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DEVELOPMENT OF EQUIVALENT DAMPING RATIO FOR CONCRETE/STEEL MIXED STRUCTURES CONSIDERING SOIL-STRUCTURE INTERACTION

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ABSTRACT

Identification of damping properties for a mixed structure and its interaction with underlying soil is a challenge for structural designers. Current codes and available commercial software packages do not provide analytical solutions for such structural systems. Due to irregular damping ratios, dynamic response of each part of a mixed structure differs significantly. In addition, when the structure is subjected to seismic loads, the soil-structure interaction effects cannot be neglected. To manage these issues, this paper proposes an equivalent damping ratio for mixed structures by means of a semi-empirical error minimization method which considers soil-structure interaction. The results of numerical simulations indicate that the use of the equivalent damping ratios makes the results of dynamics analyses closer to the ones obtained by the actual damping ratios. Consequently, proposed method provides a much better approximation than the case in which the conservative overall ratio of 2% or 5% is used.

Keywords: Mixed structure, equivalent damping ratio, soil-structure interaction, semi-empirical error minimization.

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1. INTRODUCTION

Many studies have been developed for the design of the structures in order to obtain the most

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sustainable, and economical design [1-5]. To increase the resistance of the structures against the earthquakes, new kinds of structural systems are recently investigated [6]. Structures consisting of concrete and steel are introduced as mixed structures, a lower part called primary structure or substructure and an upper part, known as secondary structure or superstructure. There are inherent differences in the nature of each part since damping properties and material laws of the two parts are different. Therefore, dynamic analysis of these structures when it is subjected to stimulation earthquake can be very complicated. In this paper, the substructure and superstructure of mixed structures are composed of reinforced concrete and steel respectively. Also, the damping ratio is considered 5% for concrete and 2% for steel.

For common analyses, it is usual to assume that the structure has a rigid support, but in practice, the structure is constructed on the soil, a material of low stiffness and high damping, due to the natural periods of the system and the overall response is altered. The studies show the effect of soil–structure interaction may considerably change response in a specific load and specific condition [7-9]. In this paper, soil and structure interaction is considered and the soil under the structure is modeled with an equivalent dynamic system consisting of a dashpot and spring.

Current seismic design codes of the building do not have analysis methods for these structures. Several investigators have proposed methods in the past. All analysis methods are divided into two categories. In the first, introduced as the decoupled method, the structure is divided into two parts and each part is analyzed separately, but it has no significant accuracy because the interaction of two parts is neglected. The second, known as the coupled method, the structure is modeled as a whole, and the interaction of the two parts is considered, but the problem with this method is the irregular damping ratio [10-11]. Papageorgiou and Gantes [12] compared the maximum responses of coupled and decoupled time history analyses and presented in the form of error levels between the two methods. If a coupled method is chosen, the interaction of the two parts is considered, the method problem is the irregular damping matrix of these structures that are found. The Classical modal analysis does not reach the diagonal matrix and thus complex eigenmodes are required to time history analysis. The objective of this work is to treat the irregular damping distribution and proposed an overall equivalent damping ratio for obtaining its dynamic response with the readily available commercial software.

Equivalent damping ratio for mixed structures is also obtained modal strain energy method and its modifications. Shen et al. [13] performed shake table tests of reinforced concrete frames. Therefore, the strain energy method was used to predict the response of the damaged structures after applying the two sets of viscoelastic dampers. A trial-and-error procedure to obtain a uniform damping ratio for MDOF structure, which the lower degrees of freedom were made of concrete and the upper ones were made of steel, is proposed [14-15] then they proposed the equivalent structure with aid of the first mode characteristics of each substructure. An expansion their work is introduced by Papageorgiou and Gantes [16] where an equivalent damping ratio is obtained by means of semi-empirical error minimization procedure. The equivalent damping ratio is estimated by harmonic excitation, which ignores the full frequency content of the ground motion. The present work is based on the approach of by Papageorgiou and Gantes [16] aiming at proposing an equivalent damping ratio for mixed structures, as described above. However, it is attempted to express this method with soil and structure interaction (SSI) effects.

