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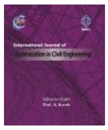

A NEW APPROACH BASED ON FINITE ELEMENT MODEL UPDATING FOR STRUCTURAL DAMAGE IDENTIFICATION

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ABSTRACT

In this study, the finite element model updating was simulated by reducing the stiffness of the members. Due to lack of access to the experimental results, the data obtained from an analytical model were used in the proposed structural damage scenarios. The updating parameters for the studied structures were defined as a reduction coefficient applied to the stiffness of the members. Parameter variations were calculated by solving an unconstrained nonlinear optimization problem. The objective function in the optimization problem was proposed based on the Multi-Degree-of-Freedom (MDOF) equations of motion as well as the dynamic characteristics of the studied structure. Only the first natural frequency of the damaged structure was used in the proposed updating process, and only one vibration mode was used in the updating problem and damage identification procedure. In addition, as elimination of high-order terms in the proposed formula introduced errors in the final response, the variations of natural frequency and vibration mode for higher-order terms were included in the free vibration equation of the proposed objective function. The Colliding Bodies Optimization (CBO) algorithm was used to solve the optimization problem. The performance of the proposed method was evaluated using the numerical examples, where different conditions were applied to the studied structures. The results of the present study showed that, the proposed method and formulation were capable of efficiently updating the dynamic parameters of the structure as well as identifying the location and severity of the damage using only the first natural frequency of the structure.

Keywords: Damage identification; Model updating; Stiffness matrix; Colliding Bodies Optimization (CBO).

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1. INTRODUCTION

It is essential to investigate and identify the structural damages at their initial stages in many scientific fields, such as civil and mechanical engineering. The structural damages are made intentionally or unintentionally by different factors, such as environmental, constructional, and usage-related factors over time. Their negative effects are reflected by the current behavior and/or future performance of the structure. As a result, damage identification has a significant effect on the usage and service life of the structure. Generally, dynamic parameters of the damaged structures, such as stiffness and mass cannot be directly determined by the conventional structural damage identification methods. As a result, such dynamic properties including the stiffness must be factored while identifying the damage location and extent. In some cases, these assumptions reduce the accuracy and speed of the structural damage identification process. On the other hand, the damage location and extent in the updating methods can be determined by specifying the dynamic parameters of the structure. The Finite Element (FE) model updating methods, which are based on the structural simulation approaches are employed to identify the differences between the analytical model and the real structure. In other words, the updating method can be used to identify any existing damage in a given structure based on the experimental data obtained from that structure by expressing the relevant structural characteristics. Therefore, results from updating the analytical model of the structure reflect the possible damages and can be used as an alternative to common inspection methods (destructive tests, visual inspection, etc.), as well as structural damage identification methods [1-3]. In general, the finite element model updating methods, which are based on comparison of the experimental and analytical data can be classified into two groups: direct methods and iterative methods, and each method is capable to identify a certain level of damage in the given structure [2-3]. The direct methods require accurate modeling and measurement of the studied structure. These methods produce erroneous damage identification results in the presence of measurement errors. In addition, no physical meaning can be associated with the updated parameters in these methods. The limitations of the direct methods have been partially resolved in the sensitivity-oriented model updating methods [4-7]. On the other hand, the main process in the iterative updating methods involves solving an optimization problem using an iteration procedure. In these methods, the differences between the dynamic characteristics of the analytical model and the measured characteristics of the actual structure are obtained by assuming the updating parameters as unknown variables [8-9]. In general, FE model updating methods require initial information such as natural frequencies of the structure and structural vibration modes. However, obtaining such information imposes various costs in terms of construction, time, etc. Therefore, the best updating method should be based on a process, in which minimum initial data are used, and an updated model of the structure is produced and consequently, the location and extent of damage is identified.

Different studies have been conducted to improve the performance of the updating process. Imregun et al. in 1995 used the frequency response function of the structure to update their structural model. They subsequently used the actual data obtained from a 500-Degree of Freedom (DOF) planar beam to evaluate the efficiency of their proposed method. They showed that, the updated model obtained based on the assumed criteria was probably not unique [10]. In another study, Smith in 1998 proposed an iterative method to estimate the stiffness matrix in the finite element model updating process. In this study, the location and severity of the possible damage were identified in the structural elements based on an iterative procedure by obtaining the stiffness matrix from the experimental data [11]. Zhang et al. in 2000 updated the dynamic characteristics of complex structures based on the eigenvalue sensitivity. Applying first-order Taylor series expansion to the eigenvalues using an iterative procedure based on a constrained optimization problem, they updated the structural model [12]. Yang and Chen proposed a new method (based on the measured modal data) for updating the finite element model of a structure. They assumed that, the required quantitative data obtained for the relevant dynamic characteristics were available. Using the first vibration mode of the structure and observing the principle of orthogonality of modes, they calculated the natural frequencies of the studied structure and performed the updating process based on the results. Given the practical difficulties in determination of dynamic properties of the structure, such as its natural frequencies, the authors proposed a method, which despite its simplicity, could conduct the updating process only using the first mode of structures with high degrees of freedom [13]. Khanmirza et al. in 2011 used meta-heuristic methods to study the updating processes applied to the mass, stiffness, and damping matrices of multi-story shear structures. The authors used two updating methods, in the first method, the updating process was based on the neural networks and forced vibration response of a shear structure. In the second method, the updating process was based on the direct technique along with modification of equations governing the movement as well as using neural networks without employing modal analysis. Their findings indicated relatively accurate prediction of stiffness, mass and damping in the presence of 10% noisy data. However, the proposed method has been found to require more powerful filters in the updating process for data with a noise level of higher than 10% [14]. Song et al. in 2012 used the nonlinear behavior of materials for updating the finite element model and identifying the damage severity in a structure. The possibility of detecting dynamic parameters at lower vibration frequencies was among the advantages of their proposed method [15]. Kaveh & Maniat in 2014 modeled damage detection based on vibrational data using the charged system search (CSS) algorithm. They employed natural frequencies and mode shapes for developing the objective function and applied penalization to incorporate noise into vibrational data. The proposed method was found to be effective in determining the location and extent of damage [16]. Kaveh & Mahdavi in 2016 employed the colliding bodies optimization (CBO) and enhanced colliding bodies optimization (ECBO) algorithms to detect damage to truss structures. Analyzing the three numerical examples demonstrated the advantages of ECBO over CBO [17]. Hernandez et al. in 2016, proposed a damage identification method based on the incomplete modal data. In their method, the damage was identified by minimizing the frequency difference between the damaged and undamaged structures. The ability to identify the location and extent of damage in many damaged elements despite limited information on dynamic properties of the structure was among the characteristics of this method. On the other hand, this method is highly capable of identifying the location and extent of damage in the presence of error and noisy data. Since the efficiency of this method has been evaluated only for small structures, its performance for large-scale structures is uncertain [18]. Kaveh & Zolghadr in 2016 introduced the cyclical parthenogenesis algorithm (CPA) for structural damage detection through a modal strain energy-based index. The objective function was defined based on the generalized flexibility matrix (GFM). CPA was compared to other metaheuristic algorithms, and the damage detection model was shown to be efficient [19]. Kaveh & Dadras in 2017 studied damage detection using thermal exchange optimization (TEO) and enhanced thermal exchange optimization (ETEO) through noisy and noiseless vibrational data. Damage detection was defined as an inverse problem, and the results suggested that ETEO outperformed TEO in determining the location and extent of damage [20]. Kaveh et al. in 2018 proposed an objective function based on frequencies and mode shapes and evaluated damage detection in skeletal structures. They employed water evaporation optimization (WEO) to optimize the damage detection process. The objective function was found to reduce the number of assessment scenarios in detecting the location and extent of damage [21]. Zhang and Aoki proposed a new seismic-damage identification method for multi-story shear structures. Using an iterative method, they calculated the stiffness of each story before obtaining the dynamic characteristics of the studied structure. Suitable performance in dealing with noisy data was the key characteristic of the proposed method. Given that accuracy of the story stiffness highly depends on the identification accuracy of natural frequencies, the proposed method may be prone to errors at high noise levels. On the other hand, the location of the device to record vibration responses is important in identifying the location and extent of damage [22]. Kaveh et al. studied vibrating particles system (VPS) and enhanced vibrating particles system (EVPS) in detecting damage to truss structures with noisy and noiseless data. EVPS was found to outperform VPS in detecting the location and extent of damage [23]. Kaveh et al. introduced the boundary strategy (BS) for damage detection through metaheuristic algorithms. They used the shuffled shepherd optimization algorithm (SSOA) to investigate a number of structures and evaluate the model. Furthermore, SSOA was compared to other metaheuristic algorithms in damage detection. The model reduced search space dimensionality and accelerated the optimization convergence in determining the location and extent of damage. The model was effective with noisy vibration data and for large-scale structures [24]. Kaveh et al. in 2020 evaluated the location and extent of damage to structures using the water strider algorithm (WSA). They also used noisy vibrational data and evaluated the model. Kaveh et al. (2019) proposed a two-stage damage detection approach based on the graph-theoretic hierarchical (GHM) method and the modal strain energy-based index (MSEBI) [25-26]. Kaveh et al. in 2020 proposed plasma generation optimization (PGO) to detect damage to skeletal structures. Damage detection was formulated as an inverse optimization problem. They employed a hybrid objective function to reflect the extent of damage to structural members as a design variable in optimization. The model showed excellent damage detection performance even under noisy vibrational data [27].

