SHAPE OPTIMIZATION OF CONCRETE GRAVITY DAMS CONSIDERING DAM–WATER–FOUNDATION INTERACTION AND NONLINEAR EFFECTS

M. Khatibinia[*, †], H. Chiti, A. Akbarpour and H.R. Naseri
Department of Civil Engineering, University of Birjand, Birjand, Iran

ABSTRACT

This study focuses on the shape optimization of concrete gravity dams considering dam–water–foundation interaction and nonlinear effects subject to earthquake. The concrete gravity dam is considered as a two–dimensional structure involving the geometry and material nonlinearity effects. For the description of the nonlinear behavior of concrete material under earthquake loads, the Drucker–Prager model based on the associated flow rule is adopted in this study. The optimum design of concrete gravity dams is achieved by the hybrid of an improved gravitational search algorithm (IGSA) and the orthogonal crossover (OC), called IGSA–OC. In order to reduce the computational cost of optimization process, the support vector machine approach is employed to approximate the dam response instead of directly evaluating it by a time–consuming finite element analysis. To demonstrate the nonlinear behavior of concrete material in the optimum design of concrete gravity dams, the shape optimization of a real dam is presented and compared with that of dam considering linear effect.

1. INTRODUCTION

Concrete gravity dams in comparison with other concrete structures are distinguished as critical structures because of their size and their interactions with the reservoir and foundation. In addition, the nonlinear behavior of dam concrete affecting the dynamic

---

[*]Corresponding author: M. Khatibinia, Assistant Professor, Department of Civil Engineering, University of Birjand, Birjand, Iran
[†]E-mail address: m.khatibinia@birjand.ac.ir (M. Khatibinia)

response of concrete gravity dams subject to earthquake ground motions is considered as the main factor. Hence, the effects of dam–water–foundation interaction and the nonlinear behavior of dam concrete play important roles in the earthquake design of new dams and the earthquake safety evaluation of existing dams. The failure of dams due to the effects could also result in heavy loss of human life and substantial property damages.

Finding a proper shape design of concrete gravity dams has been considered as great interest in engineering. To achieve this purpose in practice, several alternative schemes with various patterns should be selected and modified to obtain a number of feasible shapes. Thus, the proper shape of concrete gravity dams considering the economy and safety of design, structural considerations, *etc*. is selected as the final shape [1]. In the recent years, optimization techniques have been effectively utilized for finding an optimal shape of dam instead of this try and error procedure.

During the last years, various studies related to design optimization of arch dams with hydrodynamic effects were reported [1-7]. Furthermore, the optimal shape design of concrete gravity dams including dam–water–foundation rock interaction has been attracted by few researchers. Salajegheh *et al*. [8] introduced the shape optimal design of concrete gravity dams including hydrodynamic effects. In this work, the shape optimal design of concrete gravity dams was achieved using a hybrid of gravitational search algorithm (GSA) and particle swarm optimization (PSO). In the work of Slajegheh and Khosravi [9], the shape optimal design of concrete gravity dams including the dam–water–foundation rock interaction was obtained using a hybrid of GSA and PSO. Khatibina and Khosravi [10] proposed a hybrid approach based on an improved gravitational search algorithm (IGSA) and orthogonal crossover (OC) in order to efficiently find the optimal shape of concrete gravity dams. Deepika and Suribabu [11] used the differential evolution technique for the optimal design of gravity dam. Recently, Kaveh and Zakian [12] have presented the shape optimization of a gravity dam imposing stability and stress constraints.

The main aim of this study is to introduce the shape optimization of concrete gravity dams considering dam–water–foundation interaction and nonlinear effects subject to earthquake. The geometry and material nonlinearity effects of dam are considered in the analysis procedure of dams. In order to model the nonlinear behavior of concrete material under earthquake loads, the Drucker–Prager model based on the associated flow rule is adopted in this study. The hybrid of an improved gravitational search algorithm (IGSA) and the orthogonal crossover (OC) proposed by Khatibinia and Khosravi [10] is utilized for finding the optimum design of concrete gravity dams. In order to reduce the computational cost of the optimization process complicated by numerous nonlinear dynamic analyses, the support vector machine (SVM) approach is employed to approximate the nonlinear dynamic responses of dams. To demonstrate the nonlinear behavior of dam in the optimum design of concrete gravity dams, the shape optimization of a real dam is presented for two conditions of the dam behavior.

2. GEOMETRICAL MODEL OF CONCRETE GRAVITY DAMS

In this study, the geometrical model of concrete gravity dams can be assigned by seven parameters. Based on the model of concrete gravity dam depicted in Fig. 1, the shape of

concrete gravity dam is defined by the seven parameters as follows:

$$X = \{b \ b_1 \ b_2 \ b_3 \ H_2 \ H_4 \ H_5\} \tag{1}$$

where b and H_1 are two parameters required to defined crest and free board of gravity dam, respectively. H_3 depends on H_4 and reservoir water level (H).
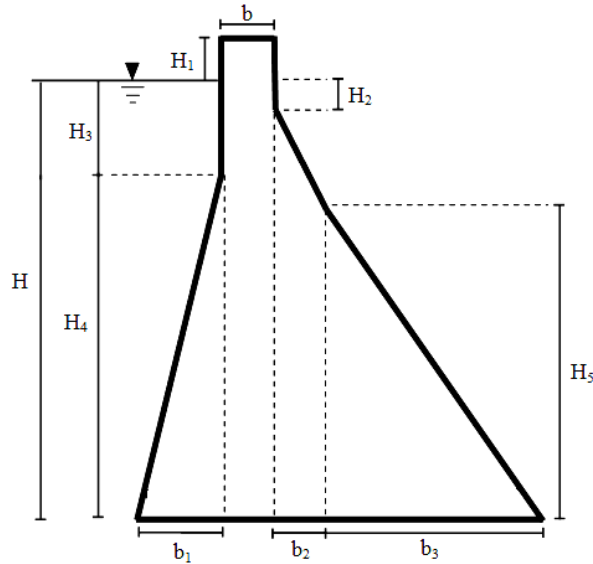

Figure 1. Geometrical model of concrete gravity dam

3. PROBLEM FORMULATION

The optimization problem of concrete gravity dam considering dam–water–foundation interaction and nonlinear effects subject to earthquake is stated as follows:

$$
\begin{aligned}
&\text{Find} \quad && X^L \leq X \leq X^U \\
&\text{Minimize} \quad && f(X) \\
&\text{Subject to} \quad && g_i(X,t) \leq 0 \qquad i = 1, 2, ..., m \ , t = 0, ..., T
\end{aligned}
\tag{2}
$$

where f and g_i are the objective function and the constraints, respectively. X^L and X^U are the lower bound and the upper bound of the design variables, X, respectively. t and T are the time step and the earthquake duration, respectively.

