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# TWO-STAGE METHOD FOR DAMAGE LOCALIZATION AND QUANTIFICATION IN HIGH-RISE SHEAR FRAMES BASED ON THE FIRST MODE SHAPE SLOPE

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## ABSTRACT

In this paper, a two-stage method for damage detection and estimation in tall shear frames is presented. This method is based on the first mode shape of a shear frame. We demonstrate that the first mode shape slope is very sensitive to the story stiffness. Thus, at the first stage, by using the grey system theory on the first mode shape slope, damage locations are identified in shear frames. Damage severity is determined at the second stage by defining the damage detection problem as an optimization problem by using grey relation coefficients. The optimization problem is solved by a socio-politically motivated global search strategy which is the imperialist competitive algorithm. The efficiency and robustness of the proposed method for the identification and estimation of damages in tall shear frames were studied by using two numerical examples. In addition, the capability of the presented method in real conditions was demonstrated by contaminating of modal data with different levels of random noises. All the obtained results from the numerical studies are shown the good performance of the presented method in the damage localization and quantification of tall buildings.

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KEY WORDS: Damage detection; shear frame; first mode shape slope; grey system theory; imperialist competitive algorithm

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## **1. INTRODUCTION**

Early damage detection in aerospace, mechanical, and civil engineering structures for the purpose of structural health monitoring has been considered more attention in the last two decades. In order to monitor the existence condition of structures, some researchers attempted to develop vibration-based damage detection methods for the localization and quantification of damages [1-3]. The basic idea of the vibration-based methods is that modal parameters are functions of the physical properties of structure such as mass, stiffness, and damping. Therefore, changes in the physical properties cause changes in the modal data. The most important modal data that have been employed for damage diagnosis are natural frequencies and mode shapes. Complete review of these methods can be found in [4, 5].

Some researchers utilized only natural vibration frequencies for damage identification [6-8]. The advantages of these approaches are that the test of a structure is relatively easy which can be done by only few sensors and has relatively higher precision than those based on other parameters; because the natural frequencies have least statistical variation from random error sources than other modal data. The most important problems of these methods are the low sensitivity to damage occurring and the high sensitivity to random noises. Other researchers used mode shapes or the derivative of mode shapes to develop some effective methods for damage identification [9-14]. Based on mode shapes, some damage indexes for damage localization have been proposed, namely MAC and COMAC. The application of the first mode shape slope sensitivity in damage localization is studied by Zhu *et al.* [14]. They defined the first mode shape slope as a sensitive parameter in occurring single damage scenarios, and then developed this hypothesis for multiple damage occurrences.

Although natural frequencies or mode shapes are suitable for damage localization, but the requirement for quantifying damage severity causes to develop some mixed methodologies for damage diagnosis by means of the eigenvalue and eigenvector of modal analysis [15-19]. Liu [15] proposed a method for damage identification in trusses based on the inspection of the modal data. His method was based on the finite element analysis, and he used the statistical bases for more reliability. To determine the best dynamical characteristics, Zhao and Dewolf [20] studied the sensitivity coefficients of the natural frequencies, mode shapes, and modal flexibilities with respect to each element of the stiffness matrix. For determination of damage magnitudes in a simple way, Yang [18] attempted to organize a direct methodology for damage localization and quantification at two stages and validated the method by studying a truss structure.

In addition to the mentioned researches, there are some studies that are based on an optimization formulation for damage detection [21-30]. Cobb and Liebst [21] identified damages in the structural elements by a nonlinear optimization scheme based on an incomplete set of modal data. Fallahian and Seyedpoor [29] developed a two-stage method for structural damage prognosis by defining multiple damage location assurance criterions through changes in natural frequencies as a correlation-based index. They employed an adaptive neuro-fuzzy inference system and the particle swarm optimization. Ghodrati Amiri *et al.* [30] presented an optimization-based damage detection method in the plate-like structures by means of the genetic and pattern search algorithms. Recently, Bagheri *et al.* [31] developed an optimization strategy for solving an inverse problem to estimate and detect damages in different types of structures by defining a cost function based on modal data and the free vibration scheme of structures. Torkzadeh *et al.* [32] localized damages in large-scale structures by the kinetic strain energy and quantified their severities by using the heuristic particle swarm optimization algorithm for

## TWO-STAGE METHOD FOR DAMAGE LOCALIZATION AND QUANTIFICATION ... 655

solving an objective function organized by the modal strain energy.

