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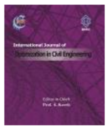

DAMAGE DETECTION IN THIN PLATES USING A GRADIENT-BASED SECOND-ORDER NUMERICAL OPTIMIZATION TECHNIQUE

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ABSTRACT

The purpose of the present study is the damage detection in the thin plates in terms of the wide application of such structures in various branches of engineering such as structural, mechanical, aerospace, shipbuilding, etc. using gradient-based second-order numerical optimization techniques. The technique used for optimization in this study is the second-order Levenberg-Marquardt algorithm (SOLMA). Using the acceleration response in a number of structural nodes under dynamic excitation, identification of the location and extent of damage in the plate elements are obtained by the proposed algorithm over an iterative cycle and by updating the sensitivity matrix. The damage has been assumed in the form of decreased modulus of elasticity in linear mode. A numerical problem has been solved and presented in order to verify and compare the proposed damage detection method with other methods. Also several numerical problems have been solved and its results have been presented in order to evaluate different scenarios such as one or more damages, small or large damage extent, absence or presence of noise with different levels, number of measured responses (number of sensors), position of measured points and the dynamic analysis time of the damage detection problem with the proposed method. The results show the appropriate accuracy, efficiency and performance of the proposed damage detection method.

Keywords: damage detection; inverse problem; thin plates; second-order gradient technique; dynamic excitation; acceleration response.

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1. INTRODUCTION

Structural health monitoring (SHM) is essential to ensure the safety performance of the structure over its useful life. Plates are an important part of the components of engineering structures whose ratio of thickness to other dimensions is less than 0.1. Due to the need to prevent unintended damage, it is important to develop Structural health monitoring methods such as plates for their widespread use in various branches of engineering such as structures, mechanics, aerospace, shipbuilding and so on. During operation, all plates are affected by damage which may cause structural failures such as cracks that lead to structural failure over time. Changes in structural responses occur due to structural damage such as cracks. By examining these changes, the location and extent of damages can be identified. Therefore, it is necessary to evaluate the status of the structure by appropriate techniques to remove structural damages. The damage detection in these methods is based on the measured responses in the structure, i.e., the types of static and dynamic responses. A number of researchers have studied the damage detection of plates by combining mode shapes or curves of mode shapes with wavelet transform theory: Chang and Chen [1] investigated rectangular plate damage detection using spatial wavelet analysis with mode shapes after damage. The method used by them was extremely sensitive to damage extent. Loutridis et al. [2] used two-dimensional wavelet transforms to detect cracks in plates using vibratory modes. Their method was only able to determine the location and extent of a damage. Fan and Qiao [3] presented a new method for plate damage detection by using mode shapes data and continuous two-dimensional wavelet transform data. Xiang et al. [4], first identified the location and then the extent of the damage for conical shells using a two-step method. They first identified the damage location with the help of mode shape curvature and the wavelet finite element method, then, in the second step, the damage extent was determined using the Support Vector Machine (SVMs) method. Xiang and Liang [5] introduced a two-step approach to identify multiple damages in thin plates. They first determined the damage location by applying two-dimensional wavelet transform to the mode shapes to identify singularities. In the next step, the particle swarm optimization algorithm was used to identify the damage extent. They also examined the results in terms of noise effects. Xu et al. [6] identified plate damages using two-dimensional mode shape curvature based on wavelet transform and energy conversion. Some other researchers have only investigated the damage detection using the modal features such as mode shapes, frequencies, modal strain energy, mode shape curvature, frequency response function (FRF) or a combination of them: Eraky et al. [7], identified the damage location of a cantilever beam and a plate by comparing the modal strain energy in different damage indicator conditions by relying on the Damage Indicator Method (DIM) and laboratory investigations. Good agreement was obtained between the results of the DIM method and the experiment. Navabian et al. [8] presented a new damage indicator based on modal data such as mode shapes and derivatives of mode shapes with numerical examples with and without noise for damage detection of plates. Hoseini Vaez and Fallah [9] investigated the damage detection of thin plates in the cases with and without noise by combining Genetic Algorithm and Particle Swarm Optimization (GA-PSO) based on modal data including frequencies and mode shapes. They first obtain the damage detection results of the models using genetic algorithms and particle swarm separately. They were then compared them with the results of damage detection for the combination of genetic algorithm and particle swarm optimization. The effect of using a combination of the two methods is especially evident in the case with noise. Eraky et al. [10], identified the damage location based on the residual force vector by comparing different mode shapes. Their method for symmetric damage scenarios has a higher accuracy in identifying the location of damage on the plates. Fu et al. [11] used a two-step method by modal strain energy and sensitivity analysis response to detect damage in isotropic plates with moderate thickness. Rucevskis et al. [12] investigated damage detection in plate structures based on the mode shape curvature differences in damaged and undamaged structures. The advantage of their method is the use of structural information only in the damaged state. They approximated the mode shape curvature of the undamaged structure using a polynomial term. Wu and Law [13,14] proposed the damage identification in the plate structures using uniform load curvature changes. Their method has fast convergence except for high percentage noise and has high accuracy even in the cases with low number of modes. Li et al. [15] investigated damage identification in plate structures using two indicators of damage sensitivity, namely bending moment and residual strain mode shape. Despite simplicity, their method could not detect the damage in a number of elements. Bayissa and Haritos [16] used spectral strain energy analysis to detect damage in the plates. They proposed a method not based on mode shape using modal strain energy (SSE). The SSE was obtained using two damage sensitivity parameters, namely the bending moment response of the power spectral density and the power spectral density curvature. Kazemi et al. [17] proposed a two-step approach based on modal soft changes to detect damage in thin plates. Wei et al. [18] introduced a two-step technique based on strain energy variations and plate sensitivity analysis (MSECR). Fu et al. [19] investigated the identification of damage in steel plates by reducing the modulus of elasticity due to numerical modeling under external load and vibrational response of the structure using a finite element model update in time domain. Huh et al. [20] presented a two-dimensional damage indicator based on the vibratory power of thin plates. The vibratory power of the thin plates was estimated based on the response of arbitrary excitations such as acceleration. Yam et al. [21] studied the sensitivity of damage detection parameters with static and dynamic methods by determining damage indicators in static and dynamic states for plate structures. Two indicators based on the mode shape curvature and the strain frequency response function were proposed in the dynamic method. Then, the effect of the change in the number of modes selected as well as the shift of natural frequencies on these indicators were investigated. Zhang et al. [22] investigated a method based on the frequency shift surface curvature, which is one of the vibratory parameters of the structure on the plates. Their proposed method was validated by numerical and experimental studies. The results showed that the curvature of the frequency transduction surface is highly sensitive to local damages in contrast to the natural frequencies and, it works more successfully to identify local damages. Pedram et al. [23] studied the damage detection of plate and shell structures using the finite element model updating method in the frequency domain. Some of the other researchers' studies of plate damage detection using wavelet and curvelet transform methods alone are as follows: Rucka and Wilde [24] used continuous wavelet transform to identify the location of damage on a steel plate and a cantilever beam without knowing the specification and mathematical model of the structure. They proposed two-dimensional formulas based on wavelet transform for damage detection problem. Huang et al. [25] investigated the damage detection using continuous two-dimensional wavelet transform for plates. Douka et al. [26] studied the identification of depth and location of cracks in rectangular plates using one-dimensional wavelet theory. Rucka and Wilde [27] studied the damage detection in rectangular plates using two-dimensional wavelet theory. Wilde and Rucka [28] studied the identification of damage location in rectangular plates using continuous two-dimensional Gaussian wavelet. Beheshti-Aval et al. [29] identified damage in the plates using harmonic class loading and wavelet transform technique. Their approach only required the features of the plate in the damaged state and did not require healthy structural information. Bagheri et al. [30] developed a new method based on curvelet transform theory for plate damage detection. Other researchers have investigated the damage detection for the plates in specific cases using static and dynamic responses and newer methods: Lin and Yuan [31] investigated plate damage detection using migration technique. Torkzadeh et al. [32] investigated the damage detection of plate structures using the smart alternative model. Their method was implemented in a two-step process using artificial neural network optimization. In the first step, they identified the locations of damage with the help of the concepts of moment curvature and the derivation of moment curvature, then, in the second step, the damage extent was determined using BAT algorithm. Lu et al. [33] investigated the selection and effect of a weight matrix for sensitivity-based responses in damage detection of a plate frame and a rectangular plate with considering the measurement and modeling errors. Qiao et al. [34] proposed a new hybrid technique of static and dynamic techniques for damage detection in layered composite plates. They applied dynamic excitation after static load to determine damage in the plates. Bayissa and Haritos [35] studied the damage detection of plates using spectral density bending moment response. They identified the damage in the two-dimensional plate structures using a new damage sensitivity parameter, namely the bending moment response of the spectral power density. Moore et al. [36] investigated the detection of crack locations in simple support vibratory plates using Bayesian parameter estimation.

