



OPTIMAL DESIGN OF REINFORCED CONCRETE ONE-WAY RIBBED SLABS USING IMPROVED TIME EVOLUTIONARY OPTIMIZATION

Sh. Bijari^{1*,†} and M. Sheikhi Azqandi²

¹*Department of Civil Engineering, University of Birjand, Birjand, Iran*

²*Department of Mechanical Engineering, University of Birjand, Birjand, Iran*

ABSTRACT

In this paper, a new robust metaheuristic optimization algorithm called improved time evolutionary optimization (ITEO) is applied to design reinforced concrete one-way ribbed slabs. Geometric and strength characteristics of concrete slabs are considered as design variables. The optimal design is such that in addition to achieving the minimum cost, all design constraints are satisfied under American Concrete Institute's ACI 318-05 Standard. So, the numerical examples considered in this study have a large number of design variables and design constraints that make it complicated to converge the global optimal design. The ITEO has an excellent balance between the two phases of exploration and extraction and it has a high ability to find the optimal point of such problems. The comparison results between the ITEO and some other metaheuristic algorithms show the proposed method is competitive compared to others, and in some cases, superior to some other available metaheuristic techniques in terms of the faster convergence rate, performance, robustness of finding an optimal design solution, and needs a smaller number of function evaluations for designing considered constrained engineering problems.

Keywords: metaheuristic algorithm; reinforced concrete slab; optimum cost design; improved time evolutionary optimization.

Received: 18 January 2022; Accepted: 25 March 2022

1. INTRODUCTION

In the last two decades, several metaheuristic optimization algorithms have been developed to solve engineering optimization problems. Generally, depending on the type of design

*Corresponding author: Department of Civil Engineering, University of Birjand, Birjand, Iran

†E-mail address: shbijari@birjand.ac.ir (Sh. Bijari)

variables, two clusters of optimization algorithms, including continuous and discrete design variables, are analyzed [1]. However, the standard size of sections obtainable in the market has discrete values; most research has focused on the optimum design of engineering problems with continuous variables. Furthermore, analysis of the problems with discrete design variables is far more complicated than continuous design variables [2-3]. A common method is that researchers utilize mathematical procedures such as rounding on continuous solutions for solving engineering optimization problems with discrete design variables. This technique may lead to weakness, including violation of the constraints and deviate in the infeasible domain. To solve these weaknesses, researchers analyze engineering optimization problems by utilizing metaheuristic optimization algorithms with discrete variables. In general, metaheuristic optimization methods have two phases named exploration and exploitation [4]. At the exploration phase, algorithms should detect feasible space to explore new areas, and at the exploitation phase, they exploit the best places at the same moment, to reach the global optimal design. On the other hand, balance between these two phases is the vital parameter for achieving the best performance of powerful metaheuristic algorithms. Among the metaheuristic optimization methods that have been presented in recent years, the following methods can be mentioned: colliding bodies optimization (CBO) is motivated based on the physics laws of momentum and energy that govern the collisions accrued between several bodies [5], Numbers cup optimization (NCO) algorithm is inspired by the sports competitions [6], Charged system search (CSS) is based on Coulomb law and Newtonian laws [7]. Momentum Search Algorithm (MSA) established on laws of momentum conservation and kinetic energy conservation between bodies (Newton's laws) [8], Gravitational Search Algorithm (GSA) based on the law of gravity and mass interactions [9], artificial ecosystem-based optimization (AEO) mimics three unique behaviors of living organisms, including production, consumption, and decomposition [10], arithmetic optimization algorithm (AOA) simulates the distribution characteristics of the basic arithmetic operations of addition, subtraction, multiplication, and division [11], Escaping Bird Search (EBS) inspired by aerial escaping strategies of a bird to avoid being captured by the hunter [12], Dolphin Echolocation (DE) [13] mimics dolphin's sonar abilities to detect the prey, Vibrating Particles System (VPS) based on free vibration of one degree of freedom systems with viscous damper [14], Color Harmony Algorithm (CHA) utilizes the Munsell technique and harmonic templates to produce new colors [15], Tug of War Optimization (TWO) inspired by the game of tug of war [16] and etc.

Recently, Sheikhi Azqandi et al. have been proposed a new robust metaheuristic optimization algorithm called Time Evolutionary Optimization (TEO) [17]. Up to now, the TEO has been applied for solving some real-world optimization problems, including constrained engineering optimization problems [18] and fuzzy multi-objective structural optimization [19].

