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ENHANCING WEIGHTED UNIFORM SIMULATION FOR STRUCTURAL RELIABILITY ANALYSIS

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ABSTRACT

Weighted Uniform Simulation (WUS) is recently presented as one of the efficient simulation methods to obtain structural failure probability and most probable point (MPP). This method requires initial assumptions of failure probability to obtain results. Besides, it has the problem of variation in results when it conducted with few samples. In the present study three strategies have been presented that efficiently enhanced capabilities of WUS. To this aim, a progressively expanding intervals strategy proposed to eliminate the requirement to initial assumptions in WUS, while low-discrepancy samples simultaneously employed to reduce variations in failure probabilities. Moreover, to improve the accuracy of MPP, a new simple local search method proposed and combined with the simulation that strengthened the method to obtain more accurate MPP. The capabilities of proposed strategies investigated by solving several structural reliability problems and obtained results compared with traditional WUS and common reliability methods. Results show that proposed strategies efficiently improved the capabilities of conventional WUS.

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KEY WORDS: Weighted uniform simulation, Failure probability, Low discrepancy samples, Expanding interval strategy, Most probable point, Local search

1. INTRODUCTION

In recent decades, reliability analysis of structures has become a topic with great interest between the engineers and designers [1-7]. This type of analysis inserts the probabilistic uncertainties in load and resistance model of structures that always exist but often neglected during conventional deterministic analyses and design methods [8]. A fundamental problem

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in structural reliability theory is the computation of the multi-fold probability integral that provides the failure probability (P_f) of structure as:

$$P_f = \Pr ob[\, G(X) \le 0\,] = \int_{G(X)\le 0} f(X)\, dX \tag{1}$$

where $X=[X_1,\ldots,X_n]^T$ is a vector of random variables representing uncertain structural quantities such as loads, environmental factors, material properties and structural dimensions [9]. The functions G(X) and f(X) denote the limit state function (LSF) and the joint Probability Density Function (PDF) of X respectively, such that:

- if G(X) < 0 the system is in a failure state,
- if G(X) = 0 the system is in a limit state (this state is considered as a failure state),
- if G(X) > 0 the system is in a safe state [10].

Exact evaluation of this equation is difficult for common structures, hence during the two last decades intensive researches have been carried out to provide methods to solve Equation (1). Among proposed methods, First Order Reliability Methods (FORM) have been considered as the most acceptable computational methods of approximation attempt [11]. FORM works by linearizing LSF, searching Most Probable Point (MPP) in standard normal space, referred to as "U space" and approximating P_f based on reliability index instead of calculating it directly [12, 13]. These methods are encountered with serious drawbacks for the case of nonlinear LSFs. They may be listed as follows:

1- To obtain the reliability index, one may need to solve a constrained optimization problem [10]. Therefore, a selection of proper parameters such as step size and initial search point significantly affects their accuracy and efficiency [14, 15]. Thus, researchers must have enough experience and skill in the field of optimization since selecting improper parameters may result in converging optimization algorithm to an improper answer. Moreover, most of classic optimization algorithms encounter a problem with multiple design points [13, 16].

2- The linearization of LSF will lead to a loss of precision in the reliability evaluation of nonlinear problems. If the safety domain is concave, FORM overestimates the P_f, while in the opposite case, it will be underestimated. Therefore, even if the method converges to an accurate design point, the result of FORM for P_f may be not reliable. By using Second Order Reliability Methods (SORM) this reliability evaluation may be more accurate, but it could make the formulation of the problem more complicated and fuzzier because curvature of the failure region must be obtained [17].

3- In FORM, it is assumed that all random variables have normal distribution. However, the basic variables of environmental actions and structural resistances for most of engineering structures are not normally distributed [17, 18]. For various probability distributions, transformation of random variables from original space to standard normal space requires numerous nonlinear mappings. These transformations can produce additional nonlinearity of safety margins in the equivalent normal space, even for linear LSF. According to Ref. [18] the transformations of the exponential or Gamma resistance variables can generate 24% errors in the FORM failure probability, and the transformation of Frechet action variables could generate a 31% drift. If the LSF is linear and MPP is determined, still there is no certainty of FORM results. Over the past three decade several researchers improved the accuracy of the FORM method, also using higher order moments to overcome the limitations of this method has been developed [19-21].

