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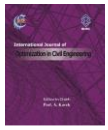

RELIABILITY-BASED MULTI-OBJECTIVE OPTIMAL DESIGN OF SPATIAL TRUSSES USING GAME THEORY AND GA

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ABSTRACT

This paper introduces a reliability-based multi-objective design method for spatial truss structures. A multi-objective optimization problem has been defined considering three conflicting objective functions including truss weight, nodal deflection, and failure probability of the entire truss structure with design variables of cross sectional area of the truss members. The failure probability of the entire truss system has been determined considering the truss structure as a series system. To this end, the uncertainties of the applied load and the resistance of the truss members have been accounted by generating a set of 50 random numbers. The limitations of members' allowable have been defined as constraints. To explain the methodology, a 25-bar benchmark spatial truss has been considered as the case study structure and has been optimally designed using the game theory concept and genetic algorithm (GA). The results show effectiveness and simplicity of the proposed method which can provide Pareto optimal solution. These optimal solutions can provide both safety and reliability for the truss structure.

Keywords: reliability-based optimal design; multi-objective optimization; spatial truss; game theory; genetic algorithm.

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1. INTRODUCTION

One of the most important civil engineering structures are truss structures which have been extensively used in different applications such as bridges, transmission towers, outriggers, roofs, and etc. Optimally design of truss structures is still a challenging task to satisfy several criteria related to cost, safety, and reliability. To this end, several researches have been addressed to optimal design of truss structures [1-10]. In the majority of previous

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studies, limiting the nodal displacements and axial stress of the members as a safety criterion as well as minimizing the truss weight as an economic criterion have been intended. The optimization techniques have been used to design truss structures considering only the truss response or weight in a deterministic framework. Notwithstanding, different sources of uncertainties exist within the design variables, material properties, and applied load which can significantly increase the failure probability of the truss elements or the entire truss system.

In the light of previous studies, the major drawback is neglecting the reliability of the truss system as a design criterion. However, an appropriate and reliable design procedure could be provided by considering the minimization of failure probability as a design criterion. Some studies have addressed this issue and limited the failure probability of the truss structure majorly as constraints in optimization-based procedures. Papadrakakis et al. [11] have considered the objective function of minimization of weight of the structure while satisfying the probabilistic constraints. Yadav and Ganguli [12] optimized truss structures and laminated composite plates by considering failure probability as a constraint. They have used Monte Carlo simulation to obtain the probability of failure. Also, regarding the consideration of uncertainties in design process of structures, the robust design optimization has also been introduced. In this method, the objective is to minimize the probabilistic properties of the objective function such as expectation value or standard deviation. Doltsinis and Kang [13] converted a multi-objective optimization problem to a single objective by introducing weighting coefficients for expectation value and standard deviation of the objective function. Lee and Park [14] designed truss and frame buildings by using weighting coefficients and they linearized the constraint function using Taylor's series first-order approximation. Thus, the robust optimization problem was converted to a deterministic optimization problem. Sandgren and Cameron [15] have used Monte Carlo simulation to determine expectation value and standard deviation of the constraint function. They have used this method to topology optimization of a truss structure and an automotive inner body panel.

In these studies, the minimization of the failure probability of truss structures has not been considered as the objective function. Therefore, this paper aims to design truss structure considering multiple objectives of failure probability along with weight and deflection of the truss. These objectives conflict with each other and a multi-objective optimization problem have been defined and solved using game theory procedure and genetic algorithm.

2. RELIABILITY OF TRUSS STRUCTURES

2.1 Failure definition

Failure definition is an essential task in determining the failure probability of each component and the entire structure as well. According to the reliability theory, the failure could be defined by performance function or limit state function as follows [16]:

$$g = R - Q \tag{1}$$

in which, R denotes the resistance and Q represents the load effect. Both R and Q are random variables. When $g<0$, the load effect exceeds the resistance, then the performance is undesirable and the component is failed. Conversely, when $g\geq 0$, the performance is desirable and the component is safe. Consequently, the probability of failure, P_f, is the probability that the undesired performance occurs and could be expressed as:

$$P_f = P(R - Q < 0) = P(g < 0) \tag{2}$$

In this paper, the uncertainties have been accounted for both the applied load and the resistance of the truss members by generating random numbers. The uncertain applied loads cause the random stress demand in each elements of the truss structure. On the other hand, the randomness has also been considered for yielding and buckling stress capacity of truss elements.

