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DISCRETE SIZE AND DISCRETE-CONTINUOUS CONFIGURATION OPTIMIZATION METHODS FOR TRUSS STRUCTURES USING THE HARMONY SEARCH ALGORITHM

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ABSTRACT

Many methods have been developed for structural size and configuration optimization in which cross-sectional areas are usually assumed to be continuous. In most practical structural engineering design problems, however, the design variables are discrete. This paper proposes two efficient structural optimization methods based on the harmony search (HS) heuristic algorithm that treat both discrete sizing variables and integrated discrete sizing and continuous geometric variables. The HS algorithm uses a stochastic random search instead of a gradient search so the former has a new-paradigmed derivative. Several truss examples from the literature are also presented to demonstrate the effectiveness and robustness of the new method, as compared to current optimization methods.

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KEY WORDS: structural optimization, harmony search, heuristic algorithm, truss structures, discrete optimization, configuration.

1. INTRODUCTION

Structural design optimization is a critical and challenging activity that has received considerable attention in the last two decades. Typically, structural optimization problems involve searching for the minimum of a stated objective function, usually the structural weight. This minimum design is subjected to various constraints with respect to performance measures,

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such as stresses and displacements, and also restricted by practical minimum cross-sectional areas or dimensions of the structural members or components. If the design variables can be varied continuously in the optimization, the problem is termed “continuous”; while if the design variables represent a selection from a set of parts, the problem is considered “discrete”.

Many gradient-based mathematical programming methods have been developed, and they are frequently used to solve structural optimization problems. The majority of these methods assume that cross-sectional areas (*i.e.*, the sizing variables) are continuous. In most practical structural engineering design problems, however, sizes have to be chosen from a list of discrete values due to the availability of components in standard sizes and constraints caused by construction and manufacturing practices. This leads to discrete optimization problems, which are somewhat difficult to solve. Although conventional mathematical methods can consider discreteness by employing round-off techniques based on continuous solutions, the rounded-off solutions may yield results that are far from optimum, or they may even become infeasible as the number of variables increases. Because most available optimization methods treat design variables as continuous, they are inadequate in the presence of discrete design variables. A few methods based on mathematical programming techniques have been developed to handle the discrete nature of design variables [1-8].

They provide useful strategies when solving limited problems, but every method has its drawbacks. These include low efficiency, limited reliability, and the problem of readily becoming trapped at a local optimum [6, 9].

Over the last decade, new optimization strategies based on heuristic algorithms, such as the simulated annealing algorithm and the genetic algorithm (GA), have been devised to obtain optimal designs for discrete structural systems and to overcome the computational drawbacks of conventional mathematical optimization methods. The GA-based discrete optimization methods, in particular, have been studied by many researchers, including Rajeev and Krishnamoorthy [10, 11], Lin and Hajela [12], Wu and Chow [9, 13], Camp *et al*. [14], Pezeshk *et al*. [15], and Erbatur *et al*. [16]. The GA was originally proposed by Holland [17] and was further developed by Goldberg [18] and by others. It is a global search algorithm that is based on concepts from natural genetics and the Darwinian survival-of-the-fittest code. Heuristic algorithm-based discrete optimization methods for structures, including GA-based methods, have occasionally overcome several deficiencies of conventional mathematical methods. However, structural engineers are still concerned with seeking a more powerful, effective, and robust method for discrete structural optimization problems.

The main purpose of this paper is to propose an efficient discrete sizing and integrated discrete sizing and continuous geometric optimization methods for truss structural systems. In our previous research [19, 20], a new optimization method for structures with continuous variables was proposed, based on the harmony search (HS) heuristic algorithm, and its effectiveness and capability were verified from comparisons with GA-based and conventional mathematical optimization techniques. The recently developed HS algorithm is based on natural musical performance processes that occur when musicians search for a better state of harmony, such as during jazz improvisation [21]. Compared to conventional mathematical optimization algorithms, the HS algorithm imposes fewer mathematical requirements to solve optimization problems, and the probability of becoming entrapped in a local optimum is reduced because this algorithm is not a hill-climbing algorithm. Since the

HS algorithm uses a stochastic random search, it has a new-paradigmed derivative [22]. The algorithm considers several solution vectors simultaneously, in a manner similar to the GA. However, the major difference between the GA and the HS algorithm is that the latter generates a new vector from all the existing vectors, while the former generates a new vector from only two of the existing vectors (parents). In addition, the HS algorithm can consider each component variable in a vector independently when it generates a new vector; the GA cannot, because it has to maintain the gene structure.

This paper proposes two structural optimization methods, based on the HS heuristic algorithm: one treats pure discrete sizing variables (subsequently referred to as discrete size optimization) and the other treats integrated discrete sizing and continuous geometric variables (subsequently referred to as discrete-continuous configuration optimization). Several truss examples from the literature, including large-scale trusses under multiple loading conditions, are also presented to demonstrate the effectiveness and robustness of the new methods, as compared to current optimization techniques.

2. FORMULATION OF SIZE AND CONFIGURATION OPTIMIZATION PROBLEMS

In this study, discrete size and discrete-continuous configuration optimization methods based on the HS heuristic algorithm are introduced. The discrete size optimization of structural systems involves arriving at optimum values for discrete member cross-sectional areas $\boldsymbol{A}$ that minimize an objective function $f(\boldsymbol{x})$, *i.e.*, the structural weight W. Discrete-continuous configuration optimization involves simultaneously arriving at optimum values for continuous nodal coordinates $\boldsymbol{R}$ and discrete cross sections $\boldsymbol{A}$ that minimize the structural weight. For a given topology, the configuration optimization problem is generally considered to be more difficult, but it is also a more important task than pure size optimization because of the potential for much larger savings.

Both minimum designs must satisfy q inequality constraint functions that limit the design variable sizes and the structural responses. Thus, the problems can be stated mathematically, as minimizing the structural weight:

$$\text{Minimize } f(\boldsymbol{x}) = W(\boldsymbol{A}) \text{ or } W(\boldsymbol{R}, \boldsymbol{A}) = \gamma \sum_{i=1}^{n} L_i A_i \tag{1}$$

$$\text{subjected to } G_j^l \le G_j(\boldsymbol{A}) \text{ or } G_j(\boldsymbol{R}, \boldsymbol{A}) \le G_j^u,\ j = 1,2,\ldots,q \tag{2}$$

where $f(\boldsymbol{x})$ is an objective function, $\boldsymbol{x}$ is the set of each design variable, $\boldsymbol{A} = (A_1, A_2,\ldots, A_n)^T$ is the sizing variable vector that consists of the cross-sectional areas chosen from a list of available discrete values, and $\boldsymbol{R} = (R_1, R_2,\ldots, R_m)^T$ is the continuous nodal coordinate variable vector. Also, $W(\boldsymbol{A})$ and $W(\boldsymbol{R}, \boldsymbol{A})$ are the objective functions (*i.e.*, the structural weight) for the discrete size or the discrete-continuous configuration optimizations, respectively, γ is the material density of each member, and A_i and L_i are the cross-sectional area and length of the ith member. $G_j(\boldsymbol{A})$ or $G_j(\boldsymbol{R}, \boldsymbol{A})$, shown in Eq. (2), are the inequality constraints for the discrete size or the discrete-continuous configuration optimizations, and G_j^l and G_j^u are the lower and the upper bounds on the constraints.