In the optimization problem of concrete gravity dams, the concrete weight of gravity dam body is considered as objective function, $f(X)$, that should be minimized. The weight of concrete gravity dam can be determined as follows:

$$f(X) = W = \rho_c \, g \, V \tag{3}$$

where ρ_c and V are the mass density of the concrete and the volume of gravity dam, respectively. g is gravity acceleration.

In the present study, the behavior and stability constraints are considered as the problem constraints, $g_i(X,t)$. The behavior constraints consist on the principal stresses of each node in the body of gravity dam which must be satisfied for all time points of the earthquake interval as follows [10]:

$$g_{k,1} = \sigma_1(X,t) \le \sigma_t \; ; \; k = 1, 2, ..., nj \tag{4}$$

$$g_{k,2} = |\sigma_2(X,t)| \le \sigma_c \; ; \; k = 1, 2, ..., nj \tag{5}$$

where $\sigma_1(\cdot)$ and $\sigma_2(\cdot)$ are the compressive and tensile principal stresses of each node in time t, respectively. σ_t and σ_c are the allowable tensional and compressive stresses, respectively. nj is the total number of nodes in the dam body which created by meshing of the dam body in finite element method (FEM) framework.

The stability constraints of a gravity dam are defined in terms of its factors of safety against sliding, overturning and uplift pressure, respectively. The factor of safety against sliding is equal to the ratio of the total frictional force, F_V, which the foundation can develop to the force tending, F_H, to cause sliding as follows [13]:

$$g_s = \frac{\sum F_V}{\sum F_H} \ge 1.75 \tag{6}$$

The factor of safety against overturning about the toe is defined as the ratio of the resisting moments, M_R, to the overturning moments, M_O, which is considered as stability constraint. It is expressed as [13]:

$$g_o = \frac{\sum M_R}{\sum M_O} \ge 1 \tag{7}$$

Furthermore, the safety of concrete gravity dam against uplift pressure force is considered as follows [13]:

$$g_u = \frac{B}{6e} > 1 \tag{8}$$

where B and e are the bottom width of the dam eccentricity of the resultant force on the dam section, respectively.

Constraint handling approaches have been proposed in conjunction of problem constrain with meta–heuristic optimization methods. In the present study, the external penalty function

method as one of the most popular forms of the penalty function in the structural optimization is utilized as follows [14–17]:

$$\tilde{f}(X) = f(X)(1 + R_p Pf) \tag{9}$$

where $\tilde{f}(X)$, Pf and R_p are the modified function (fitness function), the penalty function and an penalty factor, respectively.

4. THE OPTIMIZATION ALGORITHM

In the present study, the hybrid of an improved gravitational search algorithm (IGSA) and the orthogonal crossover (OC) proposed by Khatibinia and Khosravi [10] is utilized to search the optimal shape of concrete gravity dams. The proposed hybrid approach is called IGSA–OC. In this section IGSA and OC algorithms are described at first, and then the proposed IGSA–OC is presented.

4.1 The IGSA method

Gravitational search algorithm (GSA) is one of the newest heuristic search algorithms, which mimics Newton's gravitational force laws [18]. In GSA, each agent of the population represents a potential solution of the optimization problem. The ith agent in tth iteration is associated with a position vector, $X_i(t) = \{x_i^1, \dots, x_i^d, \dots, x_i^D\}$, and a velocity vector, $V_i(t) = \{v_i^1, \dots, v_i^d, \dots, v_i^D\}$. D is dimension of the solution space. The force exerting on the object i from the object j is defined as:

$$F_{ij}^d(t) = G(t)\frac{M_i(t)M_j(t)}{R_{ij}(t) + \varepsilon}(x_j^d(t) - x_i^d(t)) \tag{10}$$

where M_j and M_i represent the gravitational mass of the objects j and i, respectively, ε is a small constant, G is a gravitational constant, $R_{ij}(t)$ is the Euclidian distance between two objects i and j. The total force $F_i^d(t)$ on the object i in the dth direction is calculated by a randomly weighted sum of the dth components of the forces from other objects:

$$F_i^d(t) = \sum_{j=1, j \neq i} rand_j F_{ij}^d(t) \tag{11}$$

where rand$_j$ is a random number from the interval [0,1].

The acceleration of the object i, $a_i^d(t)$, at time t and in the dth direction, is given as:

$$\alpha_i^d(t) = \frac{F_i^d(t)}{M_{ii}(t)} \tag{12}$$

where M_{ii} is the inertial mass of the object i. Its next velocity $v_i^d(t+1)$ and its next position $x_i^d(t+1)$ are calculated as Eqs. (13) and (14):

$$v_i^d(t+1) = rand_i \; v_i^d(t) + \alpha_i^d(t) \tag{13}$$

$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1) \tag{14}$$

In IGSA, a new moving strategy in the searching space was proposed by obeying the law of gravity and receiving guide of memory as follows [11]:

$$v_i^d(t+1) = rand_{i,1} v_i^d(t) + a_i^d(t) + c_1 \, rand_{i,2}(pbest_i^d - x_i^d(t)) + c_2 \, rand_{i,3}(gbest^d - x_i^d(t)) \tag{15}$$

where **pbest$_i$** is the best previous position of the ith agent; and **gbest** is the best previous position among all the agents. c_1 and c_2 are variables in the range [0, 1], respectively.

4.2 OC operator

In OC operator, an orthogonal array is integrated into the classical crossover operator so that chromosomes are produced by exploring alleles in parents based on combinations of an orthogonal array. Q is the number of levels, M is the number of rows and N is the number of columns of the orthogonal array $L_M(Q^K)$, respectively [19]. Leung and Wang [20] proposed a new version OC based on combination of the quantization technique and the OC operator, called QOC. In QOC, it is assumed that the search space defined by any two parents, i.e., $P_1 = \{p_1^1, p_1^2, ..., p_1^D\}$ and $\boldsymbol{P}_2 = \{p_2^1, p_2^2, ..., p_2^D\}$. $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$ define a search range $[\min(p_1^i, p_2^i), \max(p_1^i, p_2^i)]$ for ith variable of the design variable vector, $\boldsymbol{X}$. First, the search range is quantized into Q levels, $l_{i,1}, l_{i,2}, \ldots, l_{i,Q}$, for variable x^i as follows [20]:

$$l_{i,j} = \min(p_1^i, p_2^i) + \frac{j-1}{Q-1}(\max(p_1^i, p_2^i) - \min(p_1^i, p_2^i)) \;, \;\; j = 1, 2, ..., Q \tag{16}$$