2.2 Failure probability of truss element

Several methods could be used for estimating the failure probability of a truss element. As a common approach, a reliability index denoted by β has been introduced by Hasofer and Lind [17] as follows:

$$\beta = \frac{\mu_R - \mu_Q}{\sqrt{\sigma_R^2 + \sigma_Q^2}} \tag{3}$$

where μ and σ respectively denotes mean and standard deviation. By assuming normal distribution for both random variables R and Q, the probability of failure could be derived by:

$$P_f = \Phi(-\beta) \tag{4}$$

in which, Φ is the standard normal cumulative distribution function. This equation represents the failure probability of a single truss element, while the failure probability of the entire truss structure is required.

2.3 Failure probability of the entire truss system

Determining the failure probability of a truss structure is a challenging task which should be performed properly. Indeed, it is important to distinguish that the failure of a single element may or may not cause the failure of the entire structure. The truss system configuration is within series and parallel system. In a series structural system, the failure of one element leads to immediate failure of the whole system. A definite truss and an indefinite truss with brittle elements are examples of series systems. Conversely, in a parallel system, all of the elements must fail before the system fails. An indefinite truss structure with ductile elements behaves similar to a parallel system. In this paper, the case study truss structure is assumed to be an indefinite truss with brittle elements and thus it is categorized as a series structural system. The failure probability of a series structural system belongs to the following range [16]:

$$\max(P_{f-i}) \le P_{f-sys} \le 1 - \prod_{i=1}^{N_e}\left(1 - P_{f-i}\right) \tag{5}$$

where, $P_{f\text{-}sys}$ is the failure probability of the system, $P_{f\text{-}i}$ is the failure probability of the i-th element, and Ne is number of truss elements. In a series system, the failure probability of the system depends on the statistical dependence between failures of elements. The lower bound is the failure probability of the system when the all elements are fully coupled. The upper bound relates to the case that all elements are uncorrelated and statistically independent. This upper bound provides a conservative estimate of failure probability and is commonly used for series system in the literature [18-20]. In this paper, the case study truss structure is an indefinite truss with brittle elements and it is assumed that the failure of its elements to be uncorrelated. Hence, the failure probability of this system could be evaluated by the upper bound of the Equation (5) which is as follows:

$$\begin{aligned} P_{f-sys} &= 1 - \prod_{i=1}^{N_e}\left(1 - P_{f-i}\right) \\ &= 1 - \left[\left(1 - P_{f-1}\right)\left(1 - P_{f-2}\right) * \ldots * \left(1 - P_{f-N_e}\right)\right] \end{aligned} \tag{6}$$

3. MULTI-OBJECTIVE OPTIMIZATION PROBLEM OF TRUSS STRUCTURE

In many realistic engineering problems, it is required to satisfy some different objectives that conflict with each other. The multi-objective optimization is a capable method to solve such problems and represents Pareto optimal solutions instead of a single solution. The Pareto optimal solutions do not dominate each other. Generally, the definition of a multi-objective optimization problem is as follows:

$$\begin{aligned} &Find && : \mathbf{X}^* = [X_1^*, X_2^*, \ldots, X_n^*]^T \\ &Optimize && : \mathbf{f}(\mathbf{X}) = [f_1(\mathbf{X}), f_2(\mathbf{X}), \ldots, f_m(\mathbf{X})]^T \\ &Subject\ to && : g_i(\mathbf{X}) \ge 0, \quad i = 1,2,\ldots,p \\ & && \quad h_i(\mathbf{X}) = 0, \quad i = 1,2,\ldots,q \end{aligned} \tag{7}$$

where f is the vector of m number of objective functions. X denotes the vector of design variables which may have several solutions such as X*. Also, n is number of design variables and m is number of objective functions. The inequality constraints, $g_i(X)$, and the equality constraints, $h_i(X)$, with the number of p and q, respectively, should be satisfied.

