $DS^{t,t+h\ d}$.

Notice that performance changes can be calculated using findings obtained from both approaches (3) and (4).

4.   An Example and an Application of Gas Companies

4-1.   An example

In this section, it is supposed that the sustainability performance of 7 manufacturers in three years is analyzed while there are some bounded and integer measures. Input and output factors and their dimensions are shown in Table 2. Data values can be found in Table 3.

Tab. 2. Data Description

Economic (EC)	Role	Type
Labor (L)	Input	Integer
Capital (C)	Input	Real
On-time delivery (OT)	Desirable output	Bounded
Environmental (EN)	Role	Type
Water consumption (WC)	Input	Real
The number of green products (GP)	Desirable output	Integer
Waste (W)	Undesirable output	Real
Social (SO)	Role	Type
Employee training cost (ET)	Input	Real
Customer satisfaction (CS)	Desirable output	Bounded

Tab. 3. Data

Period 1 ($t1$)

#	L	C	OT	WC	GP	W	ET	CS
1	122	2473	95	121	994	47	1560	36
2	173	2621	80	50	540	54	1800	49
3	152	3712	79	68	979	28	1507	34
4	153	3004	75	127	621	27	1058	61
5	165	3775	69	86	531	21	1942	29
6	120	1330	83	140	744	50	1831	31
7	106	3752	65	135	538	31	1008	47

Period 2 ($t2$)

#	L	C	OT	WC	GP	W	ET	CS
1	255	1263	34	97	410	69	1551	59
2	207	2341	27	85	625	20	1333	58
3	186	3039	74	119	940	68	1129	50
4	165	2327	30	84	775	22	1655	83
5	131	2331	6	93	814	26	1079	73
6	209	4191	72	144	785	78	1663	52
7	280	3571	2	100	834	73	1438	51

Period 3 ($t3$)

#	L	C	OT	WC	GP	W	ET	CS
1	222	1790	72	141	944	54	1889	47
2	188	3508	31	87	624	41	1378	65
3	166	4780	49	144	983	54	1235	81
4	186	4837	92	90	603	88	1589	78
5	110	4274	94	114	633	73	1945	45
6	193	3545	30	96	939	63	1729	30
7	176	2150	91	73	721	88	1892	60

To measure the multi-period sustainability performance of manufacturers while bounded and integer-valued measures are presented, the input-oriented version is used. Results are shown in Table 4.

In addition, the performance changes between the years have been calculated and shown in Table 4. To illustrate, the performance changes for the overall sustainability and dimensions are estimated through Expressions (5) and (6).

Findings show that Manufacturer 3 with a score of 0.782 is generally more sustainable than other manufacturers.

Tab. 4. Results

#	EC			EN			SO		
	$t1$	$t2$	$t3$	$t1$	$t2$	$t3$	$t1$	$t2$	$t3$
1	1	1	0.950	0.605	0.505	0.969	0.400	0.562	0.379
2	0.636	0.464	0.277	1	0.887	0.915	0.472	0.643	0.719
3	0.671	1	0.408	1	0.914	1	0.391	0.655	1
4	0.636	0.526	0.715	0.658	1	0.681	1	0.741	0.748
5	0.539	0.122	1	0.723	0.952	0.578	0.259	1	0.353
6	1	0.866	0.264	0.426	0.627	1	0.294	0.462	0.265
7	0.792	0.023	1	0.496	1	1	0.809	0.524	0.484

#	DS^{EC}			DS^{EN}			DS^{So}		
	$t1,t2$	$t1,t3$	$t2,t3$	$t1,t2$	$t1,t3$	$t2,t3$	$t1,t2$	$t1,t3$	$t2,t3$
1	1	0.950	0.950	0.835	1.602	1.919	1.405	0.948	0.674
2	0.730	0.436	0.597	0.887	0.915	1.032	1.362	1.523	1.118
3	1.490	0.608	0.408	0.914	1	1.094	1.675	2.558	1.527
4	0.827	1.124	1.359	1.520	1.035	0.681	0.741	0.748	1.009
5	0.226	1.855	8.197	1.317	0.799	0.607	3.861	1.363	0.353
6	0.866	0.264	0.305	1.472	2.347	1.595	1.571	0.901	0.574
7	0.029	1.263	43.478	2.016	2.016	1	0.648	0.598	0.924