Therefore, Q^D points are produced after quantization since each factor has Q possible levels. Since D is often much larger than K, one cannot directly apply $L_M(Q^K)$. To overcome this problem, Leung and Wang [20] proposed that QOC divides $\{x_1, x_1, ..., x_D\}$ into K sub–vectors as:

$$\begin{cases} \bar{H}_1 = \{x_1, x_2, ..., x_{n_1}\} \\ \bar{H}_2 = \{x_{n_1+1}, ..., x_{n_2}\} \\ \quad\vdots \\ \bar{H}_K = \{x_{n_{K-1}+1}, ..., x_D\} \end{cases} \tag{17}$$

where n_1, n_2, ..., n_{K-1} are randomly generated such that $1 < n_1 < n_2 < ... < n_{K-1} < D$. QOC trears each $\bar{H}_i$ as a factor and defines the following Q levels for $\bar{H}_i$:

$$\begin{cases} L_{i1} = \{l_{t_{i-1}+1,1}, l_{t_{i-1}+2,1}, ..., l_{t_i,1}\} \\ L_{i2} = \{l_{t_{i-1}+1,2}, l_{t_{i-2}+2,1}, ..., l_{t_i,2}\} \\ ... \\ L_{iQ} = \{l_{t_{i-1}+1,Q}, l_{t_{i-2}+2,Q}, ..., l_{t_i,Q}\} \end{cases} \tag{18}$$

Then, $L_M(Q^K)$ is utilized on $\bar{H}_1$, $\bar{H}_2$, . . . , $\bar{H}_K$ to construct M solutions (i.e., combinations of levels).

4.3 The hybrid of IGSA–OC

In order to eliminate the drawback of IGSA and explore promising regions in the search space, Khatibinia and Khosravi [10] proposed the hybrid of IGSA and OC, called IGSA–OC. In the IGSA–OC, the QOC operator is utilized as local search and improves the best previous position among all the agents (***gbest***) in each iteration of IGSA. Assuming $\{\boldsymbol{pbest}_1, \boldsymbol{pbest}_2, ..., \boldsymbol{pbest}_N\}$, the process of the improving ***gbest*** using the QOC operator is executed in the following steps [10]:

Step 1: Randomly select three different numbers r_1, r_2 and r_3 from $\{1, 2, ..., N\}$.

Step 2: $\boldsymbol{U} = \boldsymbol{pbest}_{r1} + rand(\boldsymbol{pbest}_{r2} - \boldsymbol{pbest}_{r3})$.

Step 3: Combine ***gbest*** and $\boldsymbol{U}$ by using QOC based on $L_M(Q^K)$ to generate M trial vectors.

Step 4: Evaluate the fitness function values, $\tilde{f}$, of the M trial vectors.

Step 5: Select the one with the smallest fitness function to be $\boldsymbol{U}_g$.

Step 6: If $\tilde{f}(\boldsymbol{U}_g) < \tilde{f}(\boldsymbol{gbest})$, set $\boldsymbol{gbest} = \boldsymbol{U}_g$.

It is noted that the more details of the IGSA–OC can be found in [10].

5. FINITE ELEMENT MODEL OF FLUID–NONLINEAR STRUCTURE SYSTEM

In order to simulate the fluid–nonlinear structure interaction problem using the finite

element method (FEM), the discretized dynamic equations of fluid and structure should be considered simultaneously to obtain the coupled fluid–structure equation.

5.1 The discretized structure equation

In the FEM, the discretized linear dynamic equation of the structure including the gravity dam and foundation rock subject to earthquake loads are expressed as [21]:

$$\boldsymbol{M}_s \ddot{\boldsymbol{u}}_e + \boldsymbol{C}_s \dot{\boldsymbol{u}}_e + \boldsymbol{K}_s \boldsymbol{u}_e = -\boldsymbol{M}_s \ddot{\boldsymbol{u}}_g + \boldsymbol{Q}\boldsymbol{p}_e \tag{19}$$

where $\boldsymbol{M}_s$ is the structural mass matrix, $\boldsymbol{C}_s$ is the structural damping matrix, $\boldsymbol{K}_s$ is the structural stiffness matrix, $\boldsymbol{u}_e$ is the vector of the nodal displacements relative to the ground, $\ddot{\boldsymbol{u}}_g$ is the vector of the ground acceleration, $\boldsymbol{Q}\boldsymbol{p}_e$ represents the nodal force vector associated with the hydrodynamic pressures produced by the reservoir. In this work, the damping matrix of the dam body is also accomplished using Rayleigh damping.

5.2 The discretized fluid equation

In this paper, the motion of fluid is simulated by two–dimensional wave equation and assumption that the water is compressible and inviscid [22, 23]. The discretized equation of the fluid domain based on the FEM can be defined as [22, 23]:

$$\boldsymbol{M}_f \ddot{\boldsymbol{p}}_e + \boldsymbol{C}_f \dot{\boldsymbol{p}}_e + \boldsymbol{K}_f \boldsymbol{p}_e + \rho_w \boldsymbol{Q}^T (\ddot{\boldsymbol{u}}_e + \ddot{\boldsymbol{u}}_g) = 0 \tag{20}$$

where $\boldsymbol{M}_f$, $\boldsymbol{C}_f$ and $\boldsymbol{K}_f$ are the mass, damping and stiffness matrices of fluid, respectively; and $\boldsymbol{p}_e$ and $\ddot{\boldsymbol{u}}_e$ are the nodal pressure and relative nodal acceleration vectors, respectively. The term $\rho_w \boldsymbol{Q}^T$ is also often referred to as coupling matrix.

5.3 The coupled fluid–structure equation

In order to simulate the effects of dam–water–foundation interaction, Eqs. (19) and (20) are combined expressed as follows [22, 23]:

$$\begin{bmatrix} \boldsymbol{M}_s & 0 \\ \boldsymbol{M}_{fs} & \boldsymbol{M}_f \end{bmatrix} \begin{Bmatrix} \ddot{\boldsymbol{u}}_e \\ \ddot{\boldsymbol{P}}_e \end{Bmatrix} + \begin{bmatrix} \boldsymbol{C}_s & 0 \\ 0 & \boldsymbol{C}_f \end{bmatrix} \begin{Bmatrix} \dot{\boldsymbol{u}}_e \\ \dot{\boldsymbol{p}}_e \end{Bmatrix} + \begin{bmatrix} \boldsymbol{K}_s & \boldsymbol{K}_{fs} \\ 0 & \boldsymbol{K}_f \end{bmatrix} \begin{Bmatrix} \boldsymbol{u}_e \\ \boldsymbol{p}_e \end{Bmatrix} = \begin{Bmatrix} -\boldsymbol{M}_s \ddot{\boldsymbol{u}}_g \\ -\boldsymbol{M}_{fs} \ddot{\boldsymbol{u}}_g \end{Bmatrix} \tag{21}$$

Also, Eq. (21) can be written in a more compact form as:

$$\boldsymbol{M}_C \ddot{\boldsymbol{u}}_C + \boldsymbol{C}_C \dot{\boldsymbol{u}}_C + \boldsymbol{K}_C \boldsymbol{u}_C = -\boldsymbol{f}_C(t) \tag{22}$$

where $\boldsymbol{M}_{fs} = \rho_w \boldsymbol{Q}^T$ and $\boldsymbol{K}_{fs} = -\boldsymbol{Q}$.