In this study, three conflicting objectives have been defined as objective functions to be minimized simultaneously. The weight of truss has been considered as a cost criterion, the nodal deflection of the truss has been considered as a safety criterion, and the failure probability of the entire truss has been considered as reliability criterion. Thus, the multi-objective optimization problem to design the spatial truss structure is defined as follows:

$$Find \quad : \mathbf{X}^* = [A_1^*, A_2^*, \ldots, A_{nv}^*]^T \tag{8}$$

$$
\begin{aligned}
Optimize\ \ &: f_1 = W = \sum_{i=1}^{n} \gamma_i A_i L_i \\
& f_2 = \max(\delta_i)\,, \qquad i = 1,2,\ldots,p \\
& f_3 = P_{f-sys} \\
Subject\ to &: \sigma_{min} \le \sigma_i \le \sigma_{max}, \qquad i = 1,2,\ldots,q \\
& \sigma_i^b \le \sigma_i \le 0, \qquad i = 1,2,\ldots,ns \\
& A_{min} \le A_i \le A_{max}, \qquad i = 1,2,\ldots,ng
\end{aligned}
$$

in which W is the weight of the truss structure. p and q are respectively the number of node and members of the truss. Also, ns is the number of compression members and nv is the number of design variables. L_i is the length of i-th element. A_i is the cross section area of the i-th member. σ and δ are stress and nodal deflection, respectively. σ_b is allowable buckling stress when the member i-th is in compression.

4. GAME THEORY AND GA FOR MULTI-OBJECTIVE OPTIMIZATION

4.1 Game theory procedure

A game theory procedure [21-23] has been used to solve the multi-objective optimization problem discussed in the previous section. Each of the objective functions are considered as a player and their actions according to their strategies to minimize their individual gains is called game. When players do not cooperate with each other and their actions are independent, the game is called non-cooperative and the consequent solution is called Nash Equilibrium Solution. Conversely, a cooperative game is defined as a coalition of players with the aim of working together to receive an outcome better than the Nash solution. The success criterion of a cooperative game is embodied in the concept of Pareto optimal solution or Pareto front. The Pareto solutions do not dominate each other. In the game theory procedure, a single objective function is defined by a convex combination of different objective functions and the cooperative game result the Pareto optimal solutions. For solving a multi-objective optimization problem using game theory, different objectives are assumed as rational players who aims to maximize their own gains within the feasible domain. It is assumed that all players know the best and worst gains of all players including themselves. A rational bargaining model so-called supercriterion is defined based on the Nash solution for the negotiation process. Thus, all players negotiate to maximize the supercriterion function to determine the cooperative solutions. The game theory procedure for a three objective problem could be explained by the following five steps [23]:

Step 1: define reasonable conflicting objective functions, $f_1(X)$, $f_2(X)$, and $f_3(X)$ for the multi-objective optimization problem at hand.

Step 2: Select a starting feasible design vector, X_0, and normalize the objective functions by choosing the constant coefficients m_1, m_2, and m_3 as:

$$
m_1 f_1(X_0) = m_2 f_2(X_0) = m_3 f_3(X_0) = M \tag{9}
$$

in which, M is a specific constant. Therefore, all the objective functions are scaled equally at the starting design X_0. The normalized objective function for the i-th player is as follows:

$$F_i(X) = m_i f_i(X), \qquad i = 1,2,3 \tag{10}$$

Step 3: Minimize each of the objective functions separately and determine the optimum solutions X_i^* for i-th objective function.