Extensive studies have been performed on structural damage detection using nature-inspired meta-heuristic or intelligent algorithms; among which, the following studies could be cited: Zhao et al. [37] suggested a new technique, known as TCABC, for structural damage detection by using bee colony algorithm (ABC) and taboo and chaotic search methods. Their results show better performance of the TCABC algorithm compared to the traditional ABC algorithm. Daei et al. [38] provided an intelligent method to identify the location and severity of structural damages through an optimization model by using the pseudo-residual force vector (RFV) theory. The proposed method can detect damages based on only a few mode shapes of structure taking into account the noise effects. Tabrizian et al. [39] used a meta-heuristic optimization algorithm, known as charged system search (CSS), to determine the location and severity of damage in structures. This method provides acceptable results in detecting structural damage in low computational time. integrating radial basis functions (RBFs) with meta-heuristic methods, Bureerat and Pholdee [40] proposed a technique to improve the performance of solving optimization problems in monitoring the health of truss structures. The meta-heuristic method used by these authors is the differential evolution algorithm (DE). In another study, Pholdee and Bureerat [41] compared different meta-heuristic methods in structural damage detection based on changes in modal data. These methods include: differential evolution (DE), bee colony Algorithm (ABC), real-code ant colony optimization (ACOR), charged particle system search (ChSS), league championship algorithm (LCA), simulated annealing algorithm (SA), particle swarm optimization (PSO), evolution strategies (ES), teaching-learning based optimization (TLBO), adaptive differential evolution (JADE), evolution strategy with covariance matrix adaptation (CMAES), success-history based adaptive differential evolution (SHADE) and SHADE with linear population size reduction (L-SHADE). Their results show that the DE, TLBO, and L-SHADE are the best methods, among which, TLBO is more prominent for a large scale problem. They suggested that the results obtained from the TLBO method could serve as the main basis for future research into the structural damage detection using meta-heuristic techniques. Kaveh et al. have also performed extensive research on structural damage detection using meta-heuristic methods in recent years, including the following: Using enhanced vibrating particles system (EVPS) optimization with the aid of mode shape data, Kaveh et al. [42] tried to detect truss damages with or without noise. They showed that the EVPS algorithm performed better than the vibrating particles system (VPS) to detect damage. Using cyclic parthenogenesis algorithm (CPA) and the modal strain energy, Kaveh and Zolghadr [43] tried to detect structural damages. They show that the proposed method is capable of identifying structural damage using only the first few modes of the structure. Kaveh and Maniat [44] detected structural damages based on optimizing the charged system search optimization with modal incomplete data. Also, Kaveh et al. [45] used Simplified dolphin echolocation algorithm along with modal data to detect truss damages at various noise levels. Using modified charged system search (CSS) algorithm, variations in natural frequencies of structures and different mode shapes, Kaveh and Zolghadr [46] tried to detect 2D and 3D truss damage. Saberi and Kaveh [47] used a two-step method and applied a modified charged system search algorithm as well as the residual force method for the damage detection in large structures. They used a modified vector to increase the efficiency of the modal residual force method and reduce the noise effect. Using colliding bodies optimization (CBO) and enhanced colliding bodies optimization (ECBO), and applying modal data, Kaveh and Mahdavi [48] tried to detect truss damages. Studies show the superiority of the ECBO algorithm over the standard CBO algorithm. Kaveh and Maniat [49] solved the problem of structural damage detection based on the magnetic charged system search (MCSS) and particle swarm optimization (PSO) by using frequencies and mode shapes. The results show that their proposed method can reliably detect the damage in scenarios with noise and incomplete data. Kaveh et al. [50] performed structural damage detection by combining particle swarm optimization, ray optimization, and harmony search (HRPSO) techniques with the use of modal data.

Although various methods have been used for damage detection of plates, newer, more complete and comprehensive methods are still needed. The results of studies on plate damage detection indicate that most of the methods used by researchers are for plates in specific cases and in the form of single damage and noise-free and, these methods lose their performance under conditions such as increasing the number of damages, considering the noise effects, multiple damages, and are unable to detect the location and extent of the damages. Some of the other damage detection methods used by researchers are not accurate enough to detect damage, and sometimes they are only able to detect the location of the damage with unacceptable accuracy. Some other damage detection methods operate in two or more steps and have high computational cost and volume. The advantages of the proposed one-step method in this study over the other methods, considering the results obtained in the detection of plate damage, are the following: random sensor placement in active degrees of freedom of the structure, asymmetric layout pattern sensor placement, Sensor placement with limited number to record responses, ability to detect the location and extent of several damages, considering the effects of noise errors in data with acceptable accuracy, and the use of dynamic analysis in extracting input data used to detect damages. The advantages of the proposed method reduce the cost of using sensors and their accessories in structures and as a result, make the proposed method functional and practicable. The technique used in this study was performed by the gradient-based numerical optimization method, the second-order Levenberg-Marquardt algorithm (SOLMA). The proposed method is based on sensitivity analysis using response nodes of structures equipped with sensors. The acceleration response of the nodes has been extracted by performing dynamic analysis.

2. DESCRIPTION AND SOLUTION METHOD OF PLATE DAMAGE DETECTION

The problem of plate damage detection is expressed as the solution of nonlinear equations in the form of Eq. (1), where the solutions are vectors of equations (Eq. (2)) and the damage of the elements of the vector of unknowns (Eq. (4)). The damage vector is obtained by solving the equation system in the form of an inverse problem, which can be defined as the following equation system:

$$\mathbf{R_d} = \mathbf{R}(\mathbf{X}) \Rightarrow \mathbf{X} = ? \tag{1}$$

$$\mathbf{R_d} = \{r_{d1}, r_{d2}, \dots, r_{dm}\} \tag{2}$$

$$\mathbf{R}(\mathbf{X}) = \{r_1(\mathbf{X}), r_2(\mathbf{X}), \dots, r_m(\mathbf{X})\} \tag{3}$$

$$\mathbf{X} = \{x_1, x_2, \dots, x_{ne}\};\ 0 \le x_i \le 1; \quad i = 1,2,\dots,ne \tag{4}$$