Considering these shortcomings in FORM, when an accurate solution is necessary for reliability analysis of a structure, simulation methods could be considered as suitable alternative. These methods do not rely on a simplifying approximation to the shape of the LSF, so they may be more accurate comparing to some analytical methods. Monte Carlo Simulation (MCS) as a well-known simulation method may well be reliable in approximating P_f for all types of LSF. But despite high accuracy and robustness, this method is very time consuming in the evaluation of low failure probability problems as well as finite element based problems [13, 22, 23]. These obscurities have led researchers to develop variance reduction techniques such as stratified sampling [24], Latin Hypercube Sampling [25, 26], Importance Sampling [27, 28] and Directional Sampling [29-31]. During past decade, some novel techniques such as Subset Simulation [7, 32], Line Sampling [33, 34] and Local Domain Monte Carlo Simulation [35] were also proposed to treat high dimensional reliability problems. Although these techniques require much fewer samples than MCS, due to the possibility of significant errors in understanding and analysis of the problem as well as the errors of simplifications, these methods may need to be verified by MCS. Rashki et al. was presented a simulation method that eliminates some limitations of common reliability methods [36]. This method referred to as Weighted Uniform Simulation (WUS), simply approximates P_f as well as MPP with a reliable accuracy. However, two drawbacks could be observed in this method. One is a need for an initial assumption for reliability index, and the other is a tolerance in results when a low numbers of samples are employed.

In this paper three strategies are offered to enhance the accuracy and efficiency of method, which has eliminated aforementioned drawbacks. These approaches are presented in the later sections after brief review of WUS.

2. WUS FOR APPROXIMATING FAILURE PROBABILITY

WUS is the simple method that approximates P_f and MPP with few samples at the accuracy of MCS. In this method, the first step in WUS is to generate random numbers in specific intervals based on initial assumption of P_f. Based on competency of the samples, WUS then attributes a weight index, w_i, to generated samples. The indicator I, then separates the failed samples (I_i=1) from those in the safe region (I_i=0). The matrix of the generated random values (x) and the assigned weight matrix $\overline{W}$ are as follows:

$$\overline{X} = \begin{matrix} & & n{=}1 & n{=}2 & .. & n{=}n \\ First\ Variable & S=1 & x_{11} & x_{12} & .. & x_{1n} \\ Second\ Variable & S=2 & x_{21} & x_{22} & .. & x_{2n} \\ : & : & : & : & .. & : \\ i^{th}\ Variable & S=i & x_{i1} & x_{i2} & .. & x_{in} \\ : & : & : & : & .. & : \\ Lastest\ Variable & S=S & x_{S1} & x_{S2} & .. & x_{Sn} \end{matrix} \longrightarrow \overline{\mathbf{W}} = [w_1\ w_2 \dots w_n] \quad (2)$$

where:

$$w_i = \prod_{j=1}^{s} f_j(x_{ji}) \quad (3)$$

In the Equation (2), s is number of random variables and n is random numbers generated for each variable. Attributing weights to generated samples, WUS approximated P_f as:

$$P_f = \frac{\sum_{i=1}^{n} I_i . w_i}{\sum_{i=1}^{n} w_i}, \quad (4)$$

Assuming X_1 and X_2 are the two basic random variables, the required WUS steps for P_f approximation are shown in Figure 1.

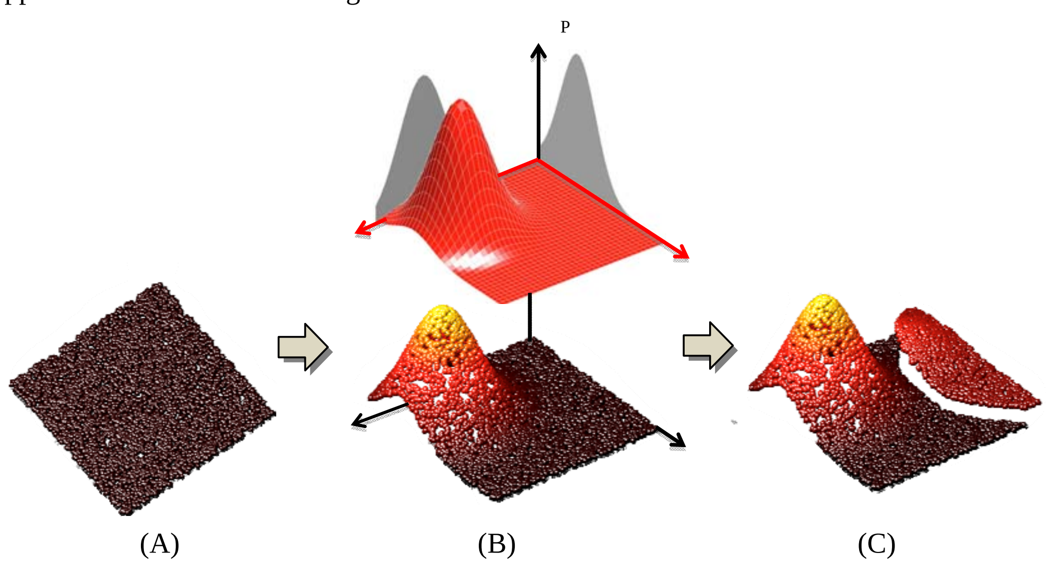

Figure 1. Failure probability estimation using WUS. (A) Sample generation, (B) Weighting of samples, (C) Failure probability estimation