For the methods presented in this paper, the lower and upper bounds on the constraint function Eq. (2) include the following: (1) nodal coordinates $(R_i^l \le R_i \le R_i^u,\ i = 1,...,m)$; (2) member cross sections $(A_i(k),\ i = 1,...,n)$; (3) member stresses $(\sigma_i^l \le \sigma_i \le \sigma_i^u,\ i = 1,...,n)$; (4) nodal displacements $(\delta_i^l \le \delta_i \le \delta_i^u,\ i = 1,...,m)$; and (5) member buckling stresses $(\sigma_i^{cr} \le \sigma_i \le 0,\ i = 1,...,n)$. Here, σ_i and δ_i are the member stresses and nodal displacements, respectively, calculated from the structural analysis; R_i^l, R_i^u, σ_i^l, σ_i^u, δ_i^l, δ_i^u, and σ_i^{cr} are the constraint limitations prescribed for optimization design purposes; and $A_i(k)$ are the available discrete cross-sectional areas, *i.e.*, $A_i(1), A_i(2),..., A_i(k)$ $(A_i(1) < A_i(2) < ... < A_i(k))$. The nodal coordinate constraints are required only for the discrete-continuous configuration optimization.

3. HS ALGORITHM-BASED SIZE AND CONFIGURATION OPTIMIZATION METHODS

The penalty approach has frequently been employed to determine the fitness measure for the constrained optimization problems, described by Eqs. (1) and (2), because the optimum solution typically occurs at the boundary between the feasible and infeasible regions [10, 13, 14, 15, 16]. However, to demonstrate the pure performance of the HS algorithm-based methods proposed in this study, a rejecting strategy for the fitness measure was adopted, *i.e.*, the optimum solution was approached only from the feasible region. Figure 1 shows the design procedure that was used to apply the HS heuristic algorithm to the discrete size and the discrete-continuous configuration optimization problems. The procedure can be divided into four steps, as follows.

(1) Step 1: Initialization

The optimization problem is first specified as $W(\boldsymbol{A})$ or $W(\boldsymbol{R}, \boldsymbol{A})$ in Eq. (1). For discrete size optimization problems, *i.e.*, $W(\boldsymbol{A})$, the number of discrete design variables (A_i) and the set of available discrete values (D), *i.e.*, $D \in \{A_i(1), A_i(2),\ldots, A_i(k)\}$ $(A_i(1) < A_i(2) < \ldots < A_i(k))$ are then initialized. For discrete-continuous configuration optimization problems, *i.e.*, $W(\boldsymbol{R}, \boldsymbol{A})$, the number of continuous geometric variables (R_i) and the possible value bounds of the continuous variables, *i.e.*, $R_i^l \le R_i \le R_i^u$ are initialized, as well as the discrete design variables.

The HS algorithm parameters that are required to solve the optimization problem are also specified in this step. These include the harmony memory size (number of solution vectors in the harmony search, HMS), harmony memory considering rate (HMCR), pitch adjusting rate (PAR), and termination criterion (maximum number of searches). The HMCR and the PAR are parameters that are used to improve the solution vector. Both are defined in Step 2. Subsequently, the “harmony memory” (HM) matrix, shown in Eq. (3), is randomly generated from the available discrete value set for size optimization problems or from the discrete size value set and the possible nodal coordinate bounds for configuration optimization problems. These sets are equal to the size of the HM (*i.e.*, HMS). Here, an initial HM is generated based on the FEM structural analysis results, subject to the constraint functions (Eq. [2]), and sorted by the objective function values (Eq. [1]).

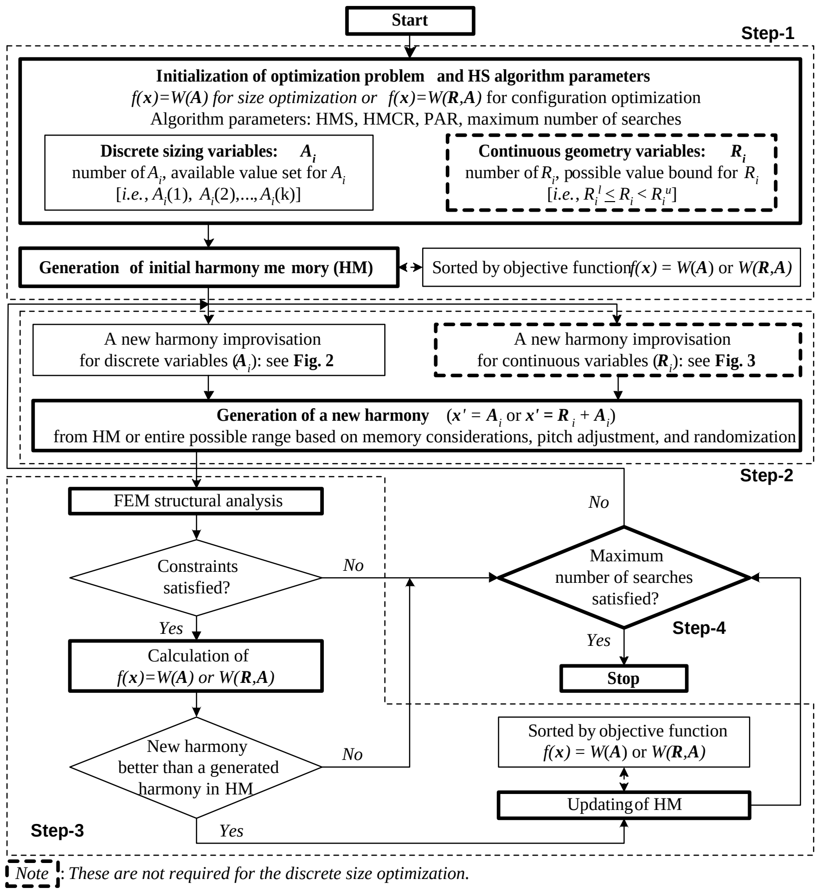

Figure 1. Design procedure for discrete size and discrete-continuous configuration optimization problems using hs heuristic algorithm

$$\boldsymbol{HM} = \begin{bmatrix} x_1^1 & x_2^1 & \cdots & x_p^1 \\ x_1^2 & x_2^2 & \cdots & x_p^2 \\ \vdots & \vdots & \cdots & \vdots \\ x_1^{HMS} & x_2^{HMS} & \cdots & x_p^{HMS} \end{bmatrix} \begin{array}{l} \Rightarrow \; f(\boldsymbol{x}^1) \\ \Rightarrow \; f(\boldsymbol{x}^2) \\ \Rightarrow \quad \vdots \\ \Rightarrow \; f(\boldsymbol{x}^{HMS}) \end{array} \tag{3}$$

In Eq. (3), $\boldsymbol{x}^1$, $\boldsymbol{x}^2$,…, $\boldsymbol{x}^{HMS}$ and $f(\boldsymbol{x}^1)$, $f(\boldsymbol{x}^2)$,…, $f(\boldsymbol{x}^{\mathrm{HMS}})$ show each solution vector for design variables ($\boldsymbol{A}$ or $\boldsymbol{R}$ and $\boldsymbol{A}$) and the corresponding objective function value (the structural weight), respectively.

(2) Step 2: Generation of a new harmony

In the HS algorithm, a new harmony vector, $\boldsymbol{x}' = (x_1', x_2', ..., x_p')$, is improvised from either the initially generated HM or the entire possible range of values. The new harmony improvisation proceeds based on memory considerations, pitch adjustments, and randomization.