2. SOIL AND STRUCTURE INTERACTION

In analyzes, one of the simple assumptions is that connection of the structure to the ground is rigid, but in practice, the structure is constructed on the soil, a material of low stiffness and high damping, this assumption makes the structure more rigid than the actual one [17].

In general, the SSI analysis approaches can be divided into two categories: the direct method and the substructure method. In the substructure analysis, the soil-structure system is divided into two substructures: the first, the structure is located on the ground and the second, the soil and foundation mechanism. The relation between two substructures is maintained by interaction forces of equal amplitude but acting in opposite directions on the two substructures. The stiffness and damping properties of the soil substructure are depended on the excitation frequency; therefore it is convenient to analyses in the frequency domain and then to obtain the response time history by transforming back to the time domain. An MDOF system with a rigid foundation plate is located on a viscoelastic half-space. Effect of soil and structure interaction are applied to the simple model presented by Wolf [18], which is used only for estimating the dynamical parameters of the first mode of the soil-structure system.

In this paper, the soil is modeled using the cone method. The cone method is an indirect method for considering the effects of soil interaction [14]. In this method, the soil under the structure is modeled with an equivalent dynamic system includes a dashpot and spring. In order to obtain the stiffness of the spring and damping coefficient, first, the soil equivalent damper is modeled by the wave propagation pathway caused by the interaction of the soil with a semi-infinite defective cone. Then, the relationship between the interaction force and displacement of the soil and structure interface is obtained using the dynamic stiffness of soil, $S(a_0)$, and calculated the stiffness and damping coefficient of the translational spring and dashpot for rigid circular disk foundations on the viscoelastic half-space, by:

$$P_0(w) = Ku(w) + C\dot{u}(w) \tag{1}$$

$$S(a_0) = K\,[k(a_0) + ia_0c(a_0)] \tag{2}$$

K= static stiffness of a disk on a half-space, w= angular frequency (rad.s^{-1}), a_0 = dimensionless frequency defined by $a_0 = \frac{wr}{v_s}$, r = foundation radius, v_s = soil shear-wave velocity, ν = soil poisson ratio. The real stiffness and damping of the translational spring and damper are expressed, by:

$$k_s = \frac{\rho A {v_s}^2}{z_0} \tag{3}$$

$$c_s = \rho v_s A \tag{4}$$

In this work due to the layering of the soil, CONAN software, introduced and evaluated in Wolf [19], have been used for soil analysis. This software models the soil using the principles of the cone method. The software calculated the stiffness and damping coefficient for equivalent spring and dashpot of the soil and transfers them to an output file. Structure modeling has been used for the shear structure model. The mass of levels in this system is concentrated.

3. CONSTRUCTION OF DAMPING MATRIX

The damping matrix of mixed structures is represented theoretically by Chopra [20]; Clough and Penzien [21]. The nonproportional damping matrix can be determined using similar methods with proportional damping matrices so that for each subsystem of the structure, the damping matrix is determined, then the system matrix is directly assembled from these matrices. This method can be used for a mixed concrete-steel structure. Damping matrices of substructures are represented by Rayleigh's method. Therefore, the damping matrix of each substructure, which is a combination of the mass and stiffness matrices of each part, can be constructed. As given by:

$$\mathbf{C}^i = a_i \mathbf{M}^i + b_i \mathbf{K}^i \qquad i = u, l \tag{5}$$

$$\binom{a_i}{b_i} = \frac{2\xi_i}{\omega_1 + \omega_2} \binom{\omega_1 \omega_2}{1} \qquad i = u, l \tag{6}$$

In Eq. (6), ω is the frequencies of natural modes of vibration and ξ_i are damping ratios. Also in this equation, l is representing the substructure and u is denoting the superstructure. As mentioned before, the substructure and superstructure of mixed structures are composed of reinforced concrete and steel respectively. The damping ratio is considered 5% for concrete and 2% for steel. After determining the damping matrices of the substructures, the damping matrix of the whole system is obtained so:

$$[\mathrm{C}] = [\mathrm{C}^u] + [\mathrm{C}^l] + [\mathrm{C}^s] \tag{7}$$

The subscripts s denoted the soil under the structure.