Although, the mentioned methods can be detected and/or quantified defects in structures, but most of these methods are not suitable for tall buildings. Therefore, it is important to develop some damage diagnosis algorithms for high-rise buildings. In this paper, a two-stage damage detection method is proposed for tall shear frames based on the first mode shape slope using the grey system theory and an optimization process. Grey system theory is a criterion for inspecting the correlation between two regular sequences by a geometrical base. This theory was used for structural damage identification by means of static displacement curvature obtained under different static loads [33, 34]. Despite of good performance of the approach, applying static loads and recording static displacements are difficult in real SHM programs. For overcoming to this limitation, in this paper, we use the grey system theory on modal data. The first stage of the proposed method is related to damage localization by using the grey relation coefficients (GRCs) on the first mode shape slopes. The second stage is devoted to quantify the damage severity by defining an optimization problem based on the GRCs obtained in stage one. The imperialist competitive algorithm is used for solving the optimization problem. One of the most important advantages of the presented method is its ability in damage localization and estimation using only the first mode shape. The proposed method was validated by two numerical examples namely 15-story and 25-story shear frame.

The paper is organized as follows. The background of the grey system theory and the imperialist competitive algorithm is described in Section 2. In Section 3, the formulation of the mode shape slope sensitivity with respect to story stiffness is proposed. It is followed by Section 4 for organizing the fundamental rules of the proposed damage detection method. Section 5 introduces the numerical examples and presents the obtained results. Lastly, the paper ends with some concluding remarks described in Section 6.

## 2. BACKGROUND

## 2.1. Grey System Theory

The grey system theory has been developed for inspecting the correlation between two regular sequences by a geometrical based comparison with reference sequences that are weak and limited measured data [35]. In addition, this analysis has good potential for statistical prediction when there are some uncertainties in recorded data. Basically, the GRCs reflect the degree of approaching two geometrical curves. It is clear that the bigger coefficients predict a big approaching between the baseline sequence and the test sequence.

Assume that the reference sequence  $X_0$  and the test sequence  $X_i$  are as follows:

$$\mathbf{X}_{0} = \left(x_{0}(1), x_{0}(2), \dots, x_{0}(n)\right)$$
(1)

$$\mathbf{X}_{i} = \left(x_{i}(1), x_{i}(2), \dots, x_{i}(n)\right)$$
(2)

where  $x_0(n)$  and  $x_i(n)$  are *n*-th points on the geometrical curves.

In the grey system theory, the GRCs can be defined as:

$$\zeta_i(n) = \frac{\min_i \min_i X + \alpha \max_i \max_i X}{X + \alpha \max_i \max_n X}$$
(3)

in which  $\xi_i(n)$  is the GRC in the *n*-th point;  $\alpha$  is a number between 0 and 1 which is the distinguishable coefficient employed to adjust the limit of the comparison environment, and to control the rank of differences of the relation coefficients; *X* is the grey variant as follows:

$$X = \left| x_0(n) - x_i(n) \right| \tag{4}$$

The comparison environment is unaltered for  $\alpha$ =1 while the comparison environment is disappeared when  $\alpha$  is equal zero. The range of GRC is between [0 1], and the GRC estimates the point-relation degree at the *n*-th point of the test sequence and reference sequence. Overall,  $\xi_i(n)>0.9$  presents that the reference point and the test point are related completely;  $0.8 < \xi_i(n) < 0.9$  shows the good relation between the two points;  $0.6 < \xi_i(n) < 0.8$  indicates that the two points are relative or irrespective possibly;  $\xi_i(n) < 0.6$  expresses that the two points are roughly irrelative [36].

#### 2.2. Imperialist competitive algorithm

The imperialist competitive algorithm is a global search optimization approach which is inspired from a socio-political event namely imperialist competition [37–39]. The objective of this algorithm is to find an optimal solution, usually the minimum, of the argument **x** of a certain function  $f(\mathbf{x})$ . Similar to other evolutionary optimization algorithms, this algorithm starts with an initial population that called *country*. The number of countries is equal to  $N_{country}$ , and they are divided into  $N_{imp}$  imperialists and  $N_{col}$  colonies. Every country is presented in a vector as follows:

$$\mathbf{country} = \begin{bmatrix} x_1, x_2, \dots, x_n \end{bmatrix}$$
(5)

$$c = f (\mathbf{country}) \tag{6}$$

The division of colonies is directly proportional with the power of every empire. The normalized cost of any imperialist can be determined as:

$$C_{j} = c_{j} - \max_{i} \left\{ c_{i} \right\} \tag{7}$$

where  $c_j$  is the cost of *j*-th imperialist. Also, the power of *j*-th empire  $p_j$  can be represented as:

$$p_{j} = \left| C_{j} \middle/ \sum_{i=1}^{N_{imp}} C_{i} \right|$$
(8)

Thus, the initial number of colonies for the *j*-th empire is as follows:

656

$$N.C._{j} = round\left(p_{j}N_{col}\right) \tag{9}$$

where *round* indicates the rounding to nearest integer number.