In the present paper, a method was proposed for updating of stiffness matrix to identify the damage location and severity based on the Colliding Bodies Optimization (CBO) algorithm as well as the MDOF equations of motion. Assuming linear structural behavior and using the least number of vibration modes (only the first mode), the authors proposed a new formula for calculation of the stiffness matrix in the damaged structure. The proposed formula is based on the effect of the damaged elements on the dynamic characteristics of the structure (i.e., frequency, stiffness, etc.). An iterative procedure (in the form of an unconstrained optimization problem) is used for calculating the stiffness matrix of the studied structure. For calculating the stiffness matrix in the optimization problem, the objective function is defined based on the variations of dynamic characteristics of the structure by expanding the MDOF equation of motion. Variations of stiffness, frequency, and modal shapes are included in this process. The proposed formulation makes it possible to update the finite element model of the structure using a single frequency (the first frequency). In the algorithm developed based on the proposed formulation, a random coefficient (within the interval of 0 to 1) termed "the damage value" is initially assigned to the structural elements. Then, an equation of motion is defined for the free vibration of the analytical model (of the assumed structure) based on the proposed formulation. In the next step, upon applying the damage coefficients and the CBO algorithm to the structure, the initially assumed damage values are corrected and the finite element model is duly updated. This process continues until the convergence condition is satisfied. To verify the efficiency of the proposed algorithm, several examples are presented where the proposed method is applied to different structures including a planar truss, a plane frame, and a 3D frame. These examples indicated that, the proposed method was both accurate and efficient.

2. STIFFNESS MATRIX OF DAMAGED STRUCTURES

2.1 Proposed formulation for calculation of stiffness matrix

Any local damage in a structure would reduce its stiffness, which would in turn lead to variations in the natural frequency and consequently, the modal shapes of the structure. Therefore, stiffness variations are essential in the study of a structure. The eigenvalue equation governing an undamped system with n degrees of freedom is expressed as:

$$\left(\left[K\right]^h - \lambda_i^h\left[M\right]\right)\left\{\phi\right\}_i^h = 0 \quad i = 1,\cdots n \tag{1}$$

In Eq. (1), $[K]^h$ and $[M]$ are the stiffness and the mass matrices of the undamaged structure, respectively. The parameters of $\lambda_i^{\ h}$ and $\left\{\phi\right\}_i^h$ are the natural frequency and the modal shape vector of the *i*th mode in the undamaged structure, respectively. A similar equation can be expressed for a damaged structure as:

$$\left(\left[K\right]^d - \lambda_i^{\ d}\left[M\right]\right)\left\{\phi\right\}_i^{\ d} = 0 \quad i = 1,...,n \tag{2}$$

Where, $\left[K\right]^d$ is the stiffness matrix of the damaged structure, $\lambda_i^{\ d}$ and $\left\{\phi\right\}_i^{\ d}$ are the natural frequency and the modal shape vector of the *i*th mode in the damaged structure, respectively. On the other hand, as damage changes the dynamic parameters of a system, the following equations can be derived for the stiffness matrix, the natural frequencies, and the modal shape vectors of the damaged structure [28].

$$\left[K\right]^d = \left[K\right]^h - \left[\Delta K\right] \tag{3}$$

$$\lambda_i{}^d = \lambda_i{}^h - \Delta\lambda_i \tag{4}$$

$$\{\phi\}_i{}^d = \{\phi\}_i^h - \{\Delta\phi\}_i \tag{5}$$

In the above equations, $[\Delta K]$ is the stiffness variation matrix, $\Delta\lambda_i$ and $\{\Delta\phi\}_i$ express the variations of the natural frequency and modal shapes for the *i*th structural vibration mode, respectively. Substituting Eqs. (3)-(5) in Eq. (2), we obtain:

$$\left[\left([K]^h - [\Delta K]\right) - \left(\lambda_i^h - \Delta\lambda_i\right)[M]\right]\left(\{\phi\}_i^h - \{\Delta\phi\}_i\right) = 0 \tag{6}$$

Expanding Eq. (6) results in the formation of the following equation.

$$\begin{aligned}&\left([K]^h - \lambda_i^h [M]\right)\{\phi\}_i^h - \left([K]^h - \lambda_i^h [M]\right)\{\Delta\phi_i\} - \left([\Delta K] - \Delta\lambda_i [M]\right)\{\phi\}_i^h \\ &+ [\Delta K][\Delta\phi]_i - [M]\Delta\lambda_i\, [\Delta\phi]_i = 0\end{aligned} \tag{7}$$

Considering Eqs. (1)-(7), we obtain:

$$\begin{aligned}&-[K]^h\{\Delta\phi\}_i + [\Delta K]\{\Delta\phi\}_i - [\Delta K]\{\phi\}_i^h + [M]\lambda_i^h\{\Delta\phi\}_i - [M]\Delta\lambda_i\{\Delta\phi\}_i \\ &+ \Delta\lambda_i [M]\{\phi\}_i^h = 0\end{aligned} \tag{8}$$

In the above equations, the terms associated with the variations of frequency, stiffness, and modal shape were included in their entirety. Though these variations are seemingly insignificant, eliminating them would result in significant effects in the accurate calculation of the stiffness matrices. For this reason, all the terms reflecting the changes in the dynamic parameters of the structure were duly included in the calculations. Rearranging Eq. (8) results in:

$$\left(-[K]^h + [\Delta K]\right)\{\Delta\phi\}_i - [\Delta K]\{\phi\}_i^h + [M]\left(\lambda_i^h - \Delta\lambda_i\right)\{\Delta\phi\}_i + \Delta\lambda_i [M]\{\phi\}_i^h = 0 \tag{9}$$

