



## MULTI-OBJECTIVE MATHEMATICAL MODELING OF AIRCRAFT LANDING PROBLEM ON A RUNWAY IN STATIC MODE, SCHEDULING AND SEQUENCE DETERMINATION USING NSGA-II

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### ABSTRACT

Runway length is usually a critical point in an airport system; so, a great interest has been created for optimal use of this runway length. The most important factors in modeling of aircraft landing problem are time and cost; while, the costs imposed on the system because of moving away from target times have different performances in terms of impact. In this paper, firstly, aircraft landing problem (ALP) and the works conducted in subject literature are briefly reviewed and presented. Then, this problem is formulated and proposed as a three-objective mathematical modeling which leads to more applicable formulation. Following this, the model introduced to solve this problem is solved for two groups including 20 and 50 aircrafts using the second version of NSGA; and the results and recommendations will be provided.

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**KEY WORDS:** multi-objective modeling of aircraft landing problem (ALP); expected landing time (ELT); scheduled landing time (SLT); NSGA-II algorithm.

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## 1. INTRODUCTION

During the past few decades, air traffic operations have experienced unprecedented and massive growth. The number of air travelers around the world increased 5.3% in the year 2011 compared to 2010 and 11.9% than its earlier year; it has reached to 5,44 billion passengers [1], also Airports Council International (ACI) preliminary traffic results, based on reports from over 1093 airports worldwide, indicate that global passenger traffic grew at a rate of 4 percent as compared to Ref. [2]. According to the statistics of International Association of Airports, it is found that total incoming aircrafts in an airport radar area is significantly high and this requires a strong management to schedule these aircrafts landing. The problem of decision making that aircraft controllers are repeatedly faced during the time is: which is the next landing aircraft and when it comes to landing? In real applications, usually finding the best solution is not the main goal but finding a feasible solution for scheduling problem of landing in a convenient time can also be enough [3]. The problem of aircraft landing is one of the important fields in air traffic operations, modeling and solution of this problem can be a good help to manage air traffic.

Aircraft landing scheduling problem has so many limitations; so, this has been reminded as a NP-hard problem. For example, aircrafts should land in a certain time interval and the best time interval between each pair of landing should be observed. Finding optimal solution is very difficult due to the characteristics related to the large size and also large numbers of limitations in aircraft landing scheduling problem. Aircraft landing scheduling problem (ALSP) is a key problem in Air Traffic Control (ATC) sector which finding a sequence of landings in a series of aircrafts is its goal. One of the key factors in determining airports' runway throughput is also the required distance between aircrafts when landing. Dependency of this separation on the front and back aircrafts and also to aircraft type should be included to this solution challenge of sorting and scheduling of this problem. Therefore, this problem has different importance from various researchers' perspectives and so far most studies have been conducted on it in order to present algorithms and models for increasing airport capacity.

In single-runway situation, decision making on the adopted landing sequence is often based on First Come First Serve (FCFS) method. The first aircraft entering the radar area should land first and the second one should land later and etc. In general, many studies have been conducted on the ALSP problem in the field of operation research. Normally, aircrafts are scheduled by FCFS method [4-6].

When multi-runway situation is talked, FCFS method is often used; so that the aircrafts land on the runway assigned to them and in sequence that they appear in the radar area.

In the works conducted on subject literature, both meta-heuristic and optimization methods including meta-heuristic method based on initial population, simple heuristic method, genetic algorithm and etc have been developed to solve ALP problem. Although, more than 60 papers in the field of optimizing aircrafts landing have been provided in the past three decades, most of the proposed methods have never been used in the studies [7].

Beasley et al. [8-9] provided a mixed integer formulation for ALP and a detailed review on the published research works in ALP subject. They offered six types of additional restrictions in order to reduce travel demand and a mixed integer formulation. Then, they solved the problem using linear schedule-based tree search for a series data including

maximum 50 aircrafts.

Jung and Laguna [10] offered a time division-based heuristic algorithm. Time horizon has been divided into temporal parts which determine ALP sub-problems. Each sub-problem has been formulated as a linear scheduling problem for integer provided by Beasley et al. [8] and then solved in sequence and optimally. Computation results have been provided for the samples including maximum 75 aircrafts and 4 runways.

Cheng et al. [11] also reviewed the application of these methods in the problems of air traffic control dependant on non-analytical event in cases such as assigning runway, determining the sequence and incoming aircrafts scheduling with multiple runways. They have developed four different formulations of genetic searching for multi-runway ALP problem. Three formulations of those apply a genetic algorithm approach. While the fourth formulation uses a genetic scheduling approach. Computation results were provided for the samples including 12 aircrafts and 3 runways.

Pinol and Beasley [12] first applied scatter search algorithm and bionomic algorithm for multi-runway ALP problem. The initial population is composed of three heuristic components based on non-decreasing order of objective time, the soonest and the latest time.