which MPP is the sample in failure region that has the maximum weight [36]. Thus,

$$MPP = \max(\bar{I}.\ \overline{W}) \tag{5}$$

It can be seen that the nature of MPP determination in WUS is basically different with FORM. However, the result of MPP provided by WUS is comparable with that of FORM.

3. PROPOSED STRATEGIES FOR ENHANCING EFFICIENCY OF WUS

As mentioned, WUS approximates P_f with few samples but with high accuracy. But, still some drawbacks could be seen in the case of initial failure probability assumption and also accuracy of MPP when few samples are employed. Besides, the method has the variation in P_f approximation, similar to that of MCS.

The following is an expanding interval strategy accommodated with a simple local search method proposed to eliminate the deficiencies of MPP and requiring initial assumptions for P_f. Besides, for the case of variance reduction of WUS results, low-discrepancy samples are employed instead of common random samples.

3.1 Expanding interval strategy to eliminate initial assumptions required

A review on reliability methods reveals that most of them require proper assumptions for their parameters to obtain reliability results. These parameters are often initial search point and assumed reliability index that affect the efficiency and accuracy. Improper selection for these values could mislead researchers by providing inaccurate solution. Furthermore for some methods such as MCS, it is also accommodates with high wasted computational costs. As an instance, consider a problem possesses a P_f of less than 10^{-7}, while one assumes it to be 10^{-4}. Based on the assumed failure probability and a 95% confidence interval, MCS requires more than 10^6 system evaluations to achieve an error of less than 20% in the failure probability calculations [37]. However, after performing MCS, the achieved result would be $P_f=0$, since the assumed reliability index was less than its real values by a great amount. This is an iterative process of leading to useless sets of computations; in particular when finite element based analysis is involved.

As it was mentioned in Section 2, requirement to assuming a conservative reliability index was also a part of WUS to determine intervals for sample generation. But since the nature of samples distribution in WUS was uniform, this problem could be easily eliminated by employing step-by-step expanding the aforementioned intervals around the mean values. As it is shown in Figure 2, if first interval had no specifications of proper interval, one could expand the interval and generate new samples in the border of two intervals.

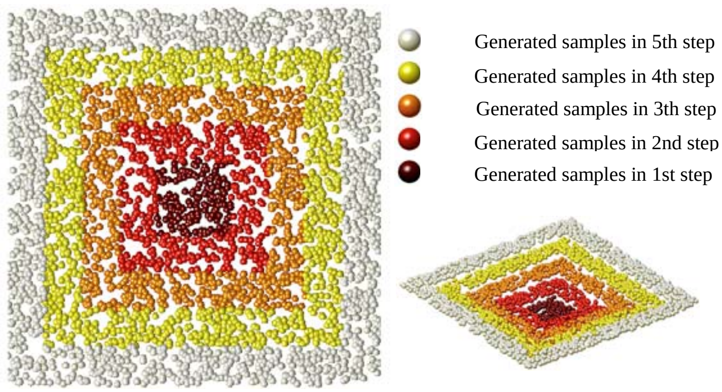

Figure 2. Gradually increasing intervals till obtain failure probability convergence

This process could continue until the obtained result satisfies desired convergence criteria for the problem. This strategy could help to conduct new WUS by employing the former samples and thus avoid wasted sample even if the initial assumption for reliability index was not adequate.

3.2 Low-discrepancy samples in WUS to reduce variances in results

The application of low discrepancy sampling methods in structural reliability problems was realized by Nie and Ellingwood for the directional simulation [38], and it was also successfully employed for importance sampling by Dai and Wang [39]. These studies reveal the capacity of low-discrepancy sequence application in structural reliability analysis. For the case of improving the convergence rate and eliminating the deviation of results, the effect of low-discrepancy of samples was also investigated in WUS. For this, two important classes of sequences of points that are well suited to multivariate integration are examined. They may be referred as lattice points and digital sequences. Hence, a Good Lattice Point (GLP), Sobol and Halton sequences have been considered alongside with the common random samples. A popular empirical method to study the uniformity of low-discrepancy sequence is to plot its two-dimensional projection [39]. Figure 3 shows two dimensional scattered plots with the sample size 100 for random, GLP set, Sobol and Halton sequences.