#	Sustainability				Ω^{*}	OS		
	$t1$	$t2$	$t3$			$t1,t2$	$t1,t3$	$t2,t3$
1	0.668	0.689	0.766	0.708		1.031	1.147	1.112
2	0.703	0.665	0.637	0.668		0.946	0.906	0.958
3	0.687	0.856	0.803	0.782		1.246	1.169	0.938
4	0.765	0.756	0.715	0.745		0.988	0.935	0.946
5	0.507	0.691	0.643	0.614		1.363	1.268	0.931
6	0.573	0.652	0.510	0.578		1.138	0.890	0.782
7	0.699	0.516	0.828	0.681		0.738	1.147	1.605

Manufacturer 6 with a score of 0.578 is subject to lower sustainability than other manufacturers. Performance changes computed from the social perspective between Periods 1 and 2 indicate that the performance has improved for five manufacturers of 1, 2, 3, 5, and 6 and has worsened in other manufacturers. Similarly, the performance changes can be analyzed in other terms, too.

Estimation of sustainability for different years shows that Manufacturer 3 is more sustainable in the second year with a score of 0.856. Manufacturers 4 and 7 are more sustainable in periods 1 and 3, respectively. In addition, A comparison of the sustainability between two years 2 and 3 reveals that the performance of two manufacturers, 1 and 7, has improved while the performance of others has worsened. Furthermore, sustainability performance between Periods 1 and 3 has improved in Manufacturers 1, 3, 5, and 7, while Manufacturers 1, 3, 5, and 6 have progressed between periods 1 and 2.

Notice that the performances of manufacturers that are equal to one are shown to be sustainable in that dimension and that year. For instance, Manufacturers 1 and 6 are sustainable from an economic point of view in the year 1, as shown in Table 4. In addition, Manufacturers 2 and 3 are environmentally sustainable in the year 1. In the next stage, the projection points of manufacturers are calculated.

Table 5 shows target points of labor as integral elements. Projection points of other integer and bounded values are equal with each other according to the model orientation ad its structure.

Now, Jablonsky's approach [14] is computed (the input-oriented model) considering all items as continuous and unbounded (and without incorporating sustainability dimensions) to show the validation of the proposed method. By solving the problem, all manufacturers are determined to be efficient with a score of 1. Therefore, the performance scores cannot be distinguished, while the suggested approach can identify the performance in a rational way and is more informative and accurate. Furthermore, integer-valued targets are obtained for integer items by the proposed approach. In the next subsection, the introduced method is utilized to analyze the sustainability performance of gas companies in several years while integer and bounded items appear.

Tab. 5. Target points of labor

#	Labor		
	t1	*t2*	*t3*
1	122	255	139
2	110	96	52
3	102	186	67
4	97	87	133
5	89	16	110
6	120	181	50
7	84	6	176

4-2. An application

Now, the offered approach is used to analyze the sustainability performance of 29 Iranian gas companies located in 29 provinces. The investigation was performed over the years 2013-2015. Performance measures and their dimensions are detailed in Table 6. Moreover, a statistical description of the performance data of gas companies over 2013-2015 is provided in Table 7. To address sustainability performance, Model (4) is utilized. Overall sustainability and the sustainability performance obtained for each year under evaluation are shown in Columns 3-6 of Table 8. Furthermore, the performance changes of sustainability over 2013-2015 are displayed in Columns 7-9. As can be seen, Kohgiluyeh and Boyer-Ahmad is a multi-period sustainable iterm, overall. It is implied that it is efficient in each dimension and each period. Thus, the performance is without any change. Kerman with a score of 0.371 has the least sustainability level. Furthermore, the sustainability level of Ardabil has only regressed between the years 2013-2014 and 2013-2015, whilst the performance of 25 companies has worsened between 2014-2015.

