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# OPTIMUM RESISTANCE FACTOR FOR REINFORCED CONCRETE BEAMS RETROFITTED WITH U-WRAP FRP

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## ABSTRACT

The use of fiber reinforced polymer (FRP) U-wrap to rehabilitate concrete beams has increased in popularity over the past few years. As such, many design codes and guidelines have been developed to enable designers to use of FRP for retrofitting reinforced concrete beams. FIB is the only guideline for design which presents a formula for torsional capacity of concrete beams strengthened with FRP. The Rackwitz-Fiessler method was applied to make a reliability assessment on the torsional capacity design of concrete beams retrofitted with U-wrap FRP laminate by this guideline. In this paper, the average of reliability index obtained is 2.92, reflecting reliability of the design procedures. This value is somehow low in comparison to target reliability level of 3.5 used in the guideline calibration and so, optimum resistance factor may be needed in future guideline revisions. From the study on the relation between average reliability index and optimum resistance factor, a value of 0.723 for the optimum resistance factor is suggested.

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KEY WORDS: fiber reinforced polymer; reliability analysis; optimum resistance factor; torsion retrofitting; concrete beams.

## 1. INTRODUCTION

The application of externally-bonded of FRP composites is now widely recognized as a viable technique for the renewal of existing structures. The lightweight and formability of

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FRP reinforcement make these systems easy to install. As the materials used in these systems are non-corrosive, non-magnetic, and generally resistant to chemicals, they are an excellent choice for external reinforcement. In special cases, FRP materials are applied to enhance the structure for changing load demands. Retrofitting with externally bonded FRP sheets has been shown to be applicable to many types of reinforced concrete structures. Currently, this method has been implemented to retrofit such structures as columns, beams, slabs, walls and tunnels. The uses of external FRP reinforcement may be generally classified as flexural retrofitting, improving the confinement and ductility of compression members, shear retrofitting and torsion retrofitting [1, 2]. To promote the responsible use of these materials, numerous design guidelines have been developed for external retrofitting of reinforced concrete structures (e.g., FIB 2001 [3], ISIS 2001 [4] and ACI 2008 [5]). However, few studies are available on the statistical characteristics of the main design variables and the reliability of the retrofit structures. Reliability-based techniques can be used to account for the randomness in important variables that affect the strength of FRP-retrofitted concrete beams. The application of such methods in structural engineering has greatly increased in the past few years as reliability-based models have become more widely accepted. There are two reasons for the applications of the theory of reliability to the structural engineering problems. First, design guidelines have been and still are being changed from the allowable stress design approach to the strength design approach. Strength design provisions in modern design guidelines are calibrated through reliability-based methods to ensure that the probability of failure ( $p_f$ ) does not exceed a target level. This approach allows designers to more rationally assess the possibility of structural collapse, whereas allowable stress design usually results in hidden reserve strength. The second reason driving the increasing popularity of structural reliability is that it makes possible a new trend in thought whereby structural systems are characterized in a probabilistic method, rather than using deterministic strength, to achieve a more rational balance between safety and life-cycle costs [6].

One of the earliest studies of the reliability of concrete structures retrofitted with CFRP was conducted by Plevris *et al*. In their approach, a virtual design space composed of a number of random parameters was created and used to study the flexural reliability of reinforced concrete beams retrofitted with CFRP. Uncertainty in member resistance was characterized using Monte Carlo Simulation considering three possible failure modes: steel yielding followed by CRFP rupture, steel yielding followed by concrete crushing, and for over-reinforced sections, catastrophic crushing of the concrete [7]. Reliability-based design of flexural strengthening was studied by El-Tawil and Okeil for prestressed bridge girders [8]. Val studied the reliability of reinforced concrete columns wrapped with FRP using existing empirical models to describe the effect of FRP confinement on reinforced concrete columns and to predict the strength of the wrapped columns. A modification to the strength reduction factor was proposed to ensure that the reliability of confined columns was at least as high as that for unconfined columns [9]. Huy Binh Pham and Riadh Al-Mahaidi studied the reliability analysis of bridge beams retrofitted with fiber reinforced polymers. They recommended that the resistance factor for flexure and intermediate span debond should be taken as 0.6 whereas the factor for end debond is 0.5 [10]. He et al. have presented reliability-based shear design for reinforced concrete beams with U-wrap FRP-

strengthening. Their study provided a reliability assessment on the shear design provisions in the Chinese Technical code [11]. Wang et al. summarized some of the available tools and supporting databases that can be used to develop reliability-based guidelines for design and evaluation of FRP composites in civil construction and illustrates their application with several practical examples involving strengthening reinforced concrete flexural members [12].

