A HYBRID ALGORITHM FOR THE OPEN VEHICLE ROUTING PROBLEM

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ABSTRACT

The open vehicle routing problem (OVRP) is a variance of the vehicle routing problem (VRP) that has a unique character which is its open path form. This means that the vehicles are not required to return to the depot after completing service. Because this problem belongs to the NP-hard problems, many metaheuristic approaches like the ant colony optimization (ACO) have been used to solve OVRP in recent years. The versions of ACO have some shortcomings like its slow computing speed and local-convergence. Therefore, in this paper, we present an efficient hybrid elite ant system called EHEAS in which a new state transition rule, tabu search as an effective local search algorithm and a new pheromone updating rule are used for more improving solutions. These modifications avoid the premature convergence and make better solutions. Computational results on sixteen standard benchmark problem instances show that the proposed algorithm finds closely the best known solutions for most of the instances in which ten best known solutions are also found. In addition, EHEAS is comparable in terms of solution quality to the best performing published metaheuristics.

1. INTRODUCTION

The Open Vehicle Routing Problem (OVRP) is a famous important extension of the vehicle routing problem (VRP) that has many applications in industrial and services (Figure 1). The

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description of this important variant of the VRP appeared in the literature over 30 years ago, but has just recently attracted the attention of scientists and researchers [1, 2]. Today, the OVRP is envisaged in a lot of practices such as the home delivery of packages and distribute newspapers. Furthermore, companies that use contractors to deliver newspapers to residential customers do not require the contractors and their vehicles to return to the depot. As a result, researcher interest in the OVRP has increased dramatically and a wide variety of new algorithms have been developed to solve the problem over the last ten years. This problem similar to VRP involves routing a homogeneous fleet of vehicles with fixed capacity Q that start to move simultaneously from the depot, but not come back to the depot after visiting customers. In other words, each route in the OVRP is a Hamiltonian path and maybe a route-length constraint in order to limit the maximum distance traveled by each vehicle. Each customer has a known demand and is serviced by exactly one visit of a single vehicle. The objective is to design a set of minimum cost routes to serve all customers [3]. In addition, we need to find the minimum number of vehicles required to deliver goods to all of the customers.
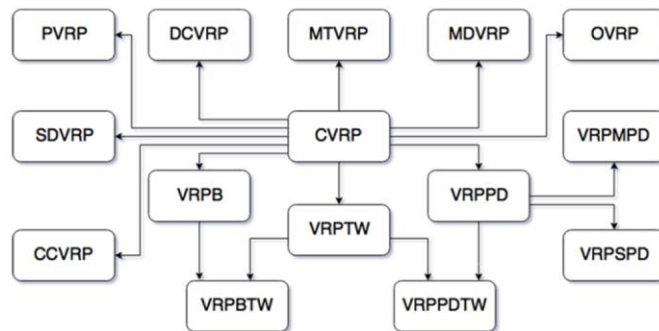

Figure 1. The versions of VRP

From the point of view of graph theory, the difference of the OVRP with the VRP is that a solution is a set of Hamiltonian paths, rather than Hamiltonian cycles. On the other hand, the OVRP turns out to be more common than the VRP, in the sense that any closed version with n customers can be converted into an open version of VRP with n customers, but transformation in the reverse direction is not possible. Figure 1 shows the feasible solutions to both the open and closed version of VRP for the same input data in which all customers have unit demands and the vehicle capacity is four units. In general, this figure shows the fact that the feasible solution for the open version of the VRP can be fairly different from that for the closed version. In this figure, the depot is represented by a square and the customers by circles.
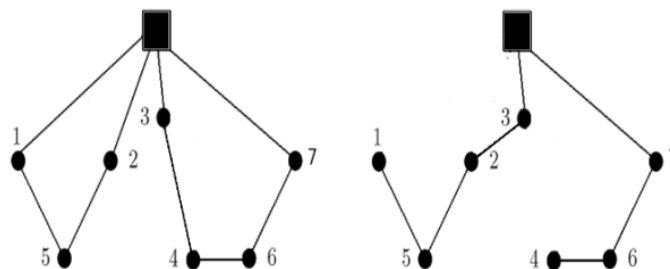

Figure 2. Two different feasible solutions for the open and close versions of VRP

As it was mentioned before, the OVRP is a main problem happening in some distribution systems, the school bus. Therefore, it has attracted significant researchers and some algorithms have been proposed in order to solve effectively. Because finding of the best Hamiltonian path for each set of customers is NP-hard [4], the OVRP is also NP-hard. According to some shortcomings like its slow computing speed and local-convergence in ACO, the basic of this algorithm cannot directly apply to the problem with acceptable performance and few researchers have proposed new methods to improve the original ACO and applied them. Besides, although the development of modern meta-heuristics has led to considerable progress, every meta-heuristic algorithm has its own weakness and strength. Therefore, in order to achieve the effectiveness and efficiency of the proposed algorithms, much researchers has tried to improve the quest for the performance of hybrid algorithms. As a result, in this work, we proposed an efficient hybrid ant colony algorithm called EHEAS in order to improve both the performance of the algorithm and the quality of so the solutions.