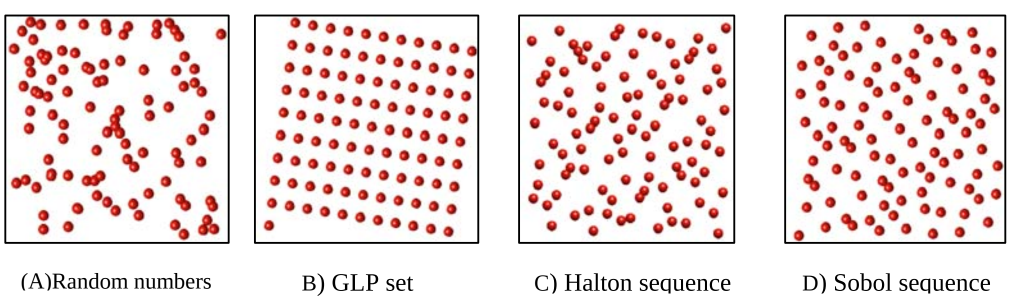

Figure 3. Two dimensional scatter plot of different point set

3.3 Improving MPP accuracy using a new local search method

For designers, it is not only important to obtain the P_f but also to know the design point of the structure [10]. As it was seen before, besides the failure probability estimation, WUS provided a new solution for MPP. However, when the failure probability approximated by using few samples, the obtained MPP may be inaccurate since design space may not be properly covered by only few samples. For this case, progressively expanding the intervals in WUS could be an advantage for accurately determining MPP. To this aim, by simultaneously employing proposed strategy together with a local search, improvement in MPP accuracy could also be feasible without using large number of samples. This is possible by employing a simple local search method when MPP appears in the simulation process. According to Figure 4, the accurate MPP could be in the vicinity of the sample with a maximum weight among others. When that sample was observed, a new random sampling using normal distribution around the MPP with the standard division of 0.2 of original standard deviation, could lead to an accurate MPP. Figure 4 signifies this manner schematically.

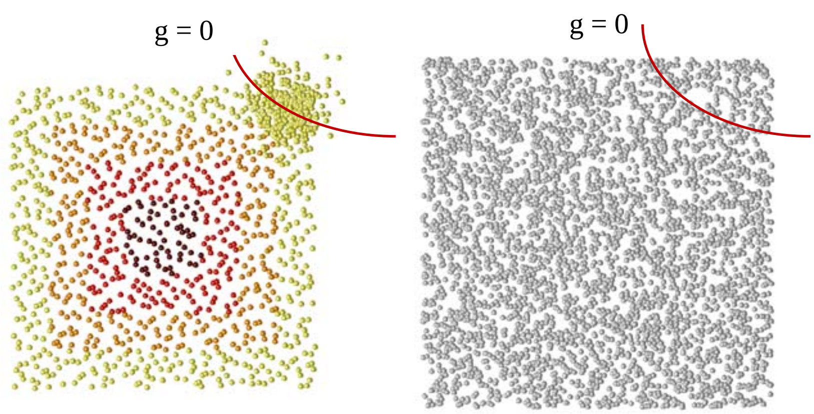

Figure 4. Enhanced WUS vs. conventional WUS

4. NUMERICAL EXAMPLES

To investigate the efficiency of proposed strategies in WUS, four examples solved by using these strategies and compared with MCS and FORM. It should be noted that the FORM algorithm which is used for computations is the improved FORM that is proposed by Der Kiureghian and Dakessian [40].

4.1 Example 1

The aim here is to investigate the effect of size of low-discrepancy samples and also the accuracy of proposed local search method for a highly nonlinear problem with variables involving four different PDFs. For this purpose, reliability analysis of a tension plate with the edge crack was investigated (Figure 5). The LSF of the problem could be considered as follows [41]:

$$g = K_{IC} - \left(1.12 - 0.23\left(\frac{c}{w}\right) + 10.56\left(\frac{c}{w}\right)^2 - 21.74\left(\frac{c}{w}\right)^3 + 30.42\left(\frac{c}{w}\right)^4\right) * S\sqrt{\pi c} \quad (6)$$

where K_{IC} is the critical mode I stress intensity factor, S is the applied normal traction and w and c are the width of plate and half length of the crack, respectively. The statistical parameters of problem are presented in Table 1. According to that, P_f for four different PDF states of random variables was investigated. c and w are considered random variables with uniform PDF while K_{IC} and, S were considered random with: 1) Normal, 2) Log-Normal, 3) Extreme type I and 4) weibul PDFs.