In the memory consideration process, the value of the first design variable (x_1') for the new vector is chosen from any value in the specified HM range {$x_1^1, x_1^2, \cdots, x_1^{HMS}$}. Values of the other decision variables (x_i') are chosen in the same manner. Here, the possibility that a new value will be chosen is indicated by the HMCR parameter, which varies between 0 and 1 as follows:

$$x_i' \leftarrow \begin{cases} x_i' \in \{x_i^1, x_i^2, \ldots, x_i^{HMS}\} & w.p. \quad HMCR \\ x_i' \in \boldsymbol{X}_i & w.p. \quad (1\text{-}HMCR) \end{cases} \tag{4}$$

where $\boldsymbol{X}_i$ is the set of the possible range of values for each design variable (***A*** or ***R*** and ***A***). The HMCR sets the rate of choosing a value from the historic values stored in the HM, and (1-HMCR) sets the rate of randomly choosing a value from the entire possible range of values (randomization process). For example, a HMCR of 0.90 indicates that the HS algorithm will choose the design variable value from historically stored values in the HM with a 90% probability, and from the entire possible range of values with a 10% probability. A HMCR value of 1.0 is not recommended, because there is a chance that the solution will be improved by values not stored in the HM.

Every component of the new harmony vector, $\boldsymbol{x}' = (x_1', x_2', ..., x_p')$, is examined to determine whether it should be pitch-adjusted (pitch adjustment process). This procedure uses the PAR parameter that sets the rate of adjusting the pitch chosen from the HM as follows:

$$\text{Pitch adjusting decision for } x_i' \leftarrow \begin{cases} Yes & w.p. \quad PAR \\ No & w.p. \quad (1-PAR) \end{cases} \tag{5}$$

The pitch adjusting process is performed only after a value has been chosen from the HM. The value (1-PAR) sets the rate of doing nothing. A PAR of 0.3 indicates that the algorithm will choose a neighboring value with 30%$\times$HMCR probability. If the pitch adjustment decision for x_i' is Yes, and x_i' is assumed to be $x_i(l)$, *i.e.*, the l-th element in $\boldsymbol{X}_i$, the pitch-adjusted value of $x_i(l)$ is

$$\begin{aligned} &x_i' \leftarrow x_i(l + c) \text{ for discrete design variables } (\boldsymbol{A}) \\ &x_i' \leftarrow x_i' + \alpha \text{ for continuous design variables } (\boldsymbol{R}) \end{aligned} \tag{6}$$

where c is the neighboring index, $c \in \{-1,\ 1\}$; α is the value of $bw \times u(-1, 1)$; bw is an arbitrary distance bandwidth for the continuous variable; and $u(-1, 1)$ is a uniform distribution between -1 and 1. Detailed flowcharts for the new harmony discrete and continuous search strategies based on the HS heuristic algorithm are given in Figures 2 and 3, respectively. Note that the HMCR and PAR parameters introduced in the harmony search help the algorithm find globally and locally improved solutions.

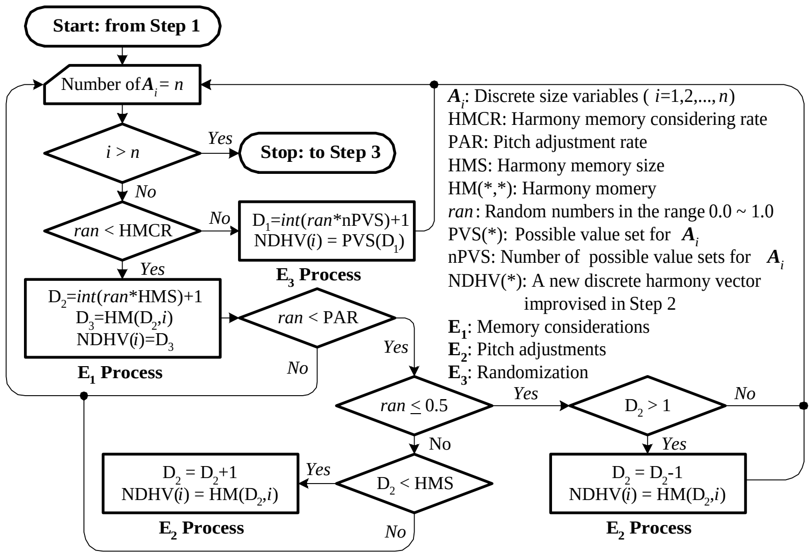

Figure 2. A New harmony improvisation flowchart for discrete sizing variables (Step 2)

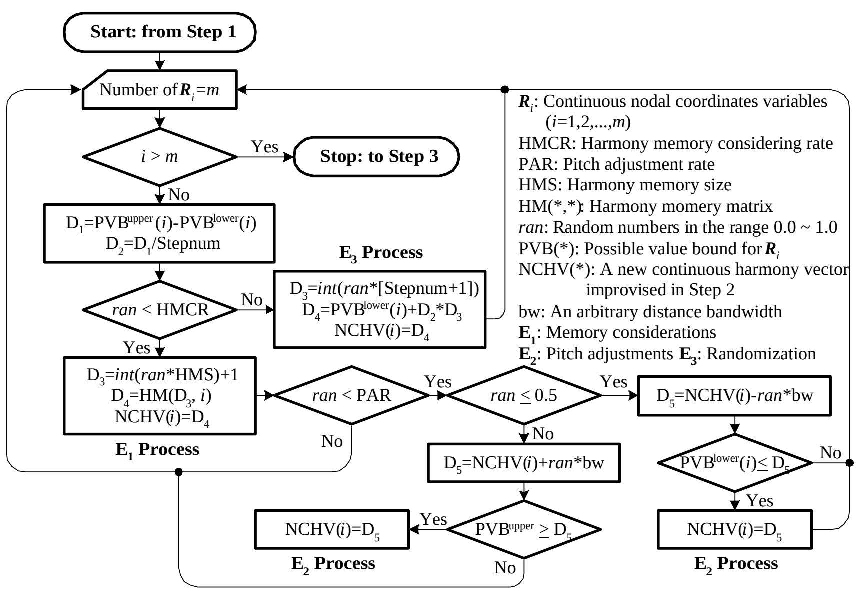

Figure 3. A new harmony improvisation flowchart for continuous nodal coordinate variables (Step 2)

(3) Step 3: Fitness measure and HM update

The new harmony improvised in Step 2 is analyzed using a FEM structural analysis method, and its fitness is determined using a rejection strategy based on the constraint function. If the new harmony vector is better than the worst harmony vector in the HM, judged in terms of the objective function value, the new harmony is included in the HM and the existing worst harmony is excluded from the HM. The HM is then sorted by the objective function value.

(4) Step 4: Repeat Steps 2 and 3 until the termination criterion is satisfied

The computations terminate when the termination criterion is satisfied. If not, Steps 2 and 3 are repeated.

4. TRUSS EXAMPLES

The previously described computational procedures were implemented in a FORTRAN computer program that was applied to discrete sizing and discrete-continuous configuration optimization problems for trusses. The FEM displacement method was used to analyze the truss structures. Standard test truss examples were considered to demonstrate the discrete search efficiency of the HS algorithm approach, as compared to current methods. The cases shown in Table 1, each with a different set of HS algorithm parameters (*i.e.*, HMS, HMCR, and PAR), were tested with all of the examples presented in this study. These parameter values were arbitrarily selected, based on the empirical findings by Geem [23], which determined that the HS algorithm performed well with $30 \le \text{HMS} \le 100$, $0.7 \le \text{HMCR} \le 0.95$, and $0.05 \le \text{PAR} \le 0.7$. The maximum number of searches was set to 30,000 for Examples 1 through 3 and 80,000 for Example 4.

