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HYBRID COLLIDING BODIES OPTIMIZATION AND SINE COSINE ALGORITHM FOR OPTIMUM DESIGN OF STRUCTURES

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ABSTRACT

Colliding Bodies Optimization (CBO) is a population-based metaheuristic algorithm that complies physics laws of momentum and energy. Due to the stagnation susceptibility of CBO by premature convergence and falling into local optima, some meritorious methodologies based on Sine Cosine Algorithm and a mutation operator were considered to mitigate the shortcomings mentioned earlier. Sine Cosine Algorithm (SCA) is a stochastic optimization method that employs sine and cosine based mathematical models to update a randomly generated initial population. In this paper, we developed a new hybrid approach called hybrid CBO with SCA (HCBOSCA) to obtain reliable structural design optimization of discrete and continuous variable structures, where a memory was defined to intensify the convergence speed of the algorithm. Finally, three structural problems were studied and compared to some state of the art optimization methods. The experimental results confirmed the competence of the proposed algorithm.

Keywords: Colliding Bodies Optimization; Sine Cosine Algorithm; Structural Design; Discrete and Continuous Optimization; Metaheuristic Algorithms.

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1. INTRODUCTION

Optimization is a pivotal context in various fields of study especially in engineering hence metaheuristic algorithms (MAs) have recently attracted the attention of a significant

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community of researchers. Considering the fact that MAs can readily be implemented in a wide range of problems needless to gradient information and they can obtain a near-optimal solution for any problem, namely continuous and discrete problems, makes them a practical optimization method[1, 2].

MAs are powerful, robust, and mostly nature-inspired optimization methods and due to their stochastic approach, a wide range of optimization problems can be tackled by employing these methods. Each MA suffers some drawbacks alongside its merits in searching the global optimum that is, one particular MA cannot be capable of solving every single optimization problem, especially since there are various types of problems. Every MA has two major phases approaching the most feasible solution in the search domain called (1) Exploration phase (diversely looking for possible feasible answers all over the search space) which takes place in the primary steps of the search process and (2) Exploitation phase (meticulously looking for any better answers in neighboring areas of the solutions found in exploration phase). The most challenging task for researchers in developing MAs is to exert a proper balance between these two phases. The desirable speed of converging to the global optimum solution plus the capability of jumping out of the local optima can severely strengthen the performance of a particular MA. Therefore, numerous algorithms are developed in order to overcome the deficits of MAs in solving different problems [3-6].

One essential key to the proper design of a structure is the optimal design of the structure. It could lead to efficient construction material consumption, considerable subsidence of material waste production, and eventually, result in a costly beneficial design. Structural optimization can be categorized as (1) sizing optimization of structural members, (2) searching for the optimal form of the structure, and (3) structural members connectivity and optimal size acquirement [7]. Kaveh and Ilchi Ghazaan applied Enhanced Whale Optimization Algorithm (EWOA) for the sizing optimization of skeletal structures [7]. Kaveh and Zaerreza utilized Improved Shaffled based JAYA algorithm (IS-JAYA) for the optimum design of the braced dome with frequency constraint [8]. Azizi *et al*. adopted Chaos Game Optimization (CGO) to optimize the shape and size of truss structures [9]. Fernandez and Masters employed hybridized Particle Swarm and Big Bang-Big Crunch optimization to explore and then exploit (ETE) the design domain of large planar frame structures [10]. Kaveh and Hosseini optimized the size of discrete and continuous large-scale truss structures exerting the Doppler Effect-Mean Euclidian Distance Threshold Algorithm (DE-MEDT) [11]. Kaveh *et al*. came up with a novel MA called Black Hole Mechanics Optimization (BHMO) in order to optimize real-size truss and frame structures [12].

Since the development of the two population-based MAs called Colliding Bodies Optimization (CBO) presented by Kaveh and Mahdavi [13] and Sine Cosine Algorithm (SCA) introduced by Mirjalili [14], they have been a hotspot for the researchers to upgrade capabilities of them and study their applications. CBO is based on Newtonian physics collision laws [13] in which two CBs collide and their positions are updated according to the laws of collision in physics. SCA is based on the random production of a population and fluctuation of them outwards or towards the best candidate solution (i.e. Destination Point) using two sine and cosine function-based mathematical models and a number of random and adaptive variables [14]. We conducted a new approach called Hybrid CBO with SCA (HCBOSCA) to insert a better correlation between global and local search to obtain a more

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reliable optimal design for discrete and continuous variable structures.

The remainder of the paper is a brief explanation of CBO and MSCA followed by instruction on HCBOSCA in Section 2; in Section 3, three structural problems with discrete and continuous variables are utilized to compare HCBOSCA with CBO and a number of its variants as well as some other well-known approaches and eventually, Section 4 presents the conclusions of this study.

2. META-HEURISTIC ALGORITHMS

In this section, after a concise overview of CBO and MSCA, we will present the hybrid variant of CBO and SCA so-called HCBOSCA.

2.1 Colliding Bodies Optimization

Colliding Bodies Optimization (CBO) is a population-based MA developed by Kaveh and Mahdavi [13] inspired by the collision phenomenon in nature. In this method, after the collision of two bodies called colliding bodies (CBs), they try to reach a minimum level of energy. This technique is notably simple to implement and does not use any memories to save any best optimum solutions or positions. Each CB is a solution candidate like x_i and each of which has a specified mass defined as:

$$m_k = \sum_{i=1}^{n} \frac{1}{fit(i)} \Big/ fit(k) \quad ; \qquad k = 1,2,\dots,n \tag{1}$$

where $fit(i)$ represents the fitness value of the ith solution candidate and n is the number of CBs.

In order to select two objects for collision, CBs will be arranged ascending according to their fitness values. Then the sorted CBs will equally be divided into two groups: (1) Stationary group, (2) Moving group. The first group (stationary group) will contain the first half of these organized CBs and the other one (moving group) will contain the second half of them. Moving objects collide to stationary ones to not only improve their own positions, but also direct the stationary objects towards a better position. Before the collision the velocity of stationary CBs is equal to zero:

$$v_i = 0 \qquad ; \qquad i = 1, 2, \dots, \frac{n}{2} \tag{2}$$

the velocity of each moving CB before collision is:

$$v_i = x_{i-\frac{n}{2}} - x_i \qquad ; \qquad i = \frac{n}{2}+1, \frac{n}{2}+2, \dots, n \tag{3}$$

the velocity of each CB in stationary group after collision (v_i') is defined by:

$$v_i' = \left(\left(m_{i+\frac{n}{2}} + \varepsilon m_{i+\frac{n}{2}}\right) v_{i+\frac{n}{2}}\right) \Big/ \left(m_i + m_{i+\frac{n}{2}}\right) \qquad ; \qquad i = 1, 2, \dots, \frac{n}{2} \tag{4}$$

and the velocity of the moving ones after the collision is specified as:

$$v_i' = ((m_i - \varepsilon m_{i-\frac{n}{2}})v_i)/(m_i + m_{i-\frac{n}{2}}) \quad ; \quad i = \frac{n}{2}+1, \frac{n}{2}+2, \dots, n \tag{5}$$