To overcome the drawbacks of the original TEO, an improved variant of the original is proposed called improved TEO (ITEO) [18]. In ITEO, Both the exploration and exploitation abilities of the original TEO are improved to overcome its drawbacks.

In this study, to evaluate the efficiency of ITEO, in designing reinforced concrete (RC) one-way ribbed slabs with discrete design variables is proposed to acquire the minimum cost. For this purpose, the code of the improved time evolutionary optimization method and the

analysis of the reinforced concrete one-way ribbed slabs are provided in MATLAB software.

2. PROBLEM DEFINITION

A one-way ribbed slab consists of a series of small, reinforced concrete T beams that are connected with girders that, in turn carried by the building column. The reinforced concrete ribbed slab is widely used in buildings where there is the necessity to avoid columns interfering the spaces or for a structure with long spans or as an aesthetic purpose.

The goal in the optimal design of reinforced concrete one-way ribbed slab is to reduce the overall cost of the slab so that the design constraints are satisfied. In Fig. 1, the schematic of a RC ribbed slab and six discrete design variables considered for modeling the RC ribbed slab are shown. These design variables include the slab thickness (ST), the ribs spacing (RS), the rib width at the lower end (RWL), the rib width at the top end (RWT), the bar diameter (BD), and the depth of rib (DR).

For optimal design of the ribbed slabs, first, random values for the variables are selected, and according to the ACI 318.08 standard [20], the dimensions are checked, then the required reinforcement for RC ribbed slab is calculated, and the strength is controlled.

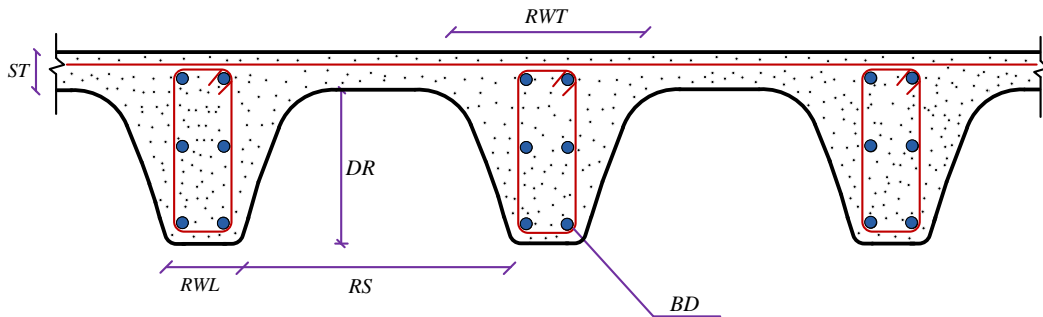


Figure 1. The schematic of a ribbed slab and the design variables

The objective function in concrete ribbed slab optimization contains the cost of concrete and steel material along with the cost of concreting and erecting the reinforcement, which must be minimized. This can be attained by determining the optimal values for decision variables ST, RS, RWL, RWT, BD, and DR. The total cost function can be written as follows:

$$\begin{aligned}
 &\text{Find} && \{X\} = [x_1, x_2, \dots, x_n] \\
 &\text{to minimize} && P_t = (V_{conc} (Pr_{vc} + Pr_c) + W_{St} (Pr_{ws} + Pr_{es})) / (RWL + RS) \\
 &\text{subjected to} && g_j(x) \leq 0, \quad j = 1 \text{ to } m \\
 &\text{where} && x_{\min} \leq x \leq x_{\max}
 \end{aligned} \tag{1}$$

where V_{conc} , and W_{st} are the volumes of the concrete and the weight of the reinforcement steel in the unit length (m^3/m , kg/m), respectively; P_{vc} , and P_{ws} are the costs of concrete and steel material ($\text{€}/m^3$ for concrete and $\text{€}/kg$ for steel), respectively; Pr_c and Pr_{es} are the costs of concreting and erecting the reinforcement, respectively. P_t is the total cost of the structure ($\text{€}/m^2$). $(RWL + RS)$ is the center-to-center rib distance. x_i and g_j are design variables and design constraints of the objective function, respectively. x_{\min} and x_{\max} are the lower band and the upper band of the optimization problem variables, respectively.