Step 4: construct the matrix [P] as:

$$[P] = \begin{bmatrix} F_1(X_1^*) & F_2(X_1^*) & F_3(X_1^*) \\ F_1(X_2^*) & F_2(X_2^*) & F_3(X_2^*) \\ F_1(X_3^*) & F_2(X_3^*) & F_3(X_3^*) \end{bmatrix} \tag{11}$$

It is evident that the diagonal arrays in matrix [P] are the minimum solutions in their corresponding column. The worst values of the objectives are calculated by:

$$F_{ju}(X) = \max_{i=1,2,3} \left(f_j(X_i^*)\right), \qquad j = 1,2,3 \tag{12}$$

It is clear that the i-th player do not expect a value better than $F_i(X_i^*)$ and a value worse than F_{iu}.

Step 5: Under this assumption that the players start negotiation from their worse value, the supercriterion (S) is formulated as follows:

$$S = \prod_{j=1}^{3} \left[F_{ju} - F_j(X)\right] \tag{13}$$

in which, the quantity in the bracket is the gain value for the j-th player. Maximizing the Supercriterion, S, is cooperative game solution that yield the design vector representing a Pareto optimal solution.

4.2 Genetic algorithm and reliability analysis

Solving a multi-objective optimization problem based on game theory procedure requires only solving some single objective optimization problems. These single objectives include minimization of separate objective functions as well as maximization of supercriterion objective function and could be solved by any metaheuristic algorithms. Several metaheuristic algorithms have been proposed for solving optimization problems such as genetic algorithm [24], particle swarm optimization [25], colliding bodies [26], search and rescue [27], and etc. The genetic algorithm, GA, is one of the most capable algorithms and is extensively used in engineering problems. This algorithm is inspired form the evolution process in the nature and has been developed first by Holland [28]. In this paper, the GA has been used to solve the optimization problems due to its effectiveness and simplicity. The GA has three main operations including selection, crossover, and mutation [29].

In this paper, the selection criterion of the individuals for mating has been considered based on the stochastic universal sampling [30]. The probability of selecting an individual is as follows:

$$P(x_i) = \frac{F(x_i)}{\sum_{i=1}^{N_{ind}} F(x_i)} \tag{14}$$

in which, $F(x_i)$ is the fitness of individual x_i and N_{ind} is the number of individuals. Then, the selected individuals generate newborns by crossover operator. The linear combination of parents genes is considered for crossover operator as follows:

$$G_{1,2} = P_1 \pm \alpha(P_2 - P_1) \tag{15}$$

in which, G_1 and G_2 are the newborn chromosome genes. P_1 and P_2 are the corresponding parent chromosome genes and α is a random scale factor within the range [-0.25, 1.25]. The mutation operator of GA is used to guarantee searching all probable solutions and to avoid local minima. The number of mutated individuals is calculated by:

$$N_{mut} = m_r N_{new} N_{var} \tag{16}$$

in which, m_r is the mutation rate. This variable is recommended to be small and the value of o.o4 has been considered for it. The N_{new} and N_{var} are respectively number of newborns and variables in each generation.

Also in recent years several metaheuristic algorithms have been proposed for reliability analysis such as using the combination of asymptotic sampling and weighted simulation for reliability estimation [31], evaluate the reliability index with the Modifed Dolphin Monitoring operator [32], calculation of the probability of failure of structural systems involves multi-dimensional integrals, which are complicated or even impossible to solve [33], applied set theoretical variants of some of population-based metaheuristic algorithms to solve frequency-constrained truss optimization problems [34], utilize four metaheuristic algorithms consisting of the improved ray optimization, democratic particle swarm optimization, colliding bodies optimization and enhanced colliding bodies optimization with the penalty function to estimate failure probability of problems [35], using the charged system search algorithm to solve aforementioned constrained optimization [36].

5. NUMERICAL ANALYSIS AND DISCUSSION

In this section, the methodology of reliability-based multi-objective optimal design of truss structures has been explained through numerical analysis. Three objective functions including the truss weight, deflection, and failure probability of the entire truss structure has been intended to be minimized. The game theory procedure along with genetic algorithm have been used to solve the multi-objective optimization problem and determine the Pareto optimal solutions. The cross section areas of the truss elements have been considered as design variables and stress of elements have been constrained.