Tab. 6. Performance data description

Role	Factor	Type	Dimension
Input	Operational cost: Expenses related to the system operation	Real	Economic
	Asset: All benefits belong to gas companies	Real	Economic
	Volume of natural gas received: The devoted gas to gas companies	Real	Environmental
	Employee: The number of staff (E)	Integer	Social
Desirable output	Number of installed branches (NIB)	Integer	Economic
	Gas subscriptions: The number of subscribers (GS)	Integer	Economic
	Income	Real	Social-economic
	Replacing gas in preference for petroleum	Real	Environmental
	Influence factor of outfitted cities and villages (IF)	Bounded	Social
Undesirable outputs	Outstanding debts (OD)	Bounded	Economic
	Environmental pollution resulting from gas leak emissions	Real	Environmental

Tab. 7. Statistical representation of data

Variable	Year	Max	Min	Mean	Standard deviation
Operational cost	2013	4035879	1847.269	877517.92	868343.36
	2014	5892482	2446.617	1290421.2	1303805.7
	2015	6342750	3247.697	1575454	1535995
Asset	2013	17712118	1386132	5176525.7	4268731.9
	2014	20488410	972393	6108664.6	5045065.8
	2015	24415670	294597	6923816.3	5859170.9
Volume of natural gas received	2013	27797	409	5321.3241	5756.5887
	2014	25911	455	5729.7931	5896.2142
	2015	26956	500	5985.4345	6040.4403
Employee	2013	3462	126	853.68966	820.51966
	2014	3145	123	843.62069	773.18665
	2015	2620	144	889.03448	725.14814
Number of installed branches	2013	41577	1890	15515.964	11284.29
	2014	29546	1121	13292.655	8368.2253
	2015	46721	1552	14645.31	10907.48
Gas subscriptions	2013	277425	1052	49109.103	53415.905
	2014	278533	1165	45065.931	52436.259
	2015	208987	1125	39967.138	40818.777
Income	2013	14426760	171430	2343164.1	3203315.9
	2014	20778626	312674	3534333.4	5148778.4
	2015	20208906	600479	4917151.3	5482697.9
Replacing gas in preference for petroleum	2013	1003	30.9	253.66897	235.70204
	2014	1042	33.3	245.38621	225.41584
	2015	1064	41	253.89655	232.53801
Influential factor of outfitted cities and villages	2013	100	1.4	74.72931	19.546851
	2014	97.4	2.6	76.098276	18.559992
	2015	98.8	6.85	81.300345	17.954088
Outstanding debts	2013	84	0.01	19.806207	17.205086
	2014	83	0.9	22.798276	14.618542
	2015	76	9	26.625517	2078368.5
Environmental pollution resultingfrom gas leak emissions	2013	99314971.1	205872.35	21018574	20834724
	2014	104647129	261617.96	22064810	21832350
	2015	108589111	286360.54	23148796	22492401

Tab. 8. Sustainability performance and changes

#	Province	Overall Sustainability	2013 sustainability	2014 Sustainability	2015 Sustainability	2013, 2014	2013, 2015	2014, 2015
1	East Azerbaijan	0.586	0.375	0.847	0.789	2.259	2.104	0.932
2	West Azerbaijan	0.716	0.497	1	0.85	2.012	1.710	0.850
3	Ardabil	0.848	0.883	0.825	0.839	0.934	0.950	1.017
4	Isfahan	0.421	0.263	0.550	0.667	2.091	2.536	1.213
5	Ilam	0.636	0.481	0.831	0.698	1.728	1.451	0.840
6	Bushehr	0.448	0.339	0.513	0.553	1.513	1.631	1.078