The main purpose of the present paper is to give a reliability evaluation of the torsional design provisions for FRP-strengthened concrete beams according to the FIB guideline. In this study, the effects of statistical variables on member resistance are examined and reliability index is determined using Rackwitz-Fiessler method. Finally, the optimum resistance factor is calculated in the framework of reliability theory- based.

## 2. DESIGN GUIDLINE

The ultimate torsional resistance of reinforced concrete beams with *U*-jacket wrapping of FRP laminate, $T_U$, consists of the resistance provided by FRP laminate, $T_{frp}$, and that provided by reinforced concrete, $T_s$, as follows,

$$T_U = T_{frp} + T_S \tag{1}$$

The contribution of the FRP to the torsion capacity of the beam, $T_{frp}$, for the case of *U*-jacket wrapping can be found as follows,

$$T_{frp} = bh\frac{t_{frp}w_{frp}}{s_{frp}}E_{frp}\varepsilon_{frpe} \tag{2}$$

where *b* and *h* are the width and the height of the cross section, respectively, $t_{frp}$ is the nominal thickness of one ply of FRP laminate, $w_{frp}$ is the width of FRP strip, $S_{frp}$ is the center-to-center distance between FRP strips, $E_{frp}$ is the elasticity modulus of FRP laminate and $\varepsilon_{frpe}$ is the effective strain of FRP laminate which is defined as follows,

$$\varepsilon_{frpe} = 0.17\left(\frac{f_{cm}^{2/3}}{E_{frp}\rho_{frp}}\right)^{0.3}\varepsilon_{frpu} \quad \text{For CFRP} \tag{3}$$

$$\varepsilon_{frpe} = 0.048\left(\frac{f_{cm}^{2/3}}{E_{frp}\rho_{frp}}\right)^{0.47}\varepsilon_{frpu} \quad \text{For GFRP} \tag{4}$$

in which $f_{cm}$ is the compressive strength of the concrete, $\varepsilon_{frpu}$ is ultimate strain of FRP

laminate and $\rho_{frp}$ is FRP reinforcement ratio with respect to concrete which can be obtained by the following relationship,

$$\rho_{frp} = \frac{2t_{frp}w_{frp}}{b_w s_{frp}} \tag{5}$$

where $b_w$ is the width of the web. $T_s$ is calculated by:

$$T_S = 2\phi_S A_0 A_t \frac{f_{yv}}{s}\cot\theta \tag{6}$$

where $\phi_s = 0.85$ is the partial safety factor of steel strength, $A_0$ is the cross sectional area bounded by the center line of the shear flow, $A_t$ is the area of one leg of the transverse steel reinforcement (stirrups), $f_{yv}$ is the yield strength of the transverse steel reinforcement, $s$ is the spacing of the stirrups and $\theta$ is the angle of torsion crack direction with respect to the horizontal line.

## 3. RELIABILITY BASIS FOR LIMIT STATE FUNCTION

The limit state function corresponding to FRP's torsional resistance is developed in this section for the reliability analysis. For analysis, it needs to define the state variables of the problem. The state variables are the basic load and resistance parameters used to formulate the performance function. For 'n' state variables, the limit state function is a function of 'n' parameters. If all loads (or load effects) are represented by the variable *Q* and total resistance (or capacity) by *R*, then the space of state variables is a two-dimensional space. Within this space, we can separate the "safe domain" from the "failure domain"; the boundary between the two domains is described by the limit state function *g(R,Q)=0*, [13].

### *3.1 Limit state function*

The following commonly-used expression governs the design FRP-retrofitted concrete beams,

$$R_d \geq \gamma_0 Q_d = \gamma_D Q_D + \gamma_L Q_L \tag{7}$$

where, $R_d$ is the factored resistance, $\gamma_0 = 1$ is the load factor, $Q_d$ is the maximum of combination of factored dead and live load effects, $Q_D$ and $Q_L$ are the characteristic load effects caused by dead load and live load, respectively; $\gamma_D = 1.35$ is the partial safety factor of dead load, $\gamma_L = 1.5$ is the partial safety factor of live load [14]. Table 1 lists the statistical data of $Q_D$ and $Q_L$ for common dead and live loads [15].