And the final proposed equation is obtained from Eqs. (3) - (5) as:

$$-\left([K]^d - \lambda_i^d [M]\right)\{\Delta\phi\}_i - [\Delta K]\{\phi\}_i^h + \Delta\lambda_i [M]\{\phi\}_i^h = 0 \tag{10}$$

Which can be rewritten in the form of the following equation:

$$\left([K]^d - \lambda_i{}^d [M]\right)\{\Delta\phi\}_i = \Delta\lambda_i [M]\{\phi\}_i^h - [\Delta K]\{\phi\}_i^h \tag{11}$$

In Eq. (11), the total stiffness matrix of the structure is calculated based on the variations of the stiffness matrix obtained for individual elements. Therefore, variation of stiffness matrix for each structural element is expressed as:

$$[k]_j^d = (1-\alpha_j)[k]_j^h \qquad j = 1,...,ne \tag{12}$$

In Eq. (12), α_j and $[k]_j^h$ are the damage value and the stiffness matrix obtained for the jth element of the undamaged structure, respectively. Moreover, $[k]_j^d$ is the stiffness matrix associated with the damaged structure, and ne is the number of structural elements [29].

2.2 Formulation of the model-updating optimization problem

As already mentioned, the basic process in the iterative updating methods includes an optimization problem solved during the iterative process. To this end, the sides of the proposed equations were written in the form of the following two vectors.

$$\{\beta\} = \left([K]^d - \lambda_i^d [M]\right)\{\Delta\phi\}_i \quad i = 1,\dots,n \tag{13}$$

$$\{\gamma\} = \Delta\lambda_i [M]\{\phi\}_i^h - [\Delta K]\{\phi\}_i^h \quad i = 1,\dots,n \tag{14}$$

Therefore, the vector form of Eq. (11) is expressed as:

$$\{\beta\} = \{\gamma\} \tag{15}$$

Based on Eq. (15), the stiffness matrix of the damaged structure must be calculated such that, the analytical frequency would be equal to the frequency received from the sensor. Considering that Eq. (15) is a vector equation, the least squares method can be used in this regard.

$$\varepsilon = \|\beta - \gamma\|^2 \tag{16}$$

Where, ε is the error resulting from the least squares method, which must be minimized during the optimization process. Eq. (16) expresses the objective function used in the updating process for calculation of the stiffness matrix in the damaged structure. To solve this equation, the problem must be defined in the form of an iterative unconstrained optimization problem. In this paper, Eq. (16) was used to evaluate the unconstrained optimization problem based on the CBO algorithm for obtaining the damage coefficients associated with each element. Herein, the purpose was obtaining an equation for the stiffness matrix of the damaged structure with a limited number of measured frequencies (i.e., the first natural frequency only). Accordingly, the following formula was obtained.

$$\begin{aligned} &Find: && \{\alpha\} = \{\alpha_1, \alpha_2, \ldots\ldots\ldots\ldots\ldots, \alpha_{ne}\}^T \\ &Minimize: && F(\alpha) = \|\beta - \gamma\|^2 \\ &Where: && 0 \le \alpha \le 1 \end{aligned} \tag{17}$$

Where, $\{\alpha\}$ is the damage variable vector (including the location and severity of damage in the structural elements). This vector was obtained by solving Eq. (17) based on the proposed optimization algorithm. The value of $\{\alpha\}$ for each element varies between 0 and 1 corresponding to the damage percentages of 0 (undamaged) and 100% (completely damaged), respectively. On the other hand, considering that damage in this paper was simulated as a reduction in the modulus of elasticity of structural elements, thus, the modified modulus of elasticity in each element was defined as:

$$E_j^d = \alpha_j E_j^h \quad j = 1, \ldots, ne \tag{18}$$

Where, E_j^d and E_j^h are the modulus of elasticity of the *j*th damaged and undamaged elements, respectively. Compared to other cross-sectional properties (e.g., moment of inertia and cross-sectional area), modulus of elasticity represents a better damage identification criterion for structural elements.

2.3 Colliding bodies optimization

The CBO algorithm is a meta-heuristic algorithm developed first by Kaveh and Mahdavi using the laws of Physics [30]. This algorithm is based on the one-dimensional collision between bodies (regarded as mass particles). It can be used to find the solutions to the optimization problems. Before colliding with another particle, each particle possesses an initial mass and velocity. After the collision, each particle separates from others at a specific velocity, traveling from its initial position to a new (secondary) position. The secondary position can have a better (or worse) fitness, compared to the initial position. The summarized procedure for this algorithm is given below [31].

In this algorithm, the number of design variables in the search space is equal to the number of structural elements, and each particle represents a single structure. Accordingly, a number of particles with randomized values were generated for the design variables. Then, a fitness value was assigned to each mass particle. The particles were subsequently sorted in a descending order in terms of their respective fitness, after which they were divided into two groups: fixed particles and moving particles. The fitness of the moving particles is smaller than that of the fixed particles. The positions of the fixed particles in the search space are changed once the moving particles collide with them. The respective velocities of the moving and fixed particles are obtained from the following equations.

$$\begin{cases} V_i = 0 & i = 1,2,3,....,\dfrac{np}{2} \\ V_i = X_{i-\frac{np}{2}} - X_i & i = \dfrac{np}{2}+1, \dfrac{np}{2}+2,...,np \end{cases} \tag{19}$$

In Eq. (19) np is the particles number, X_i is the position of the particle ith, and V_i is the particle speed of ith. Then, due to the collision between two objects in accordance with the laws of physics, the size motion of all the particles before the collision is equal to the motion size of all particles after the collision. Therefore, by equating the kinetic energy before and after the collision, the velocity of the constant and moving particles after collision ($V_i^{'}$) is obtained as the following equations.

$$\begin{cases} V_i^{'} = \dfrac{\left(mp_{i+\frac{np}{2}} + (\mu)\left(mp_{i+\frac{np}{2}}\right)\right)V_{i+\frac{np}{2}}}{mp_i + mp_{i+\frac{n}{2}}} & i = 1,2,3,....,\dfrac{np}{2} \\ V_i^{'} = \dfrac{\left(mp_i - (\mu)\left(mp_{i-\frac{np}{2}}\right)\right)V_i}{mp_i + mp_{i-\frac{np}{2}}} & i = \dfrac{np}{2}+1, \dfrac{np}{2}+2,...,np \end{cases} \tag{20}$$

In Eq. (20), mp_i is the mass of ith particle, defined as Eq. (21).

$$mp_i = \frac{\dfrac{1}{fit_i}}{\sum_{k=1}^{np}\dfrac{1}{fit_k}} \qquad i = 1,2,3,....,np \tag{21}$$

In Eq. (21), fit_i is the fitness function value of the particle ith. For better search of search space, the coefficient μ is considered as Eq. (22) in Eq. (20).

$$\mu = 1 - \frac{iter}{iter_{\max}} \tag{22}$$

In Eq. (22), $iter$ is the current repetition number, and $iter_{\max}$ is the total number of repetitions in the optimization process. Finally, each particle new position is obtained by considering the velocity after the collision in Eq. (23).

$$X_i^{new} = \begin{cases} X_i + rand\,V_i' & i = 1, 2, 3, \ldots, \frac{np}{2} \\ X_{i-\frac{np}{2}} + rand\,V_i' & i = \frac{np}{2} + 1, \ldots, np \end{cases} \tag{23}$$

In Eq. (23), $rand$ is a random number in range of zero and one, and X_i^{new} is the new position of ith particle after the collision.