7	Tehran & Alborz	0.54	0.340	0.764	0.769	2.247	2.262	1.007
8	Chaharmahal and Bakhtiari	0.551	0.404	0.729	0.626	1.804	1.550	0.859
9	South Khorasan	0.394	0.246	0.594	0.534	2.415	2.171	0.899
10	Razavi Khorasan	0.589	0.378	0.845	0.795	2.235	2.103	0.941
11	North Khorasan	0.584	0.385	0.782	0.792	2.031	2.057	1.013
12	Khuzestan	0.5	0.327	0.716	0.645	2.190	1.972	0.901
13	Zanjan	0.621	0.409	0.776	0.912	1.897	2.230	1.175
14	Semnan	0.627	0.440	0.867	0.734	1.970	1.668	0.847
15	Fars	0.598	0.383	0.843	0.822	2.201	2.146	0.975
16	Qazvin	0.697	0.461	0.904	0.974	1.961	2.113	1.077
17	Qom	0.68	0.470	0.932	0.827	1.983	1.760	0.887
18	Kurdistan	0.543	0.366	0.750	0.685	2.049	1.872	0.913
19	Kermanshah	0.612	0.432	0.890	0.683	2.060	1.581	0.767
20	Kerman	0.371	0.244	0.497	0.504	2.037	2.066	1.014
21	Kohgiluyeh and Boyer-Ahmad	1	1	1	1	1	1	1
22	Golestan	0.628	0.423	0.931	0.745	2.201	1.761	0.800
23	Gilan	0.551	0.335	0.822	0.806	2.454	2.406	0.981
24	Lorestan	0.623	0.464	0.767	0.737	1.653	1.588	0.961
25	Mazandaran	0.595	0.382	0.840	0.808	2.199	2.115	0.962
26	Markazi	0.404	0.267	0.547	0.542	2.049	2.030	0.991
27	Hormozgan	0.691	0.426	1	1	2.347	2.347	1
28	Hamadan	0.468	0.303	0.685	0.606	2.261	2.000	0.885
29	Yazd	0.394	0.248	0.533	0.585	2.149	2.359	1.098

In order to study the findings more accurately, Table 9 shows the efficiency values of any dimension of sustainability for the years 2013-2015. As can be seen, 4 provinces, i.e., Bushehr, North Khorasan, Fars, and Mazandaran, are sustainable in economic-social terms. Moreover, their changes are indicated in Table 10. The last three rows of Table 10 represent general statistics of the performance changes of dimensions. Table 11 exhibits target points of integer and bounded measures. As shown, the projecting points of integer-valued measures are obtained as integer values. Moreover, targets of bounded variables are established between the defined ranges. Notice that the projection points of outstanding debts and the number of employees are equal with each other; thus, they are not stated here.

To demonstrate the advantages of the suggested approach, Model (2) is calculated. To illustrate, all measures are considered as continuous and unbounded and also without including sustainability dimensions. The results of efficiency scores are provided in Table 12. As can be observed, approximately 73% of companies were determined to be totally efficient. Moreover, almost 76%, 90%, and 83% of companies were identified as efficient in the years 2013, 2014, and 2015, respectively. Therefore, as depicted in Figure 1, the overall performance scores of the presented approach are more distinctive and informative in comparison to those of Model (2). What's more, the target points of integer and bounded items are displayed in Table 13. It can be seen that non-integer targets are obtained for some companies. Moreover,

projection points of the influence factor (IF) for some companies such as Company 1 over the years 2013 and 2015 are not determined within the specified boundaries.

Therefore, detections show that the proposed approach can estimate the multi-period sustainability performance and the projection points of integer and/or bounded items in a reliable and accurate manner.