5.1 Twenty five-bar spatial truss

In this paper, a 25-bar benchmark spatial truss structure has been considered as the case study structure. This benchmark truss structure has been previously studied in several researches [1-7]. The topology and nodal and element numbers of this truss structure have been illustrated in Fig. 1. This truss structure has been subjected to two different load cases as represented in Table 1. The density and elasticity modulus of the material are considered the values of 0.1 lb/in^3 (2767.99 kg/m^3) and 10000 ksi (68950 Mpa), respectively.

The elements of the truss have been categorized into eight groups in terms of cross section area as: (1) A_1, (2) A_2-A_5, (3) A_6-A_9, (4) A_{10}-A_{11}, (5) A_{12}-A_{13}, (6) A_{14}-A_{17}, (7) A_{18}-A_{21}, (8) A_{22}-A_{25}. The tensile stress is constrained to be below the value of 40 ksi (275.8 Mpa) and the limitations of the compressive stress are considered according to Table 2. The cross section area varies in the range of 0.01 to 3.4 in^2 (0.6452-21.94 cm^2).

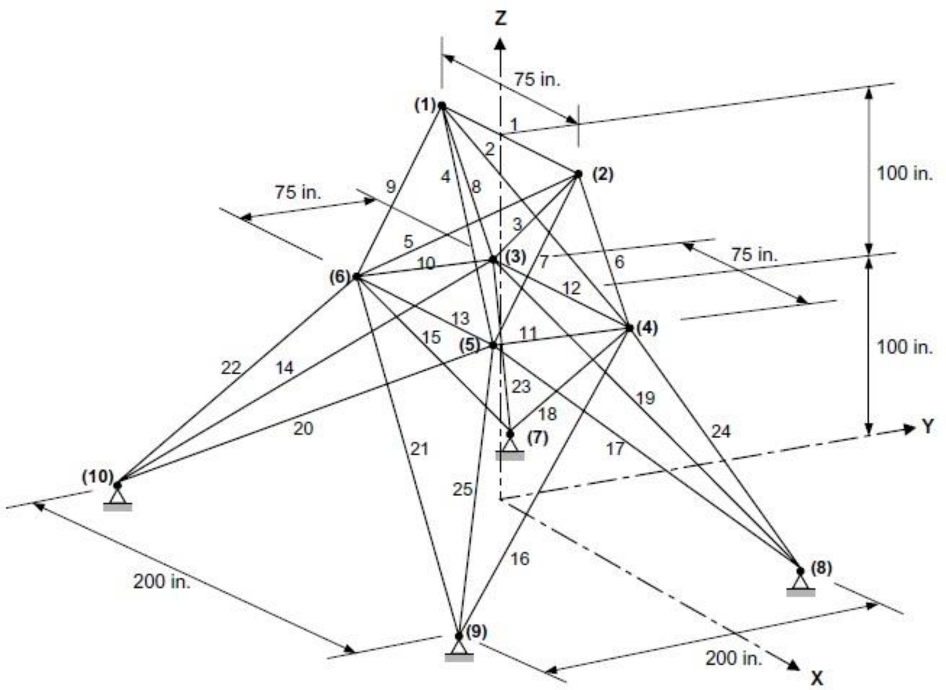

Figure 3. The 25-bar spatial truss [7]

Table 1: The load case for the spatial truss

Load case	Node	P_X kips (kN)	P_Y kips (kN)	P_Z kips (kN)
Case1	1	0	20 (89)	-5 (22.25)
	2	0	-20 (89)	-5 (22.25)
Case2	1	1 (4.45)	10 (44.5)	-5 (22.25)
	2	0	10 (44.5)	-5 (22.25)
	3	0.5 (2.22)	0	0
	6	0.5 (2.22)	0	0

Table 2: The limitations of compressive stress for the spatial truss members

Element group		Compressive Stress ksi (Mpa)
1	A_1	35.092 (241.96)
2	A_2~A_5	11.590 (79.913)
3	A_6~A_9	17.305 (119.31)
4	A_{10}~A_{11}	35.092 (241.96)
5	A_{12}~A_{13}	35.092 (241.96)
6	A_{14}~A_{17}	6.759 (46.603)
7	A_{18}~A_{21}	6.959 (47.982)
8	A_{22}~A_{25}	11.082 (76.410)