Liu [13] developed a solution procedure based on genetic local search (GLS) algorithm for ALP with the characteristics of dependency to airport runway. He provided a set of sample issues to test validity of the proposed algorithm based on five predominant samples and reviewed them along with previous studies and researches. Numerical results were obtained for five experimental cases. The results of the proposed GLS algorithm were compared with the outputs result from previous studies including gerchberg saxton (GS) algorithm, scatter search (SS) algorithm and bionomic algorithm (BA); and it was shown that these results conduct effectively the assignment of runways, sequence and incoming aircraft landing times by minimizing total delays under the restrictions of separation times.

Tang et al. [14] proposed use of multi-objective evolutionary algorithms (MOEA) to solve aircraft landing scheduling problem. ALSP was formulated as a two-objective optimization problem. A multi-Objective neighborhood search differential evolution (MONSDE) algorithm was applied to solve two-objective ALSP problem.

Soomer and Franx [15] considered single-runway situation of aircraft landing problem and provided a model that regarded airlines preferences in relation to delay costs. They solved their proposed model with a knowledge-based local approach. Numerous examples that indicated costs reducing by the proposed model were produced in this field and solved.

Hu and Paolo [16] explained applications of genetic algorithms to cope with Aircraft Sequence Scheduling (ASS) problem in multi-runway systems. For solving ASS problem, most of available algorithms were faced with the difficulties and efficiency in the design of evolutionary operators specifically crossovers. A new genetic algorithm was reported in this article which uses the relationship between aircrafts for making chromosomes. This activity makes designing of a uniform crossover operator possible which was difficult to run for those genetic algorithms that have directly been designed based on aircraft arrangements in entry queue. The main advantage of the proposed uniform crossover operator is its efficiency and profitability on substitution and preservation of common traffic sub-sequences without sacrificing the capability of diversifying chromosomes which were proved in extensive comparative simulation study.

Simple heuristic algorithm for searching the problem space provides the quickest

responses among heuristic algorithms [8]. However, the qualities of responses are unstable. For the tested examples, the worst response was about 77% far from optimal response. A heuristic method of time division provides better responses. For the same examples, distance (gap) optimality is less than 6.5%. Another great heuristic method is initial population-based heuristic method. Moreover, this method is very efficient and has been used for the problems with 500 aircrafts that are much greater than most examples of other articles.

All exact algorithms such as search tree [8] and the presented algorithm in [17] use the branch and bound method for finding optimal response of ALP integer. However, using this method, the runtime increases exponentially as the problem size increases. Thus, it is not able to optimally solve very large-scale problems and it is only used for the problems with maximum 50 aircrafts.

Due to extensive studies done in this area, since ALP problem can involve simultaneously several interdependent goals optimizations, and due to lack of optimization with more than two target functions in subject literature, the necessity of addressing this kind of optimization for better evaluation seems to be unavoidable. Such method should also have the ability to have dynamic coefficients depending on the place of using the model. Therefore, in this paper a model with three objective targets is presented and for its validation the NSGA-II is utilized to compare the results.

## 2. PROBLEM DEFINITION

Air transportation has different stakeholders including ATC, airlines, airports and government, each of them has explicit or implicit goals. Consequently, ALP formulation includes simultaneous optimization of types of goals which may be in conflict and likely cause that decision maker to consider probable substitutions. When an aircraft reaches the airport radar requires landing permit, landing time and appropriate runway (if it is available) from air traffic control. Each aircraft cannot descend earlier than a certain time because of having the maximum for certain speeds and it cannot stay in the sky longer than a specified time since fuel per aircraft is limited. A target landing time is defined within this time window that is the preferred landing time according to airline, when aircraft can land; if it lands to runway with the cruise speed (the most economical aircraft speed); the same time that is announced to the passengers. Any deviation (early or late) from target landing time will disturb airport program. Consequently, with any deviation before or after target landing time; penalty fee is considered separately for fuel, parking, apron and etc. Therefore, the goal is to minimize total time landing sequence (or maximize runway efficiency) as well as minimizing the total costs of fine including the followings:

1. Vertical and horizontal separation of standard flights that keeps them apart is one of the most important security tools for air traffic control (ATC). Minimum required separation creates minimum allowable distance between aircrafts that are approaching to runway. In general, the required separation (Wake vortex (WV)) between aircrafts depends on aircraft type. So, this problem is also related to the arrangement.

2. Each aircraft should land within a predetermined time window (the earliest landing time plus the required holding time and the latest landing time).

### 3. MATHEMATICAL FORMULATION

In this section for ALP static problem on a runway, a new mathematical formulation will be provided based on the presented problem in Ref. [7].