Tab. 9. Sustainability dimensions performance

#	Province	Economic			Environmental			Social		
		2013	2014	2015	2013	2014	2015	2013	2014	2015
1	East Azerbaijan	0.598	0.783	0.682	0.192	0.858	0.789	EC 13	EC 14	EC 15
2	West Azerbaijan	1	1	0.936	0.279	1	1	0.687	1	0.685
3	Ardabil	0.87	0.98	0.826	1	0.703	0.82	0.8	0.838	0.873
4	Isfahan	0.474	0.542	0.852	0.121	0.388	0.435	0.943	0.974	0.973
5	Ilam	1	1	0.607	0.236	0.62	0.669	1	1	0.868
6	Bushehr	1	1	1	0.146	0.26	0.292	1	1	1
7	Tehran & Alborz	1	1	1	0.147	0.519	0.526	0.948	1	1
8	Chaharmahal and Bakhtiari	0.65	0.799	0.522	0.21	0.575	0.56	0.881	0.892	0.915
9	South Khorasan	1	0.987	0.561	0.1	0.36	0.383	0.806	0.793	0.82
10	Razavi Khorasan	1	1	0.911	0.168	0.644	0.674	1	1	0.839
11	North Khorasan	1	1	1	0.173	0.545	0.56	1	1	1
12	Khuzestan	0.719	0.967	0.701	0.156	0.472	0.505	0.739	0.967	0.805
13	Zanjan	0.699	0.753	1	0.221	0.809	0.775	0.726	0.769	1
14	Semnan	1	0.991	0.62	0.208	0.695	0.71	1	0.991	0.941
15	Fars	1	1	1	0.171	0.641	0.605	1	1	1
16	Qazvin	0.797	0.863	1	0.25	1	0.926	0.797	0.863	1
17	Qom	1	1	0.748	0.228	0.821	0.826	1	1	0.926
18	Kurdistan	0.795	0.822	0.653	0.182	0.698	0.59	0.687	0.738	0.869
19	Kermanshah	1	1	0.554	0.202	0.73	0.828	1	1	0.725
20	Kerman	0.476	0.701	0.553	0.123	0.378	0.419	0.495	0.51	0.57
21	Kohgiluyeh and Boyer-Ahmad	1	1	1	1	1	1	1	1	1
22	Golestan	1	1	0.575	0.196	0.819	0.815	1	1	0.945
23	Gilan	0.929	1	1	0.149	0.606	0.581	0.87	1	1
24	Lorestan	0.915	0.993	0.759	0.254	0.633	0.662	0.702	0.756	0.805
25	Mazandaran	1	1	1	0.171	0.637	0.584	1	1	1
26	Markazi	0.855	0.584	0.511	0.111	0.422	0.391	0.932	0.713	0.978
27	Hormozgan	0.331	1	1	1	1	1	0.331	1	1
28	Hamadan	0.68	0.669	0.484	0.136	0.541	0.534	0.952	0.967	0.988
29	Yazd	0.493	0.545	0.735	0.116	0.408	0.392	0.695	0.745	0.825

Tab. 10. Performance changes of sustainability dimensions

#	Province	DS								
		13,14[*]	13,15	14,15	13,14	13,15	14,15	13,14	13,15	14,15
		EC	EC	EC	EN	EN	EN	SO	SO	SO
1	East Azerbaijan	1.309	1.140	0.871	4.469	4.109	0.920	1.024	1.052	1.027
2	West Azerbaijan	1	0.936	0.936	3.584	3.584	1	1.456	0.997	0.685
3	Ardabil	1.126	0.949	0.843	0.703	0.820	1.166	1.048	1.091	1.042
4	Isfahan	1.143	1.797	1.572	3.207	3.595	1.121	1.033	1.032	0.999
5	Ilam	1	0.607	0.607	2.627	2.835	1.079	1	0.868	0.868
6	Bushehr	1	1	1	1.781	2.000	1.123	1	1	1
7	Tehran & Alborz	1	1	1	3.531	3.578	1.013	1.055	1.055	1
8	Chaharmahal and Bakhtiari	1.229	0.803	0.653	2.738	2.667	0.974	1.012	1.039	1.026
9	South Khorasan	0.987	0.561	0.568	3.600	3.830	1.064	0.984	1.017	1.034
10	Razavi Khorasan	1	0.911	0.911	3.833	4.012	1.047	1	0.839	0.839
11	North Khorasan	1	1	1	3.150	3.237	1.028	1	1	1
12	Khuzestan	1.345	0.975	0.725	3.026	3.237	1.070	1.309	1.089	0.832
13	Zanjan	1.077	1.431	1.328	3.661	3.507	0.958	1.059	1.377	1.300
14	Semnan	0.991	0.620	0.626	3.341	3.413	1.022	0.991	0.941	0.950
15	Fars	1	1	1	3.749	3.538	0.944	1	1	1
16	Qazvin	1.083	1.255	1.159	4.000	3.704	0.926	1.083	1.255	1.159
17	Qom	1	0.748	0.748	3.601	3.623	1.006	1	0.926	0.926
18	Kurdistan	1.034	0.821	0.794	3.835	3.242	0.845	1.074	1.265	1.178
19	Kermanshah	1	0.554	0.554	3.614	4.099	1.134	1	0.725	0.725
20	Kerman	1.473	1.162	0.789	3.073	3.407	1.108	1.030	1.152	1.118
21	Kohgiluyeh and Boyer-Ahmad	1	1	1	1	1	1	1	1	1
22	Golestan	1	0.575	0.575	4.179	4.158	0.995	1	0.945	0.945
23	Gilan	1.076	1.076	1	4.067	3.899	0.959	1.149	1.149	1
24	Lorestan	1.085	0.830	0.764	2.492	2.606	1.046	1.077	1.147	1.065
25	Mazandaran	1	1	1	3.725	3.415	0.917	1	1	1
26	Markazi	0.683	0.598	0.875	3.802	3.523	0.927	0.765	1.049	1.372
27	Hormozgan	3.021	3.021	1	1	1	1	3.021	3.021	1
28	Hamadan	0.984	0.712	0.723	3.978	3.926	0.987	1.016	1.038	1.022
29	Yazd	1.105	1.491	1.349	3.517	3.379	0.961	1.072	1.187	1.107
No.	Progress	13	8	4	26	26	14	16	17	12
	Regress	4	15	17	1	1	12	3	7	9
	Fixed	12	6	8	2	2	3	10	5	8