4. EQUIVALENT UNIFORM DAMPING RATIO

3.1 Elastic systems considering SSI effects

Due to different energy depreciation, each substructure of steel, concrete, and soil is replaced by an equivalent SDOF system. An equivalent SDOF system is obtained by exploiting the first mode characteristics of each separate part. The structure matrix is defined by considering the interaction of soil and structure. The stiffness and mass matrices of the whole system, the following equations are described by:

$$\mathbf{K} = \mathbf{K}^s + \mathbf{K}^l + \mathbf{K}^u = \begin{bmatrix} K_s + K_l & -K_l & 0 \\ -K_l & K_l + K_u & -K_u \\ 0 & -K_u & K_u \end{bmatrix} \tag{8}$$

$$\mathbf{M} = \mathbf{M}^s + \mathbf{M}^l + \mathbf{M}^u = \begin{bmatrix} M_s & 0 & 0 \\ 0 & M_l & 0 \\ 0 & 0 & M_u \end{bmatrix} \tag{9}$$

A complete time history analysis is performed with the exact damping distribution, the equation is described by:

$$[\mathrm{C}] = [\mathrm{C}^u] + [\mathrm{C}^l] + [\mathrm{C}^s] \tag{10}$$

where $\boldsymbol{r} = (1 \quad 1 \quad 1)^T$, in addition $\{y\}\, and\, \{\ddot{y}\}$ are the vectors of relative displacements and accelerations. The vector of total acceleration is as follows:

$$\{\bar{\ddot{y}}\} = \{\ddot{y}\} + \boldsymbol{r}\{\ddot{x}_g\} \tag{11}$$

The soil and structure interaction are considered and soil is added as a DOF system to the structure. A semi-empirical error minimization method is used to calculate the equivalent structural damping ratio. The 3-DOF model is then assumed to have a uniform damping ratio ξ_{un} for whole parts. This uniform damping ratio is varied from $2\%\ to\ 5\%$ at intervals 0.001. Therefore, the damping matrix of the system, which is a combination of the mass and stiffness matrices of the whole structure, can be constructed as given by:

$$\xi_{un} = \{0.02{:}\,0.001{:}\,0.05\} \tag{12}$$

$$\binom{a}{b} = \frac{2\xi_{un}}{\omega_1 + \omega_2}\binom{\omega_1\omega_2}{1} \qquad i = u, l \tag{13}$$

$$\mathbf{C}' = a\mathbf{M} + b\mathbf{K} \tag{14}$$

For each uniform damping ratio, a complete time history analysis of the 3-DOF structure is performed by the following equations:

$$\mathbf{M}\{\ddot{y}'\} + \mathbf{C}'\{\dot{y}'\} + \mathbf{K}\{y'\} = -\mathbf{M}\boldsymbol{r}\ddot{x}_g \tag{15}$$

where **M**, **C'** and **K** are the mass, damping, and stiffness matrices and $\{y'\},\{\ddot{y}'\}$ is the vector of relative displacements and accelerations of vibration and total accelerations as described by the following:

$$\{\bar{\ddot{\boldsymbol{y}}}'\} = \{\ddot{\boldsymbol{y}}'\} + \boldsymbol{r}\{\ddot{x}_g\} \tag{16}$$

Maximum acceleration and displacement values are obtained in both exact and approximate methods and their difference. For each equivalent damping, the error value is calculated using the following equation:

$$e_{accl,i} = \frac{\max(|\bar{\ddot{y}}_i|) - \max\left(\left|\overline{\ddot{y}_i'}\right|\right)}{\max\left(\left|\overline{\ddot{y}_i'}\right|\right)} \quad i = l,u \tag{17}$$

$$e_{disp,i} = \frac{max(|y_i|) - \max\left(\left|y'_i\right|\right)}{\max\left(\left|y'_i\right|\right)} \quad i = l,u \tag{18}$$

where $e_{accl,i}$ and $e_{disp,i}$ are acceleration and displacement errors, and the optimal uniform equivalent damping ratio ξ_{eq} is chosen to minimize the errors. Since the results of this method are dependent on ground motion $\ddot{x}_g$, the ground motion is considered harmonic.