2.4 Proposed model-updating method

In this study, it was assumed that, the first natural frequency was the only available characteristic for calculation of the stiffness matrix in the damaged structure. The measured natural frequencies are far less in number than those obtained from the analytical model, which is a major problem in obtaining the stiffness matrix of a damaged structure (i.e., solving Eq. (16)). In other words, the number of the measured DOFs in the actual structure (for positioning the sensors) is less than that in the analytical model, attributing to the economic and executive problems associated with multi-directional vibration measurements (rotational directions and directions of coordinate axes within the structure). Accordingly, the analytical data always outnumber the measured data (obtained from the sensors). To partially overcome this problem, the authors used only the first natural frequency of the structure (obtained from the sensor) in the updating process. Accordingly, the other structural characteristics (i.e., stiffness matrix, modal shapes, etc.) of the damaged structure were approximated based on its first natural frequency alone. An iterative method was used to obtain the stiffness matrix of the damaged structure, where in the stiffness matrix was updated during each iteration via the optimization process. The iterations continued until the convergence condition was satisfied, ultimately leading to the generation of the stiffness matrix in the damaged structure. The other dynamic characteristics of the structure, including its other natural frequencies were then obtained based on this stiffness matrix. The method proposed for calculation of the stiffness matrix in the damaged structure (based on the proposed formulation) is as follows.

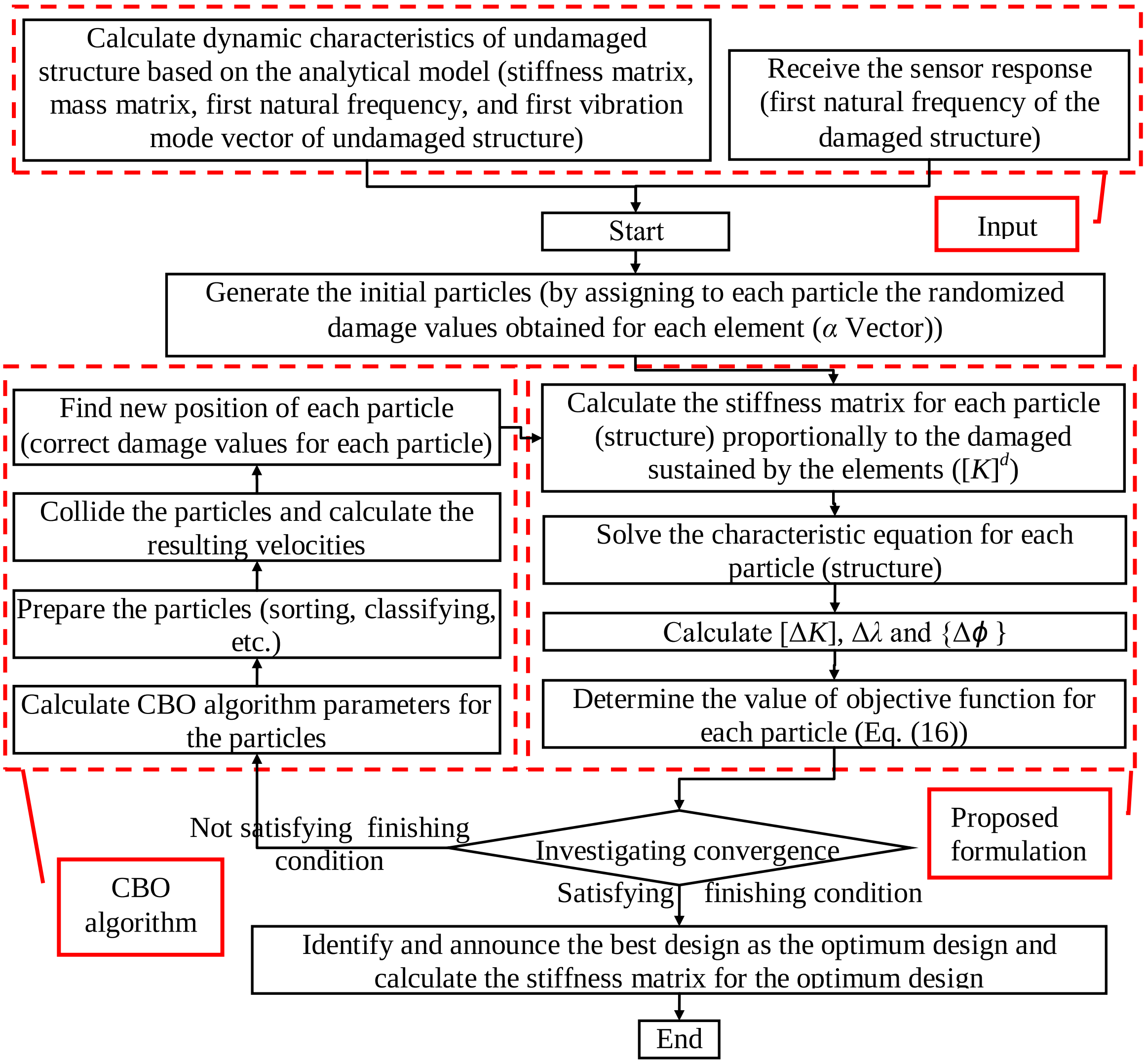

Figure 1. A Flowchart of the proposed method for calculation of stiffness matrix in the damaged structure

Assuming damage values for individual structural elements, the total stiffness matrix for the damaged structure was calculated from Eq. (12) based on coupling of the structural elements. Then, the stiffness variation matrix for the damaged structure was obtained from Eq. (3). Therefore, the stiffness matrix of the damaged structure as well as the stiffness variation matrix were found to be directly dependent on the damage value assumed for each element.

The *i*th modal shape of the damaged structure was obtained based on the natural frequency of the damaged structure, the sensor response, and using Eq. (2). Therefore, $[K]^d$, $[\Delta K]$ and $\{\phi\}_i^d$ were calculated by assuming a damage value for each element. In addition, the natural frequency and modal shape vector for the *i*th mode of the undamaged structure were obtained from Eq. (1) based on calculating the mass and stiffness matrices for the undamaged structure. The natural frequency and modal shape variations for each vibration mode were then obtained from Eqs. (4)-(5), respectively. Ultimately, the assumed damage severity values corresponding to each element were updated using the results as well as the optimization process based on the proposed objective function (Eq. (16)). Based on these updated values, the stiffness matrix for each element and subsequently, the total stiffness matrix for the structure were also updated. This process continued until the error vector in Eq. (16) reduced to 0. Fig. 1 shows the flowchart of the method proposed for calculation of the stiffness matrix in the damaged structure.