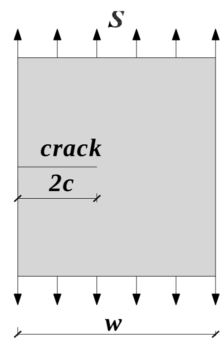

Figure 5. Finite tensile plate that includes an edge crack

Table 1: Random variables for Example 1

Random variable	Mean	S.D	PDF	
C	5 cm	$1/2\sqrt{3}$ cm	Uniform	
W	25 cm	$4/2\sqrt{3}$ cm	Uniform	
S	500 kgf/cm^2	50 kgf/cm^2	Case 1	Normal
			Case 2	Log-Normal
			Case 3	Ext. type I
			Case 4	weibul
K_{IC}	4500 kgf/cm$^{1.5}$	450 kgf/cm$^{1.5}$	Case 1	Normal
			Case 2	Log-Normal
			Case 3	Ext. type I
			Case 4	weibul

The problem was also solved with MCS using 30,000,000 samples with a failure probability being referred to as $P_{f\ accurate}$. It was then compared to those obtained by WUS using Equation (7):

$$maximum\ obtained\ relative\ error = \left| P_f - P_{f\ accurate} \right| \Big/ P_{f\ accurate} \tag{7}$$

Table 2 shows the MCS results for different number of samples for the mentioned cases. This table also presented averages and maximum obtained relative errors as in Eq. 7.

Table 2: Effects of PDFs of random variables on MCS results and corresponding errors for different number of samples for Example 1

Number of samples		10^4	5×10^4	10^5	2×10^5	5×10^5	10^6	5×10^6	10^7	3×10^7
$P_f \times 10^{-3}$	Case 1	1.100	0.860	0.460	0.490	0.558	0.569	0.587	0.592	0.606
	Case 2	1.000	0.220	0.210	0.295	0.328	0.413	0.391	0.389	0.375
	Case 3	0.200	0.960	0.910	0.620	0.660	0.655	0.751	0.752	0.737
	Case 4	4.000	2.020	2.350	2.375	2.742	2.697	2.593	2.604	2.632
Maximum error (%)		166.40	41.83	44.06	21.41	12.62	11.15	4.11	3.50	"true"
Average error (%)		93.17	34.17	25.59	16.56	8.81	7.45	2.66	2.21	"true"

It could be found that MCS required about 5×10^6 samples to obtain results with less than 5% errors. Table 3 presented P_f and their relative errors using FORM and SORM, which has revealed that FORM had about 30%-37% error in computation of P_f. the corresponding error was recorded as 14%-19% for SORM. Although these methods provide solutions with few LSF evaluations, but it is clear that they provide unacceptable accuracies for engineering purposes.

Table 3: Effects of PDFs of random variables on FORM and SORM results and corresponding errors for Example 1

		FORM	SORM
$P_f \times 10^{-3}$	Case 1	0.781	0.705
	Case 2	0.514	0.448
	Case 3	0.985	0.835
	Case 4	3.158	2.804
Maximum error (%)		36.98	19.24
Average error (%)		29.87	13.84

For the case of WUS, the reliability index was assumed to be 3.5 and low-discrepancy samples employed to obtain failure probabilities and the obtained results are presented in Table 4 and 5 for 10^4 and 50625 samples respectively. Figures 6 and 7 illustrate relationship between number of samples and relative error in MCS and WUS methods.

Table 4: Effects of PDFs of random variables on WUS results with low-discrepancy samples with 104 samples for Example 1

Types of pseudo-random samples		Random	Halton	Sobol	GLP	Uniform
$P_f \times 10^{-3}$	Case 1	0.519	0.644	0.614	0.588	0.647
	Case 2	0.394	0.336	0.354	0.347	0.388
	Case 3	0.546	0.720	0.780	0.669	0.520
	Case 4	2.359	2.565	2.746	2.660	2.740
Maximum error (%)		25.97	10.46	5.84	9.22	29.46

Average error (%)	13.91	5.39	4.29	5.25	10.89

Table 5: Effects of PDFs of random variables on WUS results with low-discrepancy samples with 154 (50625) samples for Example 1

Types of pseudo-random samples		Random	Halton	Sobol	GLP	Uniform
$P_f \times 10^{-3}$	Case 1	0.558	0.592	0.610	0.593	0.604
	Case 2	0.366	0.360	0. 381	0.364	0.371
	Case 3	0.738	0.737	0.717	0.753	0.746
	Case 4	2.416	2.595	2.633	2.612	2.633
Maximum error (%)		8.20	4.07	2.74	3.00	1.19
Average error (%)		4.75	1.97	1.21	2.03	0.68