In this system of equations, the objective is to find the damage vector of the plane elements, the vector X, using the real vector structure response vector R_d, where x_i is the damage ratio of the element i, ne is the number of elements or unknowns, m is the number of equations, and R(X) is the hypothetical damaged structure response vector which is a nonlinear function of the damage vector. The values of $x_i = 0$ and $x_i = 1$ represent perfectly healthy and damaged state of the structural elements, respectively. Given that the responses are of nodal acceleration type and because of the higher number of equations than the number of elements, damage detection problem in the proposed method is considered as a solution of a nonlinear non-linear system with m> ne. Since damage is a nonlinear phenomenon, solving the damage detection equation system in this case is direct, limited, and sometimes impossible. Therefore, the problem of plate damage detection in this study is first solved by using Taylor expansion to solve the linear equation system and then solved by optimization problems. Most studies today are based on minimizing the difference between the real and hypothetical damaged structural response obtained by updating the damage indicator at each stage of the iteration. The damage indicator can be selected as the decrease ratio of the element cross-section, reducing the element's inertia moment, reducing the modulus of elasticity of the element, or any other parameter that indicates a decrease in the element's stiffness and damage. In this study, the reduction of the modulus of elasticity of the elements according to (5) is considered as the damage indicator x^e:

$$x^e = -\frac{E^e - E_0^e}{E_0^e} \Rightarrow E^e = E_0^e(1 - x^e) \tag{5}$$

where, E_0^e and E^e are the initial modulus of elasticity of the element and the updated modulus of the element, respectively. Since the damage detection reported in this study is a kind of optimization problem that is, finding the best solution with the least error rate, where the difference between the response of hypothetical damaged structure and the real damaged structure must be reduced at each optimization step to reach zero. Therefore, the objective function must be defined so that this difference can be controlled to lead it to zero. The objective function in such problems can usually be defined as a nonlinear least squares problem in the form of a second sum differences based on the Eq. (6) (Teughels et al. [51] and Teughels and De Roeck [52]).

$$\begin{gathered} f(\boldsymbol{X}) = \frac{1}{2}\sum_{j=1}^{m} \boldsymbol{r}_j^2(\boldsymbol{X}) = \|\boldsymbol{r}(\boldsymbol{X})\|^2 \\ \Rightarrow f(\boldsymbol{X}) = \frac{1}{2}(\boldsymbol{R}(X) - \boldsymbol{R}_d)^T(\boldsymbol{R}(\boldsymbol{X}) - \boldsymbol{R}_d) \end{gathered} \tag{6}$$

where, $f(\boldsymbol{X})$ is the objective function and r(X) is the output error of the damage function. Using the Taylor expansion of the damage detection equation around zero, the problem solution of the nonlinear damage detection system, i.e. E. (1) can be transformed to the linear system solution by with approximation.

$$\Delta\mathbf{R} \cong \mathbf{S}\Delta\boldsymbol{X} \Rightarrow \Delta\boldsymbol{X} \cong \mathbf{S}^{+}\Delta\mathbf{R} \tag{7}$$

$$\Delta\mathbf{R} = \mathbf{R_d} - \boldsymbol{R}_h \tag{8}$$

$$\Delta\boldsymbol{X} = \mathbf{X} - \mathbf{0} \tag{9}$$

$$\mathbf{S} = \frac{\partial\mathbf{R}(\mathbf{0})}{\partial\mathbf{X}} \tag{10}$$

where, $\boldsymbol{R}_h$ is the structure response vector in the healthy state, S is called the sensitivity matrix or Jacobin and indicates the sensitivity of response changes (ΔR) to damage changes

(ΔX) and S^+ represents the pseudo inverse of the S matrix. In the case of gradient-based optimization method, the number of equations is greater than the number of design variables. In this study, the structure response is the nodal acceleration recorded by the sensors, which is obtained by dynamic analysis with time step nt. Therefore, the number of equations is sufficient to solve the equation system. In this case the sensitivity matrix can be expressed by Eqs. (11)-(13):

$$\boldsymbol{S} = [\boldsymbol{s}_1\ \boldsymbol{s}_2 \dots \boldsymbol{s}_l \dots \boldsymbol{s}_{Nsensor}]^T \tag{11}$$

$$\boldsymbol{s}_l = \begin{bmatrix} \frac{\partial R_l(t_1)}{\partial X_1} & \frac{\partial R_l(t_1)}{\partial X_2} & \frac{\partial R_l(t_1)}{\partial X_3} & \cdots & \frac{\partial R_l(t_1)}{\partial X_{ne}} \\ \frac{\partial R_l(t_2)}{\partial X_1} & \frac{\partial R_l(t_2)}{\partial X_2} & \frac{\partial R_l(t_2)}{\partial X_3} & \cdots & \frac{\partial R_l(t_2)}{\partial X_{ne}} \\ \frac{\partial R_l(t_3)}{\partial X_1} & \frac{\partial R_l(t_3)}{\partial X_2} & \frac{\partial R_l(t_3)}{\partial X_3} & \cdots & \frac{\partial R_l(t_3)}{\partial X_{ne}} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial R_l(t_{nt})}{\partial X_1} & \frac{\partial R_l(t_{nt})}{\partial X_2} & \frac{\partial R_l(t_{nt})}{\partial X_3} & \cdots & \frac{\partial R_l(t_{nt})}{\partial X_{ne}} \end{bmatrix}_{nt*ne} \tag{12}$$

$$\mathbf{R}_l = [R_l(t_1)\ R_l(t_2)\ R_l(t_3) \dots\ R_l(t_{nt})]^T \tag{13}$$

where, Nsensor represents the total number of degrees of freedom of the structure with sensor, $\mathbf{R}_l$ is the acceleration response vector at the lth degree of freedom of the sensor, nt is the number of steps of the acceleration response recorded and not is the number of elements of the structure. The number of S-sensitivity matrix columns is equal to the number of vector components of the update parameter (usually the number of structure elements (ne)). Also, the number of rows of the S-sensitivity matrix is equal to the sum of the total responses recorded by the sensors (nt * Nsensor). The only disadvantage of this method is that its inversability has problems in the update process, which can usually be resolved by using pseudo-inverse methods such as single SVD values analysis. One of the most important decomposition tools to facilitate the solution of large linear systems is the single value decomposition method. In this method, each matrix with any dimension can be transformed into the multiplication of three matrices, one of which is diagonal and the other two are orthogonal matrices. This decomposition is of great help in examining the behavior of matrices from single values and corresponding vectors. The matrix A_{m*n} can be decomposed into Eq. (14) (Hansen [53]).

$$\mathrm{A} = \mathrm{U\Sigma V^T} = [\mathrm{u}_1 \dots \mathrm{u}_m] \begin{bmatrix} \sigma_1 & 0 & \dots & 0 & 0 \\ 0 & \sigma_2 & 0 & \vdots & \vdots \\ \vdots & 0 & \ddots & 0 & 0 \\ 0 & \vdots & 0 & \ddots & 0 \\ 0 & \vdots & \cdots & 0 & \sigma_r \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathrm{v}_1^T \\ \mathrm{v}_2^T \\ \mathrm{v}_3^T \\ \vdots \\ \mathrm{v}_n^T \end{bmatrix} \tag{14}$$

where, always $(\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_r \geq 0)$. r represents the rank of matrix A ($r \leq \min(m.n)$). U and V matrices are matrices with orthogonal columns:

$$U^T U = V^T V = I \tag{15}$$

The U and V columns are the left and right individual vectors, respectively. The matrix Σ matrix is diagonal and nonzero entries on its diameter are defined as the singular values of matrix A and also the eigenvalues of matrices $A^T A$ and AA^T. According to the defined equations, the pseudo-inverse of matrix A is expressed as Eq. (16):

$$A^+ = V\Sigma^+ U^T \tag{16}$$

where the matrix Σ^+ is expressed by Eq. (17):

$$\Sigma^+ = \begin{bmatrix} 1/\sigma_1 & 0 & \cdots & 0 & 0 \\ 0 & 1/\sigma_2 & 0 & \vdots & \vdots \\ \vdots & 0 & \ddots & 0 & 0 \\ 0 & \vdots & 0 & \ddots & 0 \\ 0 & \vdots & \cdots & 0 & 1/\sigma_r \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \tag{17}$$

In order to bring the theoretical results to reality, we need to apply noise to the data. These noises have been applied to the responses in this study based on the Eq. (18) (Li and Law [54] and Wang and Yang [55]).

$$\boldsymbol{R}_{measured} = \boldsymbol{R}_{calculated} + E_p * \boldsymbol{N}_{noise} * \sigma(\boldsymbol{R}_{calculated}) \tag{18}$$

where, $\boldsymbol{R}_{measured}$ is the acceleration response vector with noise, $\boldsymbol{R}_{calculated}$ represents the noise-free acceleration response vector, $\sigma(\boldsymbol{R}_{calculated})$ denotes the standard deviation of noise-free acceleration response vector, E_p is the noise level (one percent, two percent, five percent, etc.) and $\boldsymbol{N}_{noise}$ represents the vector of normal distribution with mean value of zero and standard deviation of one.