3. NUMERICAL EXAMPLES

To evaluate the efficiency of the proposed method and its associated formulation, three examples were selected, where different structural conditions were applied to the structure. Due to the lack of access to the experimental results during the modeling process, a hypothetical damage scenario was assumed for each structure and the first natural frequency of the damaged structure was calculated from the relevant analytical equations. The calculated value was assumed as the sensor output for the actual structure. The updating process and calculation of the stiffness matrix were then carried out in accordance with the proposed procedure making the following assumptions: (1) damage values for the elements are unavailable, and (2) only the first natural frequency of the damaged structure (the hypothetical sensor response) is known. Ultimately, the results obtained from the updating process were compared with the initial analytical results. According to the results, the proposed formulation can be efficiently used for calculation of the stiffness matrix in the structure in all the examples. In all the examples, it was assumed that, structural stiffness was altered due to the imposed damage, and the mass of the structure remained constant.

3.1 Example 1: Planar 9-bar truss structure

In the first example, a planar truss with 9 members was evaluated based on the assumptions presented in a previous study to investigate the performance of the proposed method and its associated formula (Fig. 2). The finite element model generated for this structure comprised 6 nodes with 9 active degrees of freedom. For all the elements, E and ρ were assumed as 200 GPa and 7860 kg/m^3, respectively. The cross-sectional area for all the elements was assumed as 2.5×10^{-3} m^2, as introduced in a previous study [32].

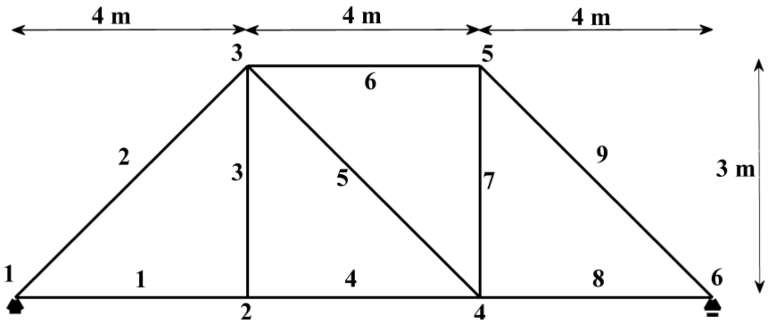

Figure 2. Schematic of the planar 9-bar truss structure

In this example, the stiffness matrix calculation process was explained in detail based on the number of degrees of freedom to elaborate the proposed method. As already mentioned, due to the lack of access to the experimental results, a damage scenario was defined for calculation of the first natural frequency of the damaged structure (Table 1).

Table 1: Damage scenario defined for the planar 9-bar truss structure

Element No.	Damage ratio
3	0.20
8	0.30

Based on the assumptions made in Table 1, a first natural frequency of 54,895 rad/s^2 was obtained for the damaged structure in the analytical model. This frequency was assumed as the sensor output in the proposed method. The following stiffness matrix, first natural frequency, and first modal shape of the undamaged structure were available for the analytical model.

$$\left([K]^h\right)_{analytical} = \begin{bmatrix} 2.5 & 0 & 0 & 0 & -1.25 & 0 & 0 & 0 & 0 \\ 0 & 1.67 & 0 & -1.67 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2.53 & 0 & -0.64 & 0.48 & -1.25 & 0 & 0 \\ 0 & -1.67 & 0 & 2.39 & 0.48 & -0.36 & 0 & 0 & 0 \\ -1.25 & 0 & -0.64 & 0.48 & 3.14 & -0.48 & 0 & 0 & -1.25 \\ 0 & 0 & 0.48 & -0.36 & -0.48 & 2.03 & 0 & -1.67 & 0 \\ 0 & 0 & -1.25 & 0 & 0 & 0 & 1.89 & -0.48 & -0.64 \\ 0 & 0 & 0 & 0 & 0 & -1.67 & -0.48 & 2.03 & 0.48 \\ 0 & 0 & 0 & 0 & -1.25 & 0 & -0.64 & 0.48 & 1.89 \end{bmatrix} \times 10^8 \left(N/m\right)$$

$$\lambda_1^h = \left(\omega_1^2\right)^h = 58020 \quad rad/s^2$$

$$\{\phi\}_1^h = [\,-0.0107 \quad 0.042508 \quad -0.0205 \quad \ldots \quad -0.01478 \quad 0.035802 \quad -0.02854]^T$$

According to the flowchart presented in Fig. 1, the modal shape and stiffness matrix of the undamaged structure were assumed as the known initial data in the updating process. Then, a random damage value was applied to the structural elements in the CBO algorithm in accordance with the proposed method. As already mentioned, each particle in the CBO algorithm corresponds to a structure. Accordingly, the total stiffness matrix was calculated for the structure based on the assumed damage sustained by the elements. The frequency and modal shape of the assumed damaged structure as well as its frequency and mode variations were then calculated by solving the free vibration equation governing each particle. In the next step, the particles were displaced in the search space in accordance with the CBO algorithm. This displacement represents the variations of damage values for each structural element. The optimization process based on the CBO algorithm continued until the value of the error vector (Eq. (16)) reduced to 0. Fig. 3 shows the convergence trend of the optimization process in the studied 9-bar truss.

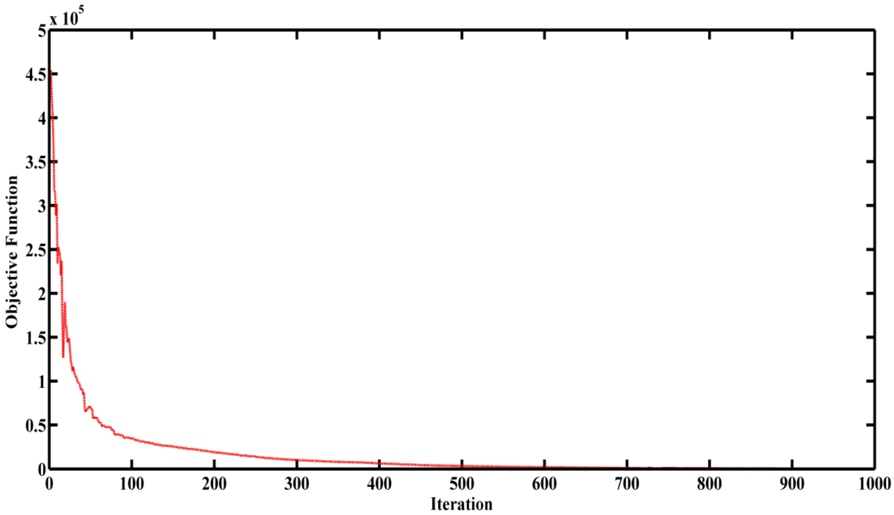

Figure 3. The convergence curves for the planar 9-bar truss structure

The best particle was identified upon completion of the optimization process once the convergence condition was satisfied. The design variables of the best particle as well as the stiffness matrix of the damaged structure were then obtained. Accordingly, the following updated stiffness matrix was obtained for the 9-bar truss.

$$\left([K]^d\right)_{updating}=\begin{bmatrix} 2.5 & 0 & 0 & 0 & -1.25 & 0 & 0 & 0 & 0 \\ 0 & 1.33 & 0 & -1.33 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2.53 & 0 & -0.64 & 0.479 & -1.25 & 0 & 0 \\ 0 & -1.33 & 0 & 2.05 & 0.479 & -0.36 & 0 & 0 & 0 \\ -1.25 & 0 & -0.64 & 0.479 & 2.76 & -0.48 & 0 & 0 & -0.87 \\ 0 & 0 & 0.479 & -0.36 & -0.48 & 2.03 & 0 & -1.67 & 0 \\ 0 & 0 & -1.25 & 0 & 0 & 0 & 1.889 & -0.48 & -0.64 \\ 0 & 0 & 0 & 0 & 0 & -1.67 & -0.48 & 2.03 & 0.48 \\ 0 & 0 & 0 & 0 & -0.87 & 0 & -0.64 & 0.48 & 1.519 \end{bmatrix}\times 10^8\ (N/m)$$

Based on the updated stiffness matrix calculated from the proposed method, a value of 54895 rad/s^2 was obtained (from Eq. (2)) for the first updated natural frequency of the structure. As can be observed, this value fully corresponds to the sensor response, while the updated frequency is exactly equal to that obtained from the sensor. The results of the previous studies indicated that, the proposed method is able to accurately identify the damaged elements while generating damage values that are in complete agreement with the initially assumed values. Fig. 3 shows the comparison of the damage location and severity values obtained from the proposed method with those obtained from the assumed values (Table 1).