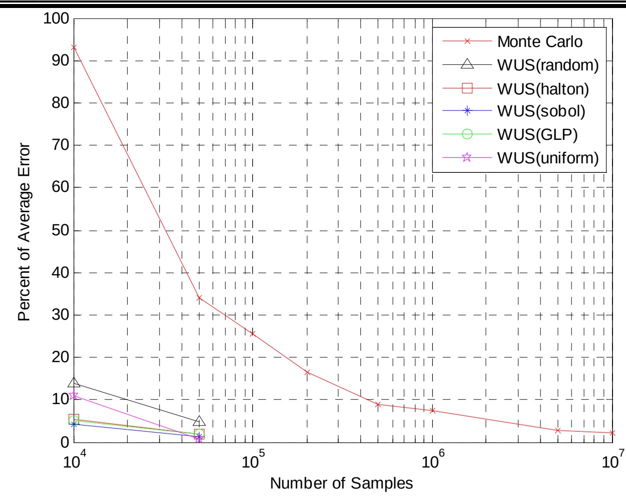

Figure 6. Relationship between number of samples and average of errors for Example 1

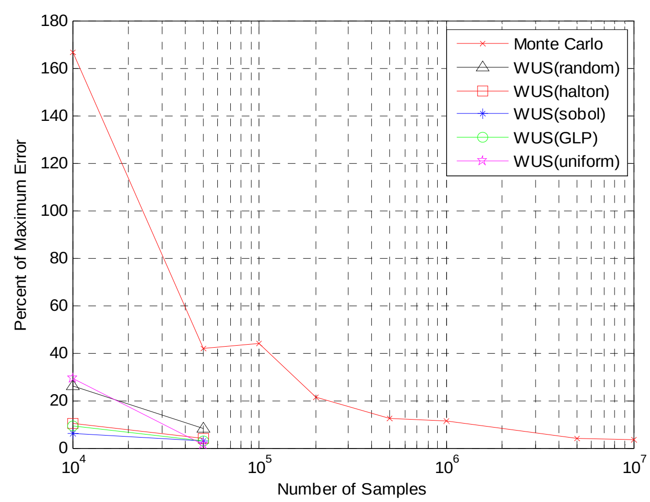

Figure 7. Relationship between number of samples and maximum of errors for Example 1

The result shows that low-discrepancy samples efficiently increases the convergence rate and reduces the errors. It can be seen that by employing low-discrepancy samples, WUS obtained reliability results with the accuracies similar to MCS while the required samples in WUS are reduced to 1%. Table 6 presented the results of proposed MPP search method for the four cases mentioned while the results of improved FORM also presented for comparison. Results show that for all four cases, MPP provided by the introduced local search method was more accurate from those extracted from the improved FORM. Therefore, it can be found that enhanced WUS is more efficient and accurate than conventional WUS, not only for the case of P_f approximation, but also for the case of MPP determination.

Table 6: MPP and weight index calculated by FORM and WUS for Example 1

	Method	MPP				Weight index
		K_{IC}	σ_0	c	w	$(\times 10^{-8})$
Case 1	WUS	3493.240	571.404	5.500	23.004	5.2221
	FORM	3360.664	581.321	5.307	24.319	1.9107
Case 2	WUS	3642.409	595.813	5.499	23.004	4.2252
	FORM	3597.478	619.246	5.328	24.228	1.4563
Case 3	WUS	3979.953	651.319	5.498	23.008	4.3651
	FORM	4048.149	703.674	5.286	24.405	1.4645
Case 4	WUS	3347.578	547.626	5.499	23.006	9.6508
	FORM	3066.605	540.088	5.226	24.599	4.1626

4.2 Example 2.

This example investigates the efficiency of expanding interval strategy for problem with high values of reliability index. The limit state of problem is a function with two independent normal random variables that is presented as:

$$g = \left(8\sqrt{2}\right) - \left|\sum_{i=1}^{2} X_i\right| \tag{8}$$

In the absence of absolute operator for the proposed LSF, the reliability index of problem is 8 [39,42], while by inserting absolute operator, the reliability index reduces to 7.92 because the absolute operator causes the generation of two separate failure regions. For the proposed level of safety, the approximation of P_f by MCS seems to be impossible by common computers since MCS requires about 3.27×10^{17} samples to obtain results with coefficient of variations less than 5% error. Besides FORM and SORM are not able to approximate P_f accurately, since obtaining result in these methods is based on MPP, which means that it just considers one of two existed failure domains. But, as it was shown in Table 7, WUS to approximate P_f accurately, requires only 449995 Samples, meaning a very minor fraction of that provided by the other method.