#### 3.1 Abbreviations

##### Decision variable

**SLT<sub>i</sub>**: The scheduled landing time of each aircraft *i*, calculated by trajectory synchronizer equipment after entering the aircraft into the radar range.

##### Parameters

**n**: The number of aircraft to be scheduled.

**X<sub>ij</sub>**: Defined to be 1 if aircraft *i* land before (not necessarily immediately) aircraft *j* and 0 other wise.

**ELT<sub>i</sub>**: The expected( or target) landing time of aircraft *i*, based on the assigned time slot which is normally specified in flight plan.

**TELT<sub>i</sub>**: Aircraft type *i* in size category based on three different types of aircraft in small, medium and large.

**Δ<sub>ij</sub>**: Be the minimum time separation between aircraft *i* and *j*, if aircraft *i* land before aircraft *j*.

**CAT<sub>i</sub>**: Be the airline cost per unit of time (except fuel factor) for landing after ELT<sub>i</sub> for aircraft *i*.

**CBT<sub>i</sub>**: Be the airline cost per unit of time (except fuel factor) for landing before ELT<sub>i</sub> for aircraft *i*.

**FCD<sub>i</sub>**: average required fuel burn cost per minute for aircraft *i* to be delayed.

**FCA<sub>i</sub>**: average required fuel burn cost per minute for aircraft *i* to be advanced.

**EAT<sub>i</sub>**: the earliest possible arrival time for aircraft *i*, subject to technical and operational restrictions.

**LAT<sub>i</sub>**: the latest possible arrival time for aircraft *i*, which is usually determined from fuel limitation, maximum allowed delay.

**Th**: be the time for a plan to circle for one loop when waiting its turn to land.

**ea<sub>i</sub>**: The allowed earliness for aircraft *i* to land before ELT<sub>i</sub>, from the moment the wheels touch the ground to reach the parking lot (including across from taxiways).

**da<sub>i</sub>**: The allowed lateness for aircraft *i* to land after ELT<sub>i</sub>, from the moment the wheels touch the ground to reach the parking lot (including across from taxiways).

**e<sub>i</sub>**: The earliness for aircraft *i*, ( $\max(0, \text{ELT}_i - \text{SLT}_i)$ ).

**d<sub>i</sub>**: The lateness for aircraft *i*, ( $\max(0, \text{SLT}_i - \text{ELT}_i)$ ).

#### 3.2 Objective functions

1. Maximizing runway throughput. Total landing times can be equally minimized instead of maximizing the numbers of aircrafts that lands on the runway; this is the same runway throughput.

$$\text{Minimize } \sum_{i=1}^n \text{SLT}_i \quad (1)$$

2. Minimizing apron and parking and other costs that are imposed on the airline by additional stay of aircraft at the airport through minimizing time delay and allowed soon reach.

$$\text{Minimize } \sum_{i=1}^n \{|\text{SLT}_i - \text{ELT}_i| - B\}A \quad (2)$$

In which:

$$A \begin{cases} \text{CAT}_i & , \text{if } \text{SLT}_i > \text{ELT}_i \\ 0 & , \text{if } \text{SLT}_i = \text{ELT}_i \\ \text{CBT}_i & , \text{if } \text{SLT}_i < \text{ELT}_i \end{cases} \quad B \begin{cases} \text{da}_i & , \text{if } \text{SLT}_i > \text{ELT}_i \\ 0 & , \text{if } \text{SLT}_i = \text{ELT}_i \\ \text{ea}_i & , \text{if } \text{SLT}_i < \text{ELT}_i \end{cases} \quad (3)$$

3. Minimizing fuel consumption cost and therefore minimizing carbon dioxide pollutions of the air. Fuel consumption depends on different factors including pilot flying techniques, height, wind speed, aircraft model, aircraft weight (including passengers' weight and cargo) and fuel inside tanks. Consequently, additional fuel cost resulting from late arrival and fuel cost saving because of early arrival should be considered.

$$\text{Minimize } \sum_{i=1}^n \{|\text{SLT}_i - \text{ELT}_i|\} C \quad (4)$$

In which:

$$C \begin{cases} \text{FCD} & , \text{if } \text{SLT}_i > \text{ELT}_i \\ 0 & , \text{if } \text{SLT}_i = \text{ELT}_i \\ \text{FCA} & , \text{if } \text{SLT}_i < \text{ELT}_i \end{cases} \quad (5)$$

### 3.3 Constraints

A variety of operational constraints can exist for ALP; having a look at the real world, the most practical of them for using in a runway are given below. All Scheduled landing times (SLT) should be determined and calculated according to the following constraints.

#### 3.3.1 Runway use restrictions

$$X_{ij} + X_{ji} = 1 \quad \forall_{ij} = 1, 2, \dots, n \quad (6)$$

Each runway can be used by only one aircraft at the same time. So, the aircraft  $i$  lands before the aircraft  $j$  or vice versa.