After the initialization process, the imperialistic countries begin to improve their colonies and attempt to absorb new colonies. This is the assimilation process which is modeled by moving all of the colonies toward the imperialist along different optimization axis [37]. To ensure that many positions are explored in search of the minimal cost, the assimilation of the colonies by the imperialists does not occur through the direct movement of the colonies toward the imperialist. A random path is induced by a random amount of deviation added to the direction of the movement. If during the assimilation process, a colony reaches a position with lower cost than the imperialist, then the imperialist and the colony switch their positions. Then, the algorithm will continue with the imperialist in the new position and the colonies will be assimilated by the imperialist in its new position.

The total power of an empire depends on both the power of the imperialist country and the power of its colonies. Therefore, the total cost of the *j*-th empire  $T.C_{,j}$  is determined by:

$$T.C._{j} = f\left(imperialist_{j}\right) + \xi \frac{\sum_{i=1}^{N.C._{j}} f\left(\text{the }i\text{-th colony of the }j\text{-th empire}\right)}{N.C._{j}}$$
(10)

where  $\xi$  is a coefficient comprised between 0 and 1. Normalized total cost *N.T.C.* of the *j*-th empire is obtained as:

$$N.T.C._{j} = T.C._{j} - \max_{i} \{T.C._{i}\}$$
(11)

Then, the possession probability  $p_p$  of the *j*-th empire is given by:

$$p_{p_j} = \left| N.T.C._j \right/ \sum_{i=1}^{N_{imp}} N.T.C._i \right|$$
 (12)

To divide the colonies among the empires, the vector **D** is defined as below:

$$\mathbf{D} = \mathbf{P} - \mathbf{R} \tag{13}$$

where the vector  $\mathbf{R}$  is formed by uniformly distributed random numbers comprised between 0 and 1, and the vector  $\mathbf{P}$  is defined as:

$$\mathbf{P} = [p_{p_1}, p_{p_2}, \dots, p_{p_{N_{imn}}}]$$
(14)

A certain colony is given to an empire whose relevant index in  $\mathbf{D}$  is maximum. The last step of the imperialist competitive algorithm consists of the elimination of the powerless empires. If one empire only is left then the optimization algorithm is terminated, otherwise the algorithm starts again from the assimilation step.

## **3. MODE SHAPE SLOPE SENSITIVITY**

The mode shape slope of the *j*-th story in the *i*-th mode for a shear frame can be written as:

$$\varphi'_{ij} = \frac{\varphi_{ij} - \varphi_{i(j-1)}}{h_j}$$
(15)

where  $\varphi'$  and  $\varphi$  are the mode shape slope and the mode shape, respectively;  $h_j$  is the height of the *j*-th story. In order to formulate the mode shape slope sensitivity with respect to the *t*-th story stiffness  $k_t$ , the first derivative of the Eq. (15) with respect to  $k_t$  leads to:

$$\frac{\partial \varphi_{ij}'}{\partial k_t} = \frac{1}{h_j} \left( \frac{\partial \varphi_{ij}}{\partial k_t} - \frac{\partial \varphi_{i(j-1)}}{\partial k_t} \right)$$
(16)

Basically, for a *n*-story shear building, there are *n* independent mode shape vector with the dimension of *n*. Therefore, we can assume that the mode shape sensitivity with respect to  $k_i$  is as:

$$\frac{\partial \mathbf{\phi}_i}{\partial k_t} = \sum_{s=1}^n \alpha_s \mathbf{\phi}_s = \alpha_1 \mathbf{\phi}_1 + \dots + \alpha_i \mathbf{\phi}_i + \dots + \alpha_n \mathbf{\phi}_n = \alpha_i \mathbf{\phi}_i + \sum_{r=1}^{n, r \neq i} \alpha_r \mathbf{\phi}_r$$
(17)

in which  $\alpha_s$  is the coefficient of communion for the *s*-th mode shape in the sensitivity expression. From Eq. (17), it is obvious that we can divide these coefficients into two groups: one is denoted by  $\alpha_i$  that shows the amount of the communion of the *i*-th mode in the sensitivity of the *i*-th mode, and other is denoted by  $\alpha_r$  that shows the amount of the communion of the *r*-th mode in the sensitivity of the *i*-th mode. In the next subsection, we compute these coefficients from the free vibration eigenvalue problem of a *n*-story shear building.