The new updated position of each CB after the collision is evaluated with respect to its velocity and the position of stationary CB. So, the new positions of stationary CBs are:

$$x_i^{new} = x_i + rnd \circ v_i' \quad ; \quad i = 1, 2, \dots, \frac{n}{2} \tag{6}$$

where x_i^{new}, x_i and v_i' are the new position, previous position and the velocity after the collision of the ith CB, respectively. rnd is a random vector distributed uniformly in [-1,1] domain. The sign "∘", depicts an element-by-element multiplication. The new position of each moving CB can be obtained from:

$$x_i^{new} = x_{i-\frac{n}{2}} + rnd \circ v_i' \quad ; \quad i = \frac{n}{2}+1, \frac{n}{2}+2, \dots, n \tag{7}$$

where ε is the coefficient of restitution (COR) and it decreases linearly from 1 to zero. It is stated as:

$$\varepsilon = 1 - \frac{gen}{maxgen} \tag{8}$$

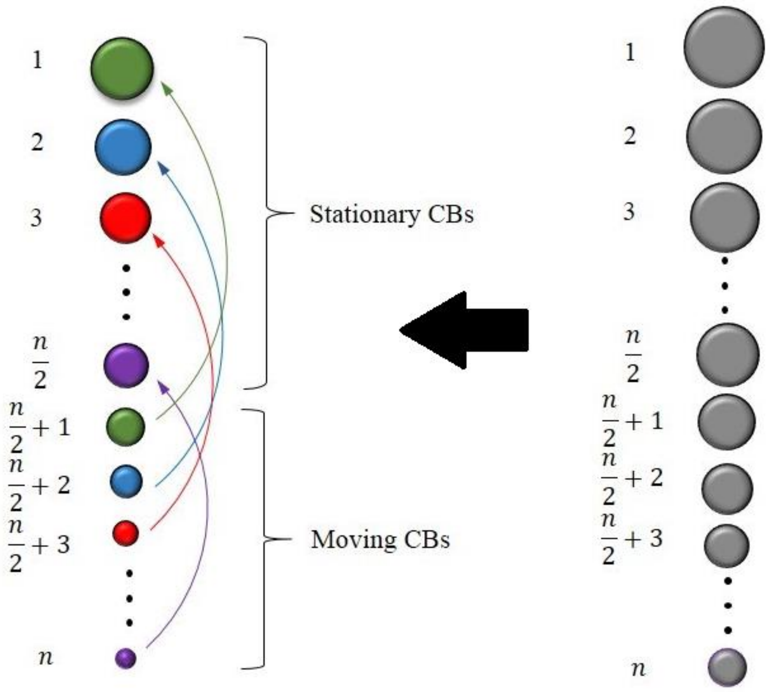

Figure 1. Schematic of colliding bodies

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where gen denotes the current generation number and $maxgen$ is the total number of generations. Fig. 1 illustrates the concept of CBO in which the CBs with higher fitness are greater in diameter compared to those with lower fitness. The reader should refer to Kaveh and Mahdavi [13] for extra details.

2.2 Modified Sine Cosine Algorithm

The Sine Cosine Algorithm (SCA) is a recently developed population-based MA [14]. The core idea is inspired by trigonometric sine and cosine functions. Modified Sine Cosine Algorithm (MSCA) is a modified version of SCA, tried to enhance the performance of the SCA [15]. In this method, the equations proposed to update the position, contains both exploration and exploitation phase of an optimization algorithm:

$$x_{i,j}^{gen+1} = \begin{cases} x_{D,j} - r_1 \sin(2\pi r_2) \left|2r_3 x_{D,j} - x_{i,j}^{gen}\right| & r_4 > 0.5 \\ x_{D,j} - r_1 \cos(2\pi r_2) \left|2r_3 x_{D,j} - x_{i,j}^{gen}\right| & r_4 \leq 0.5 \end{cases} \tag{9}$$

$$r_1 = a \times sin\left(\left(1 - \frac{gen}{maxgen}\right) \times \frac{\pi}{2}\right) + b \tag{10}$$

where $x_{D,j}$ is the jth dimension of the global best solution considered as destination point, $x_{i,j}^{gen}$ and $x_{i,j}^{gen+1}$ are the position of the ith candidate solution, $i = 1, 2, \dots, n$ and for the jth dimension, $j = 1, 2, \dots, d$ where d is the number of variables, at generation gen and $(gen + 1)$, respectively, and r_2, r_3, and r_4 are random numbers uniformly distributed in the range of (0,1). Also, a new nonlinear transition parameter r_1 was introduced as in Eq. 10 to exert a better balance between the exploration and exploitation phases.

In order to avoid the possible local optima, a new phase was added to the algorithm that contains the mutation and generation of a new solution defined as:

$$Z_i^{gen+1} = \begin{cases} X_D^{gen} \times (1 + \delta) & r_5 > 0.5 \\ X_{min} + \beta \times (X_{max} - X_{min}) & r_5 \leq 0.5 \end{cases} \tag{11}$$

$$\beta_{k+1} = c.\beta_k \times (1 - \beta_k) \tag{12}$$

where X_{min} and X_{max} are the lower and upper boundaries for the ith candidate solution respectively, X_D^{gen} is the destination point, r_5 is a random number uniformly distributed in the interval of (0,1), β is a random number derived from the Logistic Chaotic Map and c is fixed to the value 4 [15]. δ is an operator of Gaussian Mutation with a density function specified as:

$$f_{Gaussian\ (0,\sigma^2)}(\beta) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{\beta^2}{2\sigma^2}} \tag{13}$$

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Here σ^2 represents the variance corresponding to each candidate solution, so, the mutation operator utilized to update the positions can be defined as:

$$Z_i^{t+1} = X_D^t \times (1 + G(\beta)) \quad if \quad r_5 > 0.5 \tag{14}$$

The reader should study Gupta *et al*. [15] for excessive information.

2.3 Hybrid Colliding Bodies Optimization and Sine Cosine Algorithm

Considering the inadequate power of CBO in extrication of the local optima causing stagnation and in addition, its poor convergence speed; here we aim to engage the Modified Sine Cosine Algorithm (MSCA) in order to propose a new hybrid version of CBO for reliable design of structures through the enhanced balance of exploration and exploitation phases alongside the accelerated convergence speed by proper synchronization of the two aforementioned approaches.