In fact, Eq. (22) is considered as a second order linear differential equation having

unsymmetrical matrices and can be solved by means of direct integration methods. For considering the nonlinearity in the FEM, Eq. (22) is modified and stated in the incremental form as:

$$M_C \Delta \ddot{u}_C^i + C_C \Delta \dot{u}_C^i + K_C^i \Delta u_C^i = -\Delta f_C^i \tag{23}$$

where K_C^i is the stiffness matrix in the ith time step. Also, $\Delta u_C^i, \Delta \dot{u}_C^i, \Delta \ddot{u}_C^i$ and Δf_C^i are equivalent to the vectors of incremental acceleration, velocity, displacement and external load in the ith time step, respectively.

In this study, the dam–water–foundation system is simulated as a 2D model. The nonlinear behavior of dam concrete is idealized as an elasto–plastic material via an associative Drucker–Prager model [24]. In the analysis phase of gravity dams, a static analysis of gravity dam–water–foundation system is initially implemented under a gravity load and a hydrostatic pressure, and then the linear dynamic analysis of the system is performed using Newmark–Beta integration method [21]. After that, the principal stresses at the centre of dam elements are evaluated using nodal relative displacement of the gravity dam.

6. CONSTITUTIVE MODEL OF CONCRETE MATERIAL

In order to describe the nonlinear behavior of concrete material under earthquake loadings, the concrete material should be idealized using a constitutive model. For this purpose, the Drucker–Prager model [24] as a well–known model is employed in this study. This model shown in Fig. 2 is proposed for frictional materials such as soils, rock and concrete and utilized as an approximation to the Mohr–Coulomb law.
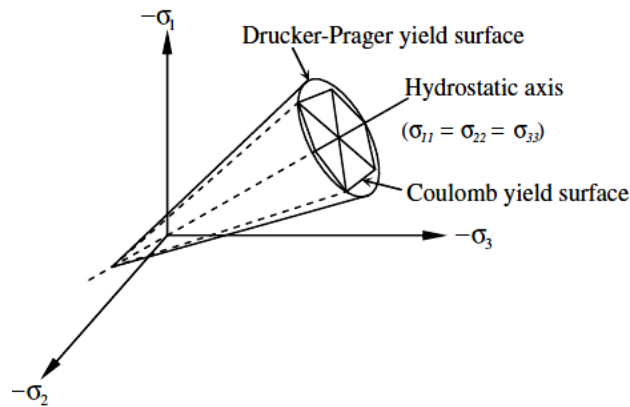

Figure 2. Drucker–Prager and Mohr–Coulomb yield surfaces [24]

In this model, the yield criterion is defined as:

$$f = 3\alpha \sigma_m + \sqrt{\frac{1}{2} \sigma_d^T M \sigma_d} - \sigma_y = 0 \tag{24}$$

where f is yield function, $\boldsymbol{\sigma}_d$ is the vector of deviatoric stresses represented by the mean or hydrostatic stress σ_m and the stress vector $\boldsymbol{\sigma}$, respectively. In the above equation, $\boldsymbol{M}$ is a matrix given by:

$$\boldsymbol{M} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix} \tag{25}$$

In addition, α and σ_y are a material constant and material yield parameter, respectively. These parameters are defined as:

$$\alpha = \frac{2\sin\phi}{\sqrt{3}(3-\sin\phi)} \tag{26}$$

$$\sigma_y = \frac{6c\cos\phi}{\sqrt{3}(3-\sin\phi)} \tag{27}$$

where ϕ and c are angle of internal friction and cohesion factor of material, respectively.

Based on the Drucker–Prager model, the yield surface does not change with progressive yielding. Hence, there is no hardening rule and the material is elastic–perfectly plastic. In the yield criterion presented in Eq. (24), when the yield function, f, is equal to zero, the material will develop plastic strains. Also, If $f < 0$ the material is elastic and the stresses will develop according to the elastic stress–strain relations. The plastic strain rate is also evaluated by the flow rule, which is defined by a scalar plastic potential function f. During plasticity, the normality plastic flow rule is applied as:

$$\Delta\boldsymbol{\varepsilon}_p = \lambda\frac{\partial f}{\partial\boldsymbol{\sigma}} \tag{28}$$

where $\Delta\boldsymbol{\varepsilon}_p$ represents the vector of incremental plastic strains; and λ is a plastic multiplier.

Using the yield criterion presented in Eq. (24), the relation between the incremental stresses and strains can be given by [25]:

$$\Delta\boldsymbol{\sigma} = \boldsymbol{D}_{ep}\Delta\boldsymbol{\varepsilon} \tag{29}$$

where $\boldsymbol{D}_{ep}$ is an elasto–plastic matrix.

7. THE SVM APPROACH FOR SIMULATING SEISMIC RESPONCES

In the meta–heuristic algorithms, the great number of function evaluations is required to obtain the optimum solution of a problem. In particular, in order to achieve the shape optimization of a concrete gravity dam considering dam–water–foundation interaction and nonlinear effects subject to earthquake, a dynamic analysis with high computational effort is implemented in each function evaluation. Therefore, the optimal problem of concrete gravity dams requires high computing time. In this study, in order to accelerate the optimization process and reduce the computational cost, the nonlinear dynamic analysis of concrete gravity dams is simulated using the weighted least squares support vector machine (WLS–SVM) regression model. The applications of the WLS–SVM have been successfully used as an excellent machine learning algorithm to many engineering problems and have yielded encouraging results [7, 10, 17, 26, 27].