The proposed algorithm used ant colony optimization algorithms (ACO) for solving OVRP and then improved the global ability of the algorithm through importing new probability function of movement for constructing solutions, updating pheromone and using tabu search as an effective local search. The EHEAS algorithm in order to be not trapped at the local optimum, discover different parts of the solution space. The results in the fourteen instances proposed by Christofides, show that the proposed algorithm can obtain high quality solutions such that the average quality of Gap is 0.26% for these instances. Besides, the average quality of Gap for the EHEAS is 0% when only the travel distance was minimized while for the two instances proposed by Golden. The algorithm is, also, compared with a number of metaheuristic, evolutionary, local search and nature inspired algorithms from the literature. The experimental results have shown that the EHEAS algorithm is to be very efficient and competitive in terms of solution quality.

The structure of the remainder of the paper is as follows. In the next section, related works on OVRP is presented and then a mixed integer linear programming of OVRP is presented. The proposed idea based on ant colony optimization (ACO) called EHEAS is specially explained in section 4. The EHEAS mainly consists of the iteration of the three steps, including each ant builds the solution independently, apply the local search algorithms to improve the solution, and update the global pheromone information. In this section, we describe each step in more details. In Section 5, the proposed algorithm is compared with some of the other algorithms on standard problems belonged in OVRP library. Some concluding remarks are given in the final section.

2. RELATED WORKS

Contrary to the VRP, the OVRP has only been considered by very limited people from the early 1980s to the late 1990s. However, several researchers have used some algorithms especially metaheuristic since 2000. So far as we know, the first author to declare the OVRP was Schrage in 1981 who dedicated to the description of realistic routing problems [5]. Nevertheless, the other earliest work that addressed to solve the OVRP seems to be that of Sariklis and Powell [6], who do not force a maximum route length. Because the OVRP

belongs to NP-hard problems, most of the practical examples of this problem cannot be solved by exact algorithms to optimality within reasonable time and the algorithms used in practice are the heuristic and metaheuristic algorithms. These approaches can find the optimal or near optimal solutions in within a reasonable computing time. For example, an efficient tabu search is proposed by Brandão [7] in which infeasibilities in middle solutions are managed through penalizing the objective function by two penalty terms including capacity violation and route length violation. A tabu search algorithm also is proposed by Fu et al. [8, 9] in which the initial solution is provided by a 'furthest first heuristic' and exchanges are based on the two-interchange generation mechanism. In this algorithm, a combination of vertex reassignment, 2-opt, vertex swap, and 'tails' swap within the same route or between two routes are used simultaneously. Tarantilis et al. offered a single-parameter metaheuristic method for solving a version of the OVRP in which the objective is to minimize the total distance covered without attempting directly to decrease the number of vehicles [10]. Li et al. [11] develop a variant of record-to-record travel algorithm for the standard OVRP that avoids the premature convergence and found very good solutions in a short computing time.

Also, Pisinger and Ropke [12] offer an effective metaheuristic based on adaptive large neighbourhood algorithm in which customers are removed randomly from the current position and reinserted in the place with cheapest possible route. Furthermore, for diversify and intensify the search, some removal and insertion heuristics are used. Moreover, several famous metaheuristics have been proposed for the versions of the OVRP involving only capacity constraints. For example, Tarantilis et al. offered a population-based algorithm and a heuristic based on threshold-accepting type for solving the OVRP in 2005 [13].

Bodin et al. [14] defined the OVRP encountered by FedEx in generating open delivery routes for airplanes. In this problem, an airplane starts to move from Memphis, makes deliveries to several cities, and does not come back to Memphis. After that, the airplane rests in the last city on the delivery route and begins its pickups from that city. Fu et al. described two further areas of the OVRP applications involving the planning of train services and a set of school bus routes. In the first problem, train starts or ends at the Channel Tunnel and in the second problem pupils are picked up at various locations and brought to school in the morning. Besides, the routes are reversed to take pupils home in the afternoon. A description of a problem of express airmail distribution in the USA is defined by Bodin et al in 1983 in which there is an open pick-up and delivery VRP with capacity constraints and time windows.

Bodin et al. describe a heuristic algorithm used by FedEx to develop an open route for each airplane. In this problem, drivers move to the FedEX depot each morning, load packages, and then make deliveries to residences, according to the couriers and vehicles which do not return to the depot after their last deliveries. Marinakis et al presented a relatively new swarm intelligence algorithm called the BBMO that simulates the mating behavior for solving the OVRP [15]. The main contribution of the work is that the equation which describes the movement of the drones outside the hive has been replaced by a local search procedure. For testing the quality of the algorithm, two sets of instances were considered and the obtained results show that the proposed algorithm found very satisfactory in most instances.

A real-world problem is proposed by an international company in Spain and modeled as a

variant of OVRP by López-Sánchez et al [16]. In this problem, the maximum time spent on the vehicle by one person must be minimized. So, a metaheuristic algorithm in order to obtain high quality solutions is proposed. In order to analyze the algorithm, 19 school-bus routing problems of the literature on 9 hard real-world instances are considered.