The objective function is considered as follows:

$$\bar{P}_t = \left(V_{conc} + W_{st} \left(\frac{Pr_{ws} + Pr_{es}}{Pr_{vc} + Pr_c} \right) \right) / (RWL + RS) \quad (2)$$

And based on the reviews and the cost estimation done [21], the value of 0.04 for Pr is considered ($Pr = \frac{Pr_{ws} + Pr_{es}}{Pr_{vc} + Pr_c}$). We have:

$$\bar{P}_t = (V_{conc} + W_{st} Pr) / (RWL + RS) \quad (3)$$

In designing the ribbed concrete slabs, the following constraints according to the ACI 318-08 [20] are considered.

2.1 Flexural constraint

The flexural constraint can be defined in the following form:

$$g(1) = M_u / (\phi_b M_n) - 1 \leq 0 \quad (4)$$

where M_u , and M_n are the ultimate design bending moment and the nominal bending moment, respectively.

2.2 Shear constraint

The shear constraint can be defined as:

$$g(2) = V_u / (\phi_v V_n) - 1 \leq 0 \quad (5)$$

where V_u is ultimate factored shear force, and V_c is the nominal shear strength of the concrete.

2.3 Serviceability constraints

The serviceability constraints are stated in terms of the limits on the steel reinforcement ratio and the bar spacing. The steel reinforcement ratio should satisfy the constraints which follow:

$$g(3) = \rho - \rho_{\max} \leq 0, \quad \rho_{\max} = 0.75\rho_b \quad (6)$$

$$g(4) = \rho - \rho_{\min} \geq 0 \quad (7)$$

where ρ_{\min} is the minimum shrinkage and temperature steel ratio, which is determined according to the following Table; but not less than 0.0014:

Table 1: The values of ρ_{\min} [20]

The grade of bars used in the slab	ρ_{\min}
40 or 50	0.0020
60	0.0018
> 60	$\frac{0.0018 \times 60000}{f_y}$

And the bar spacing should satisfy the constraints which follow:

$$g(5) = \min d_b - 25 \text{ mm} \geq 0 \quad (8)$$

where $\min d_b$ is the minimum clear spacing between parallel bars in a layer and,

$$g(6) = \max d_b - \max(5RWL, 450 \text{ mm}) \leq 0 \quad (9)$$

where $\max d_b$ is the maximum spacing between the bars.

2.4 Deflection constraints

Deflection constraints are described in terms of the thickness of the top slab as follows:

$$g(7) = ST - \left(\max \left(\frac{RS}{12}, 50 \text{ mm} \right) \right) \geq 0 \quad (10)$$

and,

$$g(8) = (ST + DR) - h_{\min} \geq 0 \quad (11)$$

where h_{\min} is minimum slab thickness depending on the support conditions, which defines based on the following Table:

Table 2. Minimum thickness [20]

Member	Simply supported	One end continuous	Both ends continuous	Cantilever
Beams or ribbed one way slabs	L/16	L/18.5	L/21	L/8

where L is the effective span length of the slab.

2.5 Geometry constraints

The geometry constraints can be described as:

$$g(9) = RW - 100\text{mm} \geq 0 \quad (12)$$

where RW is the rib width,

$$g(10) = DR - 3.5(\min RW) \leq 0 \quad (13)$$

where DR and min RW are the depth of the ribbed and minimum width of the rib, respectively and

$$g(11) = RS - 750\text{mm} \leq 0 \quad (14)$$

where RS is the clear spacing between the ribs.

3. IMPROVED TIME EVOLUTIONARY OPTIMIZATION ALGORITHM

Improved Time Evolutionary Optimization Algorithm is a new metaheuristic optimization method that it has been presented by Sheikhi Azqandi et al. in 2020 [18]. Time evolutionary optimization (TEO) is based on Darwin's theory, Evolution laws, and natural selection. Time and Environment are the two essential factors in natural selection by Darwin's theory. The environment gradually erases unfavorable features and keeps types with good characteristics.

Over time, new congenital kinds are created. By the successive influence of natural selection, a group of kids will eventually spawn one or many new features that will develop in new kinds which it's no longer similar to their ancestors. As noticed in nature, different types of animate exist in an environment. There are variations between them, such as appearance, color, size, physical strength, etc. Kids can survive that is more consistent with the environment. This indicates that they can have sufficient feed, camouflage, and live. According to this, the weak kinds were destroyed by the environment, and the excellent

types will remain and generate and raise their population.