Kaveh and Ilchi Ghazaan [16] introduced an enhanced variant of CBO called ECBO in which a memory storing some of the best-so-far solution vectors was defined named cm [16]. Also, the same is utilized in the present work to elevate the convergence speed. Accordingly, Eq. (9) will be substituted as:

$$\begin{cases} x_{i,j}^{gen+1} = cm_{1,j} - r_1 \sin(2\pi r_2) \left|2r_3 cm_{1,j} - x_{i,j}^{gen}\right| & r_4 > 0.5 \\ x_{i+\frac{n}{2},j}^{gen+1} = cm_{1,j} - r_1 \cos(2\pi r_2) \left|2r_3 cm_{1,j} - x_{i+\frac{n}{2},j}^{gen}\right| & r_4 \le 0.5 \end{cases} \tag{15}$$

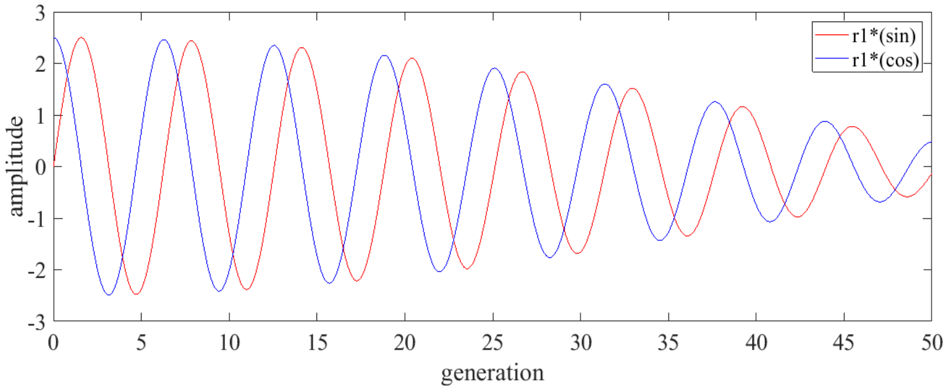

Figure 2. Subsidence pattern of the amplitude of sine and cosine ($a = 2$, $b = 0.5$)

where $i = 1, 2, \ldots, \frac{n}{2}$ and $cm_{1,j}$ is the jth dimension of the best CB vector, $j = 1, 2, \ldots, d$, obtained so far and as mentioned earlier n is the total number of CBs. Fig. 2 shows the subsidence pattern of sine and cosine functions multiplied by r_1 through 50 generations. It can be observed that in a particular generation, these two functions have notable differences in their amplitudes (e.g., see Fig. 2 at generation 50) that is, we decided to use each one of these functions as in Eq. 15. This results in a more suitable updating attitude for dimension-by-dimension updating approach since instead of alternatively using both sine and cosine functions, it benefits behavior of a particular function to update the dimensions of stationary or moving CBs (i.e. sine for stationary and cosine for moving CBs) thus, controls the randomness of oscillations and increases the possibility of generating a better CB. Additionally, the form of Eq. (11) will be defined as:

$$\begin{cases} x_{i,j}^{gen+1} = cm_{1,j} \times (1 + normrnd(0,1)) & r_5 > 0.5 \\ x_{i+\frac{n}{2},j}^{gen+1} = x_{min} + \beta \times (x_{max} - x_{min}) & r_5 \leq 0.5 \end{cases} \tag{16}$$

where $normrnd(0,1)$ is a normally distributed random number with mean equal to zero ($\mu = 0$) and standard deviation equal to 1 ($\sigma = 1$) and β is specified as [17]:

$$\beta_{k+1} = c.\beta_k \times (1 - \beta_k) \; ; \quad \beta_1 = rnd \; , \; \beta_1 \neq 0.25, 0.5, 0.75 \tag{17}$$

where number 4 was assigned to c as discussed before and rnd is a uniformly distributed random number between zero and 1.

A desirable contribution between the local and global search (i.e. exploration and exploitation phases) is founded employing Eq. (15) which illustrates that jth dimension of a stationary body or its corresponding moving body will be updated with respect to the value of r_4. Similarly, Eq. (16) employs the same updating attitude as Eq. (15) providing population diversity and enabling the ability of jumping out of the local optima. In spite of all these explanations, a compromise of these mathematical models with CBO seems crucial. To do so, an Adaptive Resolution Parameter (ARP) is introduced as follows:

$$ARP = 0.25 \times (1 - \frac{gen}{maxgen}) \tag{18}$$

which will be compared to a random number, r_6 , with uniform distribution generated in (0,1) domain in order to choose an updating method between Eq. (15) and Eq. (16). Furthermore, to refrain from the loss of information obtained by CBO and refuse to sacrifice generations, 10% of the dimensions of half (50%) of the total CBs will be updated randomly using the two updating equations which were handled by comparing r_7 to 0.5 for deciding whether to update the CB and comparing r_8 to 0.9 for deciding if the dimension of the present CB will be updated. Pseudo-code of HCBOSCA which best elucidates the implementation steps of this approach is presented in Algorithm 1.

Algorithm 1: HCBOSCA pseudo-code

Define and set values to population size (number of CBs), number of variables, lower and upper bounds (x_{min} and x_{max}) of variables, maximum number of generations ($maxgen$) and a memory (cm) and set its size to 10% of the population size.
For each CB, randomly initialize the population.
While termination criteria have not been met do:

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"CBO Process"

Define COR parameter (ε).
Evaluate $fitness\ value$ for each CB.
Sort CBs in ascending order according to their $fitness\ values$.
Replace the worst 0.1 of CBs with the ones in cm.
Rearrange CBs in ascending order according to their $fitness\ values$.
Save the best 0.1 of CBs in cm.
Calculate $mass\ values$ for each CB using Eq. (1).
Obtain the velocities of particles by Eqs (2), (3), (4), and (5).
Update the positions of CBs using Eqs (6) and (7).

"SCA and Mutation Operator"

Define $\beta_1,\ and\ r_1$
For each CB
Define r_7 equal to rnd.
If ($r_7 > 0.5$)
For each dimension of the CB
Define r_8 equal to rnd.
If ($r_8 > 0.9$)
Define $r_2, r_3, r_4, r_5\ and\ r_6$ as rnd and β_{k+1} according to Eq. (17).
If ($r_6 > ARP$)
Update the dimension of the CB utilizing Eq. (15).
Else
Update the dimension of the CB utilizing Eq. (16).
End-if
End-if
End-for
End-if
End-for
End-while

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3. EXPERIMENTAL RESULTS OF STRUCTURAL DESIGNS

In this section, the new approach presented in this paper is executed in order to tackle the optimum design of three benchmark structural problems. We have utilized MATLAB to analyze the structures employing the direct stiffness method. 20 CBs in 1000 generations are employed for design problems. To alleviate the statistical errors, each problem has been performed 20 times independently. For constraint handling, the well-known penalty approach is used. Thereafter we have done an analogy between the results of this paper and some recent studies in the literature.

3.1 The two-dimensions 200-bar truss problem

Fig. 3 illustrates the schematic of a 2-D 200-bar planar truss that contains 77 nodes. The elements are divided into 29 groups. The material parameters are defined as follows: the modulus of elasticity is 210 GPa, the material density is 7860 kg/m^3, and the lower bound of the cross-sectional area of all members is 0.1 cm^2. 100 kg fixed external loads are attached to each of the upper nodes and limitations of the three first natural frequencies of the truss must be satisfied. These constraints are defined as: $f_1 \geq 5$ Hz, $f_2 \geq 10$ Hz, and $f_3 \geq 15$ Hz.