Brito et al. proposed the close‑open VRP where the routes can be opened and closed [17]. This variant nowadays is a standard practice model in business. Furthermore, they formulate a model of this novel variant with time windows and a hybrid metaheuristic is proposed for its solutions. This algorithm is applied to a real problem with outsourcing. Finally, Erbao et al. proposed the OVRP with uncertain demands. In this paper, firstly the customer᾽s demand is described, and then an optimization model in order to achieve to minimize transportation costs is proposed. Also, they propose four strategies to handle with the uncertain demand and an improved evolution algorithm to solve the robust model. Furthermore, the performance of four different robust strategies is analyzed by considering the extra costs and unmet demand [18].

The OVRP with decoupling points (OVRP-DP) introduced by Atefi et al. [1] is faced by companies dealing with carriers to ship their goods over large territories. In this case, it may be profitable to use more than one carrier to perform a specific expedition: the first one leaves the depot and performs part of the deliveries, drops off all remaining load, and the second carrier continues from that point onwards. This drop off location is called the decoupling point of the route. This problem generalises the classical OVRP in which each route must be performed by only one carrier. Sevkli et al. [19] modeled and optimized a real-world newspaper delivery problem for a media delivery company in Turkey by reducing the total cost of carriers as real-world OVRP and proposed a new multi-phase oscillated variable neighbourhood search algorithm to solve it. Hosseinabadi et al. [20] proposed a new combinatorial algorithm named OVRP_GELS based on gravitational emulation local search algorithm for solving the OVRP. They also used record-to-record algorithm to improve the results of the GELS. Niu et al. [21] described the mathematical model of the green OVRP with time windows based on the comprehensive modal emission model and designed a hybrid tabu search algorithm involving several neighborhood search strategies to solve this problem. They also performed the experiments on realistic instances based on the real road conditions of Beijing, China and analyzed the effect of empty kilometers through comparing different cost components. Compared with closed routes, the open routes reduced the total cost by 20% with both the fuel emissions costs and the CO2 emissions cost down by nearly 30%. Also Niu et al. [22] proposed a green OVRP model with fuel consumption constraints for outsourcing logistics operations and presented a hybrid tabu search algorithm to deal with this problem. Yu et al. [23] proposed the open VRP with cross-docking in which a general example in retail is presented wherein the capital expenditure necessary in vehicle acquisition can become a burden for the retailer, who then needs to consider outsourcing a logistics service as a cost effective option. This practical scenario can be applied to create an open flow network of routes.

3. A MODEL OF OVRP

From the graph's theoretical viewpoint, the main difference between OVRP and VRP is finding Hamiltonian paths instead of Hamilton cycles. The OVRP is an extension of the basic VRP that can be described as follow:

There is a set of customers geographically dispersed within a distance radius. Demand customers should be served through a set of vehicles with limited capacity. The aim is designing the least cost open routes for delivery of goods from a depot to a set of customers with the following conditions:

- Each vehicle is not returned to the depot after visiting the last customer and the delivery process is ended as soon as the last customer is served.
- The total demands of all customers in each route should not exceed of the capacity vehicle.
- The routes must be designed such that each customer is visited only once by exactly one vehicle.
- Two different objectives are used in OVRP, the first one is the minimization of the required number of vehicles and the second one is the minimization of the corresponding total traveled distance.

Imagine an incomple graph $G = (V, E)$, where $V = \{1, 2, \dots, n\}$ is the node set in which node 1 represents the depot, and nodes $2, \dots, n$ represent the customers. $E = \{(i, j) | i \neq j \text{ and } i, j \in V\}$ is the set of edges. To present the mathematical formulation for the OVRP, we consider the below variables and parameters:

v: number of serving vehicles.

n: number of customer nodes (excluding the depot node).

q_i: demand of costumer i.

Q_k: capacity of the kth vehicle.

c_{ij}: travel $costs$ from customer i to customer j.

w_k: $cost\ of\ activation\ of\ vehicle\ k$

$$x_{ij}^k = \begin{cases} 1 & \text{if vehicle } k \text{ travels from customer } i \text{ to customer } j \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

$$z_k = \begin{cases} 1 & \text{if vehicle k is active} \\ 0 & \text{Otherwise} \end{cases} \tag{2}$$

If the company contracts its delivery or pick-up activities to external carriers, w_k represents the hiring cost of vehicle k and, in cases where company owns its own vehicle fleet, w_k is a one-time cost and is related to the fixed costs for the acquisition of vehicle k. The vehicle k is active when it services at least one customer. With these variables and parameters, the OVRP mathematical model is described as follows:

$$\min \sum_{k=1}^{v} w_k z_k \tag{3}$$

$$min \sum_{i=0}^{n} \sum_{j=0}^{n} \sum_{k=1}^{v} c_{ij} \times x_{ij}^{k} \tag{4}$$

$$\sum_{k=1}^{v} \sum_{i=1}^{n} x_{ij}^{k} = 1 \quad , \forall j = 2, \dots, n \tag{5}$$

$$\sum_{k=1}^{v} \sum_{j=1}^{n} x_{ij}^{k} = 1 \quad , \forall i = 2, \dots, n \tag{6}$$

$$x_{ij}^{k} \leq z_{k} \,, \forall i, j = 2, \dots, n \tag{7}$$

$$\sum_{i=1}^{n} x_{iu}^{k} - \sum_{j=1}^{n} x_{uj}^{k} = 0 \,, \forall k = 1, \dots, v \,, \forall u = 1, \dots, n \tag{8}$$