In this study, the improved time evolutionary optimization algorithm is presented for obtaining the optimal design in mechanical systems. Improved time evolutionary optimization algorithm has some supremacy factors compared to the initial version of TEO including an intelligent manner of kinds aids them to increase the average quality of their society. According to this, all kinds try to reach closer to their superior and obtain their characteristics. The presented optimization method, like TEO, has two phases, which are called creation and evaluation.

At first, the creation phase happens at the beginning of the presented algorithm for creating the primary population randomly.

In the evolution phase, the primary population is updated and will repeat throughout the optimization procedure [19]. Hence, in the second phase, some of the population that have the more excellent objective, are selected as elites. So, they are more consistent with the environment and can generate various children. These children are compatible with the features of their ancestors. According to this theory, the other populations with the worst objective function are erased over time, which is corresponds to natural selection. Therefore, at first, the difference between parents and children is high and then is decreased over time.

The primary population is generated randomly in the creation phase.

$$\begin{aligned} pop_{i,j} &= L_j + r \times (U_j - L_j), \quad i = 1, 2, \dots, N_{Pop}, \quad j = 1, 2, \dots, N_{Var} \\ N_{Pop} &= N_E \times N_{Ch} \end{aligned} \quad (15)$$

where N_{Pop} and N_{Var} are the numbers of the population and design variables, respectively. $pop_{i,j}$ is a member of the population, U_j and L_j are the maximum and minimum values for design variables, respectively. r is a random coefficient in the range of zero and one. N_{Ch} and N_E are the numbers of children and elites, respectively.

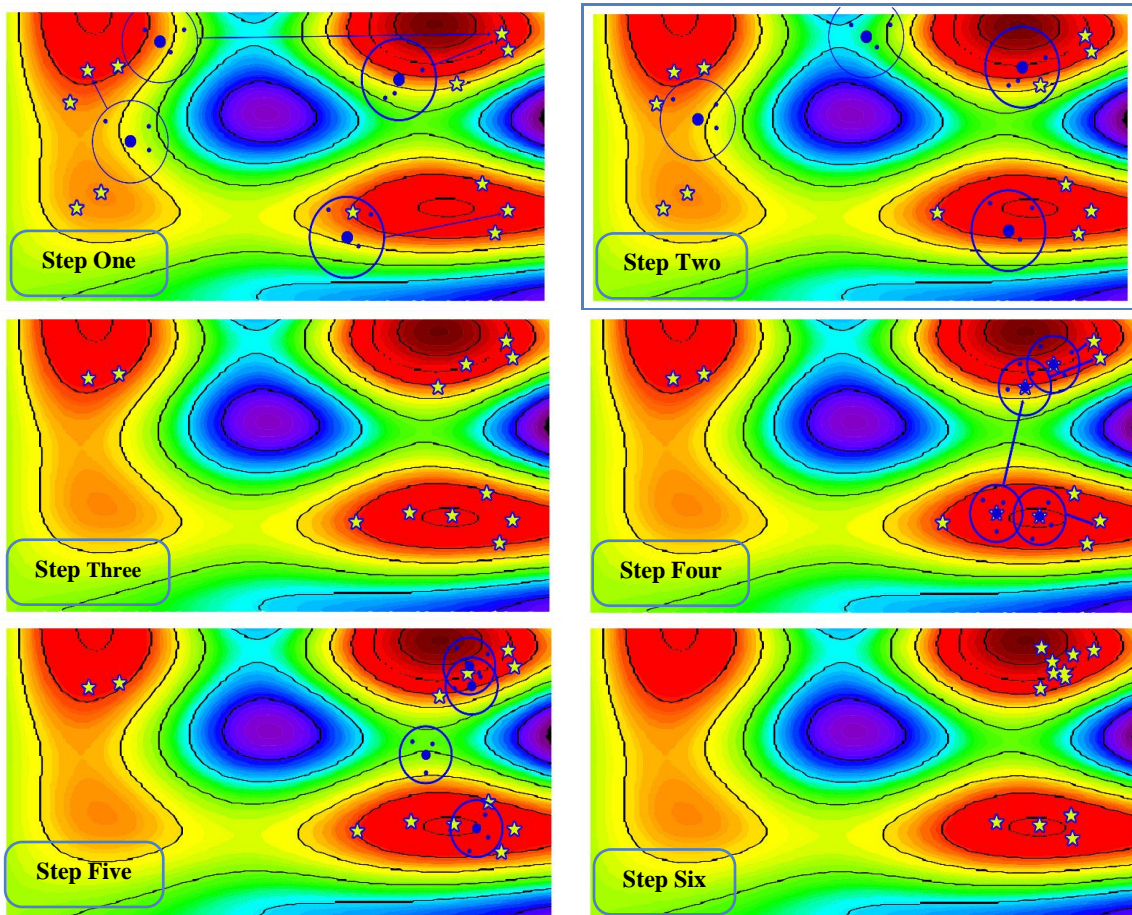
The primary society is available after the creation phase, and then the evolution phase can be started. In the evolution phase, The primary population characteristics will be improved and will eventually converge to the global optimal design.