Table 1. HS algorithm parameters used for all examples

Cases (1)	**HMS (2)**	**HMCR (3)**	**PAR (4)**
Case-1	20	0.9	0.45
Case-2	40	0.9	0.45
Case-3	30	0.9	0.4
Case-4	30	0.8	0.3
Case-5	30	0.9	0.3

5. DISCRETE SIZE OPTIMIZATION EXAMPLES

Example 1: 25-bar space truss

The 25-bar transmission tower space truss, shown in Figure 4, has been optimized using discrete size algorithms by many researchers, including Rajeev and Krishnamoorthy [10], Wu and Chow [9, 13], Adeli and Park [24], Erbatur *et al*. [16], and Park and Sung [25]. In these studies, the material density was 0.1 lb/in.^{3} and modulus of elasticity was 10,000 ksi. This space truss was subjected to the following loading condition: $P_X = 1.0$ kips and $P_Y = P_Z = -10.0$ kips acting on node 1, $P_X = 0.0$ kips and $P_Y = P_Z = -10.0$ kips acting on node 2, $P_X = 0.5$ kips and $P_Y = P_Z = 0.0$ kips acting on node 3, and $P_X = 0.6$ kips and $P_Y = P_Z = 0.0$ kips acting on node 6. The structure was required to be doubly symmetric about the *X*- and *Y*-axes; this condition grouped the truss members as follows: (1) A_1, (2) $A_2 \sim A_5$, (3) $A_6 \sim A_9$, (4) $A_{10} \sim A_{11}$, (5) $A_{12} \sim A_{13}$, (6) $A_{14} \sim A_{17}$, (7) $A_{18} \sim A_{21}$, and (8) $A_{22} \sim A_{25}$. All members were constrained to 40 ksi in both tension and compression. In addition, maximum displacement limitations of ± 0.35 in. were imposed at each node in every direction. Discrete values for the cross-sectional areas were taken from the set $D \in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.8, 3.0, 3.2, 3.4\}$ (in.^{2}), which has thirty discrete values.

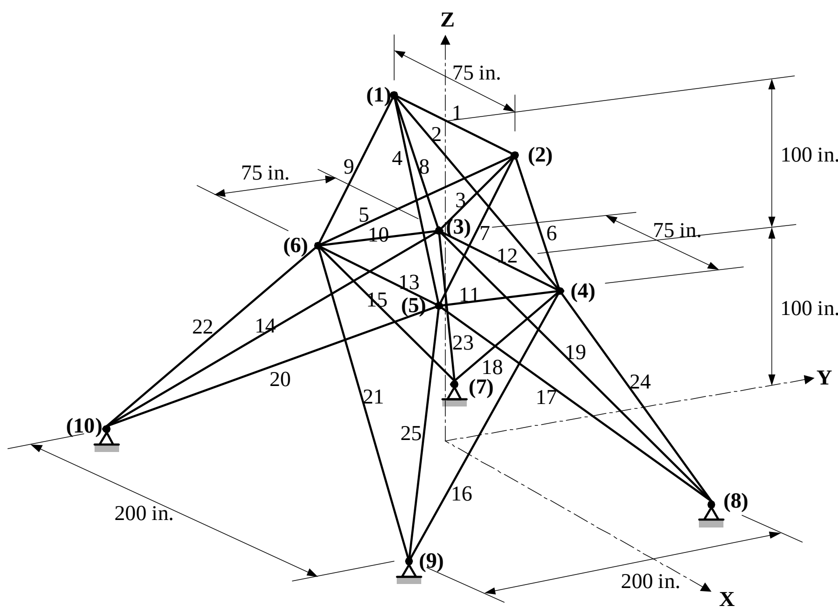

Figure 4. 25-bar space truss

The HS algorithm-based discrete size optimization approach was applied to the space truss. Table 2 lists the HS result obtained with each set of parameters given in Table 1. The results reported by Rajeev and Krishnamoorthy [10], Wu and Chow [9, 13], and Erbatur *et al*. [16], obtained with GA-based methods, by Adeli and Park [24], obtained with the neural dynamics model, and by Park and Sung [25], obtained with the simulated annealing algorithm-based method, are also included in the table. After 13,523 to 18,734 searches (FEM structural analyses), the best solution vector and the corresponding objective function value (the structural weight) were obtained for all five HS cases (see Table 2). All of the HS results were better than the values obtained in the previous investigations.

Table 2. Optimal results of 25-bar space truss (Example 1)

Design variables A_i (in.^{2}) (1)		HS results Case-1 (2)	Case-2 (3)	Case-3 (4)	Case-4 (5)	Case-5 (6)	Rajeev *et al.* (1992) (7)	Wu & Chow (1995a) (8)	Wu & Chow (1995b) (9)	Adeli & Park (1996) (10)	Erbatur *et al.* (2000) (11)	Park & Sung (2002) (12)
1	A_1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.6	0.1	0.1
2	A_2 ~ A_5	0.6	0.3	0.3	0.5	0.3	1.8	0.6	0.5	1.4	1.2	2.1
3	A_6 ~ A_9	3.4	3.4	3.4	3.4	3.4	2.3	3.2	3.4	2.8	3.2	3.4
4	A_{10} ~	0.1	0.1	0.1	0.1	0.1	0.2	0.2	0.1	0.5	0.1	0.1
5	A_{11}	1.6	2.1	2.1	1.9	2.1	0.1	1.5	1.5	0.6	1.1	2.2
6	A_{12} ~	1.0	1.0	1.0	0.9	1.0	0.8	1.0	0.9	0.5	0.9	1.1
7	A_{13}	0.4	0.5	0.5	0.5	0.5	1.8	0.6	0.6	1.5	0.4	1.0
8	A_{14} ~	3.4	3.4	3.4	3.4	3.4	3.0	3.4	3.4	3.0	3.4	3.0
	A_{17}											
	A_{18} ~											
	A_{21}											
	A_{22} ~											
	A_{25}											
Weight (lb)		485.77 [521.04][a]	484.85 [504.72][a]	484.85 [514.20][a]	485.05 [514.21][a]	484.85 [504.28][a]	546.01	491.72	486.29	543.95	493.80	537.23
Number of structural analyses		13,736 [13,445][b]	14,163 [4,414][b]	13,523 [2,160][b]	17,159 [5,226][b]	18,734 [6,850][b]	600	-	40,000	-	-	-

[a] The HS optimal results obtained after 600 structural analyses (the result of Rajeev and Krishnamoorthy, 1992).
[b] Number of analyses for the HS required to obtain a weight of 486.29 lb (the result of Wu and Chow, 1995b).