$$\sum_{i=1}^{n} q_{i} (\sum_{j=1}^{n} x_{ij}^{k}) \leq Q_{k} \quad , \forall k = 1, \dots, v \tag{9}$$

$$\sum_{j=2}^{n} x_{1j}^{k} \leq 1 \qquad \forall k = 1, \dots, v \tag{10}$$

$$\sum_{i=2}^{n} x_{i1}^{k} = 0 \qquad \forall k = 1, \dots, v \tag{11}$$

Function (3) minimizes the total number of vehicles and the total travelled distance is minimized by function (4). Constraints (5) and (6) ensure that each customer is served precisely by only one vehicle. Constraint (7) confirm that all customers are serviced by active vehicles, and Constraints (8) are the typical flow conservation equations that guarantee the continuity of each vehicle route .Constraints (9) confirm that demands of all customers on a route not   exceed the capacity of the vehicle. Equations (10) and (11) referred to the condition of the OVRP in a delivery process that ensures any vehicle that departs from the depot, in order to service a sequence of customers, will not return to the depot
.

4. THE PROPOSED ALGORITHM

The ant colony optimization (ACO) is one of the famous meta-heuristic algorithms used for solving combinatorial optimization problems that do not have a known effective algorithm. This algorithm was inspired by the behavior of real ant colonies in nature in order to find routes between their nests and food sources. As some ants travel, they deposit amount of pheromone trail that other ants are interested to follow them. This natural behavior of ants can be used to explain reasons that they can find the shortest path. Dorigo et al. used this concept and proposed the ACO to solve the combinational optimization problems in 1991 [24]. One of the most important problems that the ACO is used for solving is TSP. Figure 3 shows the steps of this algorithm in details.
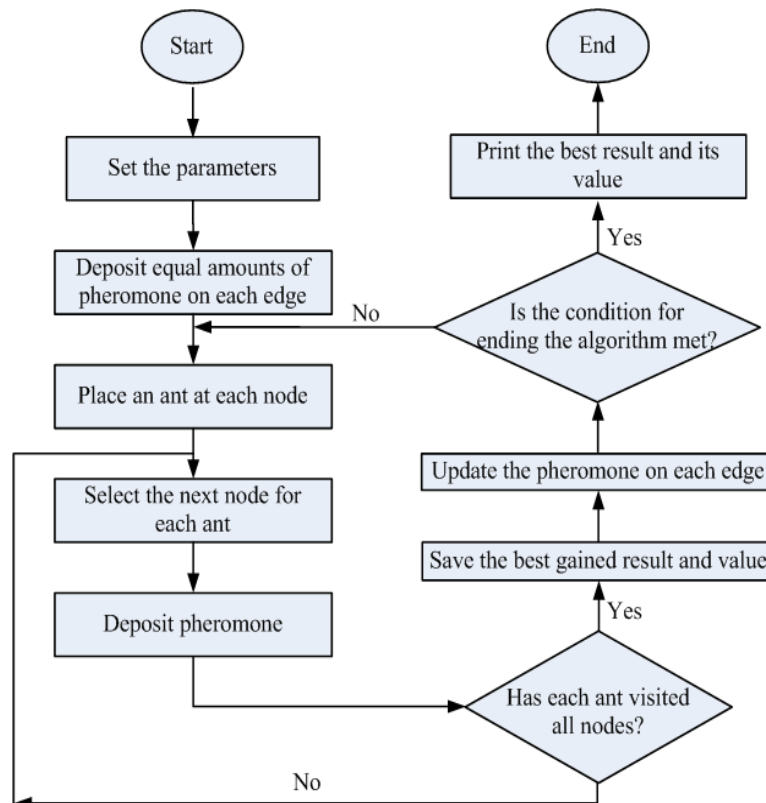
Figure 3. The steps of ACO for solving the TSP

In this section, a hybrid efficient elite ant system (EHEAS) is proposed to solve the OVRP. This EAS is strongly inspired by ant system (AS), achieves performance improvements through the introduction of new mechanisms based on ideas not included in the original AS. The proposed algorithm improved the EAS algorithm through ranking the solutions constructed by ants. Furthermore, several modifications as a method of update pheromone and using modified tabu search are used to more improve the EHEAS. These lead to avoid premature convergence and then search over the subspace. In other words, the EHEAS which uses the tabu search as an improved procedure has made three main contributions:

1. The proposed EAS presents a new transition rule in order to find the better customer for each vehicle in every iteration.
2. In addition to encourage the obtained best solution until now, the best solution in each iteration is considered and released with pheromone.
3. The vast literature on algorithms tells us in order to find high-quality solutions by metaheuristics, a powerful local search algorithm is required. Therefore, to improve the best found solution until now of the proposed EAS algorithm, the tabu search is used as a local search algorithm until the best obtained solution is not improved for five iterations.
4. The proposed tabu search algorithm comprises three kinds of neighborhood algorithms including 2-Opt, 0-1 and 1-1 exchanges. These moves are distinguished in terms of exchanges performed to convert one tour into another.
5. To improve the TS further, the size of tabu list is considered minimum and maximum

values for the diversification and intensification policies respectively.