#### 3.1. Computation of $\alpha_r$

658

The equation of the free vibration eigenvalue problem for a *n*-story shear building can be written as:

$$\left(\mathbf{K} - \omega_i^2 \mathbf{M}\right) \mathbf{\varphi}_i = \mathbf{0}_{n \times 1}$$
(18)

where **M** and **K** are the mass and stiffness matrices of the shear frame, respectively;  $\omega_i$  and  $\varphi_i$  are the *i*-th natural frequency and mode shape of the structure, respectively. In this study, the mode shape vectors are normalized to mass matrix as follows:

$$\boldsymbol{\varphi}_i^T \mathbf{M} \boldsymbol{\varphi}_i = 1 \tag{19}$$

The first derivative of Eq. (18) with respect to the *t*-th story stiffness is as:

$$\left(\frac{\partial \mathbf{K}}{\partial k_{t}} - 2\omega_{i}\frac{\partial \omega_{i}}{\partial k_{t}}\mathbf{M} - \omega_{i}^{2}\frac{\partial \mathbf{M}}{\partial k_{t}}\right)\boldsymbol{\varphi}_{i} + \left(\mathbf{K} - \omega_{i}^{2}\mathbf{M}\right)\frac{\partial \boldsymbol{\varphi}_{i}}{\partial k_{t}} = \mathbf{0}_{n\times 1}$$
(20)

By pre multiplying Eq. (20) by  $\boldsymbol{\varphi}_r^T$  and with knowing that  $\boldsymbol{\varphi}_r^T \mathbf{K} = \omega_r^2 \boldsymbol{\varphi}_r^T \mathbf{M}$ , we can write Eq. (20) as follows:

$$\boldsymbol{\varphi}_{r}^{T} \frac{\partial \mathbf{K}}{\partial k_{t}} \boldsymbol{\varphi}_{i} + \left(\omega_{r}^{2} - \omega_{i}^{2}\right) \boldsymbol{\varphi}_{r}^{T} \mathbf{M} \frac{\partial \boldsymbol{\varphi}_{i}}{\partial k_{t}} = 0, \quad r \neq i$$
(21)

Substituting Eq. (17) into Eq. (21) leads to:

$$\boldsymbol{\varphi}_{r}^{T} \frac{\partial \mathbf{K}}{\partial k_{t}} \boldsymbol{\varphi}_{i} + \left(\omega_{r}^{2} - \omega_{i}^{2}\right) \boldsymbol{\varphi}_{r}^{T} \mathbf{M} \left(\alpha_{1} \boldsymbol{\varphi}_{1} + \dots + \alpha_{r} \boldsymbol{\varphi}_{r} + \dots + \alpha_{n} \boldsymbol{\varphi}_{n}\right) = 0$$
(22)

Using the orthogonality properties of the mode shapes, we can rewrite Eq. (22) in the following form:

$$\boldsymbol{\varphi}_{r}^{T} \frac{\partial \mathbf{K}}{\partial k_{t}} \boldsymbol{\varphi}_{i} + \left(\omega_{r}^{2} - \omega_{i}^{2}\right) \alpha_{r} = 0$$
(23)

For a shear building, the stiffness matrix can be written as:

$$\mathbf{K} = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 & \dots & \dots & \dots & \dots & 0 \\ -k_2 & k_2 + k_3 & \dots & 0 & \dots & \dots & \dots & 0 \\ 0 & \dots & \dots & \dots & 0 & \dots & \dots & 0 \\ 0 & 0 & \dots & k_{t-1} + k_t & -k_t & 0 & \dots & 0 \\ 0 & \dots & 0 & -k_t & k_t + k_{t+1} & \dots & 0 & 0 \\ 0 & \dots & \dots & 0 & \dots & \dots & \dots & 0 \\ 0 & \dots & \dots & 0 & \dots & k_{n-1} + k_n & -k_n \\ 0 & \dots & \dots & \dots & \dots & 0 & -k_n & k_n \end{bmatrix}$$
(24)

Then, the first derivative of the stiffness matrix with respect to the *t*-th story stiffness can be easily computed and substituted in Eq. (23). Finally, by using Eq. (23),  $\alpha_r$  can be expressed as:

A. Zare Hosseinzadeh, A. Bagheri, and G. Ghodrati Amiri

$$\alpha_{r} = \frac{(\phi_{i1})(\phi_{s1})}{(\omega_{i}^{2} - \omega_{r}^{2})} \quad r \neq i, t = 1$$

$$\alpha_{r} = \frac{(\phi_{it} - \phi_{i(t-1)})(\phi_{rt} - \phi_{r(t-1)})}{(\omega_{i}^{2} - \omega_{r}^{2})} \quad r \neq i, 2 \le t \le n$$
(25)

## 3.2. Calculation of $\alpha_i$

The derivative of Eq. (19) with respect to the *t*-th story stiffness  $k_t$  is as follows:

$$\frac{\partial \boldsymbol{\varphi}_{i}^{T}}{\partial k_{t}} \mathbf{M} \boldsymbol{\varphi}_{i} + \boldsymbol{\varphi}_{i}^{T} \frac{\partial \mathbf{M}}{\partial k_{t}} \boldsymbol{\varphi}_{i} + \boldsymbol{\varphi}_{i}^{T} \mathbf{M} \frac{\partial \boldsymbol{\varphi}_{i}}{\partial k_{t}} = 0$$
(26)