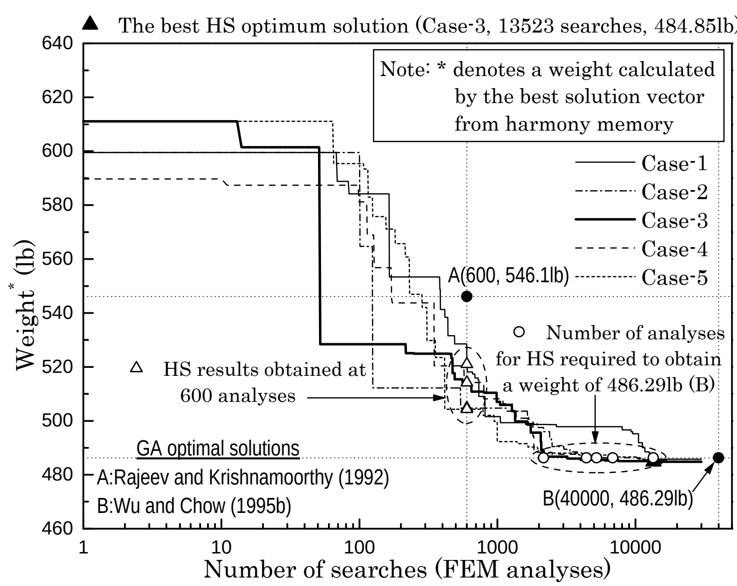

Figure 5. Convergence history of minimum weight for 25-bar space truss (Example 1)

Figure 5 shows a comparison of the convergence capability of each HS case and the GA-based approaches. While the pure GA proposed by Rajeev and Krishnamoorthy [10] obtained a minimum weight of 546.01 lb after 600 structural analyses, the HS cases obtained minimum weights of 504.28 to 521.04 lb after the same number of analyses. The steady-state GA proposed by Wu and Chow [13] obtained a minimum weight of 486.29 lb after 40,000 analyses, while all HS cases except Case 1 obtained the same weight after 2,160 to 6,850 analyses. These results suggest that the HS-based method is a powerful search and discrete size optimization technique, as compared to pure and steady-state GA-based methods, in terms of both the obtained optimal value and the convergence capability.

Example 2: 72-bar space truss

The 72-bar space truss, shown in Figure 6, is one of the most popular classical optimization design problems, and has been used as a benchmark to verify the efficiency of various optimization methods. The majority of these studies have assumed that the cross-sectional areas (size variables) were continuous. However, Wu and Chow [13] optimized this space structure with discrete cross-sectional areas using the steady-state GA-based method. In this example, the material density and modulus of elasticity were 0.1 lb/in.^{3} and 10,000 ksi, respectively. The space truss was subjected to the following two loading conditions: Condition 1, in which P_X = 5.0 kips, P_Y = 5.0 kips, and P_Z = -5.0 kips on node 17; and Condition 2, in which P_X = 0.0 kips, P_Y = 0.0 kips, and P_Z = -5.0 kips on nodes 17, 18, 19, and 20. The structure was required to be doubly symmetric about the *X*- and *Y*-axes. This condition divided the truss members into the following sixteen groups: (1) $A_1 \sim A_4$, (2) $A_5 \sim A_{12}$, (3) $A_{13} \sim A_{16}$, (4) $A_{17} \sim A_{18}$, (5) $A_{19} \sim A_{22}$, (6) $A_{23} \sim A_{30}$, (7) $A_{31} \sim A_{34}$, (8) $A_{35} \sim A_{36}$, (9) $A_{37} \sim A_{40}$, (10) $A_{41} \sim A_{48}$, (11) $A_{49} \sim A_{52}$, (12) $A_{53} \sim A_{54}$, (13) $A_{55} \sim A_{58}$, (14) $A_{59} \sim A_{66}$, (15) $A_{67} \sim A_{70}$, and (16) $A_{71} \sim A_{72}$. The members were subjected to stress limitations of ± 25 ksi, and the maximum displacement of the uppermost nodes was not allowed to exceed ± 0.25 in. for each node, in all directions. In this example, the available discrete values for the cross-sectional areas were chosen from the sixty-four discrete values listed in Table 3.

Table 3. Available discrete cross-sections

No. (1)	Areas (2)	No. (3)	Areas (4)	No. (5)	Areas (6)	No. (7)	Areas (8)
1	0.111	17	1.563	33	3.840	49	11.500
2	0.141	18	1.620	34	3.870	50	13.500
3	0.196	19	1.800	35	3.880	51	13.900
4	0.250	20	1.990	36	4.180	52	14.200
5	0.307	21	2.130	37	4.220	53	15.500
6	0.391	22	2.380	38	4.490	54	16.000
7	0.442	23	2.620	39	4.590	55	16.900
8	0.563	24	2.630	40	4.800	56	18.800
9	0.602	25	2.880	41	4.970	57	19.900
10	0.766	26	2.930	42	5.120	58	22.000
11	0.785	27	3.090	43	5.740	59	22.900
12	0.994	28	3.130	44	7.220	60	24.500
13	1.000	29	3.380	45	7.970	61	26.500
14	1.228	30	3.470	46	8.530	62	28.000
15	1.266	31	3.550	47	9.300	63	30.000
16	1.457	32	3.630	48	10.850	64	33.500

Note: cross-sectional areas are in in.^{2}.

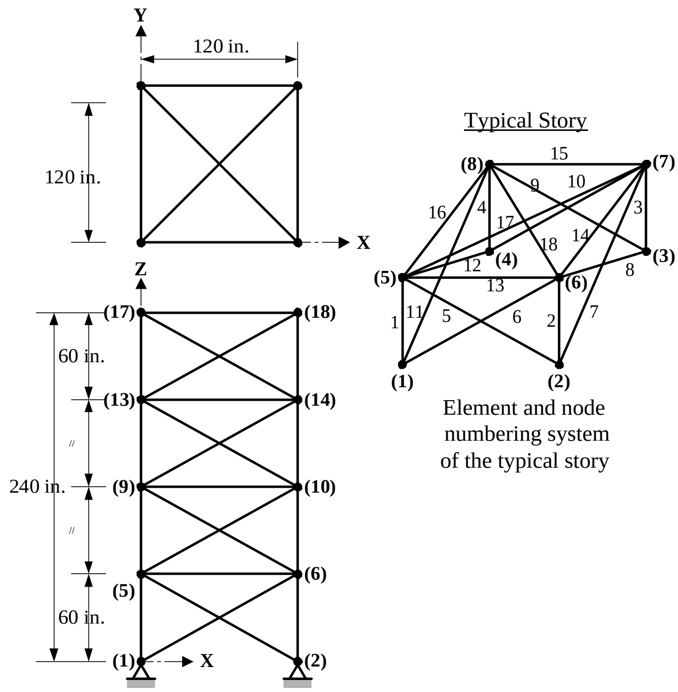

Figure 6. 72-Bar Space Truss

Table 4. Optimal results for 72-bar space truss (Example 2)