The first phase of the EHEAS is solution construction in which for n groups, m ants are initially positioned on n vertices randomly and each ant of the colony efforts to build a solution represented as a single route. Then, ants use pheromone trail and heuristic information in order to obtain feasible solutions in the process of constructing solutions. Like AS, the next node j from node i in the route is selected by ant k among the unvisited nodes J_i^k, according to the following transition rule in formula (12).

$$P_{ij}^k(t) = \frac{\tau_{ij}^\alpha(t)\kappa_{ij}^\beta(t)}{\sum_{r\in J_i^k}\tau_{ir}^\alpha(t)\kappa_{ij}^\beta(t)} \qquad \forall j \in j_i^k \tag{12}$$

where $\tau_{ij}(t)$ is the amount of pheromone on the edge joining nodes i and j and $\kappa_{ij}(t)$ is defined as the savings of combining two nodes on one tour as opposed to serving them on two different tours. The savings of combining any two customers i and j are computed as $\kappa_{ij} = c_{i0} + c_{0j} - c_{ij}$ where node 0 is the depot and c_{ij} denotes the distance between nodes i and j. Furthermore, α and β are control parameters

The pheromone updating of EAS includes local and global updating rules. Same the AS, the pheromone of all edges belonging to the route obtained by ants called local updating will be updated in EAS. In addition local updating, the EAS uses global updating after producing the best solution of n obtained solution for the problem in the current iteration. In other words, when the best solution of current iteration is found, the best solution until now is updated and if this solution is changed, the proposed modified tabu search is used to improve this solution more. After return the solution from tabu algorithm as the best solution until now, the global updating is applied. In more details, the arcs belonging to the best route until now (T^*) and the best route obtained in the current iteration (T^c) are released with pheromone and are encouraged with the constant coefficient e in the following way; This process causes that the arcs belonging to the best route in any iteration and in current iteration are more highlighted, and to be updated according to the value of the best route L^{gb} and L^{cb}. Note that, the above operator indicates; the less the value of L^{gb} or L^{cb}, the more pheromone released on the arcs. In the proposed algorithm when the best solution until now is changed, the modified tabu search algorithm is used in order to more improve the new solution. The modified tabu search will be described in details in the next sub-section. Thus the formula (3) shows the updating pheromone in the proposed algorithm.

$$\tau_{ij}(t+1) = (1-\rho).\tau_{ij}(t) + \sum_{k=1}^{m}\Delta\tau_{ij}^k(t) + \Delta\tau_{ij}^{gb}(t) + \Delta\tau_{ij}^{cb}(t) \tag{13}$$

where

ρ: A parameter in the range [0, 1] that regulates the reduction of pheromone on the edges.

$\Delta \tau_{ij}^{k}(t)$: The formula of local updating the pheromone which ants passing over the arc between nodes i and j, release some pheromone on it. The value of released pheromone is one over the value passed yet.

$T^{*} \, and \, T^{c}$:The collection of arcs passed over by the ant with the best solution until now (the best solution in current iteration).

$$\Delta \tau_{ij}^{gb}(t) = \begin{cases} e \, / \, L^{gb}(t) & (i, j) \in T^{*} \\ 0 & (i, j) \notin T^{*} \end{cases} \qquad (14)$$

$$\Delta \tau_{ij}^{vb}(t) = \begin{cases} e' \, / \, L^{cb}(t) & (i, j) \in T^{c} \\ 0 & (i, j) \notin T^{c} \end{cases} \qquad (15)$$

e and e' : constants coefficient determined by the ser.

At this stage, the final condition is checked and if it is met, the algorithm ends. Otherwise, the algorithm is iterated by returning to transition rule step. To end the loop, two conditions must be met: the iteration of algorithm n times or the best found solution until now is iterated 10 time. These conditions are checked at the end of each algorithm iteration. If any one of the conditions is met, the algorithm ends and the obtained results and values up to now are considered as the best values and results of the algorithm. The Figure 4 shows the pseudo code of the EHEAS.

Initialize pheromone trails;

For u:=1 to nn do   // nn=number of nodes of the s^{*} //

Begin

    For i:=1 to nn do

    Begin

        Construct a solution S_t by using formula 12; Local update pheromone trails for S_i

        If $f(s_i) < f^{*}$ then

        Begin

           Apply Tabu search in s_i

           $s^{*} = s_i$; // s^{*} is the best solution found until now by the EHEAS//

           $f^{*} := f(s_i)$; // f^{*} is value of s^{*} //

        End   // save the best so far solution //

    End;

    Global update pheromone trails for s^{*} and the best solution in current iteration.

    If the best solution until now is not changed for ten iterations the Break.

End

s^{*} and f^{*} are shown.

End // procedure //

Figure 4. Pseudo code of EHEAS

4.1 The proposed tabu search

The TS is one of the most powerful metaheuristic algorithms which is able to produce near optimal solutions within reasonable computing time. As a local search technique, TS moves from a current solution to the best solution in its neighborhood in order to escape from premature local optimum at each iteration. The main principle of this method is that the TS accepts a new solution which fails the current objective function value. The proposed TS (MTS) requires an initial solution so the best solution obtained until now in the EAS is considered as initial solution. The MTS comprises three types of neighborhood moves including 2-Opt, insert and swap moves. The most commonly encountered move is the 2-Opt used in one or multiple route. In multiple routes, edges (i,i+1), and (j,j+1) which form a criss-cross and belong to different routes are considered and the 2-Opt move is applied.