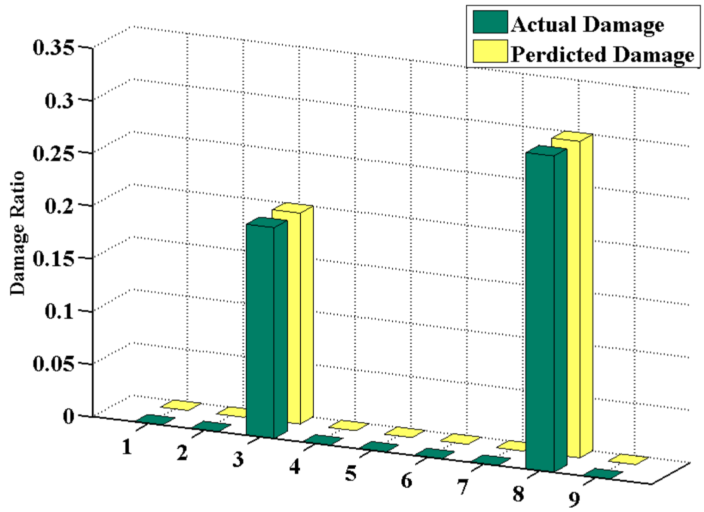

Figure 4. Identified damage elements for the planar 9-bar truss structure

In the absence of the experimental results, the stiffness matrix of the damaged structure can be obtained through the numerical analysis by assuming a damage scenario (Table 1) for the analytical model. Accordingly, the following stiffness matrix was obtained from the analytical model by assuming actual damage values:

$$\left([K]^d\right)_{analytical}=\begin{bmatrix} 2.5 & 0 & 0 & 0 & -1.25 & 0 & 0 & 0 & 0 \\ 0 & 1.33 & 0 & -1.33 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2.53 & 0 & -0.64 & 0.48 & -1.25 & 0 & 0 \\ 0 & -1.33 & 0 & 2.05 & 0.48 & -0.36 & 0 & 0 & 0 \\ -1.25 & 0 & -0.64 & 0.48 & 2.77 & -0.48 & 0 & 0 & -0.88 \\ 0 & 0 & 0.48 & -0.36 & -0.48 & 2.03 & 0 & -1.67 & 0 \\ 0 & 0 & -1.25 & 0 & 0 & 0 & 1.89 & -0.48 & -0.64 \\ 0 & 0 & 0 & 0 & 0 & -1.67 & -0.48 & 2.03 & 0.48 \\ 0 & 0 & 0 & 0 & -0.88 & 0 & -0.64 & 0.48 & 1.52 \end{bmatrix}\times 10^8\ (N/m)$$

A very slight difference was observed between the corresponding elements of the updated stiffness matrix and the stiffness matrix obtained for the analytical model. The results obtained from the proposed method demonstrated the efficiency of this method in calculating the stiffness matrix using only the first natural frequency of the structure. During the updating process applied in a previous study (Ref. [32]), two natural frequencies of the structure were used. Assuming similar structural damages in this study compared to the current example, the stiffness matrix of the analytical model of the damaged structure (in which the damage values were applied to the member's matrix and the formation of the overall structure matrix) and the stiffness of the updated model are different in some elements. Considering the stiffness matrix in this study, these elements have significant values, indicating the lack of full compliance of the updated stiffness matrix.

$$\left([K]^{d}\right)_{updating}=\begin{bmatrix} 2.5 & 0 & 0 & 0.0001 & -1.25 & 0.0001 & 0 & 0 & 0 \\ 0 & 1.67 & 0.0002 & -1.67 & 0.0001 & -0.0014 & 0 & -0.001 & 0.0002 \\ 0 & 0.0002 & 2.53 & 0.0003 & -0.64 & 0.48 & -1.25 & 0.0002 & 0 \\ 0.0001 & -1.67 & 0.0003 & 2.38 & 0.48 & -0.36 & 0 & -0.0015 & 0.0003 \\ -1.25 & 0.0001 & -0.64 & 0.48 & 3.14 & -0.48 & 0 & 0.0002 & -1.25 \\ 0.0001 & -0.0014 & 0.48 & -0.36 & -0.48 & 2.02 & 0 & -1.67 & 0.0003 \\ 0 & 0 & -1.25 & 0 & 0 & 0 & 1.89 & -0.48 & -0.64 \\ 0 & -0.001 & 0.0002 & -0.0015 & 0.0002 & -1.67 & -0.48 & 2.03 & 0.48 \\ 0 & 0.0002 & 0 & 0.0003 & -1.25 & 0.0003 & -0.64 & 0.48 & 1.89 \end{bmatrix}\times 10^{8}\ (N/m)$$

Based on this inconsistency, the method proposed in the mentioned study (Ref. [32]) is not able to produce the precise damage values in the updating process and moreover, these outcomes differ from the actual extent of damage. However, the final damage values in the updated model were not mentioned in this study.

3.2 Example 2: The 2-bay 4-story frame

To investigate the efficiency of the proposed method, a plane frame with 20 elements was evaluated for the first time. Fig. 5 shows this frame comprising 2 bays and 4 stories. For all the members, the assumed E and ρ values were equal to 2.01×10^{6} kg/cm^{2} and 7860 kg/m^{3}, respectively. IPE140 sections were assumed for this structure.

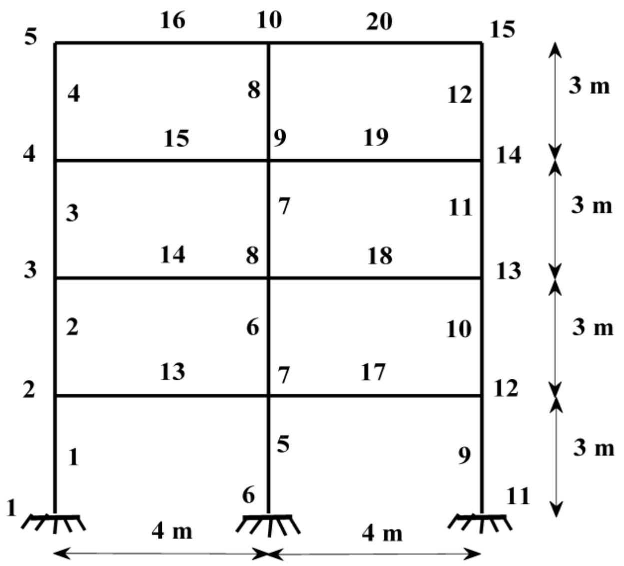

Figure 5. Schematic of the 2-bay 4-story frame

In this example, certain damage scenarios (Table 2) were assumed for calculation of the first natural frequency in the damaged structure (i.e., the sensor response). Accordingly, a value of 0.023476 rad/s^2 was calculated for this frequency. On the other hand, the first natural frequency obtained for the undamaged structure was equal to 0.023473 rad/s^2.