Fig. 2 shows the search and convergence process of improved time evolutionary optimization algorithm is presented graphically. The explanation of the steps performed in Fig. 2. is as follows [18].

- Step one: current elites reproduced and chose one of the best of them randomly to acquire closer.
- Step two: clans produce the society, and the objective function is determined.
- Step three: the natural selection occurred; at this loop, elites are specified, the best elites matrix is updated, and other spices are eliminated.
- Step four: current elites reproduce (they can be a member of the best elites matrix) and choose one of the best elites matrix randomly to get closer
- Step five: clans produce the society, and the objective function is determined again.

- Step six: the natural selection occurs again, elites of this loop are selected, the best elites matrix is updated, and other spices are eliminated.
- Step seven: the difference (radius) is decreased, reproduction and clans approach to the best elite is repeated
- Step eight: convergence has happened, and the optimal design is acquired.

The respected readers can refer to the [18] to comprehend more features of the optimization algorithm. In this reference, how to do mathematically model, exploitation, and exploration search in design space and convergence toward the optimal design are presented completely.



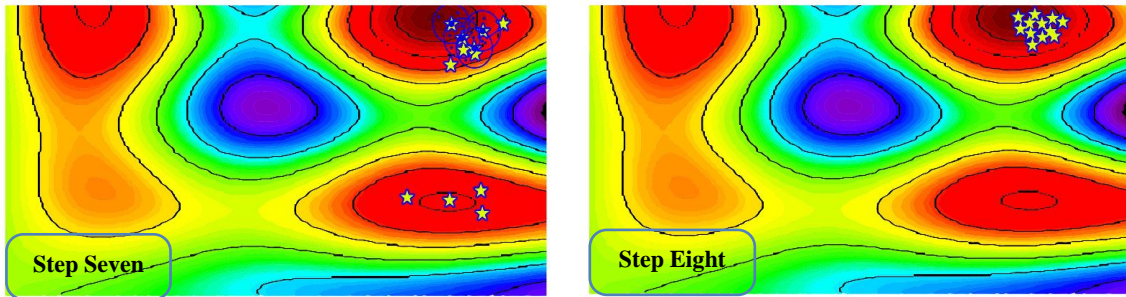


Figure 2. The search and convergence process of the improved time evolutionary optimization algorithm [18]

4. NUMERICAL EXAMPLE

In this section, two problems of a reinforced concrete one-way ribbed slab simply supported at both ends are presented. In the first problem, six design variables are considered for the optimal design is shown in Fig. 1. In the second problem, seven design variables are assessed in the optimization procedure and, f'_c as an additional design variable is added to the problem, and the slab with the optimal cost is designed. The parameters and common data required for the optimal design of a ribbed slab are: f_y is the strength of steel that is 420 MPa; DL is dead load that is 0.78 kN/m²; LL is live load that is 4 kN/m²; L is span length of the slab that is 6 m; Cover is that is 20 mm; w_s is the unit weight of bars that is 78.5 kN/m³; w_c is the unit weight of concrete that is 24 kN/m³.

4.1 Example 1. One way ribbed slab with six design variables

The design variables and the intended values for each numerical example are stated in Table 3.

Table 3: Design variables of one way ribbed slab

No.	Design variables x_i	Symbol	x_{\min}	x_{\max}	Step
1	Slab thickness (cm)	ST	2.5	10	2.5
2	Rib spacing (cm)	RS	40	75	2.5
3	Rib width at the lower end (cm)	RWL	10	25	2.5
4	Rib width at the taper end (cm)	RWT	10	30	2.5
5	Bar diameter (cm)	BD	1	2	0.2
6	Depth of rib (cm)	DR	15	75	2.5

In this problem, f'_c is the strength of concrete 28 MPa. The comparison of optimal results with several meta-heuristic algorithms such as harmony search (HS) [21], particle swarm optimization (PSO) [22], colliding bodies optimization (CBO) [22], and ITEO is presented in Table 4. In all the methods mentioned in this table, the population size equals 30. ITEO, compared to other methods, was able to achieve the optimal design in a lower number of evaluations. The value of the objective function acquired by ITEO is better than HS and PSO and is similar to CBO.