Table 7: The WUS results employing low-discrepancy samples for Example 2

Types of pseudo-random samples	Random	Halton	Sobol	GLP
$P_f \times 10^{-15}$	1.202	1.236	1.249	1.221
Reliability index	7.919	7.915	7.914	7.917

To obtain results, initial reliability index assumed to be 1 and then the proposed expanding interval strategy employed to obtain reliability results. To this aim, low-discrepancy samples employed and intervals gradually increased (10% per step) till the difference between the obtained reliability indices converged to 0.01. Results show that after 11 steps, all of the employed sampling methods converged to proper P_f without renewing sampling (see Figure 8). Therefore, if there is no premier perception about P_f of a problem, one could obtain accurate solutions by performing proposed enhanced WUS. It should be noted that if the require time to generate 449995 samples and corresponding simulation computations is about 0.1 second, the similar required computations time for 3.27×10^{17} samples (size of requires samples in MCS) is more than 2300 years.

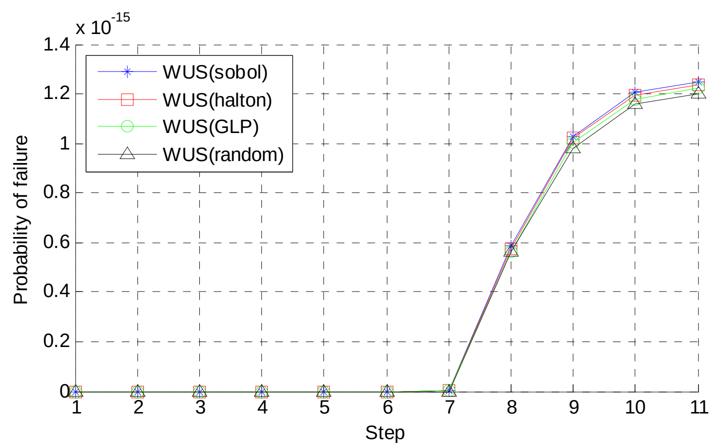

Figure 8. Increasing the number of samples and the probability of failure converging for Example 2

Example 3.

In this example, ability of extended WUS method for solving problems with island failure region investigated. For this purpose, reliability analysis of the damped single degree of freedom system with dynamic vibration absorber investigated that is subjected to harmonic force [43]. For this structure that is shown in Figure 9, the amplitude of vibration depends on R, ξ, β_1 and β_2 where R is the mass ratio of the absorber to the original system; ξ is the damping ratio of the original system; β_1 is the ratio of the natural frequency of the original system to the excitation frequency and β_2 is the ratio of the natural frequency of the absorber to the excitation frequency. By considering allowable level of vibration (y_0) equal to 27, The LSF of problem expressed as:

$$g = 27 - y \tag{9}$$

where, y is the amplitude of the original system that is normalized by the amplitude of its Quasi-static response. The amplitude, y, expressed as [43]:

$$y = \frac{\left|1 - \left(\frac{1}{\beta_2}\right)^2\right|}{\sqrt{\left[1 - R\left(\frac{1}{\beta_1}\right)^2 - \left(\frac{1}{\beta_1}\right)^2 - \left(\frac{1}{\beta_2}\right)^2 + \left(\frac{1}{\beta_1\beta_2}\right)^2\right]^2 + 4\xi^2\left[\left(\frac{1}{\beta_1}\right)^2 - \frac{1}{\beta_1\beta_2{}^2}\right]^2}} \tag{10}$$

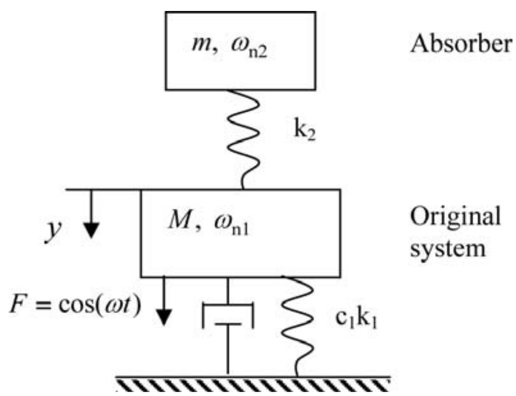

Figure 9. Tuned vibration absorber [43]

To evaluate the reliability of system, it is assumed that β_1 and β_2 are random variables with normal distribution N(1,0.025), while $R = 0.01$ and $\xi = 0.01$ are deterministic. According to Figure 10, this problem has island failure region that highly reduces the accuracy of FORM and SORM for the case of failure probability estimation.