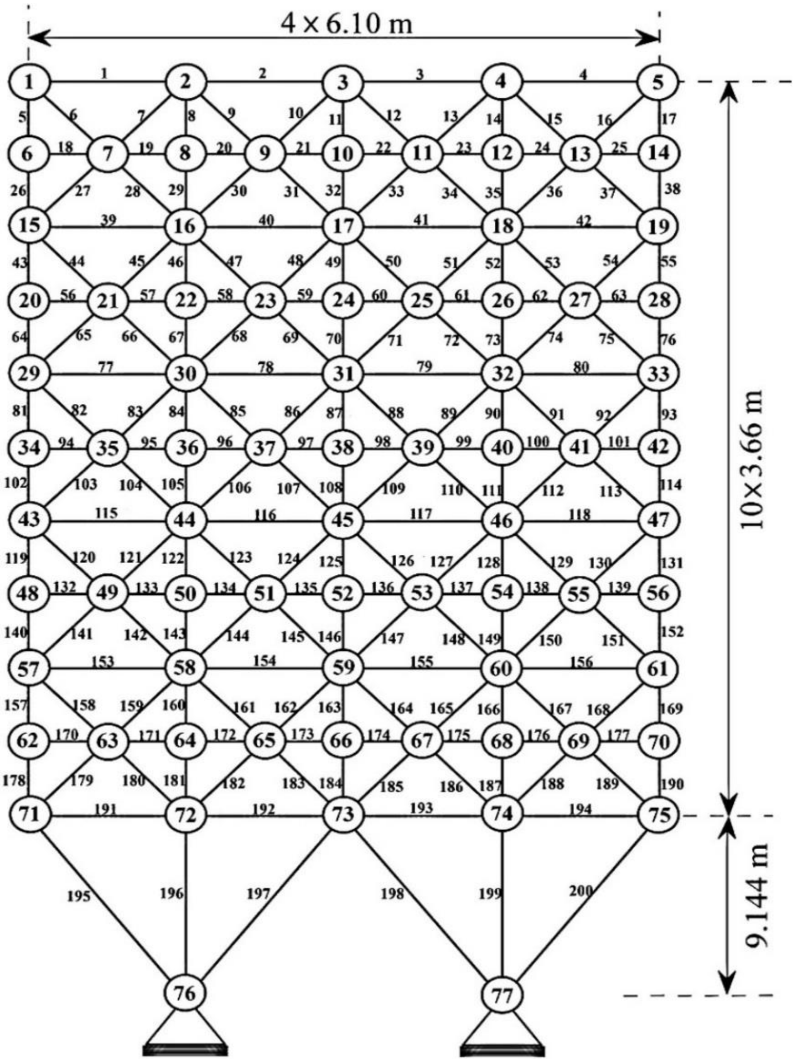

Figure 3. Schematic of the 200-bar planar truss [21]

Table 1 illustrates the optimal design details encompassing the minimum weight (Best), mean weight (Mean), and standard deviation (SD) of the repeated tests corresponding to the League Championship Algorithm with tie concept (LCA-Tie-2) [18], Differential Evolution (DE) [19], Adaptive Hybrid Evolutionary Firefly Algorithm (AHEFA) [19], Hybrid Arithmetic Optimization Algorithm and Differential Evolution (ADE) [20], CBO [13],

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ECBO [16], CBO with Morlet wavelet (MW) mutation and quadratic interpolation (QI) (MWQI-CBO) [21], and the presented new algorithm (HCBOSCA).

Table 1: Details of the optimum results of the 200-bar planar truss

Element group	Areas (cm^2)						
	LCA-Tie-2 [18]	DE [19]	AHEFA [19]	ADE [20]	ECBO [21]	MWQI-CBO [21]	HCBOSCA
1	0.30891160	0.3035	0.2993	0.3048	0.2993	0.2966	0.2952
2	0.48871045	0.4528	0.4508	0.4598	0.4497	0.4657	0.4700
3	0.10162941	0.1000	0.1001	0.1000	0.1000	0.1008	0.1000
4	0.10657586	0.1000	0.1000	0.1000	0.1	0.1002	0.1001
5	0.54794212	0.5162	0.5123	0.5075	0.5137	0.5077	0.5313
6	0.81394811	0.8203	0.8205	0.8207	0.7914	0.8253	0.8116
7	0.11532799	0.1004	0.1011	0.1001	0.1013	0.1001	0.1000
8	1.29042334	1.4393	1.4156	1.4204	1.4129	1.4194	1.4345
9	0.11282050	0.1003	0.1000	0.1000	0.1019	0.1002	0.1000
10	1.56294014	1.5918	1.5742	1.5620	1.6460	1.6222	1.5948
11	1.14548904	1.1641	1.1597	1.1583	1.1532	1.1746	1.1660
12	0.18455251	0.1319	0.1338	0.1274	0.1000	0.1013	0.1476
13	2.92990485	2.9561	2.9672	2.9828	3.1850	2.9609	2.9224
14	0.11534915	0.1003	0.1000	0.1000	0.1034	0.1006	0.1005
15	3.29811115	3.2491	3.2722	3.2612	3.3126	3.2534	3.1992
16	1.60489863	1.5949	1.5762	1.5791	1.5920	1.5706	1.5804
17	0.29433408	0.2525	0.2562	0.2555	0.2238	0.2417	0.2905
18	5.25387865	5.1567	5.0956	5.1095	5.1227	5.154	5.1806
19	0.10219255	0.1004	0.1001	0.1004	5.1227	5.154	0.1000
20	5.44086222	5.4938	5.4546	5.4613	5.3707	5.46	5.4220
21	2.02955602	2.1094	2.0933	2.1078	2.0645	2.1291	2.1273
22	0.57323199	0.6731	0.6737	0.6722	0.5443	0.656	0.6425
23	7.47936823	7.6922	7.6498	7.6301	7.6497	7.4562	7.6238
24	0.28990234	0.1150	0.1178	0.1019	0.1000	0.1616	0.1245
25	7.85261204	8.0035	8.0682	7.9284	7.6754	8.0675	7.9871
26	7.85261204	2.7794	2.8025	2.7951	2.7178	2.8185	2.7481
27	10.38474350	10.5173	10.5040	10.5555	10.8141	10.4169	10.5977
28	21.59152982	21.2292	21.2935	21.3836	21.6349	21.3471	21.4246
29	10.25871058	10.7286	10.7410	10.5765	10.3520	10.4155	10.2717
Best (kg)	2159.96	2160.7747	2160.7445	2160.7263	2158.08	2157.06	2156.96
Mean (kg)	2168.21	2162.2495	2161.0393	2160.8514	2159.93	2159.88	2158.62
SD (kg)	9.51	3.0003	0.1783	0.0946	1.57	2.94	1.3864

It can be perceived that HCBOSCA has outperformed all the algorithms with the least design weight equal to 2156.96 kg and the average design weight of 2158.62 kg which also is less than the mean weight of the other algorithms. Moreover, the standard deviation of HCBOSCA is less than other algorithms according to this Table 1. The natural frequencies of the best designed structures are represented in Table 2. It is apparent that all the constraints have been satisfied. Additionally, the convergence curves of the optimum solutions of CBO, ECBO, MWQI-CBO, and HCBOSCA are depicted in Fig. 4. The number of analyses required to extract the optimum design for each of these algorithms is 10,500, 14,700, and 15,060 respectively [21], while that of HCBOSCA is 14,460. It is worthy of note that the present algorithm achieved the optimum design of CBO, ECBO, and MWQI-CBO after 7920, 10,980, and 12,740 analyses.