Table 4: Comparison of optimal design of one way ribbed slab (example 1)

	ST (cm)	RS (cm)	RWL (cm)	RWT (cm)	BD (cm)	DR (cm)	Cost (1/m ²)	NEFs
HS [21]	5	60	10	10	1.4	35	1.3626	6000
PSO [22]	5	60	17.5	17.5	1.4	32.5	1.3184	6000
CBO [22]	7.5	67.5	10	10	1.4	30	1.2927	6000
ITEO	7.5	67.5	10	10	1.4	30	1.292	3000

In the optimal design obtained for RC ribbed slab, the values of the concrete volume and weight of steel are 0.0881125 m³ and 7.2504811 kg, respectively. The values of constraints are reported in Table 5.

Table 5: The values of the design constraints (example 1)

constraints	Symbol	Value
Flexural Constraint	g(1)	-0.883019<0
Shear Constraint	g(2)	-0.892676<0
Serviceability Constraints	g(3)	-0.0178929<0
Serviceability Constraints	g(4)	0.00002412>0
Deflection Constraints	g(7)	1.875>0
Deflection Constraints	g(8)	0.006566>0
Geometry Constraints	g(10)	-5<0

Fig. 3 shows the convergence curve to the optimal design using ITEO. The number of populations and the maximum iterations considered ITEO are equal to 30 and 100, respectively. As can be seen in the figure, the proposed algorithm was able to converge to the optimal design of the problem after about 40 iterations with 1200 function evaluations.

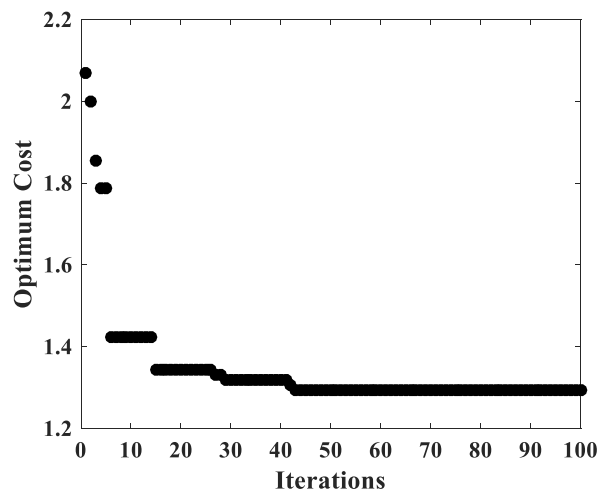


Figure 3. Convergence curves of the ITEO algorithm, for example 1

Table 6 presents the results of statistical analysis, including worst, best, mean, and standard deviation (SD) determined from 100 independent runs for problem analysis using the improved time evolutionary optimization method. The percentage relative difference between the value of the worst and the mean of the optimal objective functions with the best objective function is 3.96 and 0.97 percent, respectively.

Table 6. Statistical results obtained by ITEO, for example 1

Best	Worst	Mean	SD
1.2927	1.3461	1.3054	0.0146

4.2 Example 2. One way ribbed slab with variable concrete strength

In this part, the strength of concrete f_c' is added to the problem as the seventh variable, and the optimal design of the slab is done by the ITEO algorithm. The considered range of values for the concrete strength variable is 20 MPa to 35 MPa with step 1 MPa.

The coefficient Pr in the objective function is taken 0.04 for concrete with a strength of 28 MPa under the references. Since, in this case, the value of concrete strength is one of the design variables, the value of Pr ratio changes. According to Table 7, the relationship between the strength of concrete and the unit price of the concrete is obtained by fitting, which f_c' is in the form $Cost = 0.9527f_c' + 71.227$ where Cost is the unit price of concrete and f_c' is the strength of the concrete. Using this formula, the value of the Pr ratio, which is a function of concrete strength, is estimated and used in the objective function at equation 2.