### 3.3.2 Guarantee limit for minimum separation distance (separation WV)

$$SLT_j - SLT_i \geq \Delta_{ij} \quad (7)$$

Aircraft should be in a safe distance from other aircraft to avoid turbulence creating by aircraft ahead.

### 3.3.3 Time limit

$$EAT_i + Th \leq SLT_i \leq LAT_i \quad (8)$$

Based on technical and operational assumptions such as limited fuel, wind speed and etc; each aircraft has a minimum and maximum allowable air time, this restriction should be considered as a serious limitation. According to this problem, it is possible the runway is blocked in the time allocated to aircraft landing or the aircraft obliged to wait due to traffic saturation, poor visibility, weather conditions or lost time points; so, a soft time is added to the beginning of time interval for aircraft bypass in an air loop to avoid changing sequence scheduling in this situation.

### 3.3.4 Positivity constraint of scheduled time for each pair of aircrafts (outrider and follower)

$$(ELT_i - ELT_j)(SLT_i - SLT_j) > 0 \quad (9)$$

### 3.3.5 Restrictions related to early or late landing

$$0 \leq e_i \leq ELT_i - EAT_i, \quad 0 \leq d_i \leq LAT_i - ELT_i \quad (10)$$

Early or late landing of the aircraft  $i$  is always a function of the defined time window and also the expected landing time for this aircraft.

## 3.4 Penalty functions

Penalty function solution is a method for finding reasonable responses through valuation and also determining the role of constraints as a criteria and moving to responses with less errors and eventually to appropriate area. This method is very popular among all techniques to satisfy the constraints.

In this method, constraints violation are multiplied by penalty parameter ( $R_k = 10^6$ ) and its result added to the value assigned by each of target functions. Note that the value of  $R_k$  will be positive in minimize functions.

In the restrictions in which decision variable (SLT) exists, penalty has been determined as follows:

### 3.4.1 to limit guarantee for minimum separation distance

$$\max \left( 0, \left( \frac{\Delta_{ij} + SLT_i}{SLT_j} - 1 \right) \right)^2 \times R_k \quad (11)$$

### 3.4.2 for time limit

$$\begin{cases} \max \left( 0, \left( \frac{SLT_i}{LAT_i} - 1 \right) \right)^2 \times R_k \\ \max \left( 0, \left( 1 - \frac{SLT_i}{Th + EAT_i} \right) \right)^2 \times R_k \end{cases} \quad (12)$$

### 3.4.3 To limit positive scheduled time

$$\max \left( 0, \left( \frac{ELT_i - ELT_j}{SLT_j - SLT_i} - 1 \right) \right)^2 \times R_k \quad (13)$$

### 3.4.4 For restrictions related to early or late landing

$$\begin{cases} \max \left( 0, \left( \frac{ELT_i - SLT_i}{ELT_i - EAT_i} - 1 \right) \right)^2 \times R_k \\ \max \left( 0, \left( \frac{SLT_i - ELT_i}{LAT_i - ELT_i} - 1 \right) \right)^2 \times R_k \end{cases} \quad (14)$$

## 3.5 Problem implementation

In this problem, two linear matrixes have been used as follows for expected landing time (ELT), for two consecutive series including 20 and 50 incoming aircrafts in the sequence they enter to the radar area of the airport:

$ELT_{20} = [8.05 \ 8.00 \ 8.20 \ 8.15 \ 8.10 \ 8.45 \ 8.50 \ 8.60 \ 8.75 \ 8.80 \ 9.05 \ 9.10 \ 9.20 \ 9.35 \ 9.40 \ 9.80 \ 9.65 \ 10.00 \ 9.70 \ 9.95].$

$ELT_{50} = [8.0 \ 8.02 \ 8.05 \ 8.06 \ 8.10 \ 8.15 \ 8.20 \ 8.25 \ 8.35 \ 8.45 \ 8.50 \ 8.55 \ 8.58 \ 8.75 \ 8.80 \ 8.90 \ 8.95 \ 9.00 \ 9.07 \ 9.10 \ 9.18 \ 9.24 \ 9.29 \ 9.32 \ 9.49 \ 9.60 \ 9.65 \ 9.80 \ 9.85 \ 9.95 \ 10.0 \ 10.5 \ 10.08 \ 10.10 \ 10.15 \ 10.18 \ 10.20 \ 10.25 \ 10.30 \ 10.40 \ 10.45 \ 10.50 \ 10.55 \ 10.60 \ 10.70 \ 10.75 \ 10.80 \ 10.90 \ 10.95 \ 11.0].$

In some references such as [18] and [19], aircrafts are classified in several groups in terms of weight and separation time matrixes formed due to the group that both pairs of consecutive aircrafts belongs to them, like what is given in (Table 1). In this matrix, aircrafts have been assigned to three small (S), medium (M) and heavy (H) groups. Columns (indices i) represent follower aircraft and rows (indices j) represent outrider aircraft.