3. FORMULATION OF THE PROBLEM GOVERNING THE PLATE DAMAGE DETECTION BASED ON SECOND ORDER GRADIENT OPTIMIZATION METHOD

The proposed method for plate damage detection in this study is based on the second-order gradient-based numerical optimization method, i.e. the modified Levenberg method and, it can be expressed as a second-order Levenberg-Marquardt algorithm (SOLMA) with the following cyclic equations (Arora [56] and Marquardt [57]):

$$\mathbf{X}^{(k+1)} = \mathbf{X}^{(k)} + \Delta\mathbf{X}^{(k)}; \qquad k = 0,1,2, \ldots \tag{19}$$

$$\mathbf{X}^{(k+1)} = \mathbf{X}^{(k)} + \alpha_k \mathbf{d}^{(k)} \tag{20}$$

$$\mathbf{X}^{(k+1)} = \mathbf{X}^{(k)} - \left(\boldsymbol{H}^{(k)} + \lambda_k\, diagonal\left[\boldsymbol{H}^{(k)}\right]\right)^{-1} \boldsymbol{c}^{(k)} \tag{21}$$

$$\mathbf{X}^{(k+1)} = \mathbf{X}^{(k)} - \left(\nabla^2 f\left(\boldsymbol{X}^{(k)}\right) + \lambda_k\, diagonal\left[\nabla^2 f\left(\boldsymbol{X}^{(k)}\right)\right]\right)^{-1} \nabla f\left(\boldsymbol{X}^{(k)}\right) \tag{22}$$

$$\nabla f\left(\boldsymbol{X}^{(k)}\right) = \boldsymbol{c}^{(k)} = J^T\left(\boldsymbol{X}^{(k)}\right)\boldsymbol{r}^{(k)} \tag{23}$$

$$\nabla^2 f\left(\boldsymbol{X}^{(k)}\right) = \boldsymbol{H}^{(k)} = J^T\left(\boldsymbol{X}^{(k)}\right)J\left(\boldsymbol{X}^{(k)}\right) + Q\left(\boldsymbol{X}^{(k)}\right) \approx J^T\left(\boldsymbol{X}^{(k)}\right)J\left(\boldsymbol{X}^{(k)}\right) \tag{24}$$

$$\begin{aligned}\mathbf{X}^{(k+1)} = \mathbf{X}^{(k)} - \Big(&J^T\left(\boldsymbol{X}^{(k)}\right)J\left(\boldsymbol{X}^{(k)}\right) \\ &+ \lambda_k\, diagonal\left[J^T\left(\boldsymbol{X}^{(k)}\right)J\left(\boldsymbol{X}^{(k)}\right)\right]\Big)^{-1} J^T\left(\boldsymbol{X}^{(k)}\right)\boldsymbol{r}^{(k)}\end{aligned} \tag{25}$$

In these equations, the index k is the cycle number, $\boldsymbol{X}^{(0)}$ is the initial design, $\boldsymbol{X}^{(k)}$ is the k-th design, $\Delta\mathbf{X}^{(k)}$ is the small changes in the current design, $\boldsymbol{c}^{(k)}$is the gradient of the function $f(\boldsymbol{X})$, λ_k represents the hybrid parameter (non-negative damping coefficient), $\boldsymbol{H}^{(k)}$is the Hessian matrix of function f (X) in cycle k, and $\boldsymbol{r}^{(k)}$ is the output error of damage function. The first-order derivative matrix is known as the Jacobian matrix $J\left(\boldsymbol{X}^{(k)}\right)$ and the second-order derivative matrix is known as the Hessian matrix of $\boldsymbol{H}^{(k)}$. The Levenberg-Marquardt second-order gradient algorithm interpolates between the steepest descent gradient first-order algorithm and the Gauss-Newton second-order gradient algorithm. If the λ value is increased in the Levenberg gradient algorithm, the Hessian matrix is not used at all. Therefore, Marquardt modified the Levenberg method according to Eq. (21) and this algorithm was named Levenberg-Marquardt (Eq. (25)). Since any small movement in the negative direction of the gradient leads to the highest local reduction rate of the objective function, so the negative of the gradient vector indicates the steepest descent for the objective function.

As discussed in the previous sections, pseudo-inverse methods such as singular value decomposition can usually be used to solve the invers ability problem in the updating process in cyclic optimization:

$$\Delta\mathbf{R} = \mathbf{S}\Delta\boldsymbol{X} \Rightarrow \Delta\boldsymbol{X} = \mathbf{S}^{+}\Delta\mathbf{R} \tag{26}$$

$$\mathbf{X}^{(k+1)} = \mathbf{X}^{(k)} + \boldsymbol{H}^{+}\mathbf{S}(\mathbf{X}^{(k)})^{T}\left(\boldsymbol{R}_d - \mathbf{R}\left(\mathbf{X}^{(k)}\right)\right) \tag{27}$$

$$\begin{gathered}\boldsymbol{H}^{+} = \left[\mathbf{S}(\mathbf{X}^{(k)})^{T}\mathbf{S}(\mathbf{X}^{(k)}) + \lambda_k\, \mathrm{diagonal}(\mathbf{S}(\mathbf{X}^{(k)})^{T}\mathbf{S}(\mathbf{X}^{(k)}))\right]^{+} = V\Sigma^{+}U^{T} \\ \boldsymbol{R}_d - \mathbf{R}\left(\mathbf{X}^{(k)}\right) = \mathbf{r}^{(k)}\end{gathered} \tag{28}$$

$$\mathbf{X}^{(k+1)} = \mathbf{X}^{(k)} + (V\Sigma^{+}U^{T})\mathbf{S}(\mathbf{X}^{(k)})^{T}\mathbf{r}^{(k)} \tag{29}$$

Flowchart of the steps of the second-order gradient Levenberg-Marquardt algorithm for plate damage detection have been summarized in Fig. 1.