7.1 The WLS–SVM regression model

The WLS–SVM regression introduced by Suykens *et al.* [28] has been used for modeling the high non–linear system based on small sample. This model based on the structural risk minimization (SRM) rules is superior to artificial neural networks (ANNs), which have been developed the traditional empirical risk minimization (ERM) inductive principle [28]. Also, the problems as over learning, dimension disaster and local minimum are eliminated in the WLS–SVM regression. The WLS–SVM regression model is presented as the optimization problem in primal weight space as follows [28]:

$$\text{Minimize } J(\boldsymbol{\omega},\xi) = \frac{1}{2}\boldsymbol{\omega}^T\boldsymbol{\omega} + \frac{1}{2}\gamma\sum_{i=1}^{n} w_i\xi_i^2$$

$$\text{Subjest to } y_i = \boldsymbol{\omega}^T\varphi(\boldsymbol{x}_i) + b + \xi_i, \qquad i = 1,2,...,n \tag{30}$$

with $\{\boldsymbol{x}_i,y_i\}_{i=1}^{n}$ a training data set, input data $\boldsymbol{x}_i \in R^n$ and output data $y_i \in R$. $\varphi(.):R^n \rightarrow R^d$ is a function which maps the input space into a higher dimensional space. The vector $\boldsymbol{\omega} \in R^d$ represents weight vector in primal weight space. The symbols $\xi_i \in R$ and $b \in R$ represent error variable and bias term, respectively.

The Lagrange multiplier method is utilized for solution of the dual problem (Eq. (30)) as:

$$L(\boldsymbol{\omega},b,\xi,\boldsymbol{\alpha}) = J(\boldsymbol{\omega},\xi) - \sum_{i=1}^{n}\alpha_i(\boldsymbol{\omega}^T\varphi(\boldsymbol{x}_i) + b + \xi_i - y_i) \tag{31}$$

Based on the Karush–Khun–Tucker (KKT) conditions, after optimizing Eq. (31) and eliminating $\boldsymbol{\omega}$ and ξ, the solution is given by the following set of linear equation:

$$\begin{bmatrix} \Omega + V_\gamma & I_n^T \\ I_n & 0 \end{bmatrix}\begin{bmatrix} \boldsymbol{\alpha} \\ b \end{bmatrix} = \begin{bmatrix} \boldsymbol{y} \\ 0 \end{bmatrix} \tag{32}$$

where $V_\gamma = diag\{1/\gamma\bar{v}_1,...,1/\gamma\bar{v}_n\}$; Ω is $n{\times}n$ Hessian vector, which expression is: $\Omega_{i,j} = \langle \varphi(x_i), \varphi(x_j) \rangle_H = K(x_i, x_j)$. $K(.,.)$ is a kernel, which in this study, radial basis function (RBF) is selected as the kernel function of WLS–SVM as follows:

$$K(x_i, x_j) = \exp\left(-\frac{\|x_i - x_j\|^2}{2\sigma^2}\right) \tag{33}$$

Therefore, the resulting WLS–SVM model for the prediction of functions becomes:

$$y(x) = \sum_{i=1}^{n} \alpha_i K(x_i, x) + b \tag{34}$$

Here α and b is the solution to (32). The parameters represents the high dimensional feature spaces that is non–linearly mapped from the input space x. Furthermore, predicting value of x is obtained by the model (34).

7.2 Simulating the seismic responses of dam–water–foundation system

In order to reduce the computational cost of the nonlinear dynamic analysis of concrete gravity dams in the optimization procedure, the maximum principle stresses of concrete gravity dams as seismic responses are predicted using the WLS–SVM regression model instead of directly performing FEA of dams. To achieve this purpose, the WLS–SVM regression model is trained by using a randomly generated database which consists on the combinations of the design variables and the seismic responses of concrete gravity dams. In this study, the WLS–SVM model with the 10–fold cross–validation (CV) [28] is employed to find the optimal values of γ and σ for training the WLS–SVM model. For predicting the maximum principle stresses of concrete gravity dams, the following procedure is implemented that train and test the WLS–SVM regression model based on RBF kernel function:

(1) A database for training and testing the WLS–SVM model is randomly generated based on the vector of the design variables of concrete gravity dam defined in Section (1). This vector is considered as the input of the WLS–SVM model.

(2) For each concrete gravity dam corresponding to a design variable vector in database FEA is performed, and the maximum tensile and compressive principle stresses of dams as the seismic responses of dam are obtained. The seismic responses is considered as the output of the WLS–SVM model

(3) The provided database is divided to training and testing sets on a random basis.

(4) Two WLS–SVM models are trained and tested based on the generated sets for predicting the maximum tensile and compressive principle stresses of dams.

Moreover, several statistical methods, the mean absolute percentage error (*MAPE*), the relative root–mean–squared error (*RRMSE*) and the absolute fraction of variance (R^2), are used to compare predicted and testing values for computing the model validation. The *MAPE*, *RRMSE* and R^2 parameters are calculated as:

$$MAPE = \frac{1}{N_t} \sum_{i=1}^{N_t} 100 \times \left| \frac{y_i - \bar{y}_i}{y_i} \right| \tag{35}$$

$$RRMSE = \sqrt{\frac{N_t \sum_{i=1}^{N_t} (y_i - \bar{y}_i)^2}{(N_t - 1) \sum_{i=1}^{n_t} y_i^2}} \tag{36}$$

$$R^2 = 1 - \left(\frac{\sum_{i=1}^{N_t} (y_i - \bar{y}_i)^2}{\sum_{i=1}^{N_t} \bar{y}_i^2} \right) \tag{37}$$

where y and $\bar{y}$ are actual value and predicted value, respectively; and N_t is the number of testing samples. It is noted that the smaller $RRMSE$ and $MAPE$ and the larger R^2 are indicative of better performance generality [10]. Therefore, the framework of the IGSA–OC with the WLS–SVM model is depicted in Fig. 3.
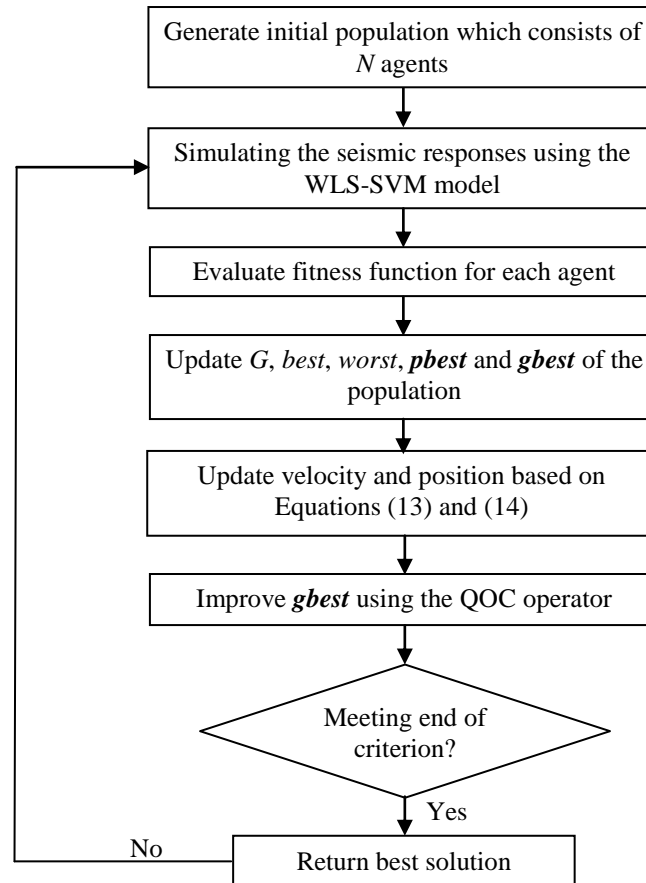