Table 1: Statistical data of dead and live loads

| Load pattern | Mean/nominal | Coefficient of variation | Probability distribution | Load factor |
|---|---|---|---|---|
| Dead | 1.05 | 0.1 | Normal | 1.35 |
| Live | 1 | 0.25 | Extreme 1 | 1.5 |

The limit state functions, *Z*, for retrofitted with *U*-jacket wrapping beam is expressed by the following equation,

$$Z = R_d - \gamma_0 Q_d = \Omega T_u - \gamma_0 Q_d = 0 \tag{8}$$

Substituting Equations (2), (6) and (7) into Equation (8) results in,

$$Z = \Omega(bh\frac{t_{frp} w_{frp}}{s_{frp}} E_{frp}\varepsilon_{frpe} + 2\phi_s A_0 A_t \frac{f_{yv}}{s}\cot\theta) - \gamma_D Q_D + \gamma_L Q_L \tag{9}$$

where $\Omega$ is the computational uncertainty factor associated with analytical method for strengthened with *U*-jacket wrapping beam. That will be assessed in section 3.2. $Q_D$ and $Q_L$ are determined through the following formula:

$$Q_D = \frac{\gamma_0 Q_d}{\gamma_D + \gamma_L \eta} \tag{10}$$

$$Q_L = \frac{\eta(\gamma_0 Q_d)}{\gamma_D + \gamma_L \eta} \tag{11}$$

in which $\eta$ is the load effect ratio ( $\eta = \frac{Q_L}{Q_D}$ ).

*3.2 Computational uncertainty factor*

The computational uncertainty factor, $\Omega$, is used to account for the uncertainties or randomness in predicting resistance. The statistics of this factor is assessed by either accurate analytical results or test data. As for the problem under consideration, $\Omega$ is defined as:

$$\Omega = \frac{T^{\exp}}{T^{pre}} \tag{12}$$

where $T^{\exp}$ is the torsional resistance of concrete beams obtained by experiment and $T^{pre}$ is the predicted value from Equation (1). The results of the calculations are summarized in Table 2.

Table 2: Statistics of the computational uncertainty factors

| Reference | Ω |
|---|---|
| Ameli *et al.* [16] | 1.10 |
| Salom *et al.* [17] | 0.92 |
| Mohamadizadeh [18] | 0.89 |
| Panchacharam and Belarbi [19] | 0.93 |
| Average | 0.96 |

## 4. DESIGN VARIABLES

As the first step in reliability analysis, the statistics of the design variables must be assigned. The reliability analysis of the retrofitted beams by Equation (9) requires probabilistic models of the important engineering variables and supporting databases to characterize the uncertainties of such variables. These statistical data should be representative of values that would be expected in a structure and should reflect uncertainties due to inherent variability, modeling and prediction, and measurement. Except dead and live load, there are ten design variables associated with the torsion resistance of retrofitted beams. Table 3 lists the statistical properties found in the literature and shows the bias (mean/nominal), coefficient of variation (COV =standard deviation/mean), and distribution type assumed by other researchers. In order to make the evaluation general, two extreme groups, i.e. A and B, are selected. The nominal value of random variables for groups A and B are adopted from Ref. [19] and Ref. [18], respectively.

Table 3: Statistics of random variables

| Design variables | Groups name | Nominal value | Mean/Nominal | Coefficient of Variation | Probability Distribution |
|---|---|---|---|---|---|
| $b(mm)$ | A<br>B | 279.4<br>150 | 1 | 0.03 | Normal [20] |
| $h(mm)$ | A<br>B | 279.4<br>350 | 1 | 0.03 | Normal [20] |
| $A_t(mm^2)$ | A<br>B | 71.29<br>50.24 | ١ | 0.015 | Normal [6] |
| $f_{yv}(MPa)$ | A<br>B | 450<br>480 | 1.12 | 0.1 | Lognormal [12] |
| $s(mm)$ | A<br>B | 152.4<br>80 | 1 | 0.06 | Normal [2] |
| $E_{frp}(MPa)$ | A<br>B | 72000<br>240000 | 1 | 0.1 | Lognormal [20] |
| $t_{frp}(mm)$ | A<br>B | 0.353<br>0.176 | 1.02 | 0.05 | Lognormal [1] |
| $W_{frp}(mm)$ | A<br>B | 114.3<br>200 | 1 | 0.02 | Normal [21] |
| $s_{frp}(mm)$ | A<br>B | 114.3<br>100 | 1 | 0.02 | Normal [21] |
| Ω | A | 0.96 | 1.05 | 0.06 | Normal [12] |