Design variables A_i (in.^{2}) (1)		HS results					Wu and Chow (1995b)				Xicheng & Guixu (1992)[c] (11)	Erbatur *et al.* (2000)[c] (12)
		Case-1 (2)	Case-2 (3)	Case-3 (4)	Case-4 (5)	Case-5 (6)	1X[b] (7)	2X[b] (8)	3X[b] (9)	4X[b] (10)		
1	$A_1 \sim A_4$	1.800	1.990	1.990	1.990	1.620	1.563	1.990	1.990	1.563	1.905	1.910
2	$A_5 \sim A_{12}$	0.602	0.602	0.442	0.602	0.602	0.307	0.602	0.563	0.766	0.518	0.525
3	$A_{13} \sim A_{16}$	0.111	0.111	0.111	0.111	0.111	0.111	0.141	0.111	0.141	0.100	0.122
4	$A_{17} \sim A_{18}$	0.111	0.111	0.111	0.111	0.111	0.111	0.111	0.141	0.111	0.100	0.103
5	$A_{19} \sim A_{22}$	1.457	1.228	1.266	1.457	1.457	2.130	0.994	1.457	1.800	1.286	1.310
6	$A_{23} \sim A_{30}$	0.563	0.563	0.563	0.391	0.391	0.602	0.602	0.602	0.602	0.516	0.498
7	$A_{31} \sim A_{34}$	0.111	0.111	0.111	0.141	0.111	0.111	0.111	0.111	0.141	0.100	0.110
8	$A_{35} \sim A_{36}$	0.111	0.111	0.111	0.111	0.111	0.111	0.307	0.111	0.307	0.100	0.103
9	$A_{37} \sim A_{40}$	0.442	0.442	0.391	0.391	0.563	0.766	0.307	0.442	0.391	0.509	0.535
10	$A_{41} \sim A_{48}$	0.442	0.442	0.602	0.602	0.563	1.000	0.602	0.766	0.391	0.522	0.535
11	$A_{49} \sim A_{52}$	0.111	0.111	0.111	0.111	0.111	0.111	0.111	0.111	0.141	0.100	0.103
12	$A_{53} \sim A_{54}$	0.141	0.111	0.111	0.111	0.111	0.111	0.563	0.141	0.111	0.100	0.111
13	$A_{55} \sim A_{58}$	0.196	0.196	0.196	0.196	0.196	0.785	0.250	0.196	0.196	0.157	0.161
14	$A_{59} \sim A_{66}$	0.563	0.563	0.563	0.602	0.602	0.602	0.766	0.442	0.602	0.537	0.544
15	$A_{67} \sim A_{70}$	0.250	0.391	0.391	0.391	0.391	0.602	0.307	0.250	0.307	0.411	0.379
16	$A_{71} \sim A_{72}$	1.000	0.563	0.563	0.563	0.785	0.602	0.391	1.000	0.766	0.571	0.521
Weight (lb)		400.63	390.62	390.30	399.23	396.38	471.98	439.77	428.00	427.20	380.84	383.12
Number of structural analyses		25,717 [7,242][a]	26,812 [7,462][a]	21,901 [3,711][a]	13,866 [4,819][a]	22,894 [3,677][a]	60,000	60,000	60,000	60,000	-	-

[a] Number of analyses for the HS required to obtain a weight of 427.2 lb (the best result of Wu and Chow, 1995b). [b] Crossover operators used by Wu and Chow (1995b). [c] The optimal results of continuous size optimization.

Figure 7 shows a comparison of the convergence capability of each HS case and the steady-state GA-based method [13]. Wu and Chow obtained a minimum weight of 427.2 lb after 60,000 structural analyses using a four-point crossover operator, while the proposed HS approach obtained the same weight after 3,711 to 7,462 analyses. The HS approach therefore outperformed the steady-state GA-based method, in terms of both the obtained optimal value and the convergence capability.

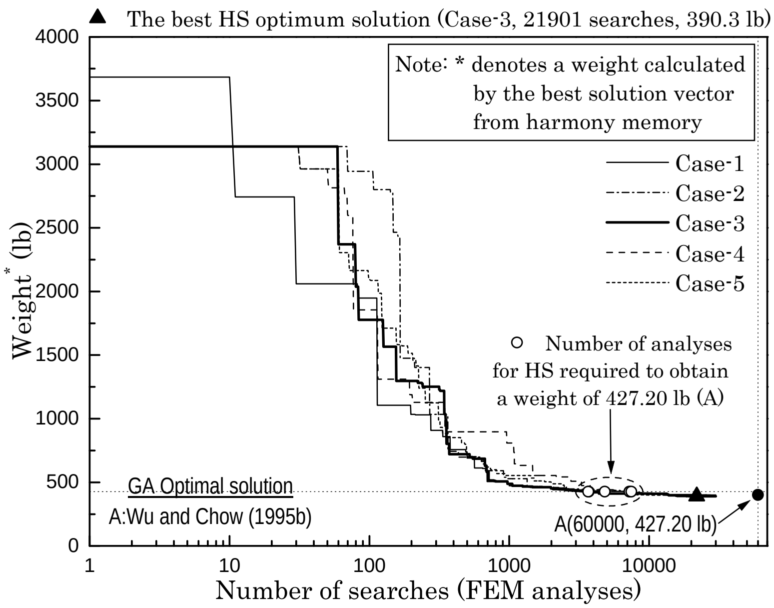

Figure 7. Convergence history of minimum weight for 72-bar space truss (Example 2)

6. DISCRETE-CONTINUOUS CONFIGURATION OPTIMIZATION EXAMPLES

Example 3: 25-bar space truss

The 25-bar transmission tower space truss shown in Figure 4, which was previously studied by Wu and Chow [9] using the GA-based method, was also analyzed to optimize both the sizes of the discrete members and the continuous geometric variables. The design details, such as the material properties, constraints, loading condition, truss member groups, and set of available discrete cross sections, were the same as those used in Example 1. For the configuration optimization, the geometric variables of the structure were selected as coordinates X_4, Y_4, Z_4, X_8, and Y_8, with symmetry required in X-Z and Y-Z planes. Hence, there were thirteen independent design variables, including the eight sizing variables given in Example 1 and five geometric variables. The side constraints for the geometric variables, *i.e.*, the lower and upper bounds on the nodal coordinates, were $20 \le X_4 \le 60$, $40 \le Y_4 \le 80$, $90 \le Z_4 \le 130$, $40 \le X_8 \le 80$, and $100 \le Y_8 \le 140$ (in.).

The HS-based discrete-continuous configuration optimization method was applied to the 25-bar space truss using each set of parameters shown in Table 1. The algorithm found the best solution vector (*i.e.*, the values of the eight sizing variables and five geometric variables) with each set of parameters within 30,000 searches. Table 5 gives the best solution and the corresponding minimum structural weight for each case, and also provides a comparison between the optimal design result reported by Wu and Chow [9] and the present work. The best minimum weight of 123.77 lb was obtained using the Case 5 parameters after 8,902 searches (structural analyses), and this minimum weight converged remarkably after only 2,000 searches. The results from each HS case were better than the previous design result reported by Wu and Chow [9], and the HS best result using the Case 5 parameters produced a weight saving of 10%, as compared to the GA-based method proposed by Wu and Chow [9].

The configuration optimization achieved an amazing optimal weight saving of 70%, as compared to the pure HS size optimization, which obtained a best minimum weight of only 484.85 lb, as shown in Table 2.