[*]In this table, years 2013, 2014, and 2015 are denoted by 13, 14, and 15, respectively.

Tab. 11. Projection points of bounded and integer measures

		IF			NIB			GS		
	Province	2013	2014	2015	2013	2014	2015	2013	2014	2015
1	East Azerbaijan	100	100	100	34619	27429	31337	135049	116654	104091
2	West Azerbaijan	95.17	67	100	14830	12642	19240	49213	49806	51948
3	Ardabil	100	100	100	13269	12128	15326	37933	29369	33267
4	Isfahan	100	100	100	58768	48387	23245	182239	132699	73622
5	Ilam	59.5	62	85.26	9346	8450	13176	12142	14364	16861
6	Bushehr	57	67.5	79	22893	18599	21366	21765	19208	21050
7	Tehran & Alborz	96.57	94.5	94	14045	24610	16204	277425	278533	208987
8	Chaharmahal and Bakhtiari	100	100	100	6943	6580	11224	25285	17334	25511
9	South Khorasan	65.73	69.36	69.81	9607	7992	14096	24123	14680	25077
10	Razavi Khorasan	77.9	83.35	100	41577	28266	18891	143870	115723	90244
11	North Khorasan	78.25	81.6	84.4	8861	9053	10575	21444	17617	15213
12	Khuzestan	100	76.38	100	48241	24577	36410	86594	59919	70665
13	Zanjan	100	100	80.2	11628	12018	12416	31867	26289	20867
14	Semnan	85.5	87.27	92.49	4912	3034	4258	18208	11584	15105
15	Fars	72.05	72.05	75.3	30511	19159	41201	81048	74680	88526
16	Qazvin	98	93.86	85.5	7545	7914	6721	30688	25683	18667
17	Qom	95.4	90.1	100	2422	3801	2726	22879	19034	20824
18	Kurdistan	100	100	100	13296	15248	14097	42350	33314	38801
19	Kermanshah	63.5	65.5	100	24078	21004	23385	35858	42928	55347
20	Kerman	100	100	100	22774	10541	33186	75280	60513	59946
21	Kohgiluyeh and Boyer-Ahmad	70.9	74.25	82	5818	5590	5603	12094	9630	9590
22	Golestan	86.6	88.9	100	13721	8016	13091	36383	29151	42311
23	Gilan	100	87.5	92.5	29801	29546	46721	65642	64223	67328
24	Lorestan	100	100	100	11114	11561	14963	34618	30923	38233
25	Mazandaran	100	92.45	94.1	39802	24580	32539	86548	65252	84916
26	Markazi	100	100	100	13605	15343	11681	37142	39853	47767
27	Hormozgan	4.23	2.6	6.85	5704	1121	1552	3175	1165	1125
28	Hamadan	100	100	100	16442	12446	19510	55329	42874	49904
29	Yazd	100	100	100	17540	12771	19064	46531	39105	37622