## 5. ANALYTICAL METHODS

*5.1 Rackwitz–Fiessler method*

The Rackwitz–Fiessler method [22] is applied to implement the reliability analysis. In limit state function, there are twelve random variables, i.e. b, d, $t_{frp}$, $E_{frp}$, $t_{frp}$, $w_{frp}$, $A_t$, $f_{yv}$, $S$, $Q_d$, $Q_L$ and $\Omega$, which are included in Equation (9). An approximate solution to the limit state function of Equation (9) can be achieved by a one-order Taylor series expansion at the design point (see the point P* in Fig. 1 for two independent random variables).

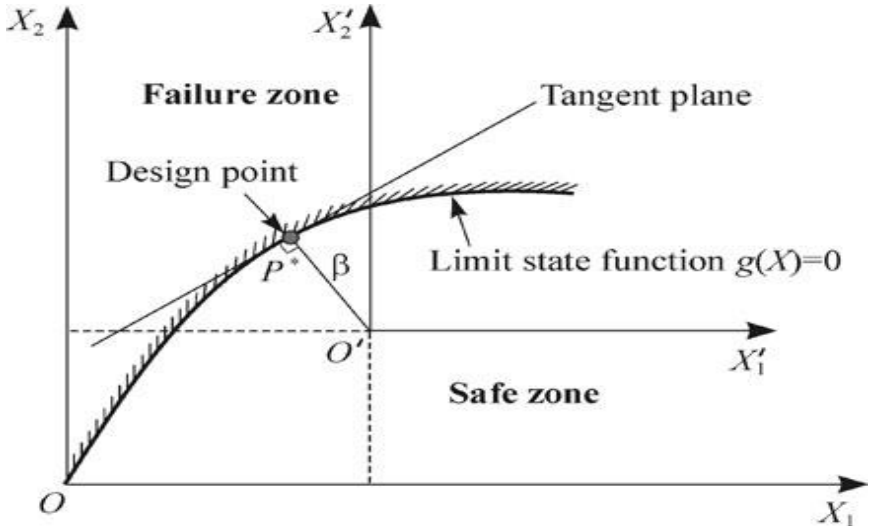


Figure 1. Geometrical definition of reliability index in standard normal space [23]

The analytical procedures in the evaluation can be outlined as follows:

Step 1. Determine the statistical data of all random design variables.

Step 2. Call the statistical data of loads. Select a load effect ratio, $\eta = \frac{Q_L}{Q_D}$.

Step 3. Develop the limit state function of concern, Z = G(X) in which $X = (x_1, x_2, ..., x_m)^T$. m is

the number of random variables.

Step 4. Assume an initial design point, $x^*(0)$, for the first iteration. Generally, $X^{*(0)} = (\mu_{x1}, \mu_{x_2}, ..., \mu_{x_m})^T$, where $\mu_{xi}$ is the mean of the $i^{th}$ random design variable.

Step 5. Determine the equivalent normal mean, $\mu_{xi}$, and standard deviation, $\sigma_{xi}$, for each non-normal distribution by the following equations, respectively.

$$\mu_{X_I'} = x_i^* - \Phi^{-1}[F_{X_i}(x_i^*)]\sigma_{X_i'} \tag{13}$$

$$\sigma_{X_I'} = \frac{\phi\{\Phi^{-1}[F_{X_i}(x_i^*)]\}}{f_{X_i}(x_i^*)} \tag{14}$$

where $\Phi(\cdot)$ is the cumulative distribution function (CDF) for the standard normal distribution; $\phi(\cdot)$ is the probability density function (PDF) for the standard normal distribution; $F_{X_i}$ and $f_{X_i}(x_i^*)$ are the CDF and PDF for the non-normal distribution under consideration, respectively.