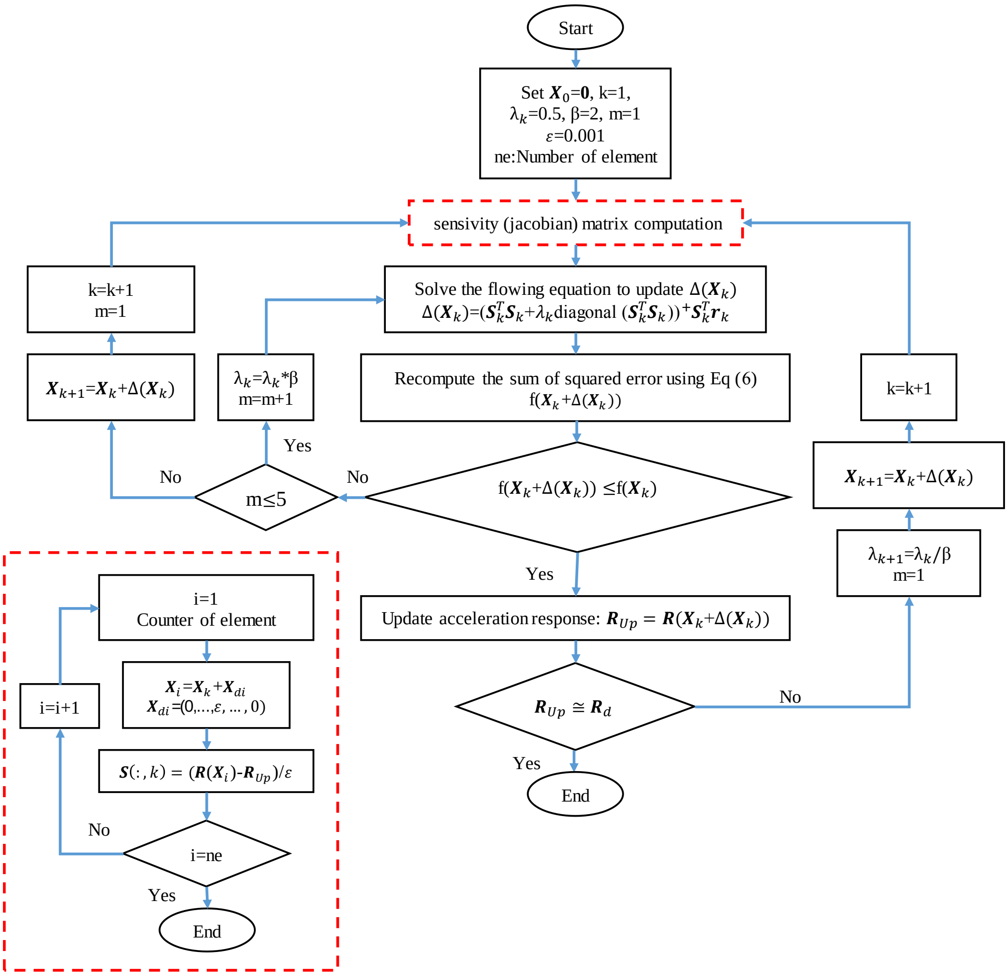


Figure 1. Algorithm of the steps of the second-order gradient Levenberg-Marquardt algorithm for plate damage detection

4. NUMERICAL EXAMPLES

In this study, in Section 4-1, a numerical example is solved and compared with the reference results for the validation of the proposed damage detection technique. By solving several numerical problems in Section 4-2, the effect of scenarios such as one or more damages, low or high damage extent, absence or presence of noise with different levels, number of measured responses (number of sensors), measured response locating (sensors) at different positions and dynamic analysis time have been presented with the proposed method for damage detection of the plates. Triangular impact loading in the vertical direction was used for all models. Using accelerated responses recorded in all problems, the structural damage detection is performed by the proposed technique in a number of nodes by updating the modulus of elasticity parameter in the finite element model of the plate.

4.1 Numerical example for the validation of the proposed damage detection technique

The rectangular plate with a simple support on both sides with dimensions of 6 cm * 150 cm * 500 cm has geometric properties as shown in Fig. 2. The finite element model of this plate consists of 30 elements and 44 nodes. The modulus of elasticity, unit weight of the plate and Poisson's ratio of plate are 25 Gpa, 2800 kg/m^3 and 0.2, respectively.

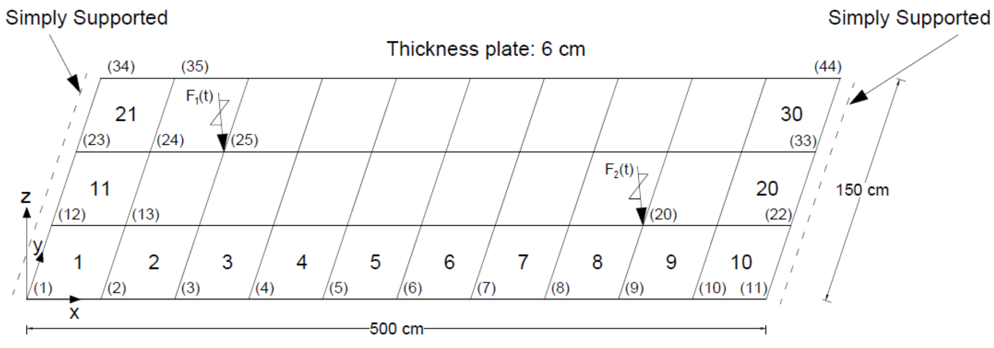
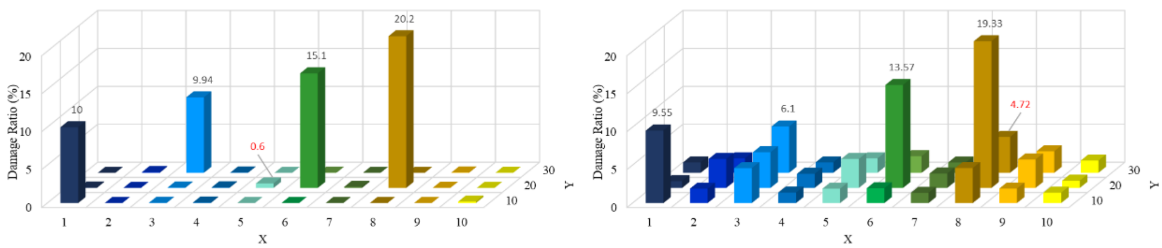


Figure 2. Geometric specifications of a rectangular plate with 30 elements and 44 nodes (Lu et al. [33])



(a) (b)

Figure 3. Comparison of the damage detection results of damage location and extent of rectangular plate with 30 elements subjected to the same damage scenario (a) Proposed method with 1% noise in the data (b) Proposed method in the reference paper (Lu et al. [33]) with 0.15% noise in the data

The dimensions of all sources (elements) are 50 cm * 50 cm. The plate at nodes 1, 11, 12, 22, 23, 33, 34 and 44 has simple support conditions. The damage is simulated by reducing the modulus of elasticity to the damage ratio. In this example, the damage scenario in elements 1, 16, 18 and 23 is assumed with the damage extent of respectively 10%, 15%, 20% and 10%. Sensor placement at nodes 2, 6, 8, 16, 25, 27, 31, 39 and 43 is assumed to measure responses with 1% noise. Gravity loading is applied at nodes 20 and 25 at 1000 N. The plate has been subjected to two triangular impact loads of $F_1(t)$ and $F_2(t)$ at nodes 25 and 20 with a period of 0.08 seconds, with a time step of 0.0005 seconds and at a time of one second according to the Eqs. (30) and (31). A comparison of the results of the proposed technique with the damage detection technique presented by Lu et al. [33] has been presented in Fig. 3.

$$F_1(t) = 1E05 \begin{cases} (t - 0.02) & 0.02 \leq t \leq 0.04 \\ (0.06 - t) & 0.04 \leq t \leq 0.08 \; (N) \\ (t - 0.1) & 0.08 \leq t \leq 0.1 \end{cases} \tag{30}$$

$$F_2(t) = 1E05 \begin{cases} (t - 1.02) & 1.02 \leq t \leq 1.04 \\ (1.06 - t) & 1.04 \leq t \leq 1.08 \quad (N) \\ (t - 1.1) & 1.08 \leq t \leq 1.1 \end{cases} \tag{31}$$

where, t is time and $F_1(t)$ and $F_2(t)$ are the triangular impact forces at time t. Comparison of the damage detection results shows the better performance of the proposed method than the method presented in the reference paper (Lu et al. [33]) for rectangular plate with 30 elements with higher noise percentage for the measured data. As can be seen from the results, the proposed method has less error in both location detection and damage extent determination than the method presented in the reference paper (Lu et al. [33]) and, it works much better under the same scenario to identify the location and extent of the damage.