The insert move transfers a node from its position in one route to another position in a different route. In the swap move, two nodes from different routes are selected and changed. The same procedure is conducted in the case of multiple routes. It is noted that the moves 2-Opt, insert and swap are approved if each one improves the objective function and satisfies in constraints. In the proposed algorithm, although all the customers are candidates to be moved, *n* numbers of neighborhoods are produced by the mentioned algorithms in which 30, 35 and 35 percent of them belong to 2-Opt, insert and swap exchanges respectively. It is noted that these moves are not equally performed in each iteration for two below reasons:
1. To diversify the search.
2. To keep the computing time at reasonable levels.

The Tabu List (TL) is one of the most important concepts of the TS. This list of the proposed MTS is used to prevent the return to the most recently visited solutions for a specific number of iterations (Tabu Tenure) in order to avoid cycling. On the other hand, some of the tabu solutions, which must now be avoided, could be of excellent quality and might not have been visited. To discount this problem, "aspiration criteria" is introduced and used. In the proposed algorithm, an aspiration criterion is defined as solution which has better quality than the current best solution. In other words, the proposed algorithm moves from the current solution to the best solution in its neighborhood that should be not in the TL or satisfies some aspiration criteria.

In order to more improve quality of the proposed algorithm, a good balance between intensification and diversification are required in the proposed algorithm. For a strong diversification technique in the proposed algorithm, the size of TL is considered as a variable. In more details, if the MTS cannot improve the best known solution for a pre-specified number of iterations, direction of the proposed algorithm should change towards a part of solution space which has not been explored yet (diversification policy). Therefore, the length of TL is increased. After the diversification policy, the search process is increased by declining the value of the TL for a number of consecutive iterations. At this stage, if the TS cannot improve the solution for five iterations, the final solution is returned to the EAS. The pseudo code of the proposed MTS is shown in Figure 5.

```
Function Output s1=(Input s)
s1=s;  //s1 is the best solution of the algorithm
Find neighborhood function N(s), tabu list T(s) and aspiration condition A(s).
Repeat   //main cycle
 Find the best feasible solution s0 in {N(s)-T(s)+A(s)};
          s=s0;                          //replace the current solution by the new one.
 If f(s) < f(s1) then s1=s;         //save the best so far solution.
  Update neighborhood function N(s), tabu list T(s) and aspiration condition A(s);
 Until the s1 is not changed for five iterations.
```

Figure 5. Pseudo-code of the proposed tabu search algorithm

5. COMPUTATIONAL EXPERIMENTS

The algorithm has been implemented in *C* programming language and run on a 3.5 GHz Intel Pentium 3 processor and 4 GB of RAM running Microsoft Windows 7 Ultimate. There are 16 test problems identified by their original number, prefixed, respectively, with the letters C and F available in the literature and they are summarized in Table 1. The fourteen problems denoted C1–C14 are taken from Christofides et al. in 1979 [3], and two problems represented F11-F12 in 1994a [14] are taken from Fisher [6]. The cost of an edge is then taken to be equal to the Euclidean distance and computed with real numbers. We had to make a decision concerning precision in the computation of these distances. In this table, some of the characteristics of problems are described in which the first row gives the instance name, the second row shows the number of customers, and the third row presents the number of used vehicles. In other words, the value of k has been estimated, the sum of all customers demands/vehicle capacity, and the value of L denotes the maximum route length. Seven of the problems have a route-length restriction. The number of customers ranges are in size from $n = 50$ to n = 199 customers. Furthermore, the problems C1–C5, C11, C12, F11 and F12 have no driving time constraint, and C6–C10, C13 and C14 are as same as C1–C5, C11 and C12, but with a travel time constraint. All problems are available online (see www.branchandcut.org).

Table 1: Characteristics of the test problems

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	F11	F12
n	50	75	100	150	199	50	75	100	150	199	120	100	120	100	71	134
k	5	10	8	12	16	5	10	8	12	16	7	10	7	10	4	7
L	-	-	-	-	-	180	144	207	180	180	-	-	648	936	-	-

To show the EAEAS's performance more clearly, we present the best known solutions (BKS) published in the related literature in Table 2. In this table, Column 2 shows the best algorithms gaining minimum vehicles with the least distance. Furthermore, the column 3 refers the best algorithms gaining minimum distance with the least number of vehicles for all of the instances. It is noted that some algorithms used different number of vehicles shown in the brackets in this table and Table 3. We compare the results obtained by the proposed algorithm on the above-mentioned instances with some algorithms as follows:
- TSF and TSR based on tabu search by Fu et al. [8, 9]

- TSAN based on tabu search by Brandao [7]
- BR based on tabu search by Tarantilis et al. [10]
- ALNS 50K based on adaptive large neighborhood search used the minimum spanning tree by Pisinger and Ropke [12]
- ORTR used large neighborhood algorithm by Li et al. [11].
- LBTA and BATA based on threshold accepting by Tarantilis et al. [10, 25]
- VNS based on a variable neighborhood Search by Fleszar et al. [26]

In more details, Fu et al. generates a starting solution in two different ways, and hence there are two variants of his TS procedure (denoted by TSR and TSF). Although Pisinger and Ropke run ALNS for 25,000 iterations and 50,000 iterations, in this table only the results of 50,000 iterations are presented denoted by ALNS 50K. Li et al. developed a record-to-record travel algorithm to handle very large instances of the standard VRP to solve the OVRP denoted by ORTR. Finally, Tarantilis et al. proposed two algorithms using threshold accepting in the name of LBTA and BATA.