For a shear building, the mass matrix is a diagonal matrix. Thus, we can have the following relation:

$$\frac{\partial \boldsymbol{\varphi}_{i}^{T}}{\partial k_{t}} \mathbf{M} \boldsymbol{\varphi}_{i} = \boldsymbol{\varphi}_{i}^{T} \mathbf{M} \frac{\partial \boldsymbol{\varphi}_{i}}{\partial k_{t}}$$
(27)

Substituting Eq. (27) and Eq. (17) into Eq. (26) leads to:

$$2\mathbf{\varphi}_{i}^{T}\mathbf{M}\left(\alpha_{1}\mathbf{\varphi}_{1}+...+\alpha_{i}\mathbf{\varphi}_{i}+...+\alpha_{n}\mathbf{\varphi}_{n}\right)=0$$
(28)

From the orthogonality properties of the mode shapes, we can find that  $\alpha_i$  is equal to zero and this result is honest for all values of *t*.

## 4. DAMAGE IDENTIFICATION METHOD

In the previous section, the sensitivity expressions of the mode shapes and mode shape slopes for a shear frame were presented. In this section, the suggested method is presented. As mentioned before, the proposed method consists of two stages for damage localization and quantification.

#### 4.1. Damage localization

Based on the calculation of the sensitivity coefficients of the first mode shape slope with respect to various story stiffness in a pristine shear frame and comparison of those for a damaged frame, it is resulted that the sensitivity coefficients obtained from the first mode shape slope follows a regular sequence which is disarranged in a given story that we want to measure its sensitivity. In this study, the grey system theory is employed for measuring the amount of correlation between a reference and test data. Therefore, by defining the GRCs for the first mode shape slope from the initial and damaged states of a shear frame, we can judge about the existence of damages in a frame.

The localization technique can be described in the following steps:

660

Step 1) Computing the first mode shape slope of the damaged and undamaged structure.

**Step 2)** Calculating the GRCs for each story of frame. The reference and test sequences are the obtained results of the first step for the undamaged and damaged structure, respectively.

**Step 3)** Identifying damaged story (or stories) by discussing on the  $\xi_i(k)$  for any story. Based on Section 2, there is an irrelative correlation between the first mode shape slopes if  $\xi_i(k) < 0.6$ , and this index can detect the damaged stories.

## 4.2. Damage severity quantification

In this section, we define the damage quantification problem as an optimization fitting data problem by the application of GRCs in the formulation of an objective function. In this method, by using evolutionary optimization approaches, we attempt to determine damage severities. As mentioned in the previous sections, the GRCs of the first mode shape slope are suitable and sensitive parameters for damage localization in shear buildings, and also they can be used for damage estimation.

The stiffness matrix for a damaged frame can be expressed as:

$$\mathbf{K}^{d} = \begin{bmatrix} (1-d_{1})k_{1} + (1-d_{2})k_{2} & -(1-d_{2})k_{2} & \dots & 0\\ -(1-d_{2})k_{2} & (1-d_{2})k_{2} + (1-d_{3})k_{3} & \dots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \dots & (1-d_{n})k_{n} \end{bmatrix}_{n \times n}$$
(29)

in which  $d_i$  is the damage severity of the *i*-th story. The GRCs for a damaged frame can be written as a vector:

$$\mathbf{GRC}_{c}(d_{1}, d_{2}, ..., d_{n}) = \left\{ \zeta_{i}(1), \zeta_{i}(2), ..., \zeta_{i}(n) \right\}^{T}$$
(30)

where  $\mathbf{GRC}_c$  is a GRC vector by the computational method for given damage severities. By using measured data, we can also obtain a GRC vector. By these two GRC vectors, a residual vector  $\mathbf{R}$  can be defined as:

$$\mathbf{R}(d_1, d_2, \dots, d_n) = \mathbf{GRC}_m - \mathbf{GRC}_c(d_1, d_2, \dots, d_n)$$
(31)

where  $GRC_m$  is a GRC vector using the measured data. To estimate damage severities, the residual vector is utilized to define the following objective function:

$$F = \left\| \mathbf{R}(d_1, d_2, ..., d_n) \right\|^2$$
(32)

in which  $\| \|$  represents the Euclidean length. For solving this objective function, we use the imperialist competitive algorithm.