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Table 2: Natural frequencies of the optimum results of the 200-bar planar truss

Frequency number	Natural frequencies (Hz)						
	LCA-Tie-2 [18]	DE [19]	AHEFA [19]	ADE [20]	ECBO [21]	MWQI-CBO [21]	HCBOSCA
1	5.000015	5.0000	5.0000	5.000	5.000	5.000	5.0000
2	12.363073	12.2301	12.1821	12.231	12.189	12.179	12.1996
3	15.173504	15.0277	15.0160	15.038	15.048	15.058	15.0787
4	16.728441	16.7054	16.6837	16.683	16.643	16.673	16.7059
5	21.576253	21.4238	21.3547	21.422	21.342	21.365	21.3606
6	21.688359	21.4435	21.4168	21.437	21.382	21.520	21.5049

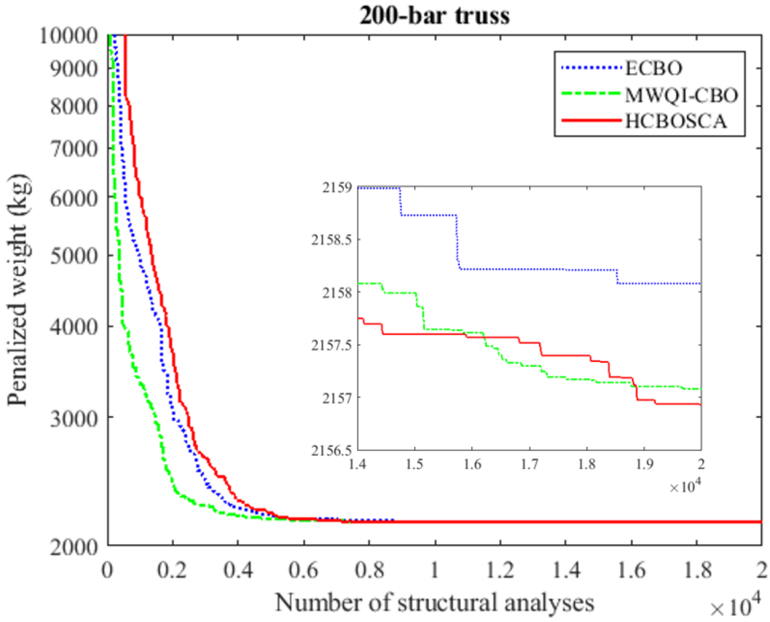

Figure 4. Convergence curves for the 200-bar planar truss

3.2 The 3-bay 15-story frame problem

The schematic, applied loads, and member group numbering for this frame is depicted in Fig. 5. This problem is consisted of 64 joints and 105 elements and is a common benchmark in structural optimization. The elements are arranged in 11 groups consisting of 10 column groups and 1 beam group. The modulus of elasticity is 29,000 ksi (200 GPa) and the yield stress is equal to 36 ksi (248.2 MPa) for the material. For a sway-permitted frame, the effective length factors of the members are calculated as $k_x \geq 0$ and the out-of-plane effective length factor is indicated as $k_y = 1.0$. Each column is considered as non-braced along its length, and the non-braced for each beam member is determined as one-fifth of the span length.

The constraints of displacement and strength are imposed according to AISC [22] specifications as follows:

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(i) Maximum lateral displacement:

$$\frac{\Delta_T}{H} - R \leq 0 \tag{19}$$

where Δ_T is the maximum lateral displacement, H is the height of the frame structure, and R is the maximum drift index specified as 1/300.

(ii) The inter-story displacements:

$$\frac{d_i}{h_i} - R_I \leq 0 \quad ; \quad i = 1, 2, \dots, n \tag{20}$$

where d_i is the inter-story drift, h_i is the story height of the ith floor, and R_I is the inter-story drift index equal to 1/300.

(iii) Strength constraints:

$$\begin{cases} \dfrac{P_u}{2\varphi_c P_n} + \dfrac{M_u}{\varphi_b M_n} - 1 \leq 0 \quad ; \quad \text{for } \dfrac{P_u}{\varphi_c P_n} < 0.2 \\ \dfrac{P_u}{\varphi_c P_n} + \dfrac{8M_u}{9\varphi_b M_n} - 1 \leq 0 \quad ; \quad \text{for } \dfrac{P_u}{\varphi_c P_n} \geq 0.2 \end{cases} \tag{21}$$

where P_u is the required tensile or compressive strength, P_n is the nominal axial tensile or compressive strength, φ_c is the resistance factor ($\varphi_c = 0.9$ for tension and $\varphi_c = 0.85$ for compression), M_u is the required flexural strengths, M_n is the nominal flexural strengths, and φ_b denotes the flexural resistance reduction factor ($\varphi_b = 0.90$).

The nominal strength P_n for yielding in the gross section is evaluated by:

$$\begin{cases} P_n = A_g F_y \quad \text{for tensile strength} \\ P_n = A_g F_{cr} \quad \text{for compressive strength} \end{cases} \tag{22}$$

$$\begin{cases} F_{cr} = \left(0.658^{\lambda_c^2}\right) F_y \quad ; \quad \text{for } \lambda_c \leq 1.5 \\ F_{cr} = \left(\dfrac{0.877}{\lambda_c^2}\right) F_y \quad ; \quad \text{for } \lambda_c > 1.5 \end{cases} \tag{23}$$

where A_g is the gross section area of the member, F_y is the yield, and F_{cr} is calculated as:

$$\lambda_c = \frac{kl}{r\pi}\sqrt{\frac{F_y}{E}} \tag{24}$$

where l is the length of the member, r is the radius of gyration, E is the modulus of elasticity, and k is the effective length factor that can be evaluated as:

$$k = \sqrt{\frac{1.6 G_A G_B + 4.0(G_A + G_B) + 7.5}{G_A + G_B + 7.5}} \tag{25}$$

where G_A and G_B are stiffness ratios of columns and girders at the two end joints A and B of the column section, respectively. Moreover, the sway of the top story is limited to 9.25 in. (23.5 cm) in this problem.