Table 7: Unit prices of the concrete [23]

Strength of the concrete (MPa)	Cost (euro/ m ³)
25	95.05
30	99.81
35	104.57
40	109.33
45	114.10
50	118.87

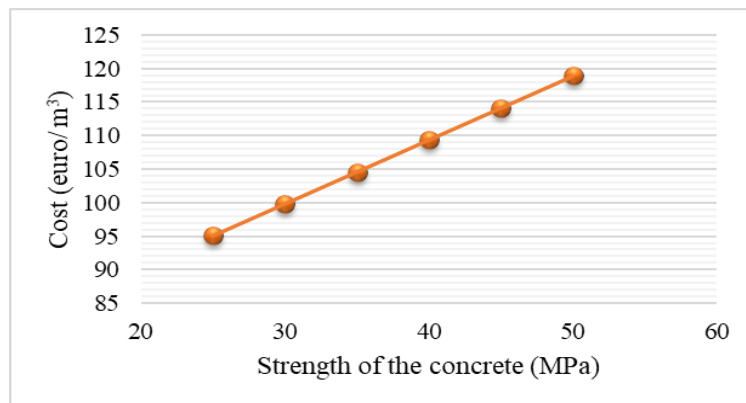


Figure 4. Unit prices of the concrete

The optimal design of one way ribbed slab is presented in Table 8. As it is known, with the variability of concrete strength, the optimal design variables have changed compared to example 1, and the cost can be further reduced.

The convergence curve of the optimal design using ITEO in one way ribbed slab with variable concrete strength is presented in Fig. 5. In this example, ITEO was converged to the optimal design of the problem after about 15 iterations with 450 function evaluations.

Table 9 presents the statistical analysis results from 100 independent runs of problem analysis using the ITEO. The percentage relative difference between the value of the mean of the optimal objective functions concerning the best objective function is 1.3 percent.

Table 8: the optimal design of one way ribbed slab (example 2)

	ST (cm)	RS (cm)	RWL (cm)	RWT (cm)	BD (cm)	DR (cm)	f_c' (MPa)	Cost (1/m ²)	NEFs
ETEO	5	65	10	10	1.4	27.5	35	1.229	3000

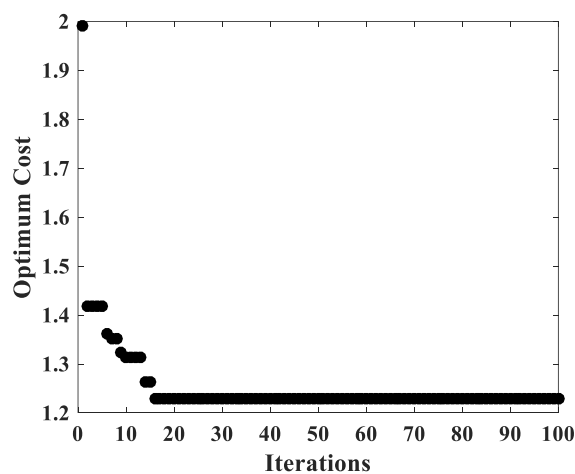


Figure 5. Convergence curves of the ITEO algorithm, for example 2

Table 9. Statistical results obtained by ITEO, for example 2

Best	Worst	Mean	SD
1.2295	1.3370	1.2459	0.0198

5. CONCLUDING REMARKS

Improved time evolutionary optimization (ITEO) is a new metaheuristic algorithm mimicking the natural selection and evolution of creatures over time. In this technique, population clustering amplified environmental factors and memory are used to save some best design variables. The optimal results obtained for ribbed concrete slabs by the ITEO algorithm show that this method has a high ability and accuracy in finding the optimal response compared to other methods.

In this research, the cost optimization of reinforced concrete one-way ribbed slabs has been studied. The objective function includes the cost of concrete and steel used in the slab. In the first example, six variables are considered to define the problem, which are the slab thickness, the ribs spacing, the rib width at the lower end, the rib width at the toper end, the bar diameter, and the depth of the rib. The optimal values for the design variables are obtained by the ITEO algorithm. Comparison of the results shows that this method has achieved good results in a lower number of evaluations of the objective function. In the second example, the strength of the concrete is added to the problem variables, and the optimal cost of the concrete slab is determined. The results show that by considering the amount of concrete strength in the acceptable range as a design variable, the cost of the slab is reduced by about 5%.