#### **5. NUMERICAL STUDIES**

In this section, the proposed method is validated by two numerical examples of tall shear frames. The first example is a 15-story shear frame. The mass, stiffness, and height of stories are presented in Table 1. In this example, at the first step, we focus on the comparison of the sensitivity coefficients for the first mode shape and first mode shape slope of the pristine structure. In order to investigate the ability of the first mode shape slope and the first mode shape for damage localization, their sensitivities were calculated with respect to the different story stiffness. Figures 1 to 3, for example, show the results for the first mode shape slope and the first mode shape slope and the first mode shape sensitivities with respect to the first, fifth, and tenth story stiffness, respectively. It is obvious that the sensitivity of the first mode shape slopes are close to the mean and median of the sensitivity coefficients rather than the same coefficients in the first mode shapes. In the other hand, the variances of the first mode shape slope can be used as a good feature for making an efficient sequence in order to localize damages with the grey system theory.





Figure 1. (a) The first mode shape sensitivity and (b) the first mode shape slope sensitivity with respect to the first story stiffness





Figure 2. (a) The first mode shape sensitivity and (b) the first mode shape slope sensitivity with respect to the fifth story stiffness



Figure 3. (a) The first mode shape sensitivity and (b) the first mode shape slope sensitivity with respect to the tenth story stiffness

To demonstrate the ability of the presented method for damage localization and estimation, four damage patterns were considered in the 15-story shear frame. These damage patterns are listed in Table 2. Damage patterns (1) and (2) are single damage scenarios and patterns (3) and (4) consist of multiple damage patterns in the structure.

Damage scenario	Damage location	Stiffness reduction (%)
1	Story 4	10
2	Story 15	20
3	Stories 2 and 7	20 and 15
4	Stories 11 and 13	15 and 15

Table 2: The considered damage scenarios in the 15-story shear frame

As mentioned in the previous sections, the presented method consists of two stages for damage detection and estimation. The first stage is damage localization and each story with a GRC lower than 0.6 is considered as a damaged story. In the stage two, the imperialist competitive algorithm is used for reaching to an optimum solution which is the result for damage severity. This algorithm starts by an initial population and continues by the imperialist competition between empires. The considered values of parameters in this optimization algorithm are as follows: the number of initial countries = 60, the number of initial imperialists = 6, and the number of iteration = 600. The selection of these parameters is based on the trial and error method, and the values of the selected numbers depend on the number of stories in a frame. A chose for the selection of the number of imperialists is about 10% of the number of countries. The maximum number of iterations is selected based on the required accuracy in the optimization.

For each single damage pattern, the obtained results of damage detection and estimation are shown in Figures 4 and 5.



Figure 4. Damage detection results for damage pattern (1) in the 15-story frame: (a) damage localization, and (b) damage severity



Figure 5. Damage detection results for damage pattern (2) in the 15-story frame: (a) damage localization, and (b) damage severity

It is clear that the method can detect the location of defect, and also the obtained value for the severity of defects is equal to the simulated damage severity. For multiple damage scenarios, Figures 6 and 7 show the localization and estimation results for damage scenarios (3) and (4), respectively. For these damage cases, the presented approach can detected and quantified perfectly the location and the value of damages.



Figure 6. Damage detection results for damage pattern (3) in the 15-story frame: (a) damage localization, and (b) damage severity



Figure 7. Damage detection results for damage pattern (4) in the 15-story frame: (a) damage localization, and (b) damage severity

Second numerical example is a 25-story shear frame. The properties of this frame are listed in Table 3. In this example, three considered damage patterns are listed in Table 4.

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r	Story number	Mass (ton)	Stiffness (MN/m)	Height (m)
	1-5	100	500	3.0
	6-10	100	400	3.0
	11-15	100	300	3.0
	16-20	100	200	3.0
	21-25	100	100	3.0

Table 3: The properties of the 25-story shear frame

Table 4: The considered damage scenarios in the 25-story shear frame

Damage scenario	Damage location	Stiffness reduction (%)
1	Story 10	10
2	Stories 4 and 15	25 and 20
3	Stories 7, 14 and 24	20, 20 and 20

Damage pattern (1) is a single damage whereas damage patterns (2) and (3) are multiple damages. The optimization parameters in this example are as: the number of initial countries =

100, the number of initial imperialists = 10, and the number of iteration = 1000. The damage localization and estimation results for damage cases are shown in Figures 8-10. The results show the good performance of the presented method in both of damage localization and estimation in high-rise frames.



Figure 9. Damage detection results for damage pattern (2) in the 25-story frame: (a) damage localization, and (b) damage severity



Figure 10. Damage detection results for damage pattern (3) in the 25-story frame: (a) damage localization, and (b) damage severity

Despite of good performance of the presented method in damage identification, it is necessary to investigate its application when the modal data are contaminated by some random noises. This issue is important in real SHM programs, because it is impossible for reaching to an error-free modal data by using attached sensors. Therefore, for the inspection of robustness of the presented method in real conditions, mode shape vectors of damaged structure are polluted by different levels of noise as:

$$\mathbf{\varphi}_{i}^{n} = \mathbf{\varphi}_{i} \left( 1 + \mu \mathbf{\eta}_{i} \right) \tag{33}$$

in which  $\mathbf{\varphi}_i^n$  is the *i*-th mode shape vector contaminated by noise,  $\mu$  is the noise level, and  $\mathbf{\eta}_i$  is a random vector which is generated by MATLAB software.