Table 1: Separation times matrix

	$S_i$	$M_i$	$H_i$
$S_i$	5	3	3
$M_i$	8	5	3
$H_i$	15	8	5



The following matrix indicates that aircrafts incoming to the radar area of the airport (in entering sequence) in terms of dimensions to what group they belong to (S, M, H).

$TELT_{20}=[$ '1S' '2M' '3H' '4S' '5M' '6H' '7S' '8M' '9H' '10H' '11H' '12S' '13S' '14S' '15M' '16H' '17H' '18H' '19S' '20M'].

$TELT_{50}=[$ '1S' '2M' '3H' '4S' '5M' '6H' '7S' '8M' '9H' '10H' '11H' '12S' '13S' '14S' '15M' '16H' '17H' '18H' '19S' '20M' '21S' '22M' '23H' '24S' '25M' '26H' '27S' '28M' '29H' '30H' '31S' '32M' '33H' '34S' '35M' '36H' '37S' '38M' '39H' '40H' '41S' '42M' '43H' '44S' '45M' '46H' '47S' '48M' '49H' '50H'].

For consecutive arrival in a series of 20 aircrafts, time interval have been considered between 8 to 10 (total time interval is 2 hours, means that a landing operation each 6 minutes) and for a series of 50 aircrafts between 8 to 11 (total time interval is 3 hours, means that a landing operation each 3.6 minutes).

## 4. MODEL SOLUTION USING THE NSGA-II ALGORITHM

### 4.1 Multi-objective evolutionary algorithms

Multi-objective evolutionary algorithms (MOEAs) have attracted the attention of many researchers in the last 20 years, and it is still a challenging area in optimization. Multi-objective optimizations have found many applications in applied science and engineering. Many researchers have worked on design and improvements of the algorithms by enhancing the operators of the evolutionary algorithms. In this direction suggestions are made by Van Veldhuizen et al. [20]. Extensions are successfully made in differential evolution by Mezura-Montes et al. [21] for single objective optimization problems. Zhou et al. [22] performed a complete review of the evolutionary algorithms in the recent 8 years.

Changing in basic evolutionary algorithms, it can be possible to maintain parts of Pareto optimum in each generation of an evolutionary algorithm. So, the necessity of multiple run in a classical method to find a Pareto response in each run can be eliminated. In other hand, a unique feature of evolutionary algorithms in solving optimization problems is access to a diverse set of non-dominated responses with appropriate distribution and utilization of an operator to establish this distribution [23].

### 4.2 NSGA-II algorithm

This algorithm was provided in Deb et al. [24] to solve multi-objective optimization problems. NSGA-II algorithm is an elitist multi-objective evolutionary algorithm. In addition to having a proper strategy to keep a better response, this algorithm has a clear mechanism to maintain the population diversity.

### 4.3 Parameters of NSGA-II algorithm

Genetic algorithms have three main operators: selection, mutation and crossover. Selection operator chooses independent parameters among different parameters; they are used in mutation and crossover operators. While, designing a good coding is important to minimize the independent parameters in search space, choosing genetic operators to comply with this coding is also important [25].

In NSGA-II algorithm, total tasks are conducted in the sections of population production,

crossover and mutation as merging and then non-dominated sorting is performed based on the rank and crowding distance; and finally additional parts are removed, Fig. 1.