Step 6. Calculate an estimate of reliability index, $\beta$, (From the geometrical point of view, reliability index, $\beta$, is defined as the shortest distance from the origin of reduced variables, e.g. $X_1'$ and $X_2'$ $\left[X_i' = \frac{X_i - \mu_{Xi}}{\sigma_{Xi}}, i = 1,2\right]$ in Fig. 1,) by

$$\beta = \frac{\mu_Z}{\sigma_Z} = \frac{-\sum_{I=1}^{m} x_i^* \frac{\partial Z_x(x^*)}{\partial X_i}\sigma_{X_I}}{\left\{\sum_{I=1}^{m}\left[\frac{\partial Z_x(x^*)}{\partial X_i}\sigma_{X_I}\right]^2\right\}^{1/2}} \tag{15}$$

Step 7. Calculate sensitivity factor, $\alpha_i$, for each random variable by

$$\alpha_i = \frac{-\frac{\partial Z_x(x^*)}{\partial X_i}\sigma_{X_I}}{\left\{\sum_{I=1}^{m}\left[\frac{\partial Z_x(x^*)}{\partial X_i}\sigma_{X_I}\right]^2\right\}^{1/2}} \tag{16}$$

where $\alpha_i$ is the $i^{th}$-axis direction cosine of the normal $O'P^*$ (see Fig. 1). All sensitivity factors must meet the following equation:

$$\sum_{i=1}^{m} \alpha_i^2 = 1 \tag{17}$$

Step 8. Determine a new design point, $X^*$, in original coordinates by:

$$x_i^* = \mu_{X_i} + \alpha_i \beta \sigma_{X_i} \text{ (i=1,2,…..m)} \tag{18}$$

Step 9. Repeat steps 5–8 until $\beta$ and the design point $X^*$ converge.

*5.2 Computation of reliability index*

The Rackwitz-Fiessler method was applied to calculate reliability index, β. Two rather extreme nominal values were selected for each design variable, as well as six load effect

ratios, $\eta = \frac{Q_L}{Q_D}$, i.e., 0.1, 0.5, 1, 1.5, 2, 2.5. Averaging all reliability indexes gives the global average reliability indexes of 2.92.

## 6. EFFECT OF DESIGN VARIABLES ON RELIABILITY ANALYSIS

Now, we investigate the sensitivity of reliability index $\beta$ with inspect to each design variable into two parts, i.e. Group A and Group B, and a local average reliability index is then calculated for each part. The sensitivity factor $\alpha_i$ is used to determine the contribution of the random variables to the reliability index. The results are illustrated in Fig. 2 from which it can be seen that yield strength of the stirrups and sectional width are the first two main influencing factors among all design variables for retrofitted beam with U-wrap. To make further investigation on the effect of $f_{yv}$, six yield strength of the stirrups were selected. The results show, as $f_{yv}$ increases, the average reliability index increases monotonically but at a slowing rate (Fig. 3). For instance as $f_{yv}$ increases from 250 MPa to 500 MPa, the average reliability index increases 28%. Design variable b is then selected for conducting a detailed parametric study of its effect on the reliability level, as shown in Fig. 4. Seven values for the sectional width, i.e. $b = 150$, 200, 250, 300, 350, 400 and 450 mm were selected. As b increases from 150 mm to 450 mm, an increase of 25% in average reliability index can be obtained for both types of the beams. In addition, load effect ratio, $\eta$, has a significant influence on reliability level, as shown in Fig. 5. As for retrofitted beam with U- wrap, if $\eta$ increases from 0.10 to 2.5, the average index, β, decreases slightly. Fig. 5 indicates, for any live load pattern, the average reliability index decreases as $\eta$ increases but at a slow rate.