The influence of noises in the proposed damage identification method was demonstrated by studying two samples of the presented damage scenarios. Figure 11 shows the damage prognosis results of the 15-story frame for damage scenario (4) when two levels of noise, namely 0.5% and 1.5%. For these two levels of noise, the obtained results for the 25-story frame for damage scenario (3) are indicated in Figure 12. From these two figures, it can be summarized that the presented method is able to localize and quantify damages when recorded modal data are polluted with different levels of noise.

## 6. CONCLUSIONS

In this paper, an effective two-stage damage detection method for the localization and determination of damage severities in tall shear frames was presented. Stage one was devoted to localize damages based on the first mode shape slope and the grey system theory. Stage two was an effort for the prediction of damage severity by solving an optimization problem based on the GRCs computed at stage one. For solving the objective function, we employed the imperialist

competitive algorithm which is a global search optimization method. One of the important advantages of the presented method is requirement only to the first mode shapes of a structure as input data for damage detection. In addition, the high speed of calculation can be summarized as another advantage of the presented method which is very important in tall buildings.

The presented method was validated by two numerical examples of shear tall frames, namely 15- and 25-story frames. Moreover, the robustness of the presented method was investigated when the computed modal data are polluted by various levels of random noises. All obtained results from the considered damage patterns were shown that the presented method is capable to localize and determine damage location and severity in high-rise shear frames.



Figure 11. Damage detection results for damage pattern (4) in the 15-story frame with noisy modal data: (a) damage localization, and (b) damage severity



Figure 12. Damage detection results for damage pattern (3) in the 25-story frame with noisy modal data: (a) damage localization, and (b) damage severity

## REFERENCES

- Bagheri A, Kourehli SS. Damage detection and de-noising of structures under earthquake excitation using discrete wavelet analysis, *Asian J Civil Eng (Building and Housing)*, 2013; 14(2): 289-304.
- 2. Ghodrati Amiri G, Zare Hosseinzadeh A, Bagheri A, Koo KY. Damage prognosis by means of modal residual force and static deflections obtained by modal flexibility based on the diagonalization method, *Smart Mater Struct*, 2013; **22**(7): 075032.
- 3. Kourehli SS, Bagheri A, Ghodrati Amiri G, Ghafory-Ashtiany M. Structural damage detection using incomplete modal data and incomplete static response, *KSCE J Civil Eng*, 2013; **17**(1): 216-23.
- 4. Doebling SW, Farrar CR, Prime MB. A summary review of vibration-based damage identification methods, *Shock Vibr Digest*, 1998; **30**(2): 91–105.
- 5. Carden EP, Fanning P. Vibration based condition monitoring: a review, *Struct Health Monit*, 2004; **3**(4): 355–77.
- 6. Lee YS, Chung MJ. A study on crack detection using eigenfrequency test data, *Comput Struct*, 2000; **77**(3): 327–42.
- 7. Kim J-T, Stubbs N. Crack detection in beam-type structures using frequency data, *J Sound Vib*, 2003; **259**(1): 145–60.
- Xia Y, Hao H. Statistical damage identification of structures with frequency changes, J Sound Vib, 2003; 263(4): 853–70.
- 9. Kim J-T, Ryu Y-S, Cho H-M, Stubbs N. Damage identification in beam-type structures: frequency-based method vs mode-shape-based method, *Eng Struct*, 2003; **25**(1): 57–67.
- Hamey CS, Lestari W, Qiao P, Song G. Experimental Damage Identification of Carbon/Epoxy Composite Beams Using Curvature Mode Shapes, *Struct Health Monit*, 2004; 3(4): 333–53.
- 11. Bagheri A, Ghodrati Amiri G, Seyed Razaghi SA. Vibration-based damage identification of plate structures via curvelet transform, *J Sound Vib*, 2009; **327**(3–5): 593–603.
- Bagheri A, Ghodrati Amiri G, Khorasani M, Bakhshi H. Structural damage identification of plates based on modal data using 2D discrete wavelet transform, *Struct Eng Mech*, 2011; 40(1): 13–28.
- 13. Nicknam A, Hosseini MH, Bagheri A. Damage detection and de-noising in twodimensional structures using curvelet transform by wrapping method, *Arch Appl Mech*, 2011; **81**(12): 1915-1924.
- 14. Zhu H, Li L, He X-Q. Damage detection method for shear buildings using the changes in the first mode shape slopes, *Comput Struct*, 2011; **89**(9–10): 733–43.
- 15. Liu PL. Identification and damage detection of trusses using modal data, *J Struct Eng* (ASCE), 1995; **121**(4): 599–608.
- 16. Ge M, Lui EM. Structural damage identification using system dynamics properties, *Comput Struct*, 2005; **83**(27): 2185–96.