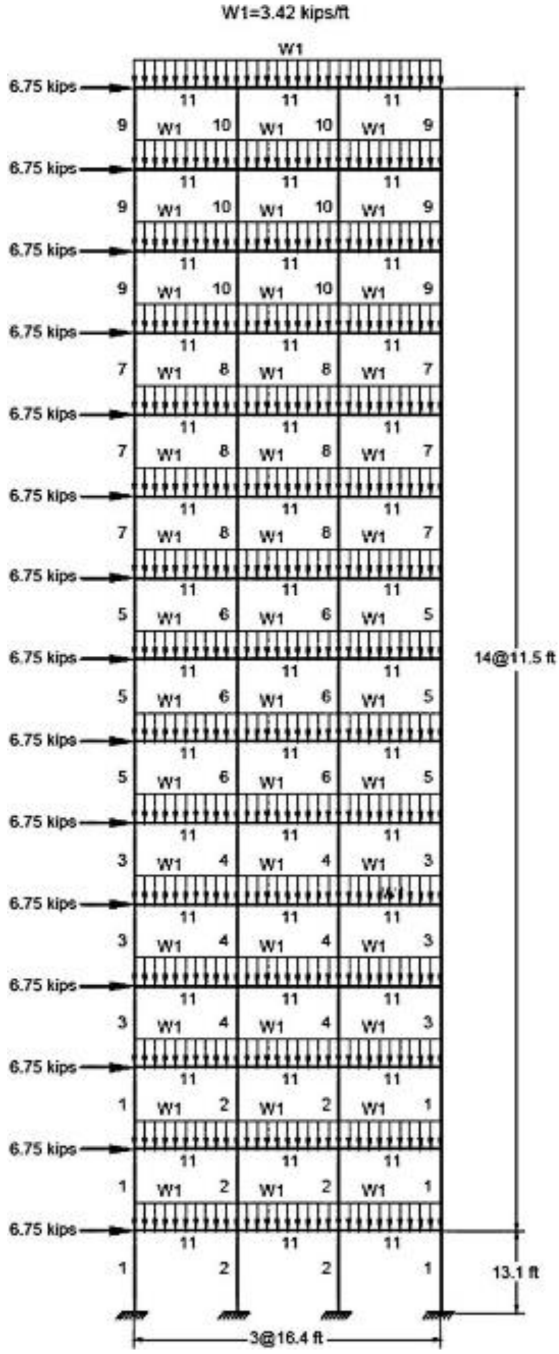

Figure 5. Schematic of the 3-bay 15-story frame [21]

The optimization results of Cuckoo Search (CS) [24], Teaching-Learning-based Optimization (TLBO) [24], Water Evaporation Optimization (WEO) [24], Quantum

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Teaching-Learning-based Optimization (QTLBO) [23], CBO [13], ECBO [16], MWQI-CBO [21], and HCBOSCA algorithms corresponding to this structure is demonstrated in Table 3. As shown in this table, the design of HCBOSCA and MWQI-CBO with the minimum weight of 86,917 lb. are the lightest designs among all listed algorithms; however, the mean design weight of HCBOSCA with a magnitude of 87,861 lb. and the lower standard deviation of it, signifies its better performance compared to the other approaches.

Table 3: Details of the optimum results of the 3-bay 15-story frame

Element group	Optimal W-shaped sections			
	QTLBO [23]	ECBO [21]	MWQI-CBO [21]	HCBOSCA
1	W24×104	W14×99	W14×90	W14×99
2	W27×161	W27×161	W36×170	W27×161
3	W18×76	W27×84	W 27×84	W27×84
4	W27×114	W24×104	W24×104	W24×104
5	W14×61	W14×61	W14×61	W14×61
6	W30×90	W30×90	W30×90	W30×90
7	W8×48	W14×48	W14×48	W18×50
8	W12×65	W14×61	W14×61	W14×61
9	W6×25	W14×30	W14×34	W8×28
10	W8×40	W14×40	W8×35	W10×39
11	W21×44	W21×44	W21×44	W21×44
Best (lb)	87,416	86,986	86,917	86,917
Mean (lb)	87,952	88,410	88,353	87,861
SD (lb)	451	N/A	1,948	900

Fig. 6 demonstrates the convergence histories of the optimum solutions of CBO, ECBO, MWQI-CBO, and HCBOSCA methods for this problem.

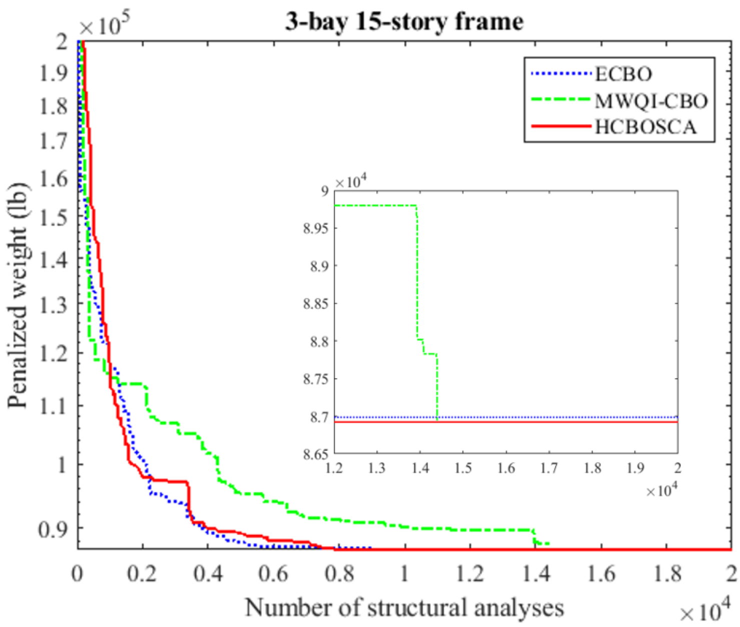
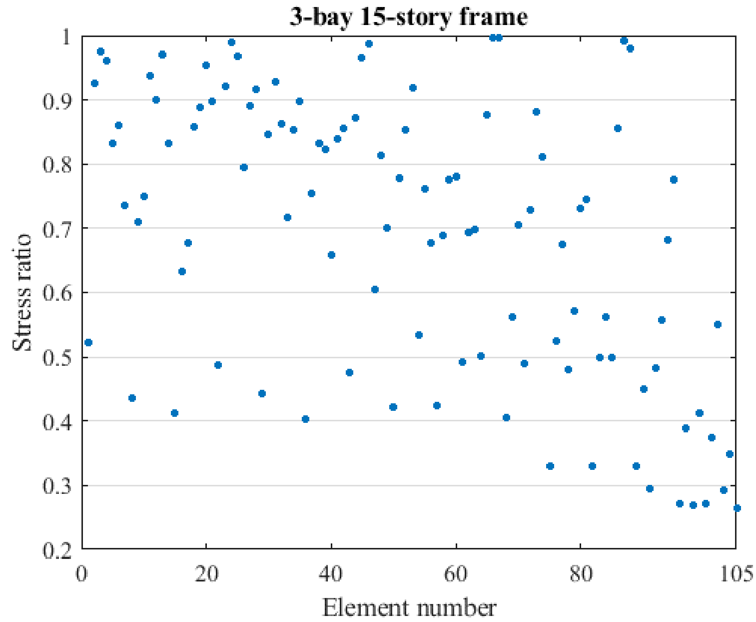


Figure 6. Convergence curves for 3-bay 15-story frame

HCBOSCA obtains the best solution after 7840 analyses, where this number for CBO, ECBO, and MWQI-CBO is 9520, 9000, and 14,420 respectively [21]. Also, the stress ratios of all the elements and inter-story drifts of the optimum design of the structure are exhibited in Fig. 7. The maximum existing value of the stress ratio equals 99.74%.



(a)

(b)

Figure 7. Constraint margins for the best design obtained by HCBOSCA for the 3-bay 15-story frame problem: (a) element stress ratio; (b) Inter-story drift

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3.3 The spatial 582-bar tower truss problem

Fig. 8 shows the geometry of the 582-bar tower truss. Due to the structural symmetry, members are linked together into 32 groups. A single loading condition is considered to be applied such that lateral loads of 1.12 kips (5.0 kN) are applied in both x- and y-directions and vertical loads of -6.74 kips (-30 kN) are applied in the z-direction to all free nodes of the tower. A discrete list of W-shaped standard steel sections was employed in order to select the cross-sectional areas of elements based on the area and radii of gyration properties. The maximum and minimum cross-sectional area of elements are 6.16 and 215 in^2 (i.e., 39.74 and 1378.09 cm^2), respectively.