5.2 Uncertainties of load and resistance

Properly accounting for the effects of uncertainties is a crucial task is reliability assessment of truss structures. The significant uncertainties involved in this problem are uncertainties of the applied load and the resistance of the truss members. The effects of load uncertainties are taken into account by modelling them as random variables. Therefore, random variables with normal distribution have been generated for all loads applied to the truss structure. According to Table 1, two load cases including 11 separate loads have been applied into the truss structure. It has been assumed that these loads are statistically independent. For each of these loads, 50 normal random numbers with the mean values according to Table 1 and different coefficient of variations (CoVs) including 0.1, 0.2, 0.3, and 0.4 have been generated.

The uncertainty in the resistance of the truss members has been considered by taking into account the allowable stress of truss members as random variables. The mean value of tensile stress has been considered 40 ksi (275.8 Mpa) and the mean values of compressive stresses have been considered according to Table 2. Different CoVs including 0.01, 0.05, and 0.1 have also been considered for them.

5.3 Reliability assessment of the truss structure

In this section, the reliability of the 25-bar spatial truss with previously designed cross-sectional area of the members has been assessed. Several studies have been addressed the optimal design of this truss structure in a deterministic framework [1-7]. In these studies, the optimization problem of the equation (8) without consideration of the second and third objective functions has been considered. Thus, only the objective function of minimization of the truss weight under the assumption of deterministic load and resistance has been intended. As sample, the results of works performed by Kaveh and Talatahari [7] have been assessed. The optimal cross section areas of the eight groups of truss members and the corresponding truss weight of these works have been reported in Table 3.

The uncertainties of the applied load and members' allowable stress have been considered by generating normal random numbers. The failure probability of each member has been calculated by equation (4) and the failure probability of entire truss has been evaluated by equation (6).

Table 3: Optimal cross-sectional area of truss members and truss weight

Element group		Kaveh and Talatahari [7]	
		in^2	cm^2
1	A_1	0.010	0.065
2	A_2~A_5	1.993	12.856
3	A_6~A_9	3.056	19.717
4	A_{10}~A_{11}	0.010	0.065
5	A_{12}~A_{13}	0.010	0.065
6	A_{14}~A_{17}	0.665	4.293
7	A_{18}~A_{21}	1.642	10.594
8	A_{22}~A_{25}	2.679	17.281
Weight		545.16 lb	2425 N

Table 4: The failure probability of each member under two load cases

Element number	P_f under load case 1	P_f under load case 2	Maximum P_f
1	3.22657e-60	1.58599e-59	1.58599e-59
2	0.00037	6.38464e-22	0.00037
3	6.02146e-46	8.34645e-27	8.34645e-27
4	1.16521e-45	9.76495e-73	1.16521e-45
5	0.00041	5.85352e-72	0.00041
6	1.01730e-51	8.42004e-35	8.42004e-35
7	9.38051e-14	3.99225e-70	9.38051e-14
8	3.99817e-14	4.77697e-34	3.99817e-14
9	1.28088e-50	3.42804e-68	1.28088e-50
10	1.47452e-76	6.70770e-77	1.47452e-76
11	1.44007e-77	1.25741e-76	1.25741e-76
12	6.93282e-71	6.92197e-65	6.92197e-65
13	4.16909e-69	1.50471e-80	4.16909e-69
14	0.00053	0.02703	0.02703
15	2.21834e-60	2.952317e-61	2.21834e-60
16	7.38021e-61	0.07453	0.07453
17	0.00109	1.01778e-63	0.00109
18	8.09802e-50	1.75182e-06	1.75182e-06
19	0.58134	2.02725e-05	0.58134
20	0.62966	1.36176e-71	0.62966
21	9.95950e-49	4.74842e-71	9.95950e-49
22	2.82709e-13	1.43733e-61	2.82709e-13
23	2.37910e-17	3.11733e-11	3.11733e-11
24	4.13807e-14	4.57208e-09	4.57208e-09
25	2.69044e-16	2.03856e-64	2.69044e-16
Failure probability of the entire truss (%)			86.07