REFERENCES

1. Sheikhi Azqandi M, Ghoddosian A. *Classical Numerical Optimization Methods for Engineering Design*, Bozorgmehr University of Qaenat Press, 2020.
2. Cheng MY, Prayogo D, Wua YW, Lukito MM. A hybrid harmony search algorithm for discrete sizing optimization of truss structure, *Automat Constr* 2016, **69**: 21-33.
3. Lee KS, Geem Z, Lee WS, Bae K. The harmony search heuristic algorithm for discrete structural optimization, *Eng Optim* 2005, **37**(7): 663-84.
4. Ghoddosian A, Sheikhi Azqandi M. *Metaheuristic Optimization Algorithm in Engineering*, Semnan University Press, 2013.
5. Kaveh A, Mahdavi V.R. Colliding bodies optimization method for optimum design of truss structures with continuous variables, *Adv Eng Softw* 2014, **70**: 1-12.
6. Riyahi Vezvari M, Ghoddosian A, Nikoobin A. Numbers cup optimization: a new method for optimization problems, *Struct Eng and Mech* 2018; **66**(4): 465-76.
7. Kaveh A, Talatahari S. A novel heuristic optimization method: charged system search. *Acta Mach* 2010, **213**: 267–86.
8. Dehghani M, Samet H. Momentum search algorithm: a new meta-heuristic optimization algorithm inspired by momentum conservation law, *SN Appl Sci* 2020; **2**(10): 1-15.

9. Rashedi E, Nezamabadi-Pour H, Saryazdi S. GSA: a gravitational search algorithm, *Inform Sci* 2009, **179**: 2232-48.
10. Zhao W, Wang L, Zhang Z. Artificial ecosystem-based optimization: a novel nature-inspired meta-heuristic algorithm, *Neural Comput Appl* 2020, **32**: 9383-9425.
11. Kaveh A, Biabani Hamedani K. Improved arithmetic optimization algorithm and its application to discrete structural optimization, *Struct* 2022, **35**: 748-64.
12. Shahrouzi M, Kaveh A. An efficient derivative-free optimization algorithm inspired by avian life-saving manoeuvres, *J Comput Sci* 2022; **57**: 101483.
13. Kaveh A, Farhoudi N. A new optimization method: Dolphin echolocation, *Adv Eng Softw* 2013, **59**: 53–70.
14. Kaveh A, Ilchi Ghazaan M. A new meta-heuristic algorithm: vibrating particles system, *Sci Iran Trans A* 2017; **24**(2): 551-66.
15. Zaeimi M, Ghoddosian A. Color harmony algorithm: an art-inspired metaheuristic for mathematical function optimization, *Soft Comput* 2020; **24**(16): 12027-66.
16. Kaveh A, Zolghadr A. A novel meta-heuristic algorithm: tug of war optimization, *Int J Optim Civil Eng* 2016; **6**(4): 469-92.
17. Sheikhi M, Delavar M, Arjmand M. Time evolutionary optimization: A new meta-heuristic optimization algorithm, *Proceedings of the 4th International Congress on Civil Engineering, Architecture and Urban Development*, Shahid Beheshti University, Tehran, Iran, December, 2016.
18. Sheikhi Azqandi M, Delavar M, Arjmand M. An enhanced time evolutionary optimization for solving engineering design problems, *Eng Comput* 2020; **36**(2): 763-81.
19. Sheikhi Azqandi M, Arjmand M. A novel fuzzy multi-objective enhanced time evolutionary optimization for space structures, *Int J Optim Civil Eng* 2020; **10**(2): 345-63.
20. ACI Committee 318, *Building Code Requirements for Structural Concrete (ACI 318-08) and Commentary (318R-08)*, American Concrete Institute, Farmington Hills, Michigan, 2008.
21. Kaveh A, Shakouri M, Abadi A. Cost Optimization of Reinforced Concrete One-Way Ribbed Slabs Using Harmony Search Algorithm, *Arab J Sci Eng* 2011; **36**: 1179-87.
22. Kaveh A, Bijari Sh. Optimum cost design of reinforced concrete one-way ribbed slabs using CBO, PSO and democratic PSO algorithms, *Asian J Civil Eng* 2014; **15**(6): 788-802.
23. Kaveh A, Mottaghi L, Izadifard A. Parametric study: cost optimization of non-prismatic reinforced concrete box girder bridges with different number of cells, *Int J Optim Civil Eng* 2022; **12**(1): 1-14.