Figure 3. Flowchart of IGSA–OC with the WLS–SVM model

8. TEST EXAMPLE

In this study, Pine Flat dam located on King's River near Fresno California is considered as a real–world structure, in order to investigate the nonlinear effects in the shape optimization of concrete gravity dams. The properties of the dam structure are 400 *ft* height with a crest length of 1840 *ft* and its construction about 9491.94 *kip* concrete [29]. The optimal shape of this dam is found subjected to the S69E component of Taft Lincoln School Tunnel during Kern country, California, earthquake (July 21, 1952) [29]. This component of the recorded ground motion is shown in Fig. 4.
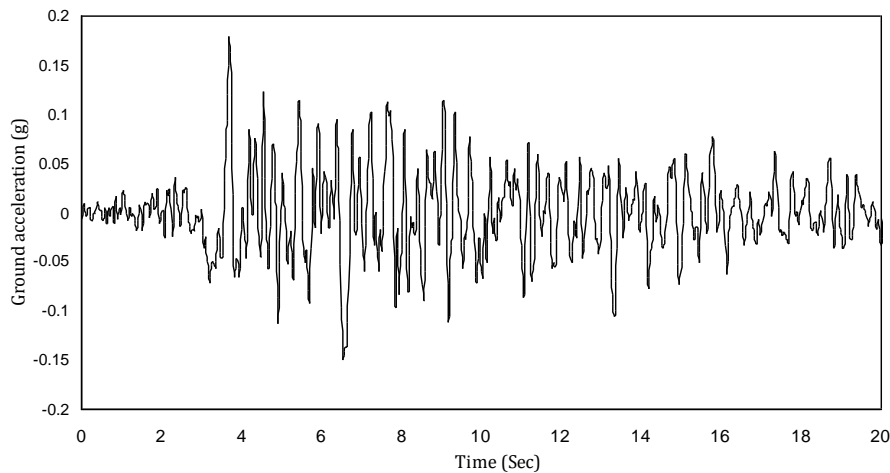

Figure 4. Ground motion at Taft Lincoln Tunnel; Kern country, California, 1952

The lower and upper bounds of the design variables (*X*) defined in Section (2) are considered to find the optimal shape of the Pine flat dam. The bounds of the variables are shown in Table 1 [29]:

Table 1: The lower and upper bounds of design variables

Design variable	Lower bound (*ft*)	Upper bound (*ft*)
B	16.67	39.34
b_1	30.232	34.166
b_3	28.413	34.727
b_4	210.6	257.4
H_2	12.6	15.4
H_4	302.32	341.66
H_5	270	330

To investigate the optimal shape of the selected dam, two cases related to dam–water–flexible foundation rock interaction problem are considered and compared as follows:

Case 1: Dam with the linear effects.

Case 2: Dam with the nonlinear effects.

8.1 Verification of finite element model of Pine Flat gravity dam

An idealized symmetric model of Pine Flat gravity dam–water–foundation rock system for full reservoir is simulated using FEM and depicted on Fig. 5.
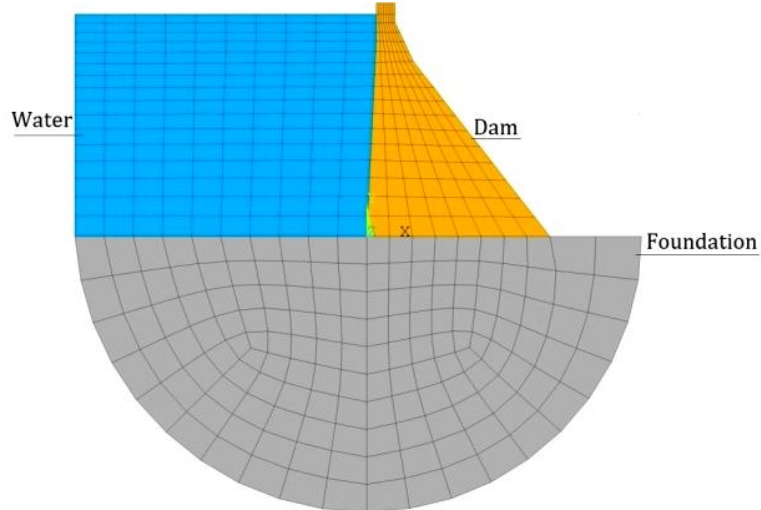

Figure 5. The finite element model of Pine Flat dam–water–foundation rock system

The properties of concrete, water and foundation are given in Table 2 [29]. In order to validate FEM with the employed assumptions in this study, the first natural frequency of FEM of the gravity dam for four cases are determined from the frequency response function.

Table 2: The properties of materials

Material	Property	Value
Concrete		
	Modulus of elasticity (*psi*)	3.25×10^6
	Poisson's ratio	0.2
	Mass density (*lb/ft³*)	155
	Cohesion factor of concrete (*kips*)	17.775
	Angle of internal friction of concrete	$45°$
Water		
	Mass density (*lb/ft³*)	62.4
	Wave velocity (*ft/sec*)	4720
	Wave reflection coefficient	0.817
Foundation		
	Moulus of elasticity (*psi*)	10^7
	Poisson's ratio	0.33

The results are compared with those reported by Chopra and Chakrabarti [29] as given in Table 3.

Table 3: A comparison of the natural frequencies from the literature with FEM

Case	Foundation rock condition	Water	Natural frequency (Hz)		Error (%)
			Chopra and Chakrabarti [29]	The present study	
1	Rigid	Empty	3.1546	3.152	0.082
2	Rigid	Full	2.5189	2.525	0.242
3	Flexible	Empty	2.9325	2.93	0.085
4	Flexible	Full	2.3310	2.383	2.18

It can be observed from the results of Table 3 that a good conformity has been achieved between the results of the present work with those reported in the literature.