Table 4 represents the failure probability of each member and the entire truss system by considering the CoV of 0.2 and 0.05 for load and resistance, respectively. The failure probabilities of the entire truss structure with different CoVs for load and resistance have also been reported in Table 5. It could be observed that the maximum failure probabilities of the truss relate to CoV of 0.2 for load and CoV of 0.1 for resistance. Also, it can be see that the minimum failure probability corresponds to the CoV of 0.1 for load and CoV of 0.05 for resistance.

Table 5: The failure probability of the entire truss (%)

		CoV of allowable stress		
		0.01	0.05	0.1
CoV of the load	0.1	81.58	81.26	81.87
	0.2	86.03	86.07	86.42
	0.3	83.69	84.05	85.23
	0.4	81.49	82.02	83.52

5.4 Reliability-based multi-objective optimal design of truss structure

In this section, the reliability-based multi-objective method has been used to design optimal cross sectional area of the 25-bar spatial truss. According to Equation (8), the three objective functions of minimization of the truss weight, minimization of nodal deflection, and minimization of the failure probability of the entire truss system have been considered in the multi-objective optimization problem. The cross sectional areas of the truss members are the considered design variables. The optimal values of the design variable are searched within the pre-defined domains in the optimization process. The upper and lower bound of these domains affect the convergence speed. However, if they include the optimal answer, they have no significant effect on the final answer. In order to provide an acceptable convergence speed, the search domain of design variables have been selected 0.01 to 3.4 in^2 (0.6452-21.94 cm^2) according to [7]. The multi-objective optimization has been solved frequently, where the parameters of the GA have been selected as presented in Table 6.

Table 6: Parameters of genetic algorithm

N_{ind}	Number of individuals in each generation	100
N_{new}	Number of newborns	20
m_r	Mutation rate	0.04
N_{max}	Maximum number of generations	1000
N_{ind}	Number of individuals in each generation	100

The values of the arrays of the starting design vector are considered to be 1.0 in^2. The values of normalized objective functions at the starting design vector are considered F_1=F_2=F_3=500. The optimization problem of designing the cross section of truss elements have been solved for each objective function separately and the results have been reported in Table 7. The design vectors for three different objectives have been illustrated in this table and all the constraints have been satisfied for them. Also, the starting point has been considered with the arrays of 1.0 in. For the design point, buckling has been occurred in the elements 2, 5, 7, 8, 19, and 20 under load case 1 and elements 23 and 24 under load case 2.

Table 7: Results of minimization of each objective function

variable	Starting point	Minimization of Weight	Minimization of deflection	Minimization of failure probability
Design vector $X(in^2)$	1	3.4	1.6378	0.01
	1	3.4	3.4	1.2375
	1	2.5437	3.4	1.1309
	1	0.01	1.2062	0.01
	1	0.01	3.4	0.2976
	1	1.3828	3.4	0.5521
	1	3.4	3.4	1.655
	1	3.4	3.4	1.2561
Weight (lb)	330.72	344.5837	1078.327	840.0104
Deflection (in)	0.777	0.6506	0.2286	0.2642
Failure probability (%)	100.0	99.8642	0.01096	0.00212
Supercriterion, S	6.25e4	2.5e-12	-0.0092	4.47e7

The values of constant coefficients are m_1=1.5118, m_2= 643.34, and m_3=5. The matrix [P] is determined as:

$$[P]=\begin{bmatrix} 520.96 & 418.57 & 499.32 \\ 1630.27 & 147.06 & 0.0548 \\ 1269.97 & 169.97 & 0.01058 \end{bmatrix} \tag{17}$$

As mentioned earlier, the diagonal arrays of matrix [P] are the smallest values in their respective column. The result illustrates that minimization of the weight resulted in 184% increase in deflection and 4,539,172% in failure probability with respect to their optimum values. Also, minimization of deflection leads to 213% increment in weight and 81% increase in failure probability. Eventually, minimization of failure probability yields 144% increase in weight and 16% increase in deflection.