4.2 Numerical example for investigating various parameters on the proposed damage detection technique

A rectangular steel plate with a fixed cantilever support with dimensions of 70 cm * 40 cm * 4 cm with the geometric specification shown in Fig. 4 has been examined. The finite element model of this plate consists of 28 elements and 40 nodes. The modulus of elasticity, unit weight of the plate and Poisson's ratio of plate are 210 Gpa, 7850 kg/m^3 and 0.3, respectively. The dimensions of all sources (elements) are 10 cm * 10 cm. The plate at the nodes 1, 9, 17, 25 and 33 has fixed supporting conditions. The damage is simulated by reducing the modulus of elasticity to the damage ratio. Gravity loading at node 40 is applied at N 100. The plate is subjected to a triangular impact load of $F_3(t)$ at node 40 with a period of 0.02 seconds, with a time step of 0.0005 seconds in accordance with Eq. (32). The damage modes, various scenarios and sensor placement modes used for damage detection in a plate with 28 elements have been presented in Tables 1-3, respectively. In the following, the results of the investigation of various parameters in the 28-element plate damage detection problem with the proposed method have been presented.

$$F_3(t) = 2E04 \begin{cases} t & 0 \le t \le 0.005 \\ (0.01 - t) & 0.005 \le t \le 0.015 \ (N) \\ (t - 0.02) & 0.015 \le t \le 0.02 \end{cases} \tag{32}$$

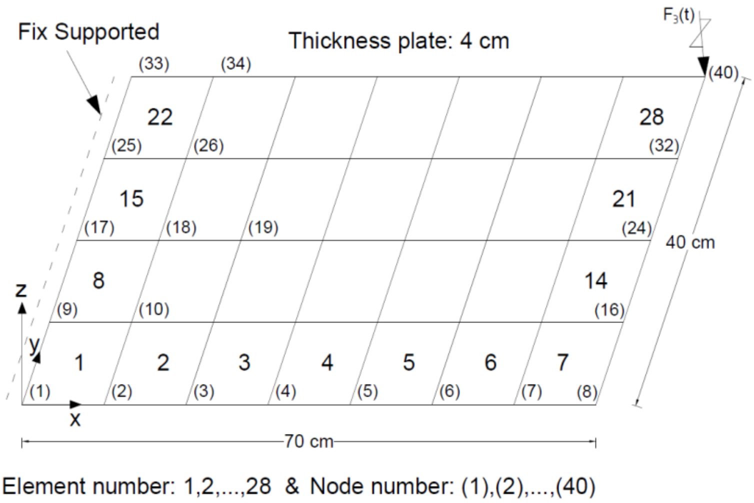


Figure 4. Geometric specifications of a rectangular plate with 28 elements and 40 nodes

Table 1: Damage modes used for damage detection in a plate with 28 elements and 40 nodes

Damage Case	Element Number	Damage Ratio (%)
1	17	5
2	2	2
	13	4
	17	5
	26	4
3	2	12
	13	10
	17	15
	26	10

Table 2: Different damage detection scenarios for the plate with 28 elements and 40 nodes

Scenario	Damage Case	Sensor Pattern	Noise Level (%)	Analysis Time (Sec)
S-1	1	a	0	1
S-2	2	a	0	1
S-3	3	a	0	1
S-4	3	a	1	1
S-5	3	a	3	1
S-6	3	a	5	1
S-7	3	b	5	1
S-8	3	c	5	1
S-9	3	d	5	1
S-10	3	e	5	1

S-11	3	f	5	1
S-12	3	a	5	2
S-13	3	b	5	2

Table 3: Sensor placement modes used for the plate with 28 elements and 40 nodes

Sensor Pattern	Number of the Sensor	Node Number of the Sensor
a	6	8,11,21,24,26,39
b	10	3,8,11,14,21,24,26,29,36,39
c	15	3,5,8,11,14,15,19,21,24,26,28,29,34,36,39
d	10	2,8,11,15,20,22,27,31,34,40
e	10	2,7,12,13,18,23,28,29,34,39
f	10	4,7,10,13,22,28,32,35,37,38

4. 2. 1 Damage detection with single low-extent damage scenario

In order to investigate a small damage on the performance of the proposed method, damage detection has been performed for the 28-element plate under the S-1 damage scenario. The results have been presented in Fig. 5. The results obtained for damage detection under the conditions of only a small damage with the proposed method have been obtained after 15 iterations and 4485 seconds. The proposed method estimates the location and extent of the damage at the damage element with 99% accuracy. The maximum error found in element 15 is 0.07%. The performance of the proposed damage detection method under the conditions of only a minor damage indicates the high accuracy of the method under these conditions.

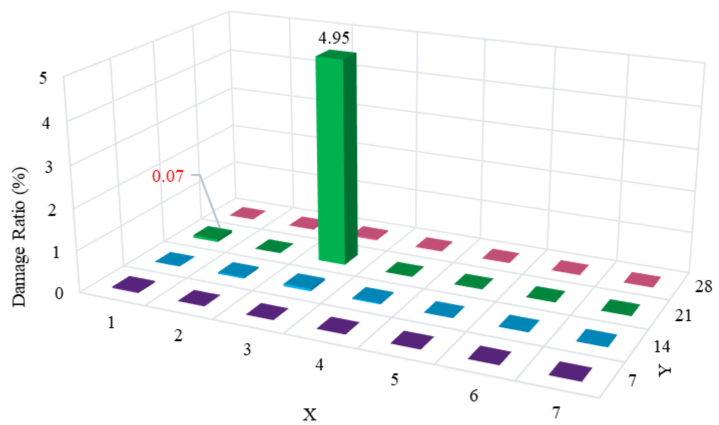


Figure 5. Detection of the damage location and extent of the 28-element plate to investigate a small damage

4.2.2 Damage detection with several low-extent damage scenario

In order to investigate several small damages on the performance of the proposed method, a damage detection has been performed for the 28-element plate under the S-2 damage scenario. The results have been presented in Fig. 6. The results for damage detection under the conditions of several small damages with the proposed method have been obtained after

15 iterations and a duration of 4475 seconds. The proposed method has estimate the location and extent of the damage in the damage elements with an accuracy of 99.3%. The maximum error found in element 15 is 0.09%. The performance of the proposed damage detection method under the conditions of several low-extent damages indicates the high accuracy of the method under these conditions.

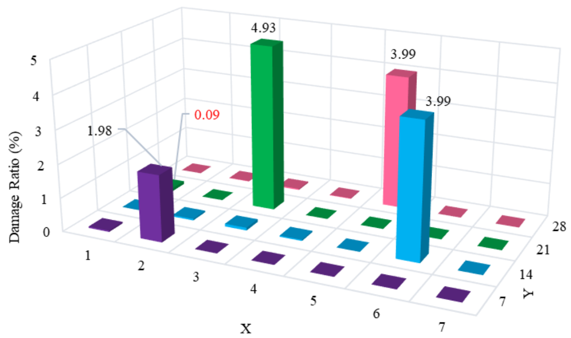


Figure 6. Detection of the damage location and extent of the 28-element plate damage to investigate several small damages

4.2.3 Damage detection with several high-extent damage scenario

To investigate several high-extent damages on the performance of the proposed method, a damage detection has been carried out for the 28-element plate under the S-3 damage scenario. The results have been presented in Fig. 7. The results for damage detection under the conditions of several large damages with the proposed method have been obtained after 15 iterations and a time of 4451 seconds. The proposed method estimates the location and extent of the damage in the damage elements with an accuracy of 99.6%. The maximum error found in element 15 is 0.3%. The performance of the proposed damage detection method under the conditions of several high-extent damages indicates the high accuracy of the method under these conditions.

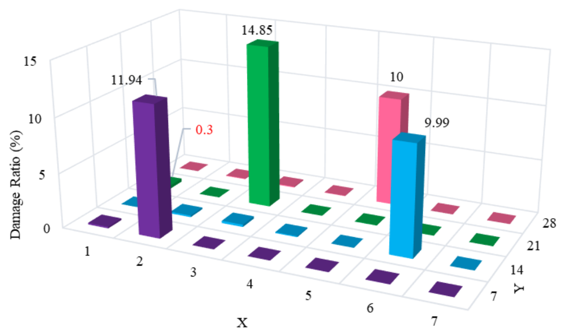


Figure 7. Detection of the damage location and extent of the 28-element plate to investigate several high-extent damages

4.2.4 Examination of damage detection with noise effect scenario

In order to investigate the effect of noise on the performance of the proposed method, damage detection for the 28-element plate has been performed under the same conditions for the S-3, S-4, S-5 and S-6 damage scenarios with noise percentages of respectively 0, 1, 3 and 5. The results have been presented in Fig. 8. The results were obtained for the damage detection under the same conditions with noise percentages of 0, 1, 3 and 5 with the proposed method after 15 iterations in 4451, 4548, 4521 and 4548 seconds, respectively. The proposed method estimates damage location and extent in damage elements at 0, 1, 3 and 5 noise percentages with accuracy of 99.6%, 98%, 93.7% and 87.7%, respectively. The maximum error detected at noise percentages 0, 1, 3 and 5 were 0.3% in element 15, 0.67% in element 9, 2.37% in element 9, and 4.02% in element 9, respectively. According to the results, the accuracy of the method decreases with increasing noise. The performance of the proposed damage detection method under the presence of noise indicates acceptable accuracy of the method under these conditions.