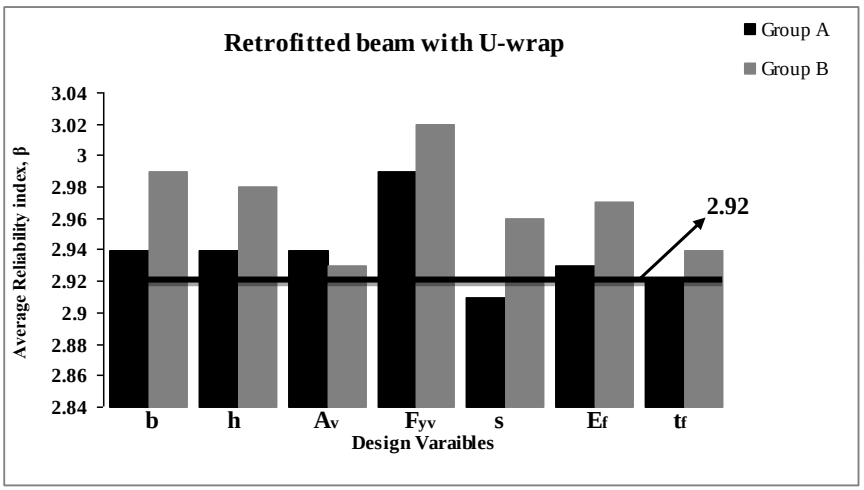


Figure 2. Effects of design variables on average reliability index for the retrofitted beam with U-wrap

## 7. DETERMINING THE OPTIMUM RESISTANCE FACTOR

Application of Equation (1) for U-wrapping suggested in the FIB guideline could lead to a significant decrease in reliability level after retrofitting (Averaging all reliability indexes gives the global average reliability index of 2.92 for retrofitted beam with U-wrap). As suggested by Szerszen and Nowak [24], the target reliability index corresponding to concrete, $\beta_c$, can be taken as 3.5.