The first player which relates to first objective cannot achieve a value lower that 520.96 for his/her objective while it is insured that it will not exceed the value of 1630.27. Such these interpretations could also be expressed for other players according to the matrix [P]. The supercriterion S in Equation (13) for the designing spatial truss should be maximized and is in the form:

$$\begin{aligned} S &= \big(F_{1u}-F_1(X)\big)*\big(F_{2u}-F_2(X)\big)*\big(F_{3u}-F_3(X)\big) \\ &= \big(F_{1u}-m_1f_1(X)\big)*\big(F_{2u}-m_2f_2(X)\big)*\big(F_{3u}-m_3f_3(X)\big) \end{aligned} \tag{18}$$

Using the worst values of F_{iu} in the matrix [P], the supercriterion S could be expressed as:

$$S=\big(1630.27-m_1f_1(X)\big)*\big(418.57-m_2f_2(X)\big)*\big(499.32-m_3f_3(X)\big) \tag{19}$$

In order to maximize S, the objective function of -S is minimized. Therefore, the multi-objective optimization problem of designing spatial truss is converted to a single objective optimization problem using the game theory concept. The single objective function is minimization of -S with design variables of cross section area of the elements. This optimization problem has been solved several times with GA. Table 8 illustrates the results of the three separate optimization runs for designing spatial truss. It seems that the applied procedure based on game theory concept and genetic algorithm have the capability to optimize the considered objectives simultaneously. The objectives of weight, deflection, and failure probability have gained rational and proper values. All the considered constraints have been satisfied and design vector of different runs are approximately equal. Also, the results show that the three optimization runs have been converged.

Comparing this Pareto solution to previously deterministic design in literature shows that the failure probability of the 25-bar truss structure under the considered load cases was about 86%, while the proposed method has introduced an optimal solution that reduces the failure probability and deflection about 83% and 10%, respectively, and only with 14% increment in weight.

Table 8: Results of designed truss based on game theory procedure for three different optimization runs

variable	Run 1	Run 2	Run 3	variable
Design vector X(in^2)	0.0999	0.0821	0.0258	Design vector X(in^2)
	2.1329	2.0696	1.9416	
	2.5129	2.5721	2.7496	
	0.01	0.01	0.01	
	0.01	0.01	0.01	
	0.8832	0.9034	0.9014	
	2.5704	2.5915	2.6518	
	2.9029	2.8568	2.809	
Weight (lb)	624.95	624.56	626.72	Weight (lb)
Deflection (in)	0.3207	0.3207	0.3183	Deflection (in)
Failure probability (%)	2.5712	2.5737	2.6852	Failure probability (%)
Supercriterion, S	7.077e7	7.083e7	7.092e7	Supercriterion, S

6. CONCLUSIONS

This paper presents a reliability-based multi-objective optimal design method to design spatial truss structures while accounting for the uncertainties of the applied load and allowable stresses of the truss members. The methodology is based on the definition of a multi-objective optimization problem and solving it using a game theory concept and genetic algorithm. It has been aimed to provide three different criteria including the truss weight as a cost criterion, nodal deflection as a performance criterion, and failure probability of the entire truss as a safety criterion. For illustration, the method has been applied to design optimal cross sectional areas of the members of a 25-bar benchmark spatial truss structure. The Pareto solution of the optimal truss have been derived using the proposed procedure. Numerical studies have shown the capability and simplicity of the applied method in designing cross sectional areas of the truss elements. This method has led to derive an optimal Pareto solution which provides both cost and safety criteria. The results show that the failure probability of the truss structure reduces by increasing the uncertainty level of load and resistance. The failure probability of the 25-bar truss structure under the considered load cases with previously deterministic design was about 86%, while the proposed method has introduced an optimal solution that reduces the failure probability and deflection about 83% and 10%, respectively, and only with 14% increment in truss weight.

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