8.2 Training and testing the WLS–SVM model

To generate a database for training and testing the WLS–SVM model, the design variable vector of earth dam with the specific height, defined in Section (1), is considered as the input vector of WLS–SVM, and the maximum compressive and tensile principal stresses in the dam body are taken as the output of the WLS–SVM model. For achieving this purpose, first, design of computer experiments is employed by generating a set of combinations of the design variables. This set is spread in the entire design variables by design of computer experiments. In this study, Latin Hypercube Design (LHD) proposed for computer experiments [30] is used for generating 150 concrete gravity dam samples. The maximum compressive and tensile principal stresses in the dam body of all dam samples subjected earthquake load are obtained using FEA. Then, the samples are selected on a random basis and from which 70% and 30% samples are employed to train and test the WLS–SVM model. The performance generality of WWLS–SVM in testing mode associated with the maximum compressive and tensile principal stresses in the dam body are shown in Figs. 6 and 7 in terms of the absolute percentage errors (APE).
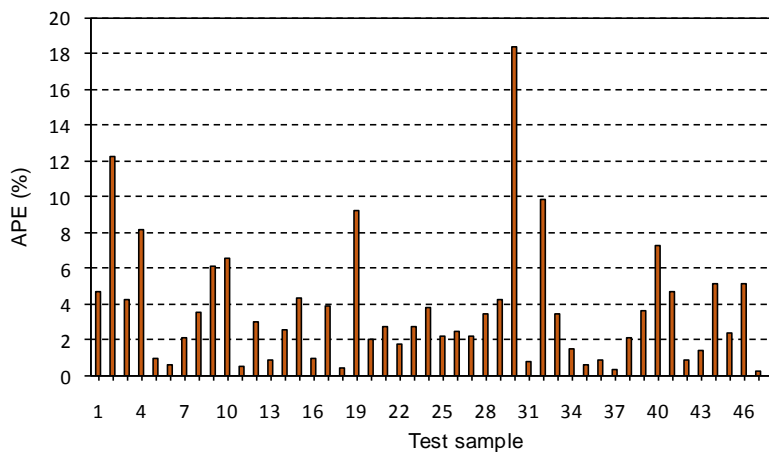

Figure 6. Absolute percentage errors associated with the maximum tensile principal stress
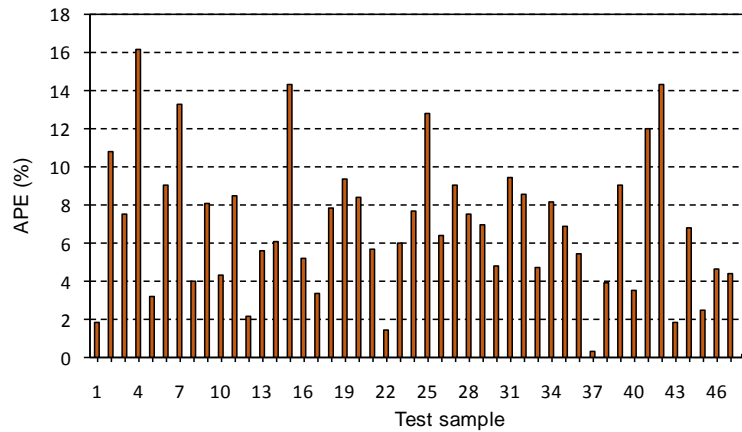
Figure 7. Absolute percentage errors associated with the maximum compressive principal stress

The displayed results in these figures demonstrate that the WWLS–SVM models achieve a good performance generality in predicting the responses of concrete gravity dam samples. Furthermore, the results of testing the performance generality of the WLS–SVM models based on the statistical values of *MAPE*, *RRMSE* and R^2 are given in Table 4:

Table 4: Statistical values for the WLS–SVM model in testing mode.

Case	Statistical values		
	MAPE	*RRMSE*	R^2
The maximum tensile principal stress	3.67	0.0292	0.9974
The maximum compressive principal stress	6.76	0.0385	0.9861

As seen from results of Table 4, the performance generality of the WLS–SVM models is good and therefore it can be incorporated into the optimization process as a powerful tool for predicting the time history responses of the structures.

8.3. *Optimization results*

In order to consider the stochastic nature of the optimization process, ten independent optimization runs are performed for the selected dam with the nonlinear and linear effects and the four best solutions are reported in Tables 5 and 6. It is noted that the optimum designs of the dam for the nonlinear and linear effects are obtained using the IGSA–OC and the WLS–SVM regression model.

By comparing the solutions obtained for the dam with the nonlinear and linear effects, it can be found that the nonlinear effects can significantly increase the optimal weight of dam. The optimal designs of the dam are also analyzed by an accurate FEA and the value of constraint for all design cases is not violated. Hence, the WLS–SMV regression model can be used reliably to predict the seismic responses of concrete gravity dams in the optimization procedure.

Table 5: Optimal solutions of concrete gravity dam for the nonlinear effects

Variables	Optimization design			
	Case 1	Case 2	Case 3	Case 4
b (ft)	34.67	33.49	38.76	33.69
b_1 (ft)	34.16	34.16	34.17	34.17
b_2 (ft)	33.60	34.73	32.96	32.6418
b_3 (ft)	257.40	250.86	248.31	257.40
H_2 (ft)	15.40	15.40	15.40	15.18
H_4 (ft)	336.33	341.66	338.00	341.66
H_5 (ft)	312.90	320.45	312.64	311.79
Concrete weight (lb)	11049	11058	11047	10928
Mean of the best solutions	11020.5			

Table 6: Optimal solutions of concrete gravity dam for the linear effects

Linear variables	Optimization design			
	Case 1	Case 2	Case 3	Case 4
b (ft)	28.55	35.00	31.78	30.46
b_1 (ft)	33.63	34.16	34.11	34.17
b_2 (ft)	34.73	34.73	34.73	31.58
b_3 (ft)	241.81	241.55	246.24	230.78
H_2 (ft)	15.40	15.40	15.40	15.11
H_4 (ft)	320.61	341.66	325.76	331.54
H_5 (ft)	296.58	309.20	314.16	302.81
Concrete weight (lb)	9945	10679	10657	9819
Mean of the best solutions	10275.23			

9. CONCLUSIONS

An efficient optimization procedure is introduced to find the optimal shapes of concrete gravity dams including dam–water–foundation rock interaction and nonlinear effects for earthquake loading. The concrete gravity dam body is treated as a two dimensional structure involving the geometry and material nonlinearity effects using the Drucker–Prager model based on the associated flow rule. The weighted least squares support vector machine (WLS–SMV) regression model is utilized to approximate the nonlinear dynamic analysis of dam instead of directly performing it by a time consuming finite element analysis.