Tab. 12. Results of Model (2)

#	Province	Efficiency			
		2013	2014	2015	Overall
1	East Azerbaijan	0.792	1	0.912	0.893
2	West Azerbaijan	1	1	1	1
3	Ardabil	1	1	1	1
4	Isfahan	1	0.744	1	0.897
5	Ilam	1	1	1	1
6	Bushehr	1	1	1	1

| 7	Tehran & Alborz	1	1	1	1
8	Chaharmahal and Bakhtiari	1	1	1	1
9	South Khorasan	1	1	1	1
10	Razavi Khorasan	1	1	1	1
11	North Khorasan	1	1	1	1
12	Khuzestan	0.862	1	0.792	0.876
13	Zanjan	0.845	1	1	0.943
14	Semnan	1	1	1	1
15	Fars	1	1	1	1
16	Qazvin	1	1	1	1
17	Qom	1	1	1	1
18	Kurdistan	0.978	1	1	0.993
19	Kermanshah	1	1	1	1
20	Kerman	0.594	0.861	0.776	0.726
21	Kohgiluyeh and Boyer-Ahmad	1	1	1	1
22	Golestan	1	1	1	1
23	Gilan	1	1	1	1
24	Lorestan	1	1	1	1
25	Mazandaran	1	1	1	1
26	Markazi	1	1	1	1
27	Hormozgan	1	1	1	1
28	Hamadan	0.944	0.9009821	0.932	0.925
29	Yazd	0.795	1	0.952	0.907
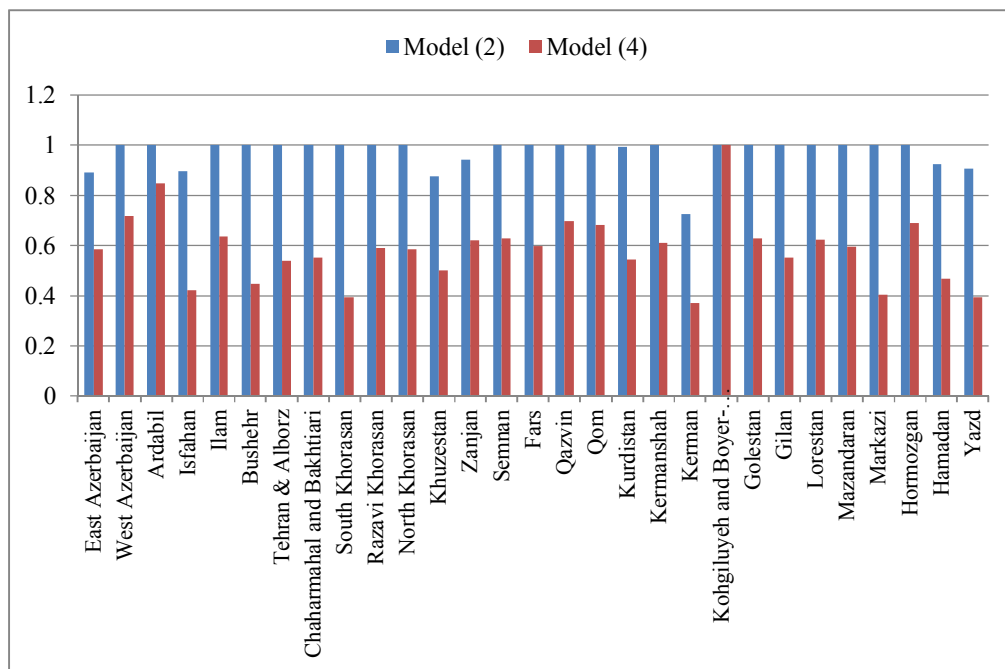
Fig. 1. The comparison of results obtained from Models (2) and (4)