5. NUMERICAL EXAMPLES

5.1. Example 1

The example is comprised of a five-story concrete frame with a vertical steel truss, as shown in Fig. 1. The sections of concrete columns have 120×30 and 60×30, and the concrete slabs provide diaphragm action. The sections of steel members have and HEB 400 and each level of the structure has one translational degree of freedom. Masses of concrete levels are assumed to be 150 Mgr and steel levels are assumed 10 Mgr. The modal characteristics of the complete MDOF structure are gathered in Table 1 and the first mode characteristics of each substructure are presented in Table 2. This structure is the same as the one analyzed in Papageorgiou and Gantes [16].

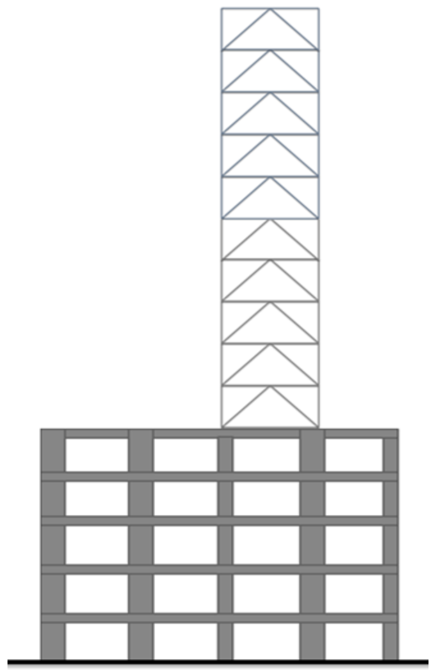

Figure 1: Irregular concrete/steel structure

Table 1: Modal characteristics of the MDOF structure

Mode	Period	Participation factor
1	0.3713	36.55%
2	0.2302	52.37%
3	0.1131	1.02%
4	0.0842	6.91%
5	0.0682	0.28%
6	0.0538	1.91%
7	0.0498	0.16%
8	0.0418	0.61%
9	0.0401	0.04%
10	0.0366	0.13%
11	0.0342	Less than 0.01%
12	0.0304	Less than 0.01%
13	0.0279	Less than 0.01%
14	0.0263	Less than 0.01%
15	0.0254	Less than 0.01%

Table 2: First mode characteristics of the two parts of the structure

Subsystem Modal	Eigen frequency (rad/s)	mass (M gr)
u	25.46	421.03
l	18.67	52.8

The structure with a rigid foundation is located on a viscoelastic half-space. The specification of the layers and their thickness are given in Table 3. Numerical studies in this paper are conducted using the horizontal impedance functions and calculated the equivalent stiffness and damping coefficients of the soil.

Table 3: The specification of soil layers

layer	foundation radius	Shear modulus	Poisson ratio	density	Damping ratio	depth
F	0.15					
L	0	1124e6	0.25	1800	0.05	6.77
L	0	562e6	0.3	1800	0.05	3.385
H	0	224e6	0.33	1600	0.05	

The error minimization method is used to calculate the equivalent damping ratio. The 3-DOF model is now assumed to have a uniform damping ratio $\xi_{un}$for whole parts. For each damping ratio, a complete time history analysis is performed, and calculated errors are shown in Fig. 2. The relevant error distribution of the substructures is irregular when the ground excitation is not considered in resonance with the structure, so the ground excitation $\ddot{x}_g$ is considered a sine wave which its frequency is set to the first mode of the equivalent 3-DOF structure and its amplitude is equal to 0.36g (m/s^2). The optimal uniform equivalent damping ratio ξ_{eq} is chosen to minimize the errors, the resulting equivalent damping ratio is $\xi_{eq} = 3.3\%$.

The preliminary study of the cone method of the soil is limited to the harmonic stimulation (frequency domain) because the soil contribution to **K** and **C** will depend on frequency. If the reflection coefficients are independent of the frequency, the analysis can directly be done in the time domain without converting from the frequency domain. In the numerical analysis, the nonperiodic load, for example, an earthquake is replaced by a periodic one. An earthquake is decomposed into a Fourier series with discrete frequency w_i (i = 1, 2, ..., n). Of course, only a limited number of expressions are processed. Each Fourier series expression is a harmonic load with a frequency w_i, the amplitudes of the harmonic loads in the frequency domain calculated at all discrete frequencies w_i are determined.