Table 5. Optimal results of 25-bar space truss (Example 3)

Design variables A_i **(in.^{2}) &** R_i **(in.) (1)**		**HS results** Case-1 (2)	Case-2 (3)	Case-3 (4)	Case-4 (5)	Case-5 (6)	**Wu & Chow (1995a) (7)**
1	A_1	0.1	0.1	0.1	0.2	0.2	0.1
2	$A_2 \sim A_5$	0.2	0.1	0.2	0.2	0.1	0.2
3	$A_6 \sim A_9$	0.9	1.0	0.9	1.0	0.9	1.1
4	$A_{10} \sim A_{11}$	0.1	0.1	0.1	0.1	0.1	0.2
5	$A_{12} \sim A_{13}$	0.1	0.1	0.1	0.2	0.1	0.3
6	$A_{14} \sim A_{17}$	0.1	0.1	0.2	0.1	0.1	0.1
7	$A_{18} \sim A_{21}$	0.1	0.4	0.2	0.1	0.2	0.2
8	$A_{22} \sim A_{25}$	1.2	0.7	0.8	1.0	1.0	0.9
1	X_4	31.64	28.54	29.51	27.94	31.88	41.07
2	Y_4	66.30	55.18	56.76	55.21	53.57	53.47
3	Z_4	102.22	127.80	130.0	123.70	126.35	124.60
4	X_8	40.00	43.02	41.74	43.63	40.43	50.80
5	Y_8	125.74	136.66	133.62	130.83	130.64	131.48
Weight (lb)		129.34 [138.10][a] [130.40][b] [129.53][c] [129.36][d]	123.81 [152.10][a] [140.63][b] [134.29][c] [124.92][d]	126.07 [154.05][a] [141.65][b] [131.71][c] [131.03][d]	126.74 [168.09][a] [146.68][b] [133.87][c] [128.16][d]	123.77 [137.79][a] [124.28][b] [123.86][c] [123.80][d]	136.20
Number of structural analyses		29,290	9,646	23,100	19,833	8,902	-

[a] The structural weights obtained after 1,000 analyses. [b] The structural weights obtained after 2,000 analyses. [c] The structural weights obtained after 3,000 analyses.
[d] The structural weights obtained after 8,000 analyses.

Example 4: 47-bar planar power line tower

The 47-bar planar power line tower design, shown in Figure 8, was the last example used to demonstrate the practical capability of the HS algorithm-based structural optimization method. This tower was previously analyzed by Felix [26] and Hansen and Vanderplaats [27] to obtain optimal continuous size and geometric variables (*i.e.*, a continuous configuration optimization). In this problem, the structure had forty-seven members and twenty-two nodes, and was symmetric about the *Y*-axis. All members were made of steel, and the material density and modulus of elasticity were 0.3 lb/in.^{3} and 30,000 ksi, respectively. This tower was designed for three separate load conditions: (1) 6.0 kips acting in the positive *X*-direction and 14.0 kips acting in the negative *Y*-direction at nodes 17 and 22, (2) 6.0 kips acting in the positive *X*-direction and 14.0 kips acting in the negative *Y*-direction at node 17, and (3) 6.0 kips acting in the positive *X*-direction and 14.0 kips acting in the negative *Y*-direction at node 22. The first condition represented the load imposed by two power lines attached to the tower at an angle. The second and third conditions represented cases that occur when one of the two lines snaps.

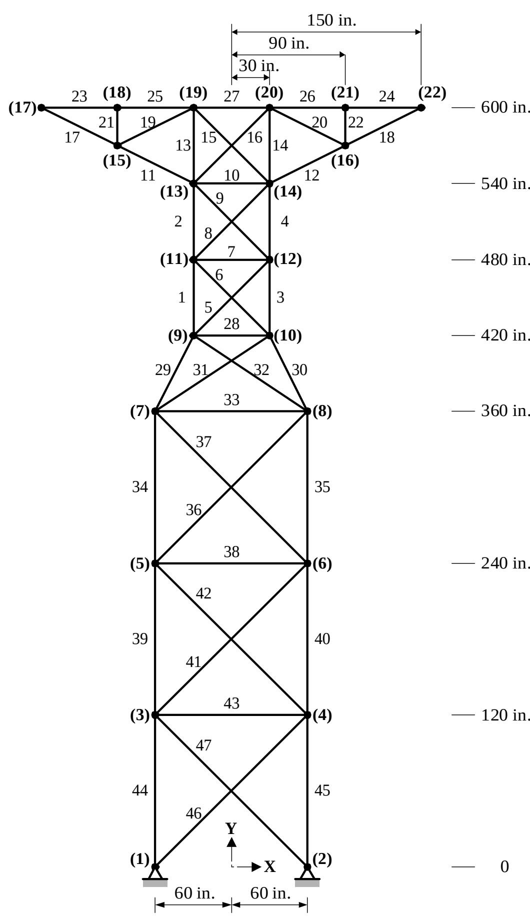

Figure 8. 47-bar planar power line tower

The structure was subjected to both stress and buckling constraints. The stress constraints were 15.0 ksi in compression and 20.0 ksi in tension. The Euler buckling compressive stress limit for each member i was used for the buckling constraints. This was computed as

$$\sigma_i^{cr} = \frac{-KEA_i}{L_i^2} \quad (i = 1,...,47) \tag{7}$$

where K is a constant determined from the cross-sectional geometry, E is the modulus of elasticity of the material, and L_i is the member length. In this study, the buckling constant was K = 3.96. The cross-sectional areas of the members were categorized into twenty-seven groups, as follows:

(1) $A_1 = A_3$, (2) $A_2 = A_4$, (3) $A_5 = A_6$, (4) A_7, (5) $A_8 = A_9$, (6) A_{10}, (7) $A_{11} = A_{12}$, (8) $A_{13} = A_{14}$, (9) $A_{15} = A_{16}$, (10) $A_{17} = A_{18}$, (11) $A_{19} = A_{20}$, (12) $A_{21} = A_{22}$, (13) $A_{23} = A_{24}$, (14) $A_{25} = A_{26}$, (15) A_{27}, (16) A_{28}, (17) $A_{29} = A_{30}$, (18) $A_{31} = A_{32}$, (19) A_{33}, (20) $A_{34} = A_{35}$, (21) $A_{36} = A_{37}$, (22) A_{38}, (23) $A_{39} = A_{40}$, (24) $A_{41} = A_{42}$, (25) A_{43} , (26) $A_{44} = A_{45}$, and (27) $A_{46} = A_{47}$.

The independent geometric variables were X_2, X_4, Y_4, X_6, Y_6, X_8, Y_8, X_{10}, Y_{10}, X_{12}, Y_{12}, X_{14}, Y_{14}, X_{20}, Y_{20}, X_{21}, and Y_{21}. The geometric variables were linked to maintain symmetry about the Y-axis. Nodes 1 and 2 were required to remain at Y = 0.0, and the coordinates of nodes 15, 16, 17, and 22 were not changed. There were forty-four independent design variables, including twenty-seven sizing variables and seventeen coordinate variables. In this example, the cross-sectional areas were chosen from the sixty-four discrete values listed in Table 3, and a pure discrete sizing variable problem (with fixed geometry) was also optimized for comparison.

Table 6 gives the optimal results obtained using each set of HS parameters for the discrete-continuous configuration optimization, along with the optimal results for the continuous configuration problem. The best pure discrete size result, which was obtained using the Case 3 parameters, is also listed in the table. After 73,257 to 76,937 searches (structural analyses), the best discrete-continuous solution vector and the corresponding objective function value were obtained for each HS case. The best minimum weight of 2,020.78 lb was obtained using the Case 1 parameters after 73,771 searches, and this minimum weight converged remarkably after 40,000 searches. The discrete-continuous configuration optimization produced a considerable weight saving of 16%, as compared to the pure discrete size optimization, which obtained a minimum weight of 2,396.8 lb.