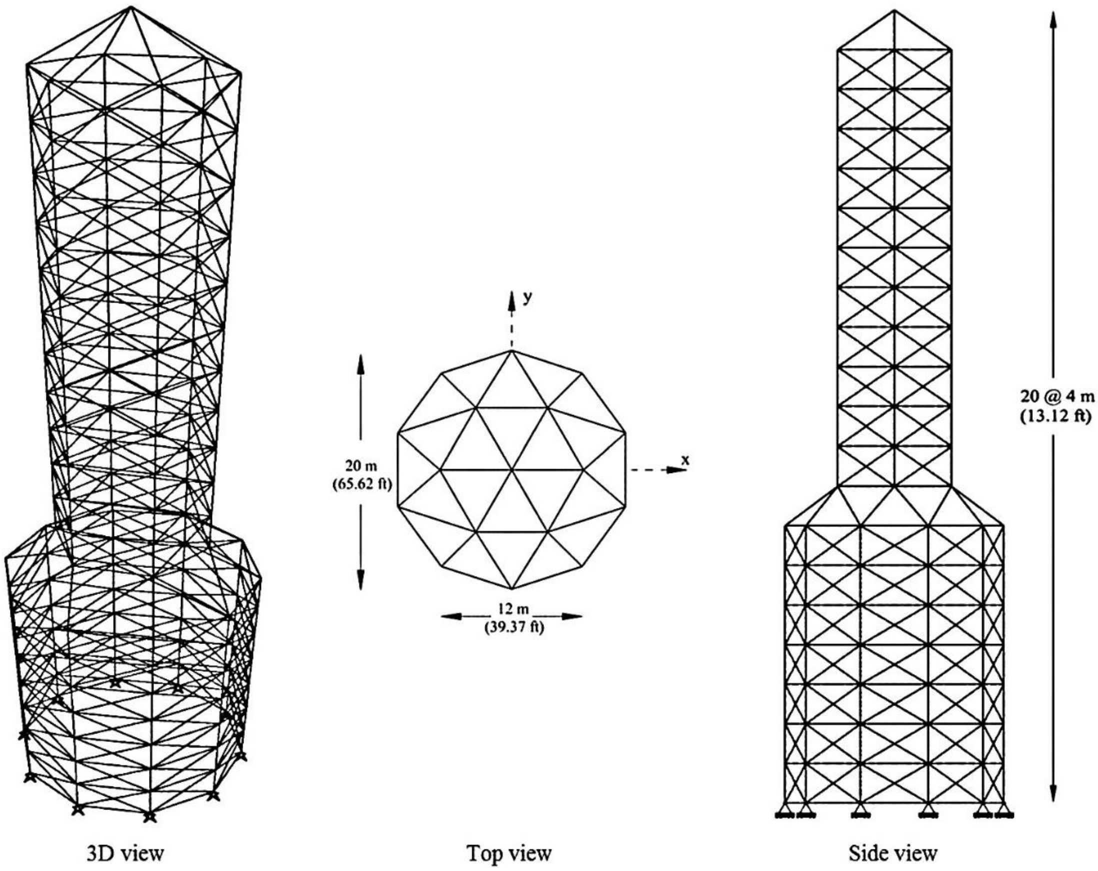

Figure 8. Schematic of the 582-bar tower [21]

Limitations on stress and stability of truss elements are imposed with respect to the provisions of AISC [22] as follows:

The allowable tensile stresses for tension members are defined as:

$$\sigma_i^+ = 0.6F_y \tag{26}$$

where F_y is the yield strength.

For compression members, the allowable stress limits are obtained considering the failure mode of members. These failure modes are elastic and inelastic buckling which can be determined as follows:

$$\sigma_i^- = \begin{cases} \left[\left(1-\frac{\lambda_i^2}{2C_c^2}\right)F_y\right] / \left[\frac{5}{3}+\frac{3\lambda_i}{8C_c}-\frac{\lambda_i^3}{8C_c^3}\right] & \text{for} \quad \lambda_i < C \\ \frac{12\pi^2 E}{23\lambda_i^2} & \text{for} \quad \lambda_i \geq C \end{cases} \tag{27}$$

in which λ_i is the slenderness ratio with a maximum magnitude of 300 for tension members where 200 is recommended for compression members. It is defined as:

$$\lambda_i = \frac{kl_i}{r_i} \tag{28}$$

that k is the effective length factor that for all truss members it is substituted by 1. l_i and r_i are length and minimum gyration radius of the i th member, respectively. C_c is the slenderness ratio that divides the elastic and inelastic buckling scopes defined as:

$$C_c = \sqrt{\frac{2\pi^2 E}{F_y}} \tag{29}$$

and E is the modulus of elasticity.

It should be noted that nodal displacements in all coordinate directions must not exceed $\pm$3.15 in. (i.e., $\pm$8 cm).

Table 4 is a summary report of the results of Particle Swarm Optimization (PSO) [25], Whale Optimization Algorithm (WOA) [7], Enhanced Whale Optimization Algorithm (EWOA) [7], CBO [13], ECBO [16], MWQI-CBO [21], and HCBOSCA in optimum design of this truss problem. According to this table, the design with the least volume is attained by HCBOSCA which the minimum volume is 1,294,516 in3 and the mean volume is 1,301,234 in3. It also can be seen that this approach excels in terms of the standard deviation over the rest. The convergence histories of CBO, ECBO, MWQI-CBO, and HCBOSCA for the best solution achieved are depicted in Fig. 9. CBO, ECBO, and MWQI-CBO need 17,700, 19,700, and 15,560 analyses respectively to reach out to their best design [21]. Meanwhile, HCBOSCA finds its optimum solution after 10,140 analyses that indicates the superiority of this algorithm over those to which it was compared. Besides, Fig. 10 illustrates the stress ratio of each element and the nodal displacement corresponding to the best design of HCBOSCA. The highest magnitude of the stress ratio of elements is 99.87% for this tower while the maximum displacements along the X and Y directions are equal to 3.1495 and 2.9848 in. respectively.