Finally, time history analyses are carried out, one with the damping ratio of lower part equal to 5% and the upper part equal to 2%, then approximate time history response calculated, one with equivalent uniform damping ratio equal to 3.3%, and one with 2% for all parts. Due to having a comparison of the behavior of the structure when the approximate damping ratio replacing the actual damping distribution. The results indicated in Fig. 3 that the proposed method gives satisfactory estimates of the equivalent damping ratios.

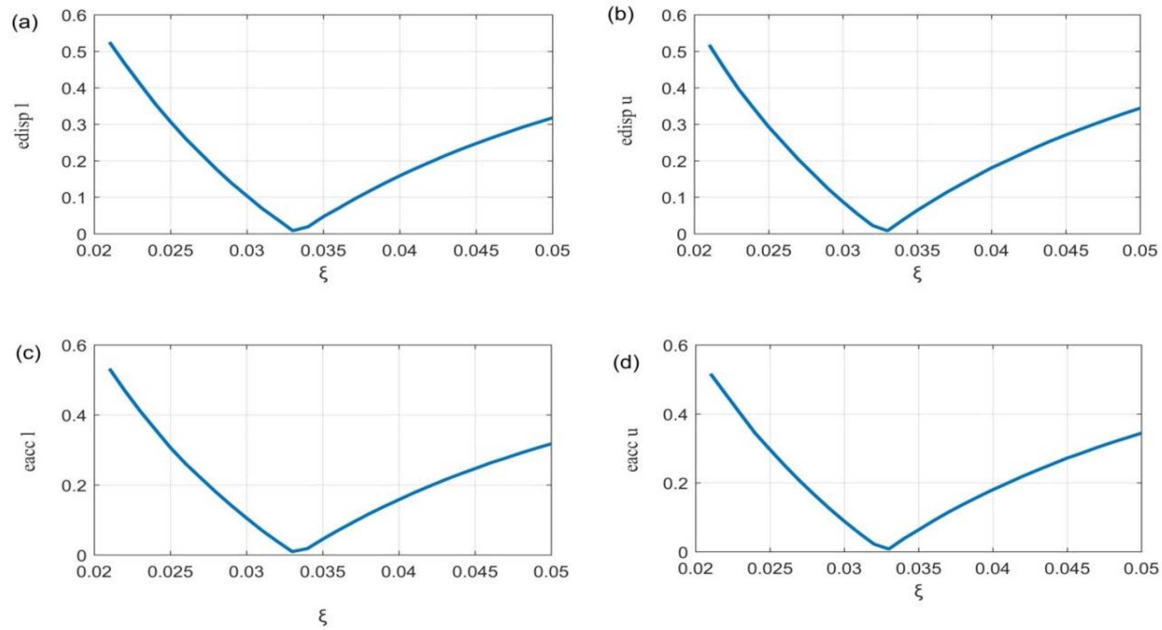

Figure 2: Variation of error with damping ratio in accelerations and displacements by the harmonic motion