Table 2: Damage scenario defined for the 2-bay 4-story frame

Element No.	Damage ratio
1	0.25
15	0.40

As observed in this example, the difference between the natural frequencies obtained for the damaged and undamaged structures is negligible (3×10^{-6}), thus it can be used as a suitable criterion to investigate the efficiency of the algorithms proposed for updating the finite element model of the structure. In this example, an efficient method is required to calculate the stiffness matrix of the damaged structure. In other words, based on the slight difference between the first natural frequencies, the initial assessment suggested that, the structure was healthy (in spite of the considerable damage incurred on Elements 1 and 15). In the proposed method, damage values for structural elements were accurately calculated using only the first natural frequency of the structure. Fig. 6 illustrates the comparison of the damage results obtained for the 20-member frame of the proposed method with the actual damage results.

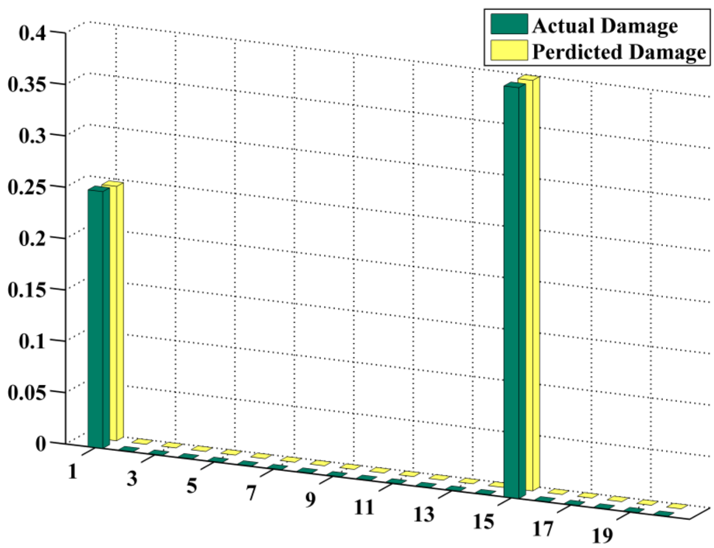

Figure 6. Identified damage elements for the 2-bay 4-story frame

Fig. 7 shows the convergence process observed in the objective function of the optimization problem. As can be observed, during the initial iterations in the proposed method, the objective function approaches 0, indicating that the formulation used for the optimization problem in the proposed method is very powerful. Considering that the proposed equation is a vector equation, and assuming that all dynamic characteristics of the structure undergo the changes governed by the free vibration equation, even the slightest difference between the obtained frequencies would influence the value of the error vector (Eq. (16)).

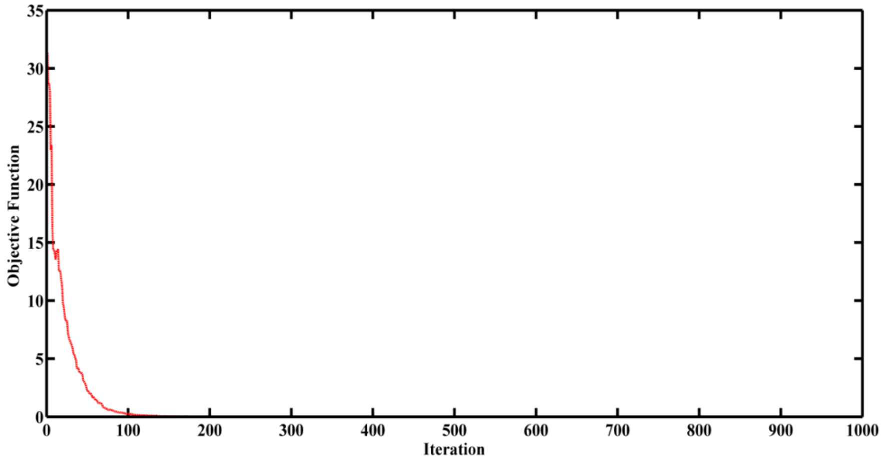

Figure 7. The convergence curves for the 2-bay 4-story frame

Ultimately, the updated stiffness matrix was used to calculate the updated first natural frequency as 0.023476 rad/s^2 (from Eq. (2)). As can be observed, this frequency is exactly equal to the frequency received from the sensor. It is noteworthy that, the updated stiffness matrix is in complete agreement with that obtained from the analytical model. However, due to the large dimensions of the stiffness matrix, the values of its individual elements are not presented here.

3.3 Example 3: 3D frame with 84 elements

To study the validity of the results obtained from the proposed method, a 3D moment frame was simulated (Fig. 8).

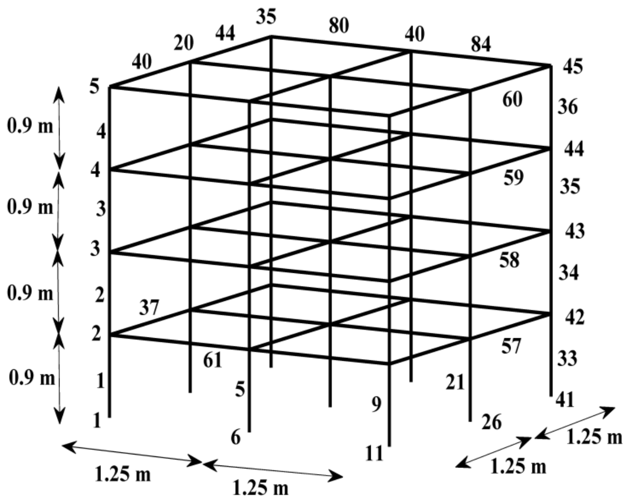

Figure 8. Schematic of the 3D frame with 84 elements

The E and ρ values for the structural elements were assumed as 2×10^{11} Pa and 7850 kg/m^3, respectively. B100×9 and S75×11 cross-sections were assumed for the frame columns and beams, respectively [33]. Table 3 shows the damage scenario assumed for calculation of the first natural frequency of the damaged structure.

Table 3: Damage scenario defined for the 3D frame with 84 elements

Element No.	Damage ratio
1	0.25
65	0.15

Based on the applied damage scenario, a first natural frequency of 9760037.44 rad/s^2 was obtained for the damaged structure using the numerical simulation method. In the proposed updating method, this frequency was assumed to be equal to the sensor response. On the other hand, a first natural frequency of 9776217.531 rad/s^2 was obtained for the undamaged structure. The first natural frequency for the updated structure was measured as 9760418.58 rad/s^2. Fig. 9 shows the damage values obtained for the structural elements.