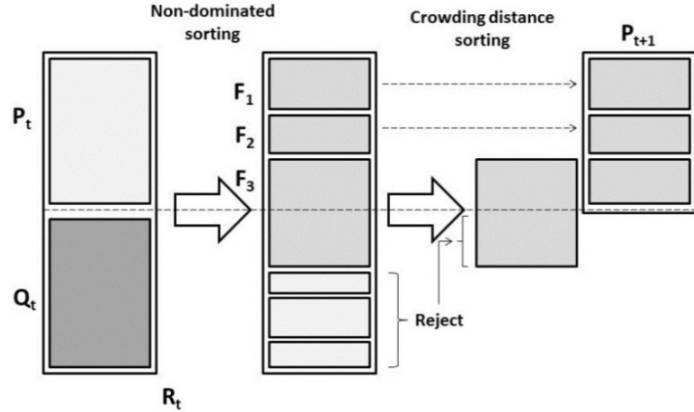


Figure 1. The steps conducted in each iteration of the NSGA-II algorithm

In Fig. 2 the pseudo code of non-dominated sorting and in Fig. 3 the calculation of crowding distance are shown in Fig. 2.

```

    or each  $p \in P$ 
         $S_p = \emptyset$ 
         $n_p = 0$ 
    for each  $q \in P$ 
        if ( $p < q$ ) then
             $S_p = S_p \cup \{q\}$ 
            If  $p$  dominates  $q$ 
            Add  $q$  to the set of solutions dominated by  $p$ 
        else if ( $q < p$ ) then
             $n_p = n_p + 1$ 
            Increment the domination counter of  $p$ 
        if  $n_p = 0$  then
             $P_{rank} = 1$ 
             $p$  belongs to the first front
             $F_1 = F_1 \cup \{p\}$ 
            Initialize the front counter
             $i = 1$ 
            Used to sort the members of the next front
        while  $F_i \neq \emptyset$ 
             $Q = \emptyset$ 
            for each  $p \in F_i$ 
            for each  $q \in S_p$ 
                 $n_q = n_q - 1$ 
                if  $n_q = 0$  then
                     $q$  belongs to the next front
                     $q_{rank} = i + 1$ 
                     $Q = Q \cup \{q\}$ 
                     $i = i + 1$ 
             $F_i = Q$ 

```

Figure 2. Pseudo code of non-dominated sorting in NSGA-II algorithm [24]

$l = |\tau|$  number of solutions in  $\tau$   
 for each  $i$ , set  $\tau[i]_{distance} = 0$  initialize distance  
 for each objective  $m$   
 $\tau = sort(\tau, m)$  sort using each objective value  
 $\tau[1]_{distance} = \tau[l]_{distance} = \infty$  so that boundary points are always selected  
 for  $i = 2$  to  $(l - 1)$  for all other points  
 $\tau[i]_{distance} = \tau[i]_{distance} + (\tau[i + 1].m - \tau[i - 1].m) / (f_m^{max} + f_m^{min})$   
 Figure 3. The calculation of the crowding distance for members of a group in NSGA-II algorithm [24]

In the pseudo code for calculation of crowding distance,  $\tau$  indicates the current front and  $\tau[i].m$  refers to the  $m$ th objective function value of the  $i$ th individual in the set  $\tau$  and the parameters  $f_m^{min}$  and  $f_m^{max}$  are the maximum and minimum value of  $m$ th target function per all available response in this front, as shown in Fig. 4.

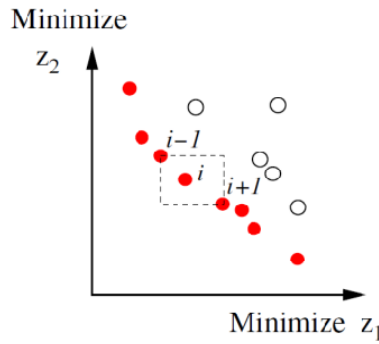


Figure 4. Calculation of crowding distance for the answer  $i$

#### 4.4 Implementing parameters of NSGA-II algorithm for the problem

Maximum number of algorithm iteration for a series of 20 aircrafts is equal to 1000 and the number of initial population (nPop) is equal to 50; and maximum number of algorithm iteration for a series of 50 aircrafts is equal to 1500 and the number of initial population is considered equal to 80. Percentage of the population taking part in crossover (pCrossover) is equal to 0.7 and the mutated population (pMutation) equal to 0.4; mutation rate is also considered equal to 0.02.

## 5. RESULTS

Pareto optimal regions obtained for each series of 20 and 50 aircrafts are shown in Fig. 5 and Fig. 6, respectively.

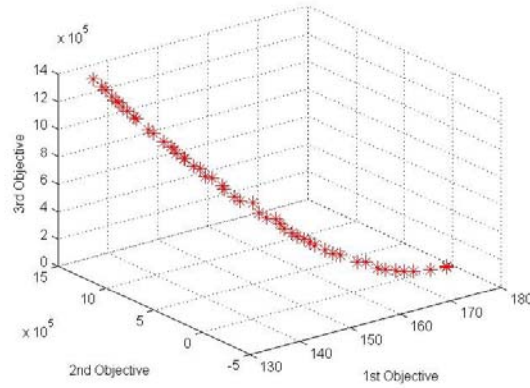


Figure 5. Pareto optimal region obtained for a series of 20 aircraft

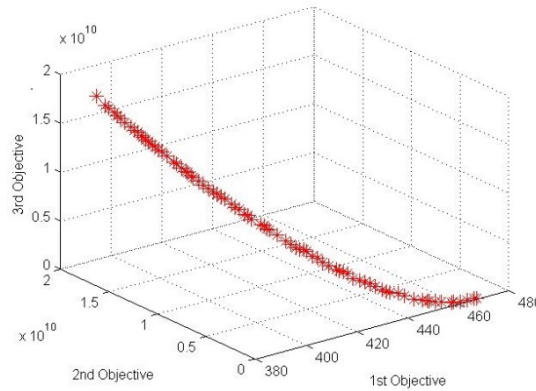


Figure 6. Pareto optimal region obtained for a series of 50 aircraft

The obtained expected landing times (ELT) and the scheduled landing times (SLT) for sequencing of 20 and 50 aircraft are shown in Tables 2 and 3, where  $\alpha$  indicates absolute value of the difference between these two times.