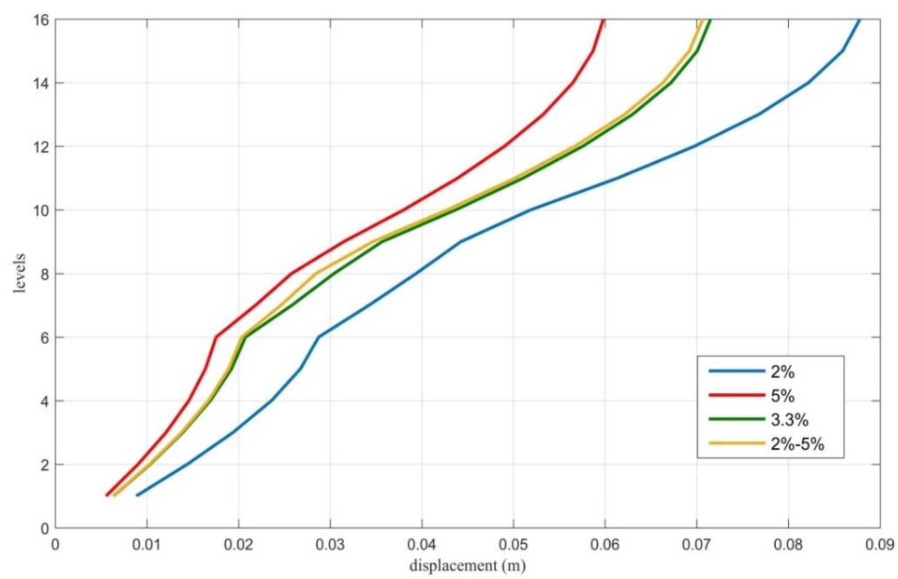

Figure 3: Displacement of MDOF structure by El Centro earthquake

5.2. Example 2

A The MDOF structure is consisted of three levels of concrete and the two levels of steel, shown in Fig.4. Each level of the frame has one translational degree of freedom. The modal characteristics of the MDOF structure and the first mode characteristics of each substructure are shown in Table 4 and Table 5. The structure supported through a rigid foundation resting on a linear elastic half-space. The specification of the soil type is given in Table 3. Numerical studies in this paper are conducted using the horizontal impedance functions [13].

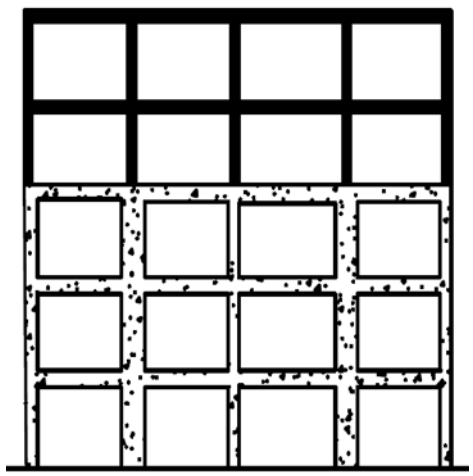

Figure 4: Steel/concrete structure

Table 4: Modal characteristics of the complete structure

Mode	**period**	**Participation factor**
1	0.5271	85.37%
2	0.2022	10.15%
3	0.1245	3.28%
4	0.0972	0.75%
5	0.0858	0.45%

Table 5: First mode of the two parts of the structures

Subsystem Modal	Eigen frequency (rad/s)	mass (M gr)
U	23.76	207.29
L	17.4	368.23

Due to the equivalent stiffness and damping coefficients of the soil are the function of the discrete value of the frequency, the Conan software calculated the stiffness and damping coefficient for equivalent spring and dashpot of the soil and transfers them to an output file. The

3-DOF model is now assumed to have a uniform damping ratio for whole parts. For each damping ratio, a complete time history analysis is performed, and calculated errors are shown in Fig. 5, therefore the ground excitation $\ddot{x}_g$ is considered a sine wave which its frequency is set to the first mode of the equivalent 3-DOF structure and its amplitude is equal to 0.36g (m/s^2).