Table 6. Optimal results of 47-bar planar power line

Design variables A_i (in.^{2}) & R_i (in.) (1)		HS results						Felix** (1981) (8)	Hansen & Vanderpl-aats**(1990) (9)
		Pure Size Case-3* (2)	Case-1 (3)	Case-2 (4)	Case-3 (5)	Case-4 (6)	Case-5 (7)		
1	$A_1 = A_3$	3.840	2.620	3.550	3.130	3.090	2.930	2.73	2.42
2	$A_2 = A_4$	3.380	2.630	3.090	3.090	2.880	2.630	2.47	2.35
3	$A_5 = A_6$	0.766	1.228	0.766	1.000	0.994	1.228	0.73	0.82
4	A_7	0.141	0.196	0.141	0.111	0.141	0.141	0.21	0.10
5	$A_8 = A_9$	0.785	1.000	1.000	0.994	1.228	0.994	0.94	0.86
6	A_{10}	1.990	1.620	1.228	1.228	1.620	1.800	1.08	1.15
7	$A_{11} = A_{12}$	2.130	1.800	1.990	2.130	2.380	2.380	1.69	1.77
8	$A_{13} = A_{14}$	1.228	0.785	1.000	0.785	0.602	0.602	0.69	0.67
9	$A_{15} = A_{16}$	1.563	1.000	1.228	1.228	1.228	0.994	1.06	0.86
10	$A_{17} = A_{18}$	2.130	1.563	1.800	1.990	1.620	1.620	1.41	1.24
11	$A_{19} = A_{20}$	0.111	0.391	0.602	0.785	0.563	0.602	0.26	0.33
12	$A_{21} = A_{22}$	0.111	0.766	0.994	0.994	1.457	1.228	0.81	1.22
13	$A_{23} = A_{24}$	1.800	1.228	1.457	1.457	1.228	1.228	1.06	0.93
14	$A_{25} = A_{26}$	1.800	1.228	1.457	1.457	1.228	1.228	1.05	0.86
15	A_{27}	1.457	1.228	1.228	1.000	1.457	1.457	0.82	0.69
16	A_{28}	0.442	0.196	0.250	0.111	0.141	0.196	0.30	0.15
17	$A_{29} = A_{30}$	3.630	2.930	2.880	2.880	3.130	3.130	2.77	2.46
18	$A_{31} = A_{32}$	1.457	0.994	1.228	0.994	0.994	0.766	0.66	0.90
19	A_{33}	0.391	0.111	0.111	0.141	0.111	0.111	0.21	0.10
20	$A_{34} = A_{35}$	3.090	3.470	2.880	3.130	3.380	3.550	2.90	2.74
21	$A_{36} = A_{37}$	1.457	1.000	1.000	1.228	1.000	1.000	0.27	0.92
22	A_{38}	0.196	0.111	0.111	0.307	0.111	0.111	1.41	0.10
23	$A_{39} = A_{40}$	3.840	3.380	3.130	3.380	3.470	3.380	3.43	2.94
24	$A_{41} = A_{42}$	1.563	1.228	1.266	1.000	1.228	1.000	0.99	1.13
25	A_{43}	0.196	0.111	0.111	0.111	0.250	0.111	0.17	0.10
26	$A_{44} = A_{45}$	4.590	3.380	3.470	3.630	3.470	3.470	3.65	3.12
27	$A_{46} = A_{47}$	1.457	0.994	1.563	1.266	0.994	1.266	1.01	1.10
1	$-X_1=X_2$	60.0*	98.9	89.4	85.9	97.7	91.6	90.0	107.1
2	$-X_3= X_4$	60.0*	80.9	83.1	80.9	80.9	80.9	90.0	91.2
3	$Y_3=Y_4$	120.0*	114.8	111.7	115.4	114.1	122.9	123.4	122.8
4	$-X_5=X_6$	60.0*	62.8	74.4	61.6	60.1	61.6	83.4	74.2
5	$Y_5=Y_6$	240.0*	236.9	234.9	233.9	225.2	238.4	244.5	241.4
6	$-X_7=X_8$	60.0*	51.3	59.5	55.4	49.2	47.6	70.5	65.5
7	$Y_7=Y_8$	360.0*	315.9	339.3	319.4	323.1	327.9	355.1	324.6
8	$-X_9=X_{10}$	30.0*	47.9	40.7	46.9	44.4	41.1	60.0	57.1
9	$Y_9=Y_{10}$	420.0*	387.4	429.6	409.9	392.1	394.1	425.0	400.4
10	$-X_{11}=X_{12}$	30.0*	50.3	35.2	35.3	38.1	42.7	58.2	49.3
11	$Y_{11}=Y_{12}$	480.0*	477.3	455.3	471.8	477.3	476.6	478.0	472.3
12	$-X_{13}=X_{14}$	30.0*	41.4	34.4	36.6	40.1	42.4	59.6	47.4
13	$Y_{13}=Y_{14}$	540.0*	521.4	505.7	504.9	519.2	504.7	519.5	507.5
14	$-X_{18}=X_{21}$	90.0*	92.5	83.9	84.8	91.3	84.7	96.9	83.3
15	$Y_{18}=Y_{21}$	600.0*	615.3	609.2	606.8	620.9	615.5	633.7	636.0
16	$-X_{19}=X_{20}$	30.0*	14.3	18.4	17.1	6.9	3.2	15.0	3.9
17	$Y_{19}=Y_{20}$	600.0*	596.5	586.4	582.0	580.7	569.6	607.6	586.5
Weight (lb)		2,396.8 [2,471.1][a] [2,434.3][b] [2,407.7][c]	2,020.78 [2,428.62][a] [2,198.13][b] [2,066.73][c]	2,116.14 [2,608.26][a] [2,339.84][b] [2,195.27][c]	2,091.21 [2,580.55][a] [2,361.17][b] [2,189.57][c]	2,096.35 [2,735.43][a] [2,421.92][b] [2,225.39][c]	2,056.77 [2,468.82][a] [2,269.06][b] [2,165.42][c]	1,904.0	1,850.4
Number of analyses		45,557	73,771	76,937	74,721	76,828	73,257	-	-

* Coordinate is stationary. ** The results of continuous configuration optimizations.
[a] Structural weights obtained after 10,000 analyses. [b] Structural weights obtained after 20,000 analyses.
[c] Structural weights obtained after 40,000 analyses.

7. CONCLUSIONS

Pure discrete size and integrated discrete size and continuous configuration optimization methods for structural systems, based on the HS algorithm, were proposed in this paper. Several standard truss examples from the literature were also presented to demonstrate the effectiveness and robustness of the proposed method. The results were compared to those obtained using current discrete optimization methods, especially GA-based techniques. The illustrative examples revealed that the HS optimal results were better than those obtained from all previous investigations. Also, the convergence capability of the proposed HS approach outperformed that of the GA-based methods. Therefore, our study suggests that the new HS-based method is a potential powerful search and optimization technique for solving structural optimization problems with discrete sizing variables.

The recently developed HS heuristic algorithm is simple and mathematically less complex than the GA. The HS algorithm generates new vectors, based on the harmony memory considering rate and the pitch adjusting rate, after considering all of the existing vectors, while the GA generates a new vector from only two of the existing vectors (parents). These features increase the flexibility of the HS algorithm and allow it to find better solutions. Furthermore, the HS algorithm adopted a parameter-setting-free adaptive feature, enabling the algorithm users not to perform tedious parameter setting process [28, 29].

The HS algorithm-based method proposed in this study is not limited to truss structural optimization problems. Besides trusses, the HS algorithm can also be applied to other types of structural optimization problems, including frame structures, plates, and shells.

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