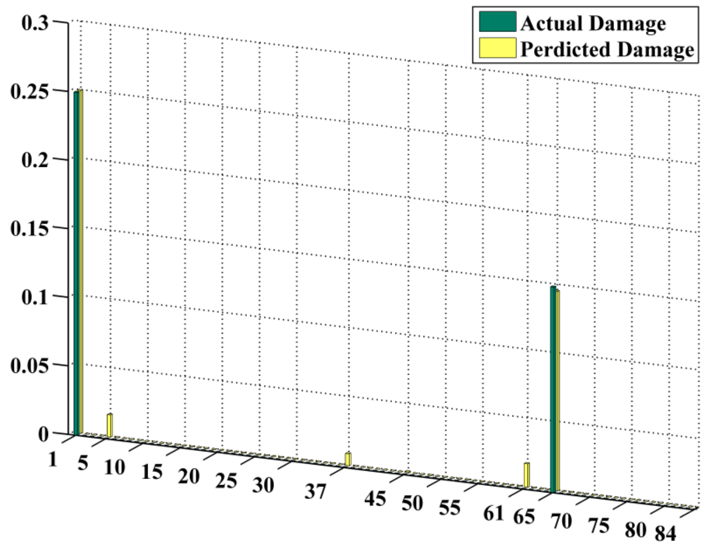

Figure 9. Identified damage elements for the 3D frame with 84 elements

As shown in Fig. 9, the difference between the analytical and actual damage values is very slight. The damage values obtained for elements 1 and 65 are almost equal to the assumed values, and very slight damage values (less than 0.02) are obtained for elements 5, 37, and 61. This difference is due to the dimensions of the search space as well as the number of the DOFs (both rotational and translational) in the studied structure. In other words, the dimensions of the stiffness matrix increase considerably (upon applying a 216×216 boundary condition), causing a corresponding increase in the search space of the optimization problem. Ultimately, the search space dimensions introduced a slight error in the optimization process. Considering the nature of the proposed formulation, a slight error would result in a considerable increase in the value of ε. Therefore, ε must be significantly reduced, if the termination condition is to be satisfied in the proposed optimization algorithm. In other words, the proposed formulation would generate a unique objective function in the optimization problem, which is highly sensitive to the final response. Fig. 10 shows the convergence trend obtained for the optimization process.

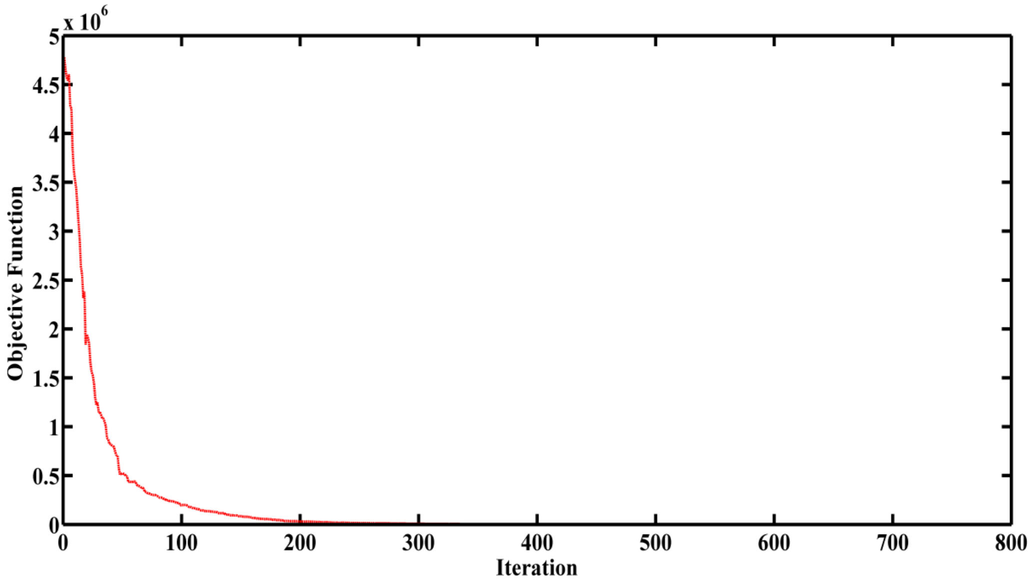

Figure 10. The convergence curves for the 3D frame with 84 elements

As illustrated in Fig. 10, the optimization algorithm approaches 0 during the initial iterations. However, the proposed method failed to generate an exact error vector of 0 after a certain number of iterations, and this introduced a slight error in the obtained damage values as well as the updated model. Therefore, the first natural frequency of the damaged structure (the sensor response) was also found to be slightly different (circa 0.004) from that obtained for the updated structure. This slight difference had a significant effect on the objective function value generated in accordance with the proposed method. In other words, in relation to the frequency and modal shape differences obtained from the proposed formulation, the final objective function in the optimization problem was highly sensitive to the complete agreement between the updated and the actual models. Therefore, the proposed formulation can be regarded as a very powerful one for calculating the dynamic characteristics of the actual model.

4. CONCLUSION

Using the CBO algorithm, in the current study, a method was proposed based on the variations of frequency, modal shape, and stiffness matrix of the structure as well as the MDOF equation of motion. This method is also based on the sensor response obtained for the damaged structure. In addition to calculating the stiffness matrix of the damaged structure, the proposed method is also able to identify the location and severity of the damaged structure (with acceptable level of accuracy). In the conventional updating methods used to calculate the dynamic characteristics of a structure, an information is generally required regarding the vibration modes of that structure. However, in an actual model, receiving information related to the higher vibration modes from the sensors is both costly and time consuming. Therefore, the proposed method does not need a full knowledge of the damaged structure for monitoring the health of that structure. Using minimal structural data, the proposed method is able to identify the location and severity of the damage as well as the stiffness matrix of the damaged structure. In the proposed method, it is assumed that, only the first natural frequency of the damaged structure is available. Accordingly, assuming that structural damage changes the dynamic characteristics of a structure (i.e., its frequency, modal shape, and stiffness), an updating formula was proposed based on the MDOF equations of motion as well as considering the variation of dynamic characteristics. A randomized damage value was assigned to each structural element to calculate the stiffness matrix of the damaged structure. Then, using the proposed formulation and defining an unconstrained optimization problem (solved through the use of the CBO algorithm), the structural model was updated and the damage values for the structure were calculated. Ultimately, the stiffness matrix of the damaged structure was calculated. The independency of the method from the number of natural frequencies of the structure is an important feature of the proposed method. Accordingly, using only the first natural frequency of a damaged structure, the proposed method is able to provide a correct approximation of other dynamic characteristics of a structure. In addition, as variations of all the dynamic characteristics are considered in the final equation, the objective function in the proposed optimization problem is highly sensitive to the uniqueness of the obtained response. In other words, even the slightest variations of dynamic parameters of the studied structure are included in the proposed formulation, making it suitable for applications with large search spaces. To evaluate the efficiency of the proposed method, different examples were presented. The results showed that, the stiffness of the damaged structures were correctly calculated by the proposed algorithm.

According to the latest example, an increase in the search space in the proposed method, or an increase in the number of structural elements and degrees of freedom may result in a slight error in the final solution. This slight error results from the high precision of the proposed formulation and subsequently the objective function of the optimization problem. Accordingly, the proposed method may be associated with a slight error while dealing with large search spaces, highlighting the necessity for further studies. On the other hand, general information received from the actual model sensors are noisy and can result in errors. Thus, the proposed method may fail in accurate identification of damage in the presence of noisy data, since it merely uses the first natural frequency of the structure. This necessitates the evaluation of the proposed method in the presence of noisy data. It is worth noting that, the updating process in the presence of noisy data is based on the statistical information and requires a considerable amount of data from the real model. However, it may result in reduced efficiency of the proposed model (using only one frequency), necessitating further studies. Finally, to remedy the disadvantages of the proposed method (large search space and noisy data), it is recommended to determine the high-order natural frequencies based on the first natural frequency in each optimization iteration in the damage identification process in order to fully establish the dynamic characteristics of the structure. However, this recommendation also needs further investigations.

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