Table 2:  $ELT_i$  and  $SLT_i$  obtained for a series of 20 aircraft

Aircraft number	$ELT_i$	$SLT_i$	$\alpha_i$	Aircraft number	$ELT_i$	$SLT_i$	$\alpha_i$
1	8.05	8.05	0	11	9.05	9.04	0.01
2	8.00	8.00	0	12	9.10	9.10	0
3	8.20	8.19	0.01	13	9.20	9.19	0.01
4	8.15	8.15	0	14	9.35	9.34	0.01
5	8.10	8.10	0	15	9.40	9.38	0.02
6	8.45	8.45	0	16	9.80	9.79	0.01
7	8.50	8.49	0.01	17	9.65	9.65	0
8	8.60	8.60	0	18	10.00	9.98	0.02
9	8.75	8.75	0	19	9.70	9.72	0.02
10	8.80	8.79	0.01	20	9.95	9.94	0.01

The sequence obtained for a series of 20 aircraft is shown in Fig. 7.

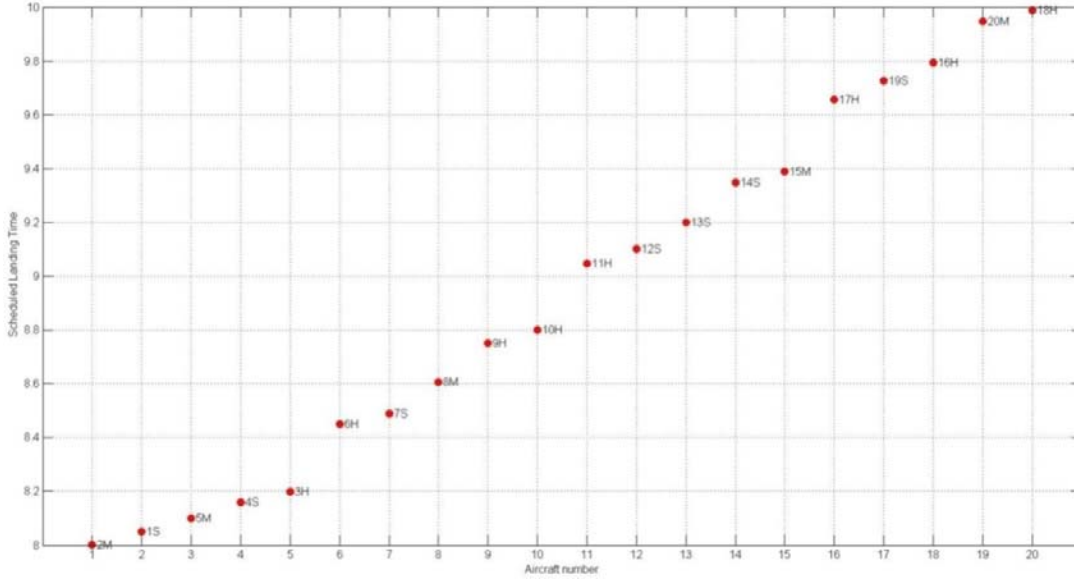


Figure 7. The sequence obtained for a series of 20 aircraft

Table 3:  $ELT_i$  and  $SLT_i$  obtained for a series of 50 aircrafts

Aircraft number	$ELT_i$	$SLT_i$	$\alpha_i$	Aircraft number	$ELT_i$	$SLT_i$	$\alpha_i$
1	8.00	7.95	0.05	26	9.60	9.65	0.05
2	8.02	7.98	0.04	27	9.65	9.70	0.05
3	8.05	8.01	0.04	28	9.80	9.80	0
4	8.06	8.08	0.02	29	9.85	9.84	0.01
5	8.10	8.11	0.01	30	9.95	9.91	0.04
6	8.15	8.13	0.02	31	10.00	9.95	0.05
7	8.20	8.21	0.01	32	10.05	10.00	0.05
8	8.25	8.26	0.01	33	10.08	10.05	0.03
9	8.35	8.34	0.01	34	10.10	10.06	0.04
10	8.45	8.45	0	35	10.15	10.09	0.06
11	8.50	8.47	0.03	36	10.18	10.11	0.07
12	8.55	8.55	0	37	10.20	10.14	0.06
13	8.58	8.59	0.01	38	10.25	10.19	0.06
14	8.75	8.70	0.05	39	10.30	10.25	0.05
15	8.80	8.74	0.06	40	10.40	10.36	0.04
16	8.90	8.85	0.05	41	10.45	10.40	0.05
17	8.95	8.94	0.01	42	10.50	10.45	0.05
18	9.00	8.96	0.04	43	10.55	10.49	0.06
19	9.07	9.06	0.01	44	10.60	10.57	0.03
20	9.10	9.10	0	45	10.70	10.65	0.05
21	9.18	9.19	0.01	46	10.75	10.70	0.05
22	9.24	9.27	0.03	47	10.80	10.75	0.05
23	9.29	9.32	0.03	48	10.90	10.85	0.05
24	9.32	9.36	0.04	49	10.95	10.91	0.04
25	9.49	9.57	0.08	50	11.00	10.96	0.04

The sequence obtained for a series of 50 aircrafts is shown in Fig. 8.