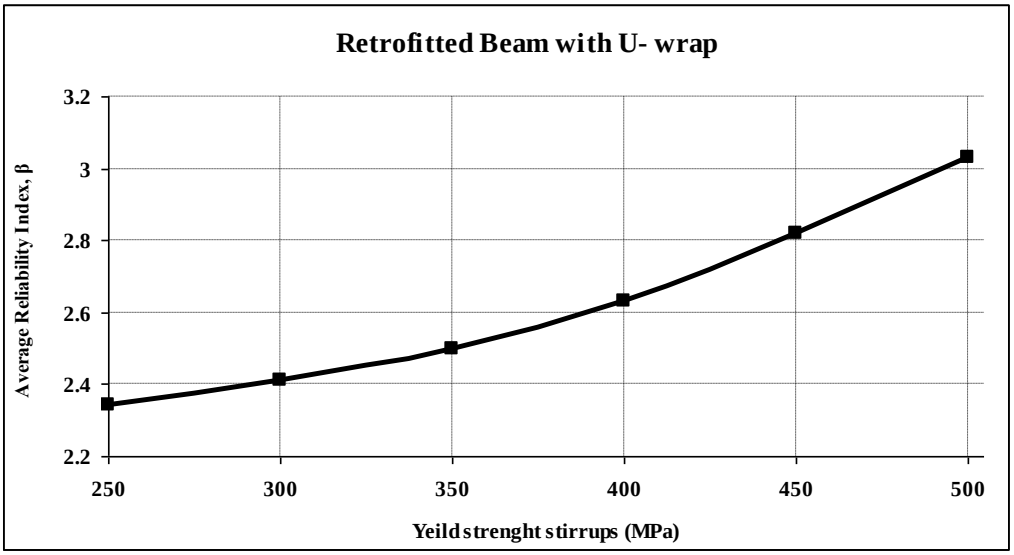


Figure 3. Effect of yield strength of the stirrups on average reliability index

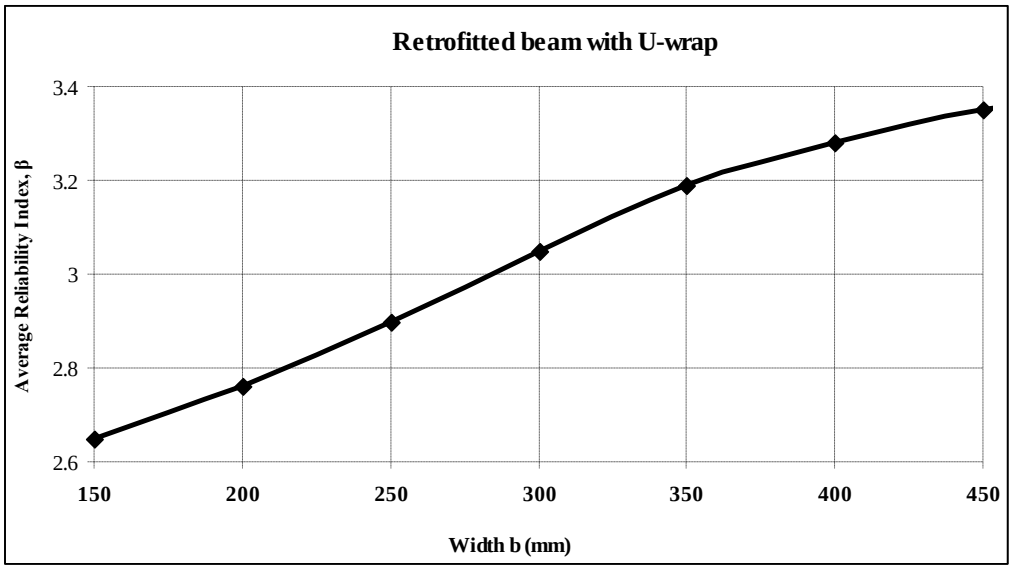


Figure 4. Effect of sectional width on average reliability index

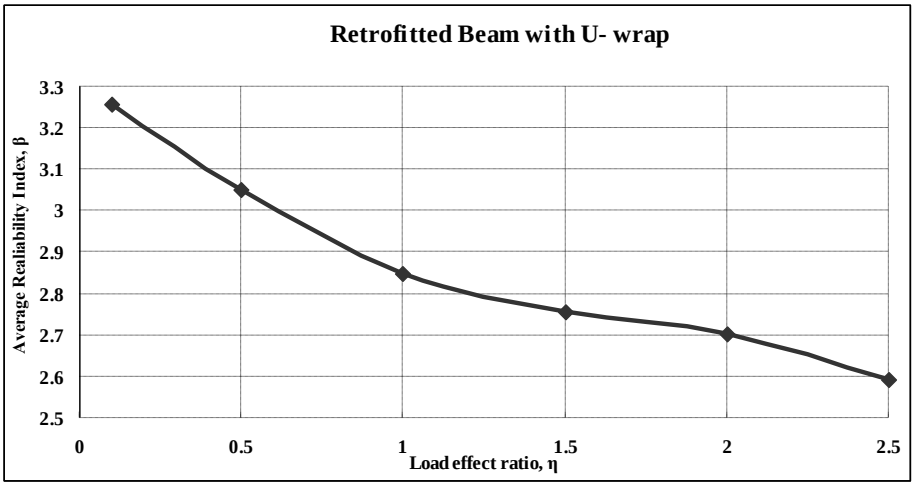


Figure 5. Load effect ratio for retrofitted beam with U-wrap

For achieving a higher reliability level after retrofitting, an optimum resistance factor ( $\phi$ ) must be applied. In this section, $\phi$ is calibrated based on a target reliability. As illustrated in Fig. 6, approximate linear relations between average reliability indexes, $\beta$ , and optimal resistance factor, $\phi$ , could be obtained for retrofitted beam with U-wrap. For $0.5 \le \phi \le 1$ , $\beta$ is determined. The factors corresponding to $\beta_c = 3.5$ are found to be 0.712 and 0.734, for groups A and B, respectively (see Fig. 6 and Fig. 7). The average of these two factors is used to determine the modified resistance factor $\phi$ =0.723.

In this section, a relationship between $\beta$ and ϕ obtained from the parametric study shows that ϕ could be taken as 0.723 for keeping the consistency in reliability level ( $\beta_c = 3.5$ ) of FRP torsional retrofitting beams with U- wrap.

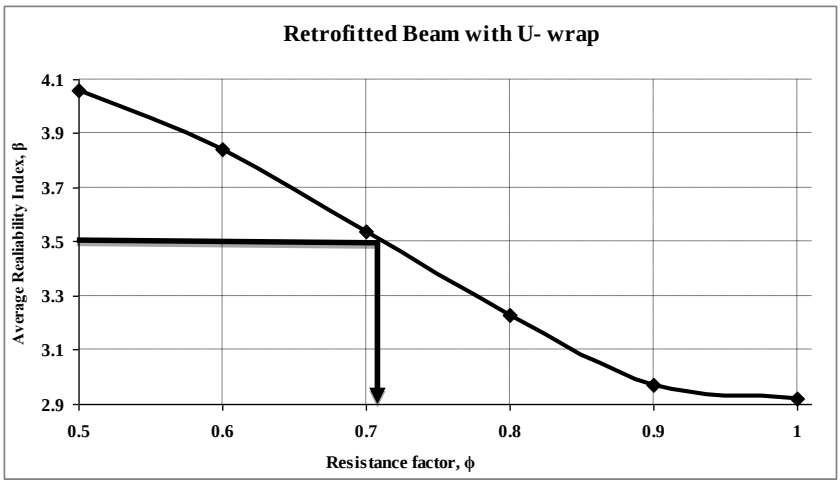


Figure 6. Reliability index versus optimum resistance factor for retrofitted beam with U-wrap, group A