Table 2: Characteristics of the test problems

Instance	k	Minimum vehicles with least distance		Minimum distance with least number of vehicles	
C1	5	408.5	TSF	408.5	TSF
C2	10	567.14	ALNS, ORTR, VNS	564.06 [11]	BR, BATA, LBTA, TSR
C3	8	617	TSR	617	TSR
C4	12	733.13	ALNS, ORTR, VNS	733.13	ALNS, ORTR, VNS
C5	16	879.37	BATA	870.26 [17]	LBTA
C6	5	400.6 [6]	TSF	400.6 [6]	TSF
C7	10	583.19	ALNS	560.4 [11]	TSR
C8	8	638.2 [9]	TSF	638.2 [9]	TSF
C9	12	757.84 [13]	ALNS	752.0 [14]	TSR
C10	16	875.67 [17]	ALNS, VNS	875.67 [17]	ALNS, VNS
C11	7	682.12	ALNS, VNS	682.12	ALNS, VNS
C12	10	534.24	BR, BATA, LBTA, ALNS, VNS, ORTR, TSAN	534.24	ALNS, VNS, ORTR, TSAN
C13	7	909.80 [11]	ALNS	896.50 [12]	BR, BATA, LBTA, ORTR
C14	10	591.87 [11]	ALNS, VNS, ORTR	591.87 [11]	ALNS, VNS, ORTR
F11	4	175.0	TSR	175.0	TSR
F12	7	769.66	ORTR, VNS	769.66	ORTR, VNS

Table 3: Results of the EHEAS compared to other metaheuristic algorithms

Instance	n	TSF [9]		TSAN [8]		ORTR [12]		BKS
		Cost	Time	Cost	Time	Cost	Time	
C1	50	408.5	170.2	438.2	1.7	416.06	6.2	408.5
C2	75	587.8	202.1	584.7	4.9	567.14	31.3	564.06 [11]
C3	100	644.3	719.9	643.4	12.3	639.74	39.5	617
C4	150	734.5	1610.3	767.4	33.2	733.13	128.6	733.13
C5	199	878.0 [17]	2060.5	1010.9	116.9	924.96	380.6	870.26 [17]
C6	50	400.6 [6]	128.0	416.0	1.4	412.96	10.3	400.6 [6]
C7	75	565.7 [11]	292.4	581.0 [11]	3.4	568.49 [11]	32.2	560.4 [11]
C8	100	638.2 [9]	987.8	652.1	10.4	644.63	53.2	638.2 [9]
C9	150	758.9 [14]	1635.2	827.6 [14]	25.2	756.38 [14]	195.1	752.0 [14]
C10	199	891.3 [18]	1922.2	946.8	100.1	876.02	363.5	875.67 [17]
C11	120	753.8	735.8	713.3	15.7	682.54	121.6	682.12
C12	100	549.9	413.4	543.2	7.8	534.24	32.9	534.24
C13	120	943.0 [12]	741.1	994.3	25.8	896.50 [12]	120.3	896.50 [12]
C14	100	586.8 [12]	463.2	651.9 [12]	8.1	591.87	62.9	586.8 [12]
F11	71	178.0	256.0	179.5	5.7	177.00	19.5	175.0
F12	134	789.7	1044.8	825.9	32.7	769.66	158.2	769.66

Instance	n	ALNS 50K [13]		VNS [22]		EHEAS	
		Cost	Time	Cost	Time	Cost	Time
C1	50	416.06	230	416.06	17.6	408.5	5.21
C2	75	567.14	530	567.14	29.0	567.14	15.38
C3	100	641.76	1280	639.74	239.6	617	35.71
C4	150	733.13	2790	733.13	585.0	738.90	65.81
C5	199	896.08	2370	905.96	292.1	879.37	95.59
C6	50	412.96	310	412.96	75.8	407.96	6.74
C7	75	583.19	330	596.47	22.3	560.4 [11]	18.63
C8	100	645.16	1140	644.63	587.6	638.2	35.11
C9	150	757.84	1850	760.06	1094.1	752.00 [14]	59.93
C10	199	875.67	2240	875.67	1252.4	875.67	73.52
C11	120	682.12	1410	682.12	231.6	682.12	38.98
C12	100	534.24	1180	534.24	163.7	534.24	32.51
C13	120	909.80	1160	904.04	1820.1	896.50 [12]	41.34
C14	100	591.87	750	591.87	389.0	586.8 [12]	35.51
F11	71	177.00	1040	178.09	140.2	175.00	11.31
F12	134	770.17	3590	769.66	1237.5	769.66	45.31

The objective of the computational experiments is to test the performance of the EHEAS in terms of quality of the solutions and compare its performance with several famous metaheuristic algorithms. As a result, 16 instances are considered in Table 3 and the efficiency and performance of the EHACS is compared with some different meta-heuristic algorithms given in the literature for solving the OVRP. In this table, each algorithm consists of two sub-column including the best gained solution and CPU time. All the CPU

times reported in the Table are in seconds. Furthermore, the last column shows best known solution (BKS) by the various algorithms until now. Since papers on the OVRP tend to report results only for the floating point versions, in this paper we do the same.