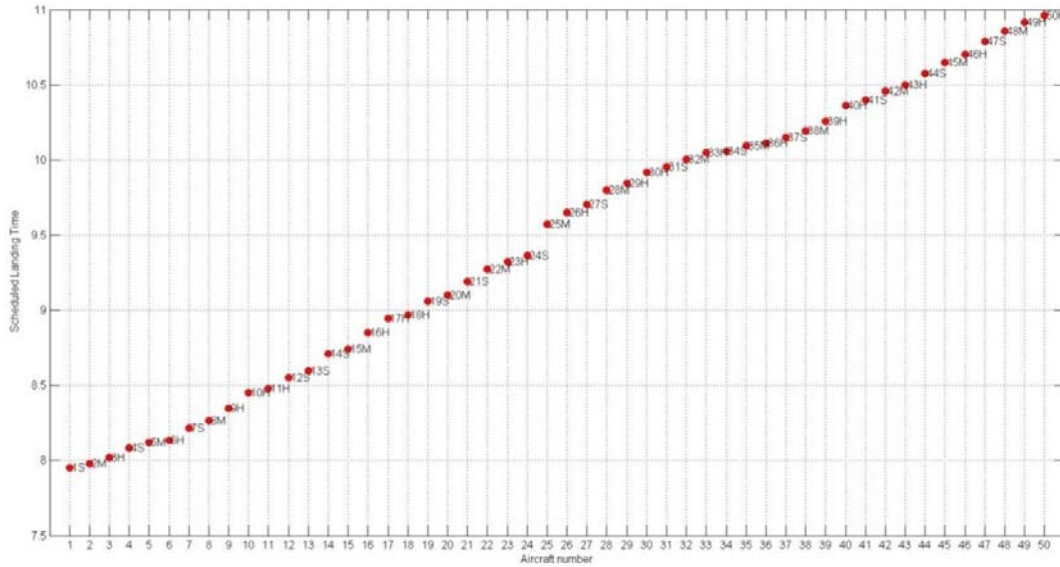


Figure 8: The sequence obtained for a series of 50 aircrafts

Values of each target function taken from first members of the first front are also as follows:

$$\text{For a series of 20 aircrafts } \begin{cases} f_1 = 178.72 \\ f_2 = 0.1666 \\ f_3 = 10.103 \end{cases}$$

$$\text{For a series of 50 aircrafts } \begin{cases} f_1 = 473.80 \\ f_2 = 4.00 \\ f_3 = 73.82 \end{cases}$$

## 6. CONCLUDING REMARKS

Due to existing 20 sequences for five first aircrafts and five last aircrafts in expected times, there are times substitution in radar area of the airport in arrival arrangement. Based on results obtained for landing times in Table 2, it is clear that many aircrafts change its scheduled landing time compared to the entry number. This shows that time scheduling is based on target landing times and does not follow First Come First Serve law for sequencing. This kind of sequences resulted in better sequencing and also reducing the costs imposed on system and fuel costs. Looking at the sequence obtained in Fig. 8, it is observed that a correct sequence in an appropriate time was achieved solving problem model with NSGA-II algorithm; while the scheduled times have minimal possible difference with the expected landing times. The worst response obtained in 20 sequences 2% and in 50 sequences only in one case 8% were far from optimal response. Looking at the values of target functions, fuel and apron costs can also be estimated for each sequence.

This research shows that NSGA-II algorithm can find a good solution for limitations in

solving landing scheduling problem of aircraft and lead the optimization process. In presenting this work, it has been tried to cover many basic problems and factors emphasized by air traffic controllers. Parts of it are as follows [26]:

1. Taking into account operational and functional limitations in order to achieve a practical and not merely a theoretical model.
2. Obtaining proper sequence in less than one minute. Optimal solutions that arise from long computation times have little use in practice.
3. Selection of a proper target function for ALP is controversial and stakeholders likely have conflicting criteria. Therefore, the first important step for the running model is the selection of multiple objectives which can meet interests of all parties or provide an acceptable compromise.

In our future studies, the presented three-objective model with other multi-objective algorithms will be solved and the results will be compared. Also, the model presented for the present problem will be developed in landing and take-off conditions.

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