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Table 4: Details of the optimum results of the 582-bar truss

Element group	Optimal W-shaped sections			
	EWOA [7]	ECBO [21]	MWQI-CBO [21]	HCBOSCA
1	W8×21	W8×21	W8×21	W8×21
2	W14×90	W14×90	W14×90	W14×90
3	W8×24	W8×24	W8×24	W8×24
4	W10×60	W14×61	W14×58	W10×60
5	W8×24	W8×24	W8×24	W8×24
6	W8×21	W8×21	W8×21	W8×21
7	W14×48	W10×49	W10×45	W14×48
8	W8×24	W8×24	W8×24	W8×24
9	W8×21	W8×21	W8×21	W8×21
10	W10×49	W14×43	W10×54	W14×48
11	W8×24	W8×24	W8×24	W8×24
12	W16×67	W12×72	W12×65	W10×68
13	W18×76	W12×72	W12×74	W12×72
14	W10×49	W10×54	W10×49	W10×49
15	W18×76	W12×65	W14×74	W14×74
16	W8×31	W8×31	W8×31	W8×31
17	W14×61	W10×60	W14×61	W14×61
18	W8×24	W8×24	W8×24	W8×24
19	W8×21	W8×21	W8×21	W8×21
20	W14×34	W14×43	W8×40	W12×40
21	W8×24	W8×24	W8×24	W8×24
22	W8×21	W8×21	W8×21	W8×21
23	W8×21	W8×21	W8×28	W8×24
24	W8×24	W8×24	W8×24	W8×24
25	W8×21	W8×21	W8×21	W8×21
26	W10×22	W8×21	W8×21	W8×21
27	W8×24	W8×24	W8×24	W8×24
28	W8×21	W8×21	W8×21	W8×21
29	W8×21	W8×21	W8×21	W8×21
30	W8×24	W8×24	W8×24	W8×24
31	W8×21	W8×21	W8×21	W8×21
32	W8×24	W8×24	W8×24	W8×24
Best (in^3)	1,295,738	1,296,776	1,295,562	1,294,516
Mean (in^3)	1,310,836	1,306,728	1,305,095	1,301,234
SD (in^3)	N/A	7536	5320	5081

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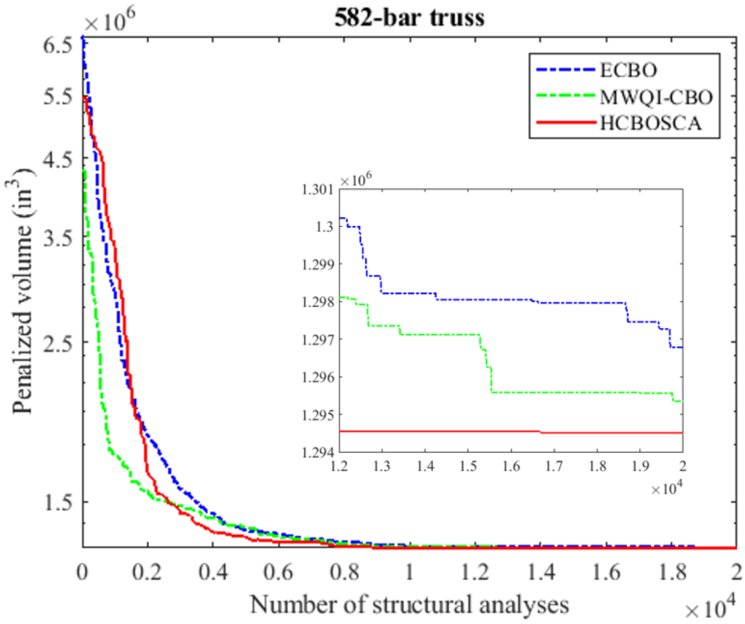
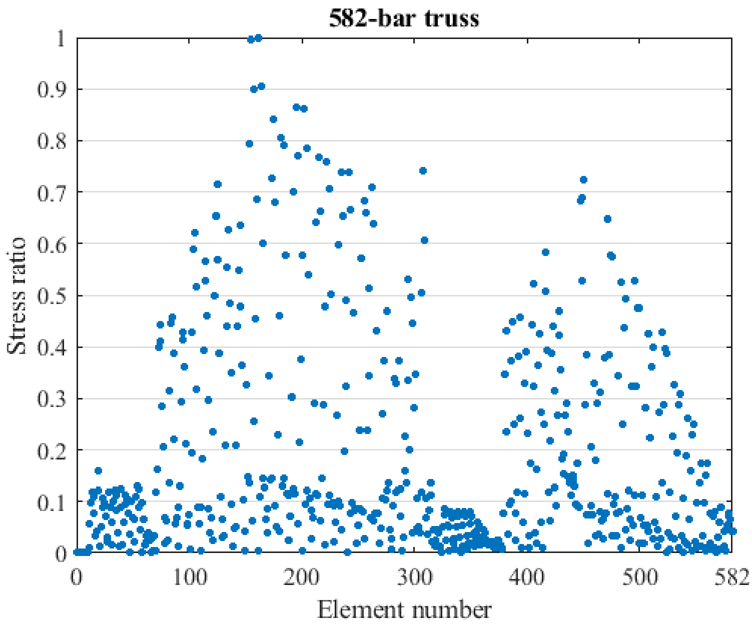
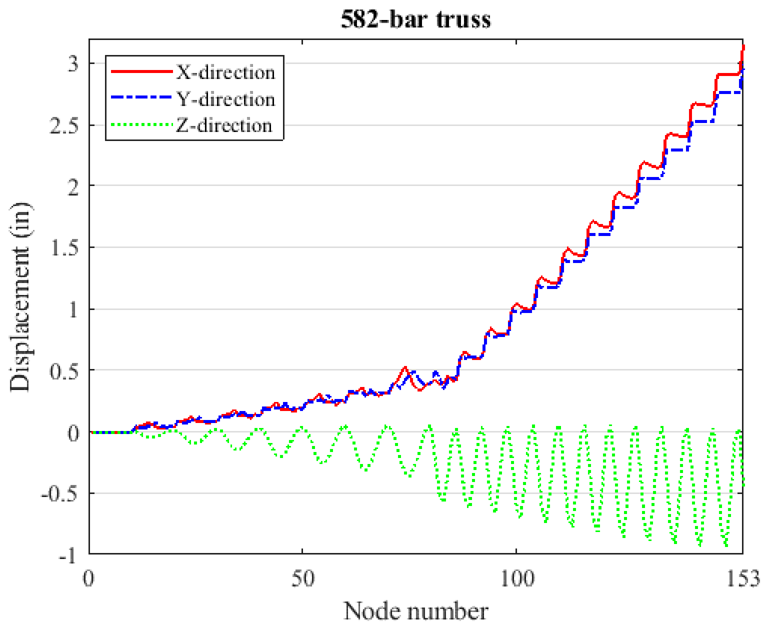


Figure 9. Convergence curves for the 582-bar tower



(a)



(b)

Figure 10. Constraint margins for the best design obtained by HCBOSCA for the 582-bar tower problem: (a) element stress ratio; (b) nodal displacements

4. CONCLUSION

In this study, we introduced a new meta-heuristic algorithm based on CBO and SCA so-called HCBOSCA. Due to the intrinsic malfunction of standard CBO in the exploration phase and its desire for premature convergence, a better composition of the exploration and exploitation phases is made using SCA. Furthermore, a logistic chaotic map in the contribution of a mutation operator based on normal distribution was added to arise the capabilities of diversification and escaping from local optima. These mechanisms were exerted on some randomly chosen variables of a random half of each generation so that the searching attitude of CBO is not been significantly distracted. Three benchmark structural problems with discrete and continuous variables were tested using HCBOSCA and thereafter, the results were compared to some recent works. The proposed method demonstrated excellent performance compared to the above-mentioned algorithms. In summation, HCBOSCA practically is a reliable approach with a high convergence speed and ability to escape the local optima which denotes its competence with other state of the art metaheuristic algorithms.

Declarations

Competing interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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