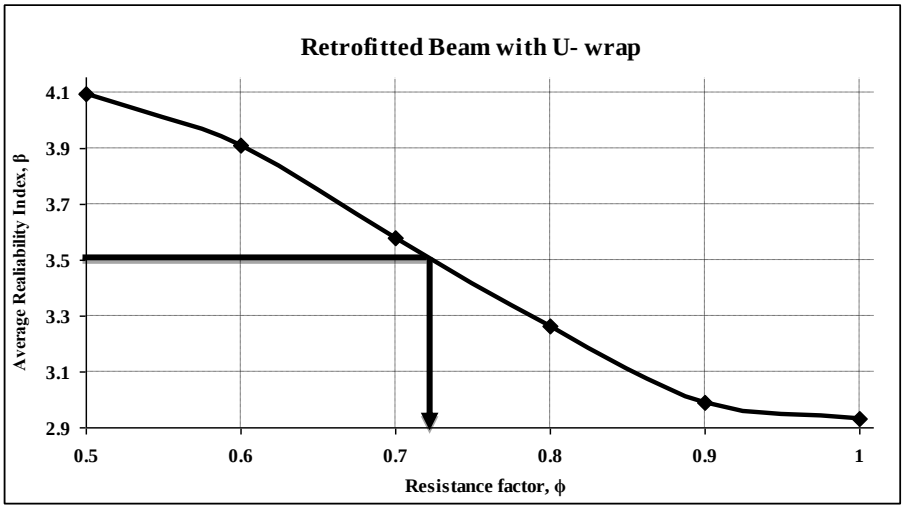


Figure 7. Reliability index versus optimum resistance factor for retrofitted beam with U-wrap, group B

## 8. CONCLUSIONS

This paper has shown the possibility of developing a probability-based limit state function for design and assessment of reinforced concrete structural members, with strength enhanced by installation of externally bonded FRP composite laminate. The main purpose of the present paper is to give a reliability evaluation of the torsional design provisions for FRP-retrofitted concrete beams according to the FIB guideline. The Rackwitz- Fiessler reliability method has been applied to make a reliability evaluation and, the effects of some design variables on the reliability level are also assessed. Some results can be drawn through the assessment as follows:

1. The Rackwitz-Fiessler method was applied to calculate reliability index, β. Reliability indexes were calculated for different load effect ratios ( $\eta = \frac{Q_L}{Q_D}$ ), i.e., 0.1, 0.5, 1, 1.5, 2, 2.5. Averaging all reliability indexes gives the global average reliability index of 2.92 for retrofitted beams with U-wrap. Therefore design provisions in the FIB guideline seems to be unconservative.

2. Yield strength of stirrups, $f_{yv}$, and sectional width, b, are dominant influencing factors among all the design variables for beams retrofitted with U-wrapping. As $f_{yv}$ increases from 250 MPa to 500 MPa, the average reliability index increases 28%. Also, while the sectional width, b, increases from 150 MPa to 450 MPa, the average reliability index increases 25%. The parametric study also indicates that load effect ratio, $\eta$, has a significant influence on the reliability level. As load effect ratio increases from 0.1 to 2.5, the average reliability index could decrease at a slow rate.

3. Application of the resistance factor $\phi$=1 for U-wrapping suggested in the FIB guideline could lead to a decrease in reliability level after strengthening. For achieving a

higher reliability level after retrofitting, a optimum resistance factor, $\phi$, must be applied. A study of the effect of the target reliability index, β, on the value of optimum resistance factor, $\phi$, is presented. As a result of the study, the modified value of 0.723 for $\phi$ is suggested. In design practice, $\phi$ = 0.7 can be used for simplicity.

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