This table shows that the proposed algorithm can be used to solve the OVRP effectively because among the 16 test problems, the EMEAS finds 10 optimal solutions published in the literature and obtains nearly the BKS for instances C2, C4, C5 and C6. Furthermore, the maximum relative error is 1.84% for the instance C6 and the average relative error is 0.26%. It is noted that for each problem of this table, the proposed algorithm used the minimum number of vehicles as specified by the lower bound of K. in addition, a simple criterion to measure the efficiency and the quality of an algorithm is to compute the number of optimal solutions found in specific benchmark instances by algorithm. As can be seen from table 3, the best algorithm except the EHEAS is VNS that finds the optimal solution for 5 out of 16 problem instances published in the literature. Besides, TSF, TSAN, ORTR and ALNS 50K can find 4, 0, 4 and 4 optimal solutions thorough of these instances. These results indicate that EHEAS is a competitive approach compared to mentioned algorithms and are much better than the result of these algorithms.

Another criterion for testing efficiency of the proposed algorithm is to compute mean Gap and compare it to other algorithms. The Gap is computed by using formula (16) where the BKS is the best solution found by the algorithm for a given instance on the Web. A zero gap indicates that the best known solution of instance is found by the algorithm.

$$Gap=((the\ best\ solution\ found\ by\ an\ algorithm-BKS)/BKS)*100 \qquad (16)$$

As it can be seen from Table 2, although the mean Gap for the EHEAS is 0.26%, this criterion is 2.27%, 6.38%, 1.29%, 1.37%, 1.58% for the TSF, TSAN, ORTR, ALNS and VNS respectively. As a result, the proposed algorithm is a competitive approach compared to mentioned algorithms and has a consistent performance, since the average gap between the best obtained solutions and the average obtained solutions was better than five other algorithms. Moreover, the ORTR performs better than ALNS and the ALNS obtains much better solution than VNS. Therefore, The algorithms in terms of their performance of mean Gap from the worst to the best are: TSAN, TSF, VNS, ALNS, ORTR and EHEAS. Figure 6 shows the Gap for each instances obtained by our algorithm and five other metaheuristic algorithms. As see in this figure, With the exception of C2, C4, C5 and C6, the EHEAS obtains the gap with zero value for instances. However, in other instances, the proposed algorithm finds nearly the BKS, i.e. the Gap is, relatively as high as 1%. The performance Comparison of results shows that the proposed algorithm clearly yields better high quality solutions than the others for more instances. However, as noted in [27], direct comparisons of the required computational times cannot be conducted, as they strictly depend on several factors including the processing power of the computers, the coding abilities of the programmers, the programming languages, the compilers and the running processes on the PC.

6. CONCLUSION

The OVRP is different from most variants of vehicle routing problems from the literature in

that the vehicles do not return to the depot after delivering the last customer. The practical importance of the OVRP has been established some years ago, but it has received very tiny attention from scientists and researchers. In this research we created an effective hybrid EAS called EHEAS that is able to find very good solutions for the OVRP in a very short computing time. We have introduced some modification to improve the algorithm such as using modified tabu search algorithm. Besides, we compare its performance with other meta-heuristic algorithms designed for the same purpose, which has been published recently. The result shows the proposed algorithm is the efficiency for the OVRP. We are convinced that this technique would be applied in some versions of vehicle routing problems such as vehicle routing problem with pickup and delivery or general vehicle routing problem in the future.
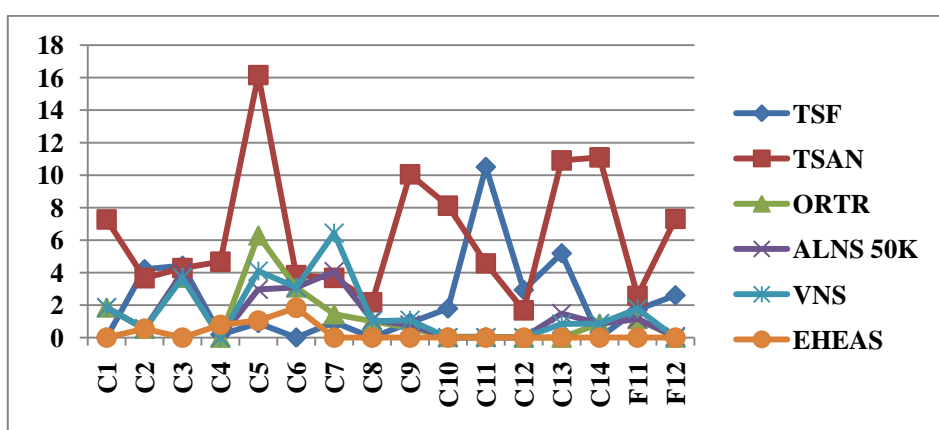

Figure 6. Comparison Gap of the metaheuristic algorithms

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