Using MCS with 1.5×10^7 samples, the accurate P_f of problem computed as 0.01097. For the case of WUS, expanding intervals strategy with low-discrepancy samples employed to approximate the P_f, and the obtained results are presented in Figure 11. The expanding interval procedure started with 260 samples in first step, while convergence criteria considered being 0.01 of differences in reliability indices. It can be seen that by employing GLP, Haltom and Sobol sampling, enhanced WUS properly converged to the solution in five steps with 1742 samples while the relative error values are 1.66%, 1.42% and 0.66%, respectively.

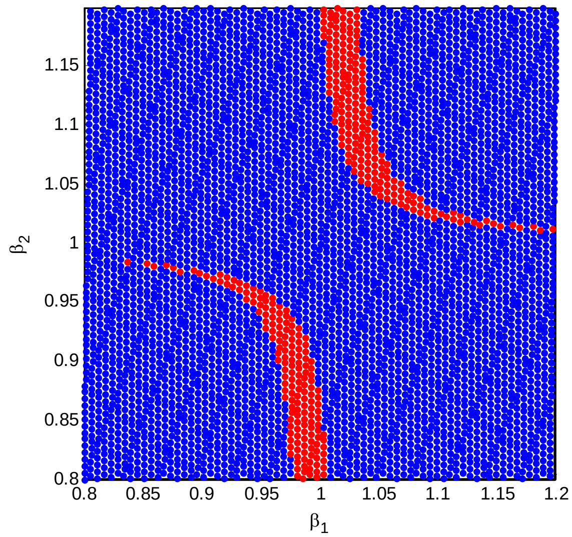

Figure 10. Failure region of Example 3

To demonstrate the variation in results for conventional WUS, 100 separate simulations were conducted by using common random samples while the samples size for each simulation was equal to that performed by using low discrepancy methods. This was led to achieve failure probabilities with the minimum error of 0.12% and maximum of 28%. It means that performing WUS with common random samples sometimes provided high accurate results (with 0.12% error) and sometimes provided failure probabilities with large errors (with 28.44% errors). Hence, enhanced WUS has been eliminated this variation with acceptable accuracy that is less than 2% error.

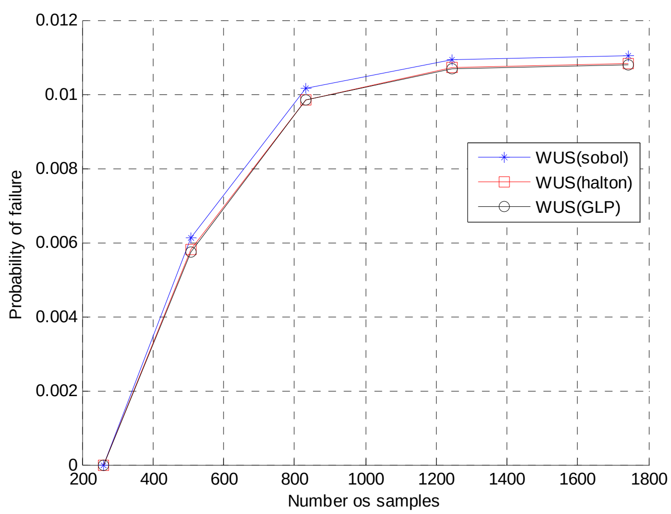

Figure 11. Increasing the number of samples and the probability of failure converging for Example 3

Although the computed failure probabilities by using improved FORM and SORM gave great error (33.6% and 52.9% respectively), the MPP provided by improved FORM compared with that provided by proposed WUS based local search methods (see Table 8). Results show that similar to previous example, proposed local search method provided more accurate solution compared to improved FORM.

Table 8: MPP and weight index calculated by FORM and WUS for Example 3

method	MPP		Weight index
	β_1	β_2	
WUS	0.9632	0.9537	15.584
FORM	1.0407	1.0455	1.288

5. CONCLUSIONS

In this study, three different strategies were introduced to enhance the accuracy and efficiency of WUS. Firstly, the need for a primary estimation of reliability index in WUS was eliminated by proposing an expanding intervals strategy. To reduce the variation of computed failure probabilities, low-discrepancy samples were then employed successfully instead of common random samples. Finally, they were accommodated with a new MPP local search method that works by distributing new samples around the former MPP, resulted from WUS, during simulation process by which more certain MPP results were determined. The proposed strategies were examined by solving several analytical and engineering problems. Results indicated that by employing all three approaches simultaneously, not only the requirement for WUS initial assumptions was eliminated, but also the methods provided more accurate MPP result. This was the case even though P_f of the problem was approximated by low number of samples compared to that of conventional WUS.

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