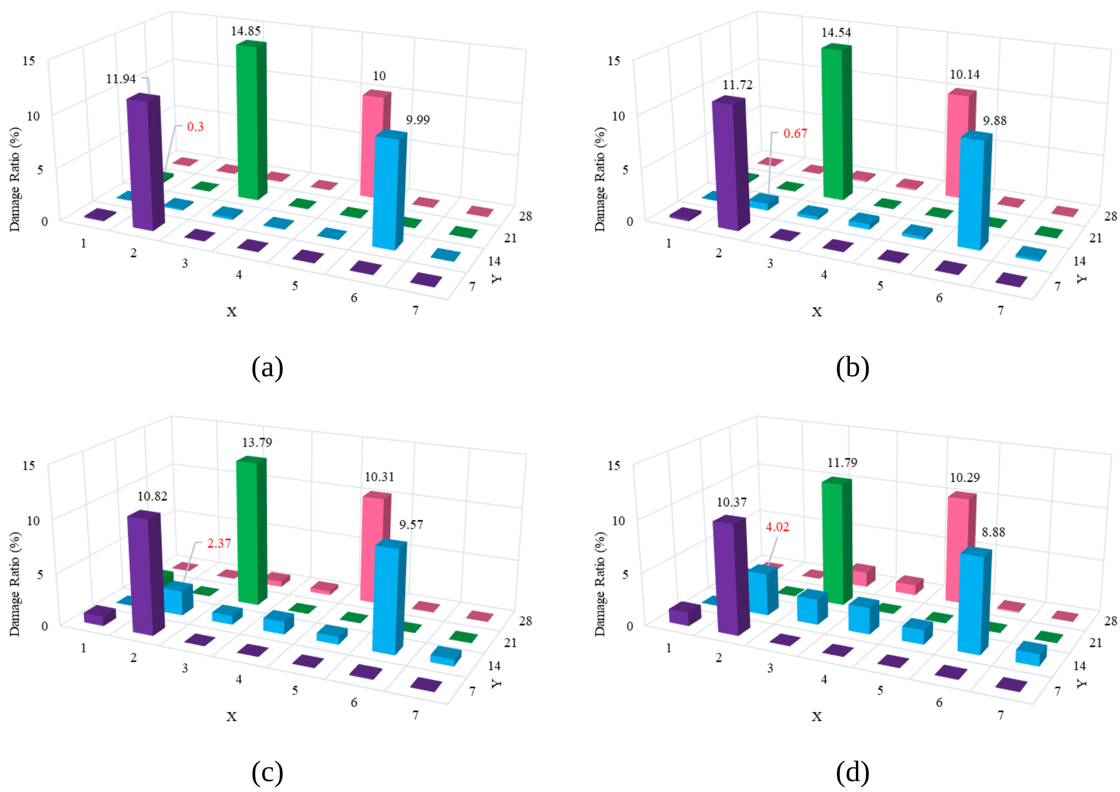
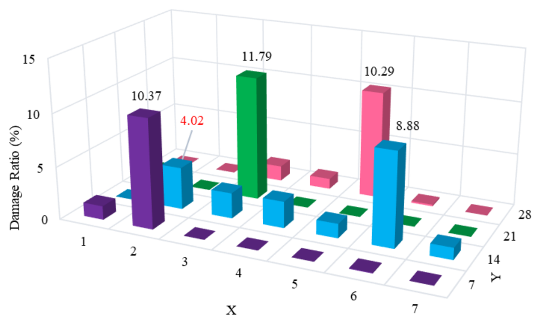


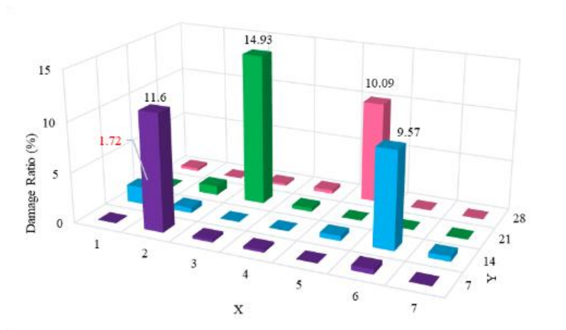
Figure 8. Detection of the damage location and extent of the 28-element plate damage to investigate noise a) 0% noise b) 1% noise c) 3% noise d) 5% noise

4.2.5 Damage detection with scenario of number of measured response (number of sensors)

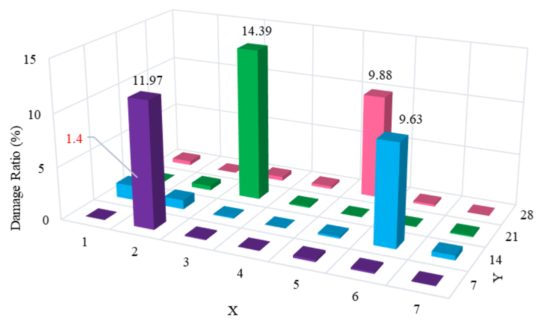
To evaluate the number of measured responses (number of sensors) on the performance of the proposed method, damage detection has been conducted for 28-element plate under identical loading conditions, number of damages (four damages) and noise (5% noise level) for damage scenarios S-6, S-7 and S-8 with 6, 10 and 15 sensors, respectively. The results have been presented in Fig. 9. The results have been obtained for damage detection under the same conditions with percentages with number of measured points (number of sensors) of 6, 10 and 15 with the proposed method after 15 iterations in 4548, 4518 and 6213 seconds, respectively. The proposed method estimated the location and extent of the damage in the damage elements with the number of measured points (number of sensors) of 6, 10 and 15 with 87.7%, 91% and 90.8% accuracy, respectively. The maximum error detected with the number of points measured (number of sensors) of 6, 10 and 15 in element 9 is 4.02%, for element 8 is 1.72% and for element 8 is 1.4%. According to the results, the accuracy of the method increases with increasing number of measured points. The performance of the proposed damage detection method under a low number of measurement points indicates acceptable accuracy of the method under these conditions.



(a)



(b)