670

- 17. Yang QW, Liu JK. Structural damage identification based on residual force vector, *J Sound Vib*, 2007; **305**(1–2): 298–307.
- 18. Yang QW. A numerical technique for structural damage detection, *Appl Math Comput* 2009; **215**(7): 2775–80.
- 19. Ge M, Lui EM, Khanse AC. Non-proportional damage identification in steel frames, *Eng Struct*, 2010; **32**(2): 523–33.
- 20. Zhao J, DeWolf JT. Sensitivity study for vibrational parameters used in damage detection, *J Struct Eng (ASCE)*, 1999; **125**(4): 410–16.
- 21. Cobb RG, Liebst BS. Structural damage identification using assigned partial eigenstructure, *AIAA J*, 1997; **35**(1): 152–8.
- 22. Chou J-H, Ghaboussi J. Genetic algorithm in structural damage detection, *Comput Struct*, 2001; **79**(14): 1335–53.
- 23. Perera R, Torres R. Structural Damage Detection via Modal Data with Genetic Algorithms, *J Struct Eng (ASCE)*, 2006; **132**(9): 1491–501.
- 24. Perera R, Fang S-E, Huerta C. Structural crack detection without updated baseline model by single and multi-objective optimization, *Mech Syst Signal Process*, 2009; **23**(3): 752–68.
- 25. Kourehli SS, Ghodrati Amiri G, Ghafory-Ashtiany M, Bagheri A. Structural damage detection based on incomplete modal data using pattern search algorithm, *J Vib Control*, 2012; **19**(6): 821–33.
- 26. Gerist S, Naseralavi SS, Salajegheh E. Basis pursuit based genetic algorithm for damage identification, *Int J Optim Civil Eng*, 2012; **2**(2): 301–19.
- 27. Kaveh A, Zolghadr A. An improved charged system search for structural damage identification in beams and frames using changes in natural frequencies, *Int J Optim Civil Eng*, 2012; **2**(3): 321–39.
- 28. Perera R, Marina R, Ruiz A. Static–dynamic multi-scale structural damage identification in a multi-objective framework, *J Sound Vib*, 2013; **332**(6): 1484–500.
- 29. Fallahian S, Seyedpoor SM. A two-stage method for structural damage identification using an adaptive neuro-fuzzy inference system and particle swarm optimization, *Asian J Civil Eng (Building and Housing)*, 2010; **11**(6): 795–808.
- 30. Ghodrati Amiri G, Seyed Razaghi SA, Bagheri A. Damage detection in plates based on pattern search and genetic algorithms, *Smart Struct Syst*, 2011; 7(2):117–32.
- 31. Bagheri A, Razeghi HR, Ghodrati Amiri G. Detection and estimation of damage in structures using imperialist competitive algorithm, *Shock Vib*, 2012; **19**(3): 405–19.
- 32. Torkzadeh P, Goodarzi Y, Salajegheh E. A two-stage damage detection method for largescale structures by kinetic and modal strain energies using heuristic particle swarm optimization, *Int J Optim Civil Eng*, 2013; **3**(3): 465–82.
- 33. Chen X-Z, Zhu H-P, Chen C-Y. Structural damage identification using test static data based on grey system theory, *J Zhejiang Univ Sci, A* 2005; **6**(8): 790–6.
- 34. Abdo M A-B. Parametric study of using only static response in structural damage detection, *Eng Struct*, 2012; **34**: 124–31.

- 35. Deng JL. Introduction to grey system theory, J Grey Syst, 1989; 1(1): 1–24.
- 36. Fu CY, Zheng JS. Application of grey relational analysis for corrosion failure of oil tubes, *Corros Sci*, 2001; **43**(5): 881–9.
- 37. Atashpaz-Gargari E, Lucas C. Imperialist competitive algorithm: an algorithm for optimization inspired by imperialistic competition, *IEEE Congress on Evolutionary Computation*, Singapore, 2007.
- Rajabioun R, Atashpaz-Gargari E, Lucas C. Colonial competitive algorithm as a tool for Nash equilibrium point achievement, *Lect Notes Comput Sci* 2008; **5073**: 680–95.
- 39. Kabbazi A, Atashpaz-Gargari E, Lucas C. Imperialist competitive algorithm for minimum bit error rate beam forming, *Int J Bio-Inspired Comp*, 2009; **1**(1–2): 125–33.