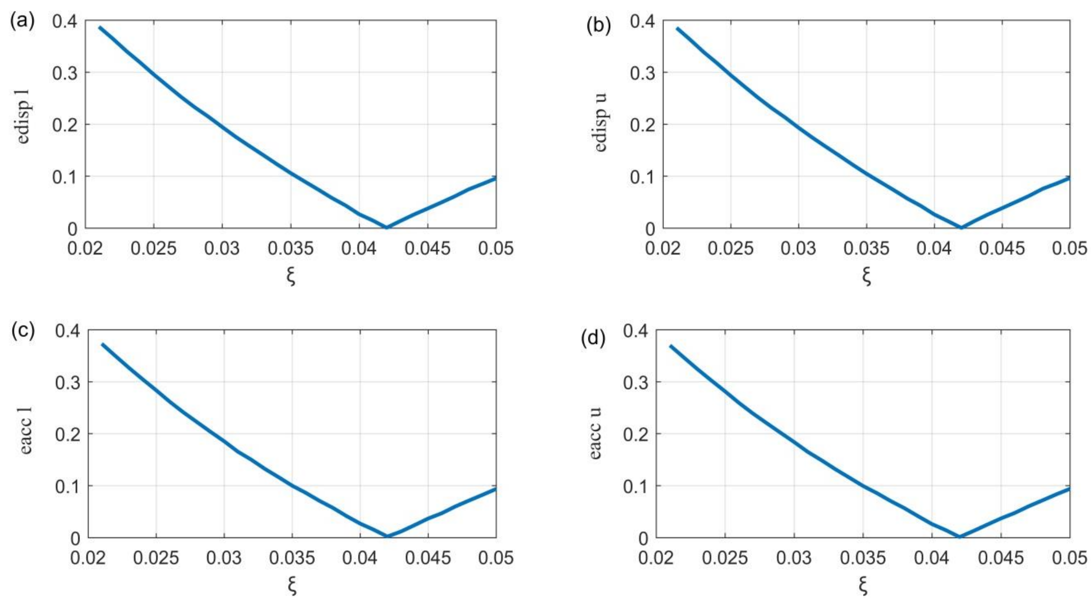

Figure 5: Variation of error with damping ratio in accelerations and displacements of structure by the harmonic motion

As another part of the test, the ground motion is considered a hybrid motion which is a combination of two harmonic motions in resonance with the two modes of the 3-DOF system. Displacement and acceleration errors, obtained in this case, are shown in Fig. 6. The obtained results, in this case, are almost similar to the case when harmonic motion is considered.

The optimal uniform equivalent damping ratio ξ_{eq} is chosen to minimize the errors. Finally, time history analyses are carried out, one with the damping ratio of the lower part equal to 5% and the upper part equal to 2%, then approximate time history response calculated, one with obtained equivalent uniform damping ratio, one with 2% for all parts, and one with 5% for all parts. The results obtained from the analysis corresponding to Fig. 7, indicates which the equivalent damping ratio is a very good approximate of the actual response.

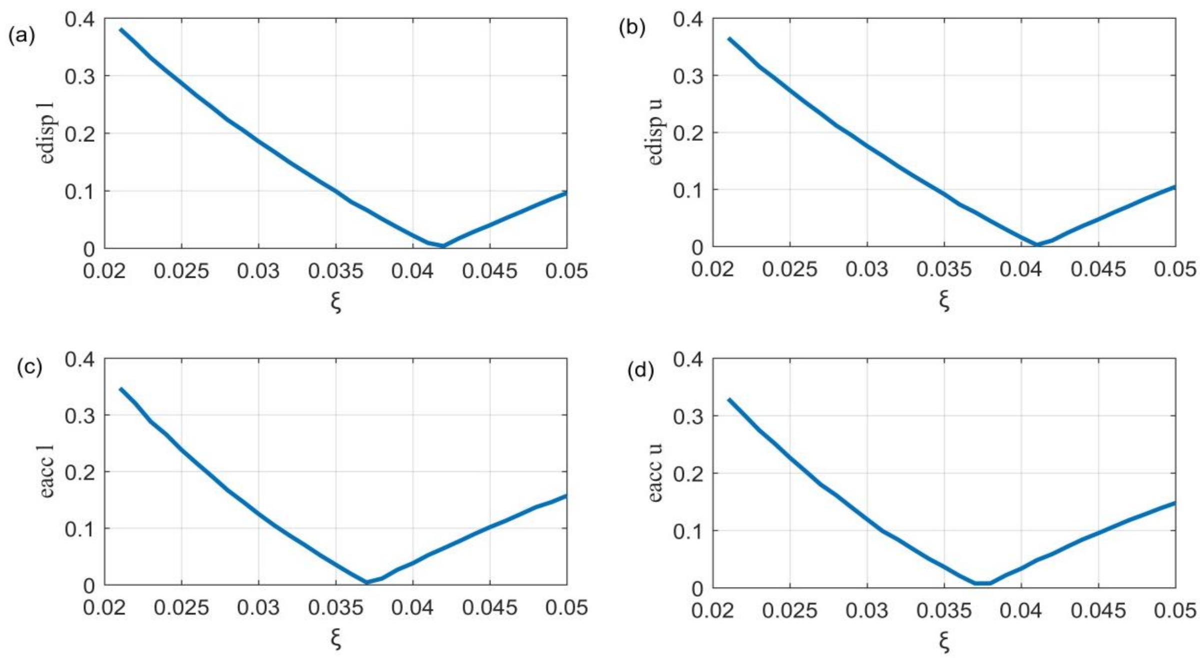

Fig. 6. Variation of error with damping ratio in accelerations and displacements by the hybrid motion