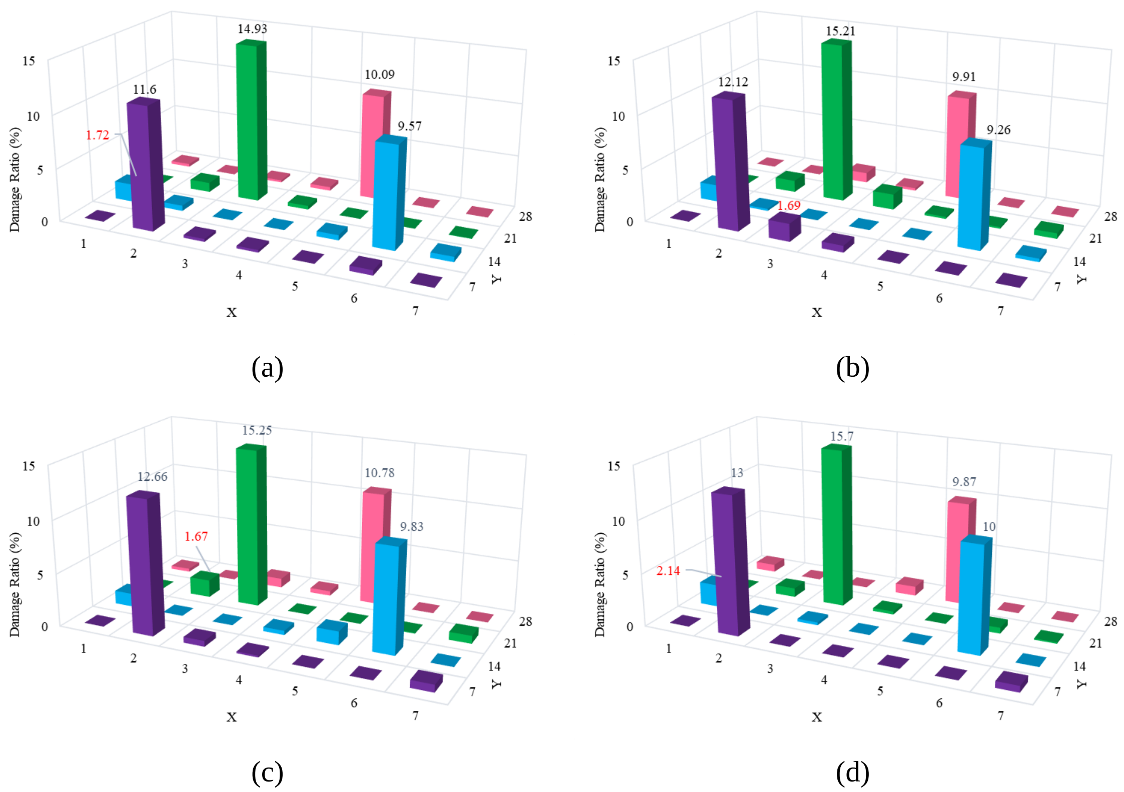


(c)

Figure 9. Detection of the damage location and extent of plate with 28 elements to evaluate the number of measured responses a) 6 sensors b) 10 sensors c) 15 sensors

4.2.6 Damage detection with the scenario of measured point location

To investigate the location of the measured points on the performance of the proposed method, damage detection has been performed for a 28-element plate under the same loading conditions, number of damages (four damages), noise (5% noise level), and number of sensors 10 for damage scenarios S-7, S-9, S-10 and S-11. The results have been presented in Fig. 10. The results have been obtained for damage detection under the same conditions for different locations of the measured points (sensor location) by the proposed method after 15 iterations at the time of 4518, 4558, 4499 and 4530 seconds, respectively. The proposed method has estimated the location and extent at damage locations for damage scenarios with identical conditions but different positions of S-7, S-9, S-10 and S-11 with the accuracy of 91%, 89.3%, 83.3% and 85.7%, respectively. The maximum error detected for different locations of the measurement points under S-7, S-9, S-10 and S-11 scenarios is 1.7% in element 8, 1.69% in element 3, and 1.67 in element 16 and 2.14% in element 8, respectively. Based on the results, the accuracy and performance of the proposed damage detection method is acceptable at all different locations investigated.



(a) (b) (c) (d)

Figure 10. Detection of the damage location and extent of the plate with 28 elements to evaluate the location of the measured points (sensor location) under the damage scenario a) S-7 b) S-9 c) S-10 d) S-11

4.2.7 Damage detection with dynamic analysis time scenario

In order to investigate the dynamic analysis time on the performance of the proposed method, damage detection has been performed for a 28-element plate for the S-12 and S-13 damage scenarios under the dynamic analysis time of 2 seconds and, the results for the similar cases for S-6 and S-7 damage scenarios under dynamic analysis time of 1 second have been compared. The results have been presented in Fig. 11. The results for damage detection during dynamic analysis with the proposed method after 15 iterations for S-6, S-7, S-12 and S-13 scenarios are obtained at 4548, 4518, 8870 and 8869 seconds. The proposed method has estimated the damage location and extent at damage locations for S-6 scenario compared to S-12 scenario with accuracy of 87.7% and 90.2% and for S-7 scenario compared to S-13 scenario, respectively with the accuracy of 91% and 95.1%. The maximum error detected for different locations of the measurement points under the S-6 scenario compared to the S-12 scenario was 4.02% for element 9 and 1.94% for element 11 and under the S-6 scenario compared to scenario S-12 is 1.72% for element 8 and 1.14% for element 19. According to the results, it is found that the accuracy and performance of the proposed damage detection method increase with increasing dynamic analysis time but the computational speed decreases.

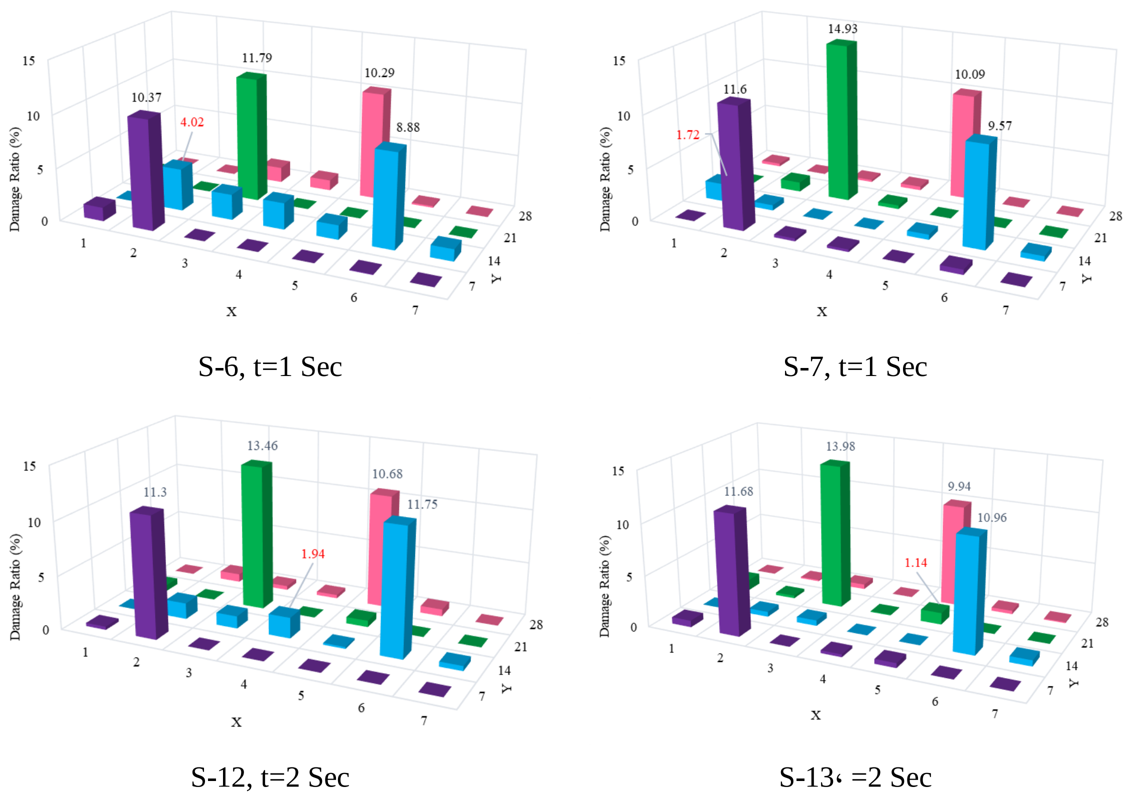


Figure 11. Detection of the damage location and extent of the 28-element plate to investigate the dynamic analysis time

5. SUMMARY AND CONCLUSION

The main objective of this study was to develop a suitable method based on second-order gradient optimization technique for thin plates. The second-order gradient optimization algorithm used consists of a sensitivity analysis-based Levenberg-Marquardt algorithm. A problem of a valid reference has been solved in order to evaluate and compare the proposed technique with other efficient methods to detect damage, the results show that the proposed method performs better than the reference method in detecting damage location and extent. The results of the damage detection with different scenarios indicate that the proposed technique under low-time dynamic analysis under various conditions such as small and large-extent damage, one or more damage, considering the noise effect at different levels, and limited number of sensors at the random locations can detect the damage location and extent with acceptable accuracy. Also, the algorithm used for damage detection in the thin plates has the ability to solve problems with low number of iterations to achieve convergence.

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