The optimum solutions obtained for the dam with the nonlinear and linear effects reveal that the nonlinear effects can significantly increase the optimal weight of dam. Therefore, the nonlinear effects should been considered in the optimal design of concrete gravity dams. The saftey can also be obtained by the optimal design of concrete garvity dams with the nonlinear effects. Finally, it is demonstrated that the best solution has been attained by using the WLS–SMV regression model, in terms of the accuracy and degree of feasibility of the solutions and therefore it can be reliably incorporated into the optimization process of the concrete gravity dams subjected to the earthquake loads.

REFERENCES

1. Akbari J, Ahmadi MT, Moharrami H. Advances in concrete arch dams shape optimization, *App Math Model* 2011; **35**: 3316-33.
2. Seyedpoor SM, Salajegheh J, Salajegheh E, Golizadeh S. Optimum shape design of arch dams for earthquake loading using a fuzzy inference system and wavelet neural networks, *Eng Optim* 2009; **41**: 473-93
3. Kaveh A, Mahdavi VR. Optimal design of arch dams for frequency limitations using charged system search and particle swarm optimization, *Int J Optim Civil Eng* 2011; **4**: 543-555.
4. Kaveh A, Mahdavi VR. Shape optimization of arch dams under earthquake loading using meta-heuristic algorithms, *KSCE J Civ Eng* 2013; **7**:1690-1699.
5. Kaveh A, Mahdavi VR. Colliding bodies optimization for optimal design of arch dams with frequency limitations, *Int J Optim Civil Eng* 2014**; 4**: 473-490.
6. Kaveh and R. Gaffarian*,* Shape optimization of arch dams with frequency constraints by enhanced charged system search algorithm and neural network, *Int J Optim Civil Eng* 2015; **13**: 102-111.
7. Mahani AS, Shojaee S, Salajegheh E. Khatibinia M. Hybridizing two–stage meta–heuristic optimization model with weighted least squares support vector machine for optimal shape of double–arch dams, *Appl Soft Comput* 2015, **27**: 205-18.
8. Salajegheh J, Salajegheh E, Khatibinia M, Khosravi Sh. A hybrid meta–heuristic method and weighted least squares support vector machine method for the optimal shape design of gravity dams, in: *Proceedings of the Eleventh International Conference on Computational Structures Technology*, B.H.V. Topping (Editor), Civil–Comp Press, Dubrovnik, Croatia, 2012.
9. Salajegheh J, Khosravi Sh. Optimal shape design of gravity dams based on a hybrid meta–heruristic method and weighted least squares support vector machines, *Int J Optim Civil Eng* 2011; **4**: 609-32.
10. Khatibinia M, Khosravi S. A hybrid approach based on an improved gravitational search algorithm and orthogonal crossover for optimal shape design of concrete gravity dams, *Appl Soft Comput* 2014; **16**: 223-33.
11. Deepika R, Suribabu C. Optimal design of gravity dam using differential evolution algorithm, *Int J Optim Civil Eng* 2015; **5**: 255-66.
12. Kaveh A, Zakian P. Stability based optimum design of concrete gravity dam using CSS, CBO, and ECBO algorithms, *Int J Optim Civil Eng* 2015; **5**: 419-31.
13. USBR, *Design of Gravity Dams, Design manual for Concrete Gravity Dams*, U.S. government printing office, 1976.
14. Rajeev S, Krishnamoorthy CS. Discrete optimization of structures using genetic algorithms, *J Struct Eng ASCE* 1992; **118**(5): 1233-50.
15. Salajegheh E, Gholizadeh S, Khatibinia M. Optimal design of structures for earthquake loads by a hybrid RBF–BPSO method, *Earthq Eng Eng Vibrat* 2008; **7**(1), 14-24.
16. Khatibinia M, Naseralavi SS. Truss optimization on shape and sizing with frequency constraints based on orthogonal multi–gravitational search algorithm, *J Sound Vib* 2014, **333**(24): 6349-69.

17. Khatibinia M, Salajegheh E, Salajegheh J, Fadaee MJ. Reliability–based design optimization of RC structures including soil–structure interaction using a discrete gravitational search algorithm and a proposed metamodel, *Eng Optimiz* 2013; **45**(10): 1147-65.

18. Rashedi E, Nezamabadi–pour H, Saryazdi S. GSA: a gravitational search algorithm, *Inform Sci* 2009; **179**: 2232-48.

19. Fang KT, Wang Y. *Number–theoretic methods in statistics*, Chapman and Hall, New York, 1994.

20. Leung YW, Wang Y. An orthogonal genetic algorithm with quantization for global numerical optimization, *IEEE T Evolut Comput* 2001; **5**: 41-53.

21. Chopra AK. *Dynamics of Structures: Theory and Applications to Earthquake Engineering*, Prentice Hall, Englewood Cliffs, 2000.

22. Kucukarslan S. Dynamic analysis of dam–reservoir foundation interaction in time domain, *Comput Mech* 2004, **33**: 274-81.

23. Kucukarslan S, Coskun B, Taskin B. Transient analysis of dam–reservoir interaction including the reservoir bottom effect, *J Fluid Struct* 2005, **20**:1073-84.

24. Akkose M, Adanur S, Bayraktar A, Dumanoglu AA. Elasto–plastic earthquake response of arch dams including fluid–structure interaction by the Lagrangian approach, *Appl Math Model* 2008; **32**(11): 2396-2412

25. Chen WF. *Plasticity in Reinforced Concrete,* McGraw–Hill, New York, 1982.

26. Khatibinia M, Fadaee MJ, Salajegheh J, Salajegheh E. Seismic reliability assessment of RC structures including soil–structure interaction using wavelet weighted least squares support vector machine, *Reliab Eng Syst Safe* 2013; **110**: 22-33.

27. Gharehbaghi S, Khatibinia M. Optimal seismic design of reinforced concrete structures under time history earthquake loads using an intelligent hybrid algorithm, *Earth Eng Eng Vibrat* 2015; **14**(1): 97-109.

28. Suykens JA, Brabanter JD, Lukas L, Vandewalle J. Weighted least squares support vector machines: robustness and sparse approximation, *Neurocomputing* 2002; **48**: 85-105.

29. Chopra AK, Chakrabarti P. Earthquake response of concrete gravity dams including hydrodynamic foundation interaction effects, UCB/EERC–80/01 Report, University of California, Berkeley, USA, 1980.

30. Mckay MD, Beckman RJ, Conover WK. A comparison of three methods for selecting values on input variables in the analysis of output from a computer code, *Technometrics* 1979; **21**, 439-45.