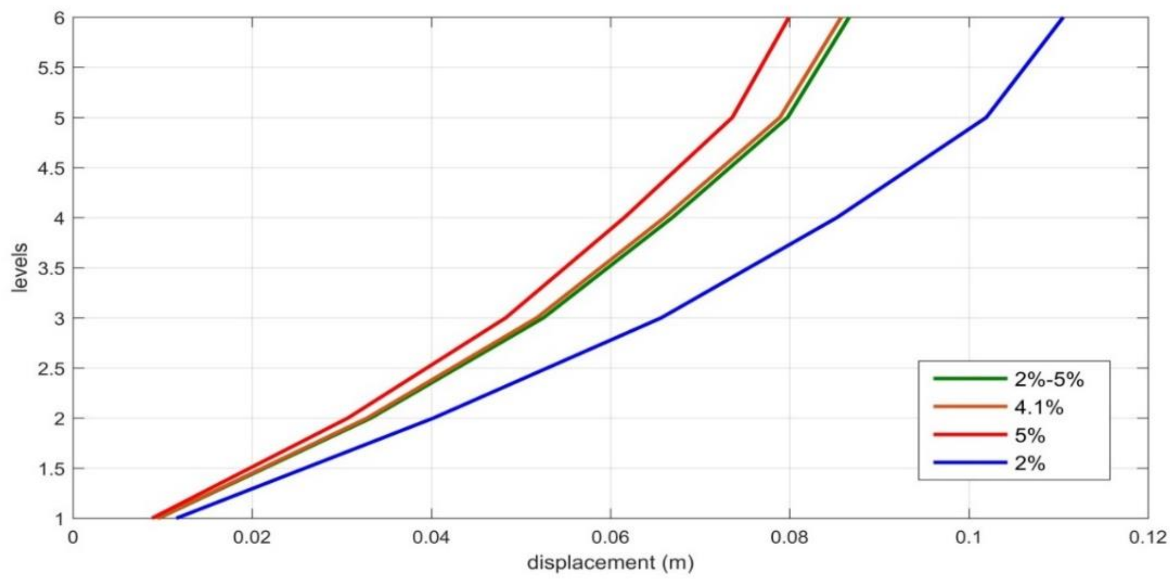

Figure 7: Displacement of MDOF structure with soil by El Centro earthquake

5. CONCLUSIONS

The effect of soil- structure interaction in the analysis of the structures is important, and it closes the response of the model to the real response of the structures. Due to the elastic dynamic response of the mixed structure is investigated by considering soil-structure interaction. Dynamic analysis of the mixed structures when it is subjected to stimulation earthquake can be very complicated. In this paper, the substructure and superstructure of mixed structures are composed of reinforced concrete and steel respectively. Also, the damping ratio is considered 5% for concrete and 2% for steel. In practical approaches, engineers assume to have an approximate uniform damping ratio of 2% or 5% for whole parts of the structure. This assumption results in errors between responses of the model and the structure. The objective of this study is to present a simple method for evaluating equivalent damping ratios of these structures. It is noted that the proposed method should be restricted to elastic structures because equivalent damping ratios depended on $\ddot{x}_g$, the relevant error distribution of the structure is irregular when the ground excitation is not considered in resonance with the structure. The results of the study show significantly different error values for the acceleration and displacement in inelastic structures. Finally, the results indicate that the use of the equivalent damping ratios makes the results of dynamics analyses closer to the ones obtained by the actual damping ratios. Consequently, the proposed method provides a much better approximation than the case in which the conservative overall ratio of 2% or 5% is used.

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