Tab. 13. Targets of integer and bounded variables obtained from Model (2)

#	Province	IF			NIB			GS		
		2013	2014	2015	2013	2014	2015	2013	2014	2015
1	East Azerbaijan	112.220	91.000	102.510	26153.89	21489	23416.27	102025.29	91392	77781.62
2	West Azerbaijan	65.350	67.000	68.500	14830	12642	18000	49213	49806	48601
3	Ardabil	80.000	83.750	87.250	11550	11890	12664	33018	28793	27489
4	Isfahan	94.350	130.880	97.300	27850	35236.82	19813	86364	96635.22	62752
5	Ilam	59.500	62.000	74.000	9346	8450	8000	12142	14364	10237
6	Bushehr	57.000	67.500	79.000	22893	18599	21366	21765	19208	21050
7	Tehran & Alborz	91.500	94.500	94.000	14045	24610	16204	277425	278533	208987
8	Chaharmahal and Bakhtiari	88.100	89.200	91.500	4513	5256	5863	16436	13846	13326
9	South Khorasan	53.000	55.000	57.250	9607	7891	7910	24123	14493	14072
10	Razavi Khorasan	77.900	83.350	83.850	41577	28266	17211	143870	115723	82218
11	North Khorasan	78.250	81.600	84.400	8861	9053	10575	21444	17617	15213
12	Khuzestan	85.730	73.850	101.670	40218.14	23764	32226.42	72191.74	57937	62544.47
13	Zanjan	85.930	76.900	80.200	9620.85	9044	12416	26365.01	19783	20867
14	Semnan	85.500	86.500	87.000	4912	3007	2638	18208	11482	9360
15	Fars	72.050	72.050	75.300	30511	19159	41201	81048	74680	88526
16	Qazvin	78.100	81.000	85.500	6013	6830	6721	24456	22165	18667
17	Qom	95.400	90.100	92.590	2422	3801	2039	22879	19034	15579
18	Kurdistan	70.240	73.850	86.950	10804.15	12533	9202	34413.37	27381	25328
19	Kermanshah	63.500	65.500	72.500	24078	21004	12957	35858	42928	30667
20	Kerman	83.340	59.210	73.410	18263.68	8577.02	23625.13	60372.16	49238.89	42675.41
21	Kohgiluyeh and Boyer-Ahmad	70.900	74.250	82.000	5818	5590	5603	12094	9630	9590
22	Golestan	86.600	88.900	94.500	13721	8016	7523	36383	29151	24316
23	Gilan	87.000	87.500	92.500	27692	29546	46721	60995	64223	67328
24	Lorestan	70.250	75.600	80.550	10173	11483	11356	31685	30714	29017
25	Mazandaran	100.000	92.450	94.100	39802	24580	32539	86548	65252	84916
26	Markazi	93.150	71.250	97.820	11632	8964	5970	31756	23283	24414
27	Hormozgan	1.400	2.600	6.850	1890	1121	1552	1052	1165	1125
28	Hamadan	100.870	107.380	106.060	11840.1	9240.06	10138.29	39844.36	31832.42	25931.87
29	Yazd	87.450	74.500	86.640	10877.49	6962	14711.18	28855.23	21318	29030.63

5. Conclusions

Performance analysis of organizations in several periods of time and their sustainability performance measurement are notable points for making decisions and developing plans. Moreover, discrete and bounded performance measures are introduced in many areas of application. Accordingly, DEA-based approaches were developed in this paper to evaluate the overall multi-period sustainability performance while discrete and bounded factors were available. Furthermore, the efficiency of each sustainability dimension for each period was estimated. In addition, internal relationships of dimensions were investigated. Afterwards, performance changes over time

were addressed for sustainability and its dimensions. The suggested approach was clarified by a numerical example and a case study of gas companies. The current findings add to a growing body of literature concerning the performance analysis of sustainability with discrete and/or bounded elements over several periods of time.

The proposed approach can be extended to situations that Likert scales and imprecise data are presented. Further, multi-period sustainability performance measurement in the presence of negative data